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Article

The Risk Premia from the European Equity Market: An Application of the Three-Pass Estimation Methodology

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Abstract

We develop an empirical application on a large dataset of European stock returns in order to estimate the risk premia. While traditional factor models often struggle with high levels of pricing errors and noisy proxies in fragmented markets, we show that the Three-Pass Estimation Method (3PEM) serves as both a robust estimator and a diagnostic tool for factor purification. By assuming the Fama–French five-factor model as the baseline model, we first show that the 3PEM yields risk premium estimates for the European market that are more economically plausible and statistically robust than those obtained using the traditional two-pass estimation method (2PEM). Moreover, our results show that the 3PEM is able to detect noise in tradable factors. Furthermore, the 3PEM is used to denoise the observed factors, providing purified versions that better capture the systematic components of risk. We also identify both noisy factors and denoised factor series that improve the estimation of stock-level exposures and expected returns.

Keywords: three-pass estimator; empirical asset pricing; PCA; large panels; European equity market

1. Introduction

Linear factor models lie at the heart of modern asset pricing. Their origins trace back to the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), which formalized the idea that expected returns compensate investors for exposure to a single source of systematic risk. The restrictive structure of the CAPM motivated the development of more flexible models, most notably the Arbitrage Pricing Theory (APT, Ross, 1976), which allows multiple factors —economic, financial, or statistical—to jointly explain the cross-section of returns.

Factor models are essential for both portfolio allocation and evaluating investment performance because they provide structured estimates of expected returns and the covariance matrix, which many optimization procedures rely on. However, classical mean-variance optimization (Markowitz, 1952) is highly sensitive to estimation errors, making factor structure assumptions a practical way to reduce dimensionality and stabilize inputs. At the same time, factor models allow practitioners to distinguish true managerial skill (alpha) from compensation for systematic risks (beta), but this requires specifying the model correctly: omitting relevant factors or including spurious ones can bias alpha estimates, distort inference, and ultimately lead to poor investment decisions (see, e.g., Chincarini & Kim, 2006; Lazzari & Navone, 2003).

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Despite their wide applicability, empirical implementation of factor models faces two enduring challenges: model specification and measurement error.

First, selecting the number and nature of factors is non-trivial. Researchers rarely observe the true underlying factors driving asset returns. If relevant systematic risks are omitted, estimated prices of risk may suffer from omitted-variable bias. Conversely, including irrelevant factors can introduce instability and spurious inference.This issue is closely related to the literature on approximate factor structures and the estimation of latent factor dimension (Bai & Ng, 2002; Onatski, 2010; Ahn & Horenstein, 2013; Gagliardini et al., 2019), but these methods do not identify which specific factors are missing nor correct for the resulting bias. Additionally, concerns about spurious or weak factors have also become central in modern empirical asset pricing. Bryzgalova (2015) shows that standard asset pricing tests can incorrectly identify factors as priced when their signal is weak relative to estimation noise.

Second, observable factors—tradable or otherwise—may suffer from measurement error since they rely on their observable proxies. Tradable factors such as the Fama–French portfolios are straightforward to use, but they remain noisy proxies constructed from finite samples and sorting rules (Racicot et al., 2011). Also, Akey et al. (2021) argue that Fama–French factors are "noisy" due to significant, retroactive, and often undocumented revisions in factor construction methodologies. These data updates, or "vintages," materially impact empirical results, changing the alpha of nearly half of analyzed mutual funds and often inflating the performance of factors. However, the authors do not propose a formal denoising procedure to address this issue. Instead, they recommend that researchers clearly report the download dates or vintages of the factor datasets used in their analyses to facilitate replication and suggest improving inference methods so that empirical tests explicitly account for the presence of factor noise. Non-tradable macroeconomic factors introduce even larger identification issues, as their premia cannot be directly estimated from time-series averages (Giglio & Xiu, 2021). Traditional methods such as the Fama–MacBeth two-pass estimator (Fama & MacBeth, 1973) assume noise-free factors and are therefore sensitive to both omitted variables and measurement error.

Recent advances have addressed these limitations. For instance, the Three-Pass Estimation Method (3PEM), introduced by Giglio and Xiu (2021), offers a unified and robust procedure for estimating risk premia when observable factors are noisy or incomplete. Leveraging the rotation invariance property of latent factor spaces, the 3PEM yields consistent estimates of the prices of risk even in the presence of omitted factors or errors-in-variables. While originally designed for non-tradable factors, the method naturally extends to tradable ones. In the case of tradable factors, however, we show that the main advantage of the 3PEM is not limited to the correction of risk premium estimates. Rather, it provides a broader empirical framework that allows researchers to diagnose factor relevance, detect weak or spurious factors, account for omitted-variable bias, and explicitly separate systematic components from measurement error through factor denoising.

Although the theoretical advantages of the 3PEM are well established (Giglio & Xiu, 2021), the full potential of its empirical applications, especially in markets characterized by high levels of idiosyncratic noise, has not yet been fully explored. Indeed, to our knowledge, empirical evidence on its performance outside the U.S. remains limited. dos Santos Rocha et al. (2023) apply the approach to the Brazilian stock market, while Mao and Xia (2023) study the Chinese A-share market, showing that the method improves the estimation of risk premia. Similarly, Borri et al. (2022) apply the 3PEM to study crypto risk premia, showing that macroeconomic risk is significantly priced in the cross-section of cryptocurrency returns.

However, these studies do not investigate factor denoising or the implications for stock-level expected returns, which are the central focus of this paper. In particular, our work distinguishes itself from the original work by Giglio and Xiu (2021) and other empirical applications above mentioned through its specific focus on tradable factors and denoising procedure within the European context. We use 3PEM as a denoising tool, providing "purified" factor series that better capture systematic risk than the observed versions. By using denoised factors, we show how to improve the estimation of stock-level exposures (betas) and expected returns, which has direct implications for portfolio management and asset allocation in the European region.

Moreover, focusing on a regional setting is particularly relevant given the evidence that factor behavior is not uniform across global markets. A large body of empirical work evaluates the performance of factor models across international markets. While the original evidence was primarily based on U.S. equities, subsequent studies show that factor premia may vary substantially across regions. For example, Fama and French (2012, 2017) document that size, value, and other factors display heterogeneous performance across international equity markets. Similarly, Griffin (2002) shows that the explanatory power of well-known factors often differs across countries, suggesting that some factors may be region-specific rather than globally priced. Evidence for European capital markets also indicates differences in market structure, accounting standards, and institutional environments (Arce and Mora, 2002), implying that the cross-section of returns may differ from that observed in the U.S. Additionally, Chaieb et al. (2021) found that 29% of European expected returns are actually "pricing errors" in standard models. Hence, these findings highlight the importance of conducting empirical analyses in different geographical settings and point to the need for separate empirical validation at the European level, where factor premia and their explanatory power may differ from those documented in U.S. markets

In a nutshell, in this paper, we empirically show that while tradable factors in the European market can be priced using conventional approaches, they are subject to significant measurement error. By exploiting the diagnostic and denoising features of the 3PEM, we show that this method is a crucial tool for tradable factor models. Applying the 3PEM on European data allows taking into account the possible instability found in European stock-level expected returns. Indeed, we distinguish between factors that are capturing systematic risk and those that are merely proxying for noise in the European cross-section. This paper fills a critical gap in the literature by providing an evaluation of the 3PEM as a diagnostic and 'denoising' tool within the European market. Our contribution is two-fold: First, we show that European factor pricing is affected by measurement error. Second, by projecting these observable factors onto a latent pricing space, we recover 'denoised' factor versions that reveal a more stable and economically plausible risk structure.

First, we conduct an extensive comparison between the 3PEM and the traditional two-pass estimator, showing that the 3PEM delivers risk premia that are substantially more robust across model specifications, subsamples, and estimation windows. Using the (Fama & French, 2015) five-factor model as a baseline, we perform the analysis that can be applied to any linear factor model. Unlike traditional methods, the 3PEM maps observable factors directly onto the latent pricing space, allowing us to identify why certain factors appear 'weak' or 'spurious' under conventional estimation. We reinforce our results by showing that the 3PEM correctly identifies and rejects deliberately introduced spurious factors—a critical feature for the 'factor zoo' (Feng et al., 2020) in a heterogeneous market. Our analysis further reveals that the 3PEM risk premium estimates are sensitive to the choice of the latent factor dimension. We find that a six-factor latent structure is required to provide a stable representation of European systematic risk. This finding is economically significant: it suggests that the five standard Fama–French factors are insufficient to span the European pricing kernel, and that an additional latent dimension is necessary to capture the regional nuances and country-specific risks documented in the international literature.

Second, we show that the 3PEM recovers economically plausible risk premia where traditional estimators fail. Most notably, the 3PEM yields a positive and statistically significant market risk premium—aligning with core asset pricing theory—whereas the standard two-pass method gives counterintuitive results. This finding illustrates that the perceived 'failure' of the market factor in Europe could be mainly explained by measurement error and omitted-variable bias, which the 3PEM successfully mitigates.

Crucially, this study is an investigation into factor 'denoising.' Even if a large body of literature recognizes that empirical asset pricing is affected by both measurement error in observable factors and the omission of relevant latent risks, research typically proceeds as if these proxies are noise-free (see, e.g., Fama & French, 2017; Griffin, 2002; Mosoeu & Kodongo, 2022). We challenge this convention by showing that these factors are subject to substantial measurement error in the European cross-section. By projecting observable factors onto a latent space, we construct 'denoised' factor series. These denoised factors yield more reliable stock-level loadings and significantly improve out-of-sample expected return predictions. These results indicate that the benefits of the 3PEM extend well beyond risk premium estimation, providing a powerful empirical framework for diagnosing factor quality and improving factor inputs in applied asset pricing.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model. Section 3 presents the data and empirical findings, including risk premium estimation, factor denoising, and stock-level implications. Section 4 presents the conclusions. Finally, Appendix A describes the estimation methodologies applied throughout the paper. Appendix B provides additional tables and figures.

2. Model Setting

This section introduces the methodological framework used in the paper and clarifies how each component of the 3PEM is designed to address the research questions outlined in the Introduction. Rather than presenting the 3PEM as a purely technical estimation procedure, we emphasize its role as an empirical tool for (i) mitigating omitted-variable bias, (ii) diagnosing the relevance and quality of observable tradable factors, and (iii) constructing denoised factor series that improve stock-level inference and expected return estimation.

Let us define r_t as the $n \times 1$ vector of excess returns for n testing assets. r_t follows the linear model:

$$r_t = \beta\gamma + \beta v_t + u_t, \tag{1}$$

with

$$f_t = \mu + v_t$$

and $\mathbb{E}(v_t) = 0$, where f_t is a vector gathering the p true latent factors, defined as the sum of two components: the constant mean μ capturing the long-run average values of the true factors, and the time-varying innovation component v_t reflecting new, unexpected information that causes deviations from the mean. Furthermore, the vector γ collects the risk premia of factors f_t. The $n \times p$ matrix β gathers the factor loadings, and each of element β_{ij} measures the sensitivity of the return on asset i to factor j. The terms $\beta\gamma$ and βv_t capture the average expected risk premium and the time-varying impact of unexpected factor shocks on returns, respectively. Lastly, u_t is the vector of idiosyncratic errors, such that $\mathbb{E}(u_t) = 0$ and $\text{Cov}(u_t, v_t) = 0$.

In general, we do not observe the p factors f_t, but we usually observe their $d \leq p$ proxies. Thus, we define g_t as the set of d observable factors (e.g., macroeconomic variables, financial market indexes, portfolios) that relate to the true unobservable factors f_t through the following measurement error model:

$$g_t = \xi + \eta v_t + z_t, \tag{2}$$

where ξ is a vector of constants, and η is a $d \times p$ loading matrix. Finally, z_t is the vector of measurement errors, such that $\mathbb{E}(z_t) = 0$ and $\text{Cov}(z_t, v_t) = 0$. The vector of d risk premia of the observable factors g_t is defined as

$$\gamma_g = \eta\gamma. \tag{3}$$

Giglio and Xiu (2021) show that the risk premia γ_g can be estimated due to the rotation invariance property. In fact, from Equation (2), the observed factors are expressed as a projection onto true latent factors, which span the true factor space. If we omit some of the true factors, standard estimators will suffer from omitted-variable bias. Nevertheless, we can apply PCA on a large panel of returns to get some rotated version of the factor space, not the exact v_t, i.e., we can recover a rotation of the latent factors $\tilde{v}_t = Hv_t$, where $H \in \mathbb{R}^{p \times p}$ is an invertible matrix such that $H'H = HH' = I_p$, $H^{-1} = H'$, and $det(H) = 1$. Thus, Equations (1) and (2) become

$$\begin{aligned} r_t &= \hat{\beta}\hat{\gamma} + \hat{\beta}\hat{v}_t + u_t, \\ g_t &= \xi + \hat{\eta}\hat{v}_t + z_t, \end{aligned}$$

where $\hat{\beta} = \beta H^{-1}$, $\hat{\gamma} = H\gamma$, $\hat{v}_t = Hv_t$, and $\hat{\eta} = \eta H^{-1}$. Noting that $\hat{v}_t$ spans the same risk space of the true factors, the estimator for the risk premia of observable factors is given by

$$\hat{\gamma}_g = \hat{\eta}\hat{\gamma} = \eta H^{-1} H\gamma = \eta\gamma.$$

Thus, even if we do not know the true H, we are able to find an estimate of the risk premia of observable factors. However, the true matrix of factor loadings η and the risk premia of latent factors, γ cannot be identified separately.[1]

3. Data and Empirical Analysis

This section mainly presents the empirical results. First, we describe the datasets employed in the analysis. Next, we present the results of our estimation of European risk premia. Furthermore, we analyze the denoised Fama–French factors, and finally investigate the risk exposures and expected returns.

3.1. Data Description

The empirical analysis conducted in this study draws on two complementary datasets: (i) a dataset of observable factors and portfolio returns constructed in accordance with the Fama–French methodology, and (ii) a dataset of individual European stock returns. Both datasets are sampled at a monthly frequency.

The first dataset is employed to estimate factor risk premia and to implement the denoising procedure. It is sourced from Kenneth R. French's Data Library and pertains to the European market. This portfolio-level dataset comprises returns for 132 characteristic-sorted portfolios, including 25 size–book-to-market portfolios, 25 size–profitability portfolios, 25 size–investment portfolios, 25 momentum portfolios, and 32 additional style-based portfolios. In addition, observations on five tradable factors—namely the market excess return (Mkt–Rf), size (SMB), value (HML), profitability (RMW), and investment (CMA)—are collected for the same sample period, from November 1990 to May 2025.

The second dataset represents the European investment equity universe. The analysis covers the period from January 2005 to May 2025. The dataset consists of time series of adjusted closing prices for 2327 European equities sourced from the Europe FactSet Market Index. The index is designed to capture a broad, investable cross-section of European stocks by including firms listed on major European exchanges that satisfy specific liquidity and capitalization requirements. Only actively traded, primary equity listings are retained, while inactive securities, secondary listings, and non-equity instruments are excluded. Furthermore, only stocks with at least five years of historical price data are included.[2]

3.2. Estimation Results and Risk Premia

We collect results on the implementation of the 3PEM and the conventional 2PEM framework on the European equity market to assess its empirical performance. Following the procedure outlined in Appendix A, the first step is to recover the latent factor space. Thus, we estimate the number of common latent factors. Figure 1 reports the eigenvalue structure and provides clear evidence of a low-dimensional factor structure in European portfolio returns. Panel A displays the distribution of the first ten eigenvalues of the sample covariance matrix. The first eigenvalue dominates the spectrum, accounting for the largest share of common variation in returns. Upon removing the first eigenvalue (Panel B), the remaining spectrum exhibits a smooth, monotonic decline. The cumulative explained variance shown in Panel C rises sharply with the inclusion of the first two principal components—surpassing 90% of total variance—and reaches roughly 95% when the first ten components are included. This pattern supports the presence of a small number of economically meaningful latent factors. Panel D compares the normalized information criteria proposed by Bai and Ng (2002). In particular, we compute the PCP1, ICP1, and BIC3.[3] All three criteria experience a pronounced drop between one and two factors, and then flatten substantially. The minima occur in the range between five and seven factors, with the Bayesian criterion (BIC3) reaching its lowest value at $\hat{p} = 6$. This result motivates the choice of six latent factors as the empirical benchmark for the subsequent estimation of risk premia. The convergence of different criteria around this dimension reinforces the robustness of the selected factor space. It provides a balanced trade-off between parsimony and explanatory power, in line with the asymptotic guidance in Bai and Ng (2002) and the empirical framework of Giglio and Xiu (2021).

A complementary perspective comes from analyzing how well the latent factor structure extracted in the first step is able to reconstruct the demeaned returns. In this regard, Figure A2 demonstrates that the latent space is able to capture a substantial share of the common variation in returns.

Moreover, a further diagnostic concerns the economic interpretability of the recovered latent space. To this end, we compute the correlations between the estimated latent factors and the observable Fama–French factors, as shown in Table 1. We observe that the first latent factor correlates almost perfectly with the market risk factor and it shows that the market component is largely recovered by the latent representation. A similarly strong correlation is observed between the second latent factor and SMB, which reflects the independence of the size premium from the remaining factor dimensions. However, HML, RMW, and CMA exhibit a different pattern. The third latent factor loads strongly on HML and only moderately on RMW and CMA. Therefore, it suggests that these factors do not correspond to clean, isolated directions in the latent space. Instead, they share common variation and occupy a more diffuse and weaker subspace of the pricing kernel. This behavior is thus consistent with their known lack of orthogonality in empirical datasets.

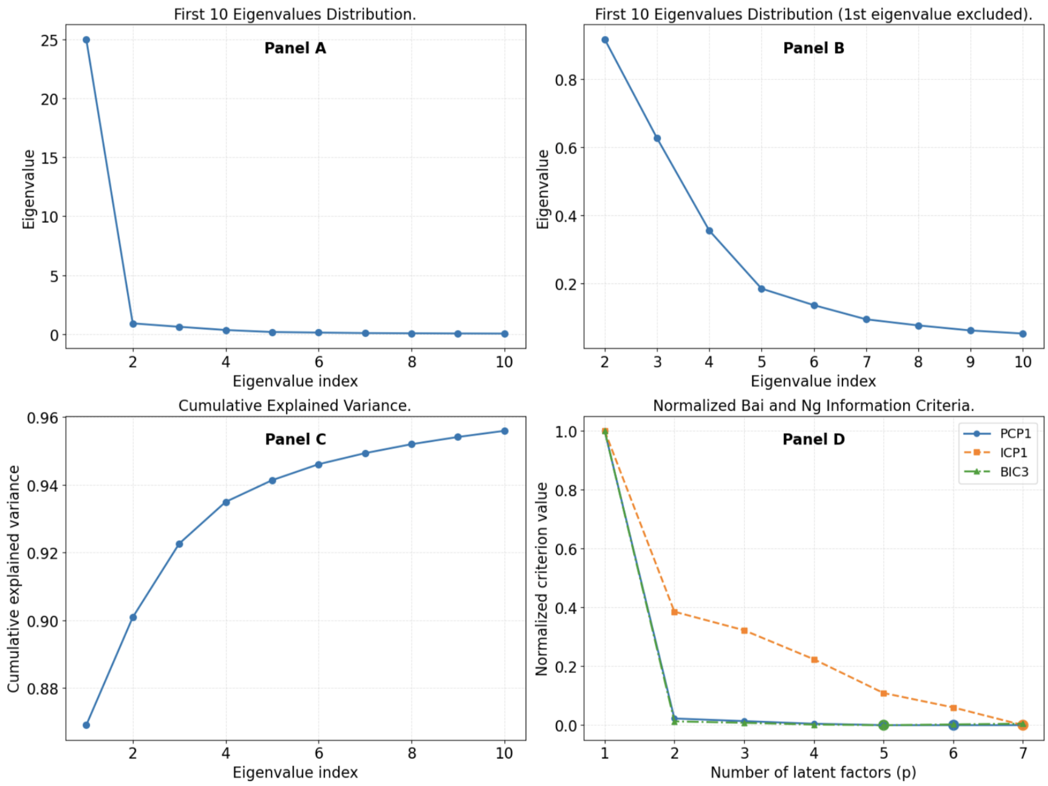

Figure 1. Eigenvalue structure and information criteria for determining the number of latent factors. *Notes:* The figure summarises the empirical eigenvalue structure of the return covariance matrix. It displays the distribution of the leading eigenvalues, the same distribution excluding the first eigenvalue, the cumulative explained variance, and the normalized Bai and Ng (2002) information criteria.

Table 1. Correlation coefficients between latent factors and Fama–French observable factors.

Latent Factor	Mkt–RF	SMB	HML	RMW	CMA
1st	0.9738	0.1378	0.2078	−0.2794	−0.2607
2nd	−0.2116	0.9726	−0.0700	0.0484	−0.0208
3rd	0.0591	−0.0803	−0.9038	0.5720	−0.6950
4th	−0.0240	0.0673	−0.2461	−0.1379	−0.3982
5th	0.0342	−0.1089	0.1692	−0.3348	−0.0551
6th	0.0277	−0.0549	−0.1197	0.0927	0.3206

Using the five-factor model of Fama and French (2015) as the baseline linear specification, Table 2 reports the estimated risk premia. Specifically, we present the estimates obtained from the traditional 2PEM and the 3PEM, and compare them to the time-series averages of the corresponding factor returns. It is important to notice that the choice of Fama–French five-factor specification serves purely as an empirical illustration of the 3PEM framework rather than a structural requirement of the method. The 3PEM can be applied to any linear factor model, including specifications based on alternative tradable or non-tradable factors. In fact, the observable factors simply act as proxies for the underlying sources of systematic risk, and the first step of the 3PEM procedure recovers the latent factor space independently of the chosen economic or fundamental model. Hence, the FF5 model is one possible application among many, selected in this work due to its extensive use in the literature and its relevance for explaining equity returns. Additionally, as we show in the subsequent analysis, the 3PEM also helps to evaluate the actual relevance of the factors included in the specification.

Table 2. Risk premia results.

	Time-Series Avg	2PEM				3PEM			
Factor	**Estimate**	**Restricted**		**Unrestricted**		**Restricted**		**Unrestricted**	
Mkt–RF	0.541	0.557	**	−0.411		0.571	***	0.371	*
SMB	0.018	0.054		0.050		0.013		0.015	
HML	0.319	0.205	*	0.250	*	0.262	*	0.264	*
RMW	0.314	0.594	***	0.529	***	0.045		0.048	
CMA	0.129	0.182	**	0.174	*	0.212	**	0.215	**

Notes: The restricted and unrestricted models correspond to a zero-beta rate that is either equal to the risk-free rate or freely estimated. Standard errors for the 2PEM estimates are computed using the Fama–MacBeth procedure with Shanken correction. For the 3PEM, the latent structure is estimated with $\hat{p} = 6$. All risk premia are expressed as percentages and computed using simple monthly returns. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Before analyzing the estimated risk premia obtained from the 2PEM and 3PEM, it is useful to examine some intermediary results from the first two steps of the traditional two-pass procedure. In particular, we inspect the cross-sectional distribution of the estimated factor loadings and their relation to average excess returns, as represented in Figures 2 and A1. Although the estimated betas show reasonably and economically plausible distributions, the beta-return scatter plots reveal that the cross-sectional pricing relationship is weak for most factors. The absence of a clear slope and the large dispersion of points is an indication of the fact that differences in factor exposures do not translate into systematic differences in average returns. As a result, the second-pass regression in the 2PEM might not perform well in determining a stable pricing signal, producing noisy and unreliable risk premium estimates.

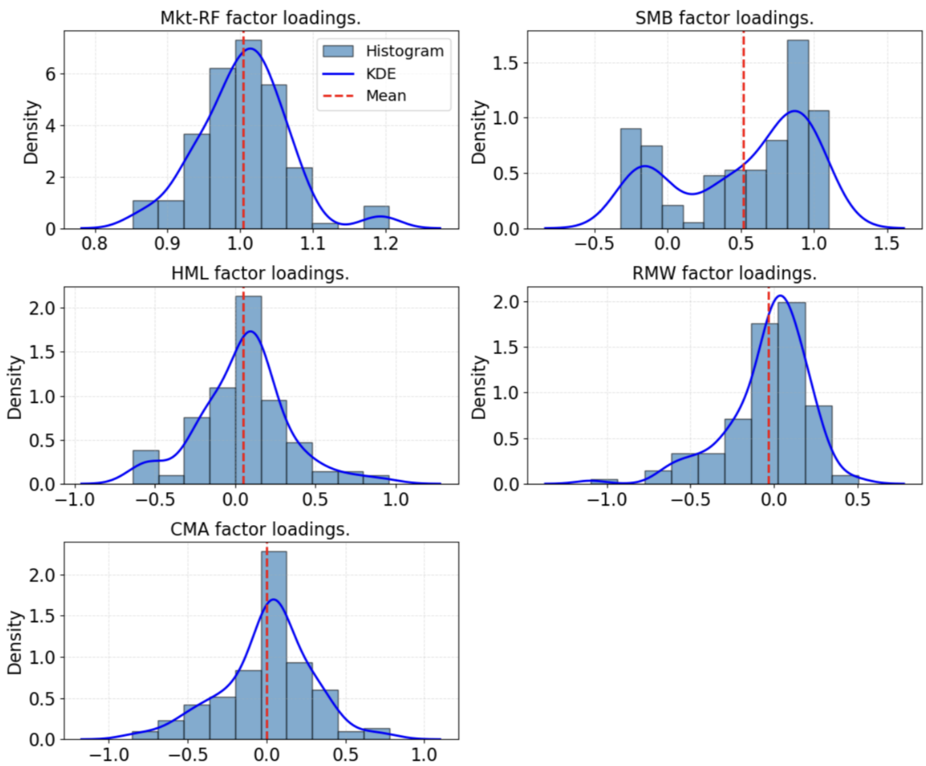

Figure 2. Cross-sectional distribution of first-pass estimated betas within 2PEM framework. *Notes:* The figure plots the empirical and kernel distributions of the estimated factor loadings obtained from the 2PEM time-series step. These factor loadings were estimated using the dataset of 132 portfolios. The mean values of the factor loadings are also shown (vertical dashed line). All estimates refer to the restricted zero-beta rate five-factors specification.

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Regarding the estimation of risk premia, and in order to illustrate the differences in detail, we examine two model specifications: one imposing the zero-beta rate to equal the T-bill rate, following Fama and French (2015), and one allowing the zero-beta rate to be freely estimated, as shown in Table 2. In order to study the significance of the estimates, we apply the inference results from Giglio and Xiu (2021) for the 3PEM estimates and Shanken (1992) for the 2PEM. Giglio and Xiu (2021) shows that $\hat{\gamma}_g$ converge to a normal distribution when the cross-sectional and time-series dimensions converge to infinity simultaneously, $n, T \to \infty$ and such that $T^{1/2}n^{-1} \to 0$.[4] We note that the asymptotic variance-covariance matrix of estimator of the risk premia of the observable factors g_t does not depend on the covariance matrix of the residual u_t or the estimation error of β. This is the main difference with the classical two-pass methodology and the Shanken correction, in which both β and Σ^u impact on the accuracy of the risk premia. See, for example, Cochrane (2009), Shanken (1992) and Gagliardini et al. (2020). The results, reported in Table 2, indicate an important difference between the two approaches. Under the traditional 2PEM, the unrestricted specification yields a negative estimate of the market risk premium. In contrast, the 3PEM produces a positive and statistically significant market risk premium. This shift reinforces the view that the 3PEM produces estimates more consistent with asset pricing theory, which predicts that the market factor should command a positive price of risk. The presence of a positive market premium under the 3PEM mirrors the findings of Giglio and Xiu (2021) for the U.S. market and is likewise confirmed here for the European equity market.

It is important to note that the alignment between the estimated and model-free risk premia is less pronounced for the profitability factor (RMW). This factor appears particularly noisy, a point that will be examined in greater detail later. Interestingly, RMW displays strong statistical significance under the 2PEM but not under the 3PEM. As demonstrated in the following section, despite the apparent significance produced by the 2PEM, the estimated factor loadings (betas) on RMW are largely insignificant across individual assets. Once these exposures are corrected using the 3PEM, the number of stocks exhibiting statistically significant loadings on the RMW factor increases, indicating that the denoising step embedded in the 3PEM facilitates a clearer and more reliable identification of true factor exposures.

To assess the robustness of the estimated risk premia to potential model misspecification, we perform additional estimations under three alternative specifications: the CAPM, the Fama–French three-factor model (FF3), and the five-factor model (FF5). Table 3 reports the estimated premia obtained using both the traditional two-pass estimation method and the 3PEM under the restricted zero-beta setting. The results indicate that the estimates of the risk premia from the 3PEM remain stable across all model specifications. Introducing or omitting observable factors does not materially alter the premia, suggesting that the 3PEM is robust to model misspecification and insulated from omitted-variable bias. This stability arises from the structure of the methodology. Indeed in the third pass of estimation approach (see Appendix A) each observable factor is projected onto the latent space extracted in the first step, ensuring that the contribution of unobserved components is already accounted for. In contrast, the 2PEM estimates exhibit substantial variation across the different model specifications.

However, while the 3PEM yields stable estimates with respect to the observable model specification, it remains sensitive to the underlying latent structure. Table A1 and Figure 3 illustrate how the estimated risk premia vary with the number of latent factors, $\hat{p}$. The market premium remains positive and stable across all configurations, confirming its robustness and strong identification. By contrast, the estimated premia and their statistical significance for the remaining factors depend on the chosen latent dimensionality.

Table 3. Risk premium estimates under 3PEM and 2PEM for different model specifications.

Factor	CAPM Estimate	CAPM SE		FF3 Factor Model Estimate	FF3 Factor Model SE		FF5 Factor Model Estimate	FF5 Factor Model SE	
Model: 3PEM, restricted ($\hat{p} = 6$)									
Mkt–RF	0.571	0.260	**	0.571	0.260	**	0.571	0.260	**
SMB				0.013	0.103		0.013	0.103	
HML				0.262	0.165	*	0.262	0.165	*
RMW							0.045	0.082	
CMA							0.212	0.116	**
Model: 2PEM, restricted									
Mkt–RF	0.571	0.248	**	0.529	0.243	**	0.557	0.258	**
SMB				0.044	0.105		0.054	0.111	
HML				0.184	0.134	*	0.205	0.142	*
RMW							0.594	0.115	***
CMA							0.182	0.099	**

Notes: All estimates refer to the restricted zero-beta rate specification for the CAPM and the FF3 factor model. Standard errors for the 2PEM estimates are computed using the Fama–MacBeth procedure with Shanken correction. For the 3PEM, the latent structure is estimated with $\hat{p} = 6$. All risk premia are expressed as percentages and computed using simple monthly returns. As implied by the 3PEM approach, the estimated risk premia remain unchanged when moving from the CAPM to the FF3 and FF5 specifications. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

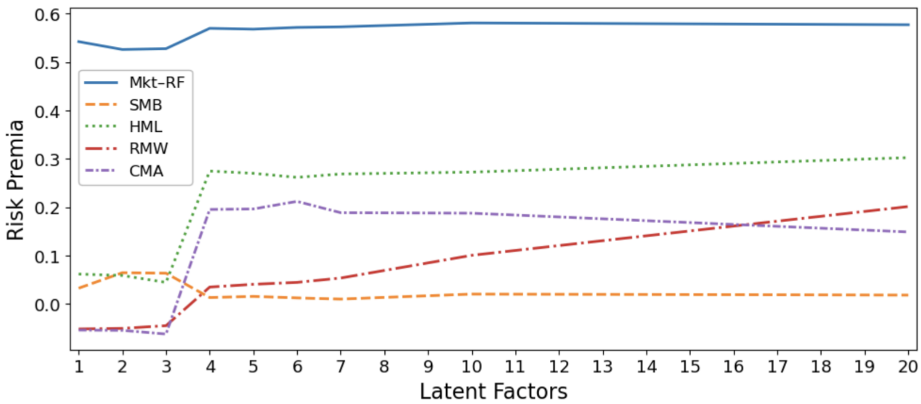

Figure 3. Risk premium estimates from the 3PEM imposing several values of latent factors $\hat{p}$. *Notes:* The figure compares the risk premium estimates for the restricted FF5 factor model by applying the 3PEM with a varying number of latent factors.

Moreover, Figures 4 and 5 examine the stability of the estimated risk premia under changes in sample composition, both across portfolios and over time. Specifically, Figure 4 assesses the sensitivity of the estimates to cross-sectional resampling, while Figure 5 presents results from a rolling-window analysis along the time-series dimension. In both cases, the distributions of the estimated premia show that the traditional two-pass approach yields wider and more dispersed estimates, indicating greater variability and weaker robustness. This effect is particularly pronounced for the factors identified as noisy under the 3PEM, such as RMW and CMA. Despite the presence of noise in certain factors, the premia estimated using the 3PEM remain substantially more stable under both cross-sectional and time-series perturbations.

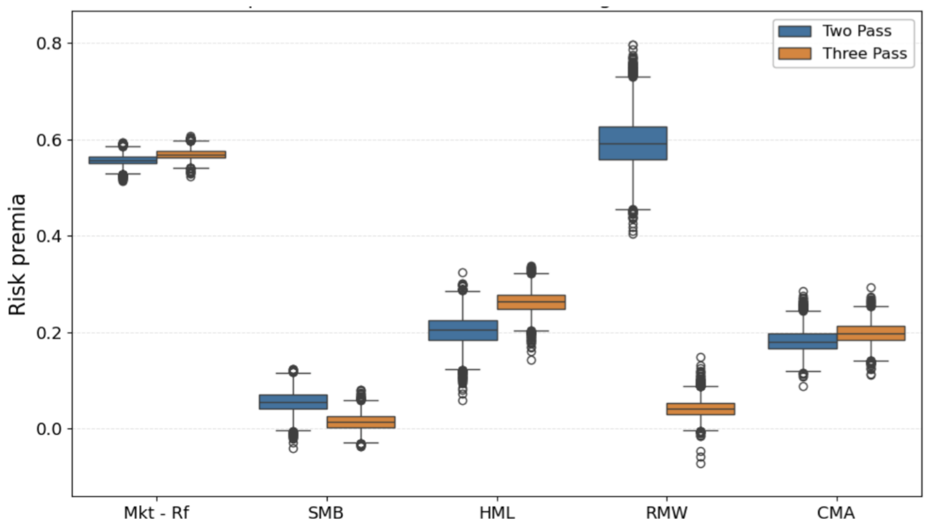

Figure 4. Risk Premia Sensitivity to Cross-Sectional Changes. *Notes:* Each boxplot shows the empirical distribution of factor risk premia obtained via cross-sectional resampling, for the restricted zero-beta rate specification of the model. At each iteration $n = 1, \ldots, 5000$, a random subset of $N = 100$ portfolios is drawn, the two-pass and three-pass estimators are computed. For the 3PEM, the latent dimension $\hat{p}$ is case-specific.

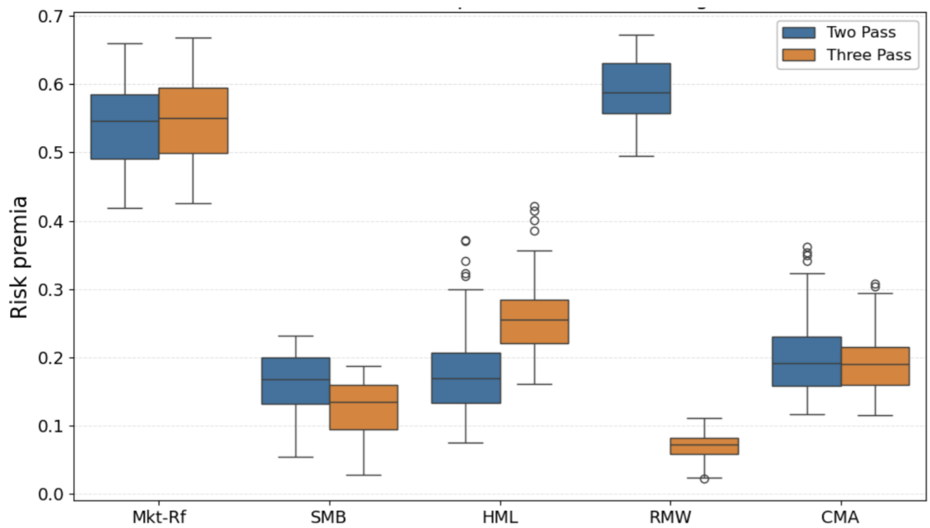

Figure 5. Risk premia sensitivity to time-series dimension. *Notes:* Each boxplot shows the time-series distribution of factor risk premia obtained through a rolling-window estimation procedure. The model is estimated over overlapping windows of 300 months, shifting by one period at each step, for the restricted zero-beta rate specification. The length of this window ensures that there is a sufficiently large time series for reliable estimation, in line with the asymptotic properties of the 3PEM approach. For the 3PEM, the latent dimension $\hat{p}$ is case-specific.

3.3. Denoised Fama–French Factors

Scholars have already raised concerns regarding measurement error in the Fama and French factors. For instance, Racicot et al. (2011) propose incorporating correction terms for factor exposures to address the errors-in-variables problem. In contrast, we advocate using the denoised version of observable factors—an approach applicable to any tradable factor, provided that the factor is priced. Indeed, given any specification of a linear factor model, 3PEM can be used as follows: (i) to detect whether the factor is weak, and if it is, to choose a model specification that excludes the weak factors, and (ii) if the factor is not weak, but it is measured with errors, that is, if it is noisy, to obtain a denoised version of it.

In the 3PEM setting, a factor g_t is weak if $\eta \to 0$, or $\eta = 0$, or equivalently, if its explanatory power with respect to a rotated version of latent factors is almost equal to noise. Giglio and Xiu (2021) propose to use the signal-to-noise ratio of each observable factor, defined as follows:

$$R^2_{g,i} = \frac{[\eta \Sigma^v \eta']_{ii}}{[\eta \Sigma^v \eta' + \Sigma^z]_{ii}},$$

where η is a $p \times d$ matrix collecting the loadings to the observed proxy factors, and Σ^v, Σ^z are the variance—covariance matrix of the innovations v_t and Z_t. In particular, if R^2_g is close to 1, then most of the variation in g is due to latent factors, meaning that g is strong: the factor is pervasive and measured with little noise. Conversely, a value close to zero signals that most of the variation in g_t is idiosyncratic and not linked to the underlying factor space.

Besides the signal-to-noise ratio, Giglio and Xiu (2021) also propose a formal statistical test of factor strength. The Wald statistic is constructed to test the null hypothesis that the observable factor is weak,

$$H_{0,i} : \eta_{i1} = \cdots = \eta_{i\hat{p}} = 0, \text{with } i = 1, ..., d, \tag{4}$$

against the alternative that it loads on at least one latent factor. The test is therefore designed to detect whether g_t has any systematic exposure to the latent factor space recovered in the first step of the 3PEM.

Thus, we compute the signal-to-noise ratios and formally test hypotheses on η. The evidence reported in Table 4 consistently points to the strength of the Fama–French observable factors. The estimated signal-to-noise ratios $\hat{R}^2_g$ are generally high, indicating that most of the variation in these factors is captured by the latent components extracted in the model. The market, SMB, and HML factors clearly behave as strong factors, while RMW shows a comparatively lower value, suggesting a greater role for idiosyncratic noise. The Wald test results reinforce this interpretation: for all five factors, the null hypothesis of weakness is decisively rejected. This provides formal evidence that each observable factor significantly loads on at least one latent component. Overall, the findings imply that all Fama–French factors are statistically and economically non-weak (that is, they are priced) in the European stock market, supporting the use of this model in subsequent empirical analysis.

Table 4. Estimated signal-to-noise ratios and Wald statistics for the five FF factors.

Factor	$\hat{R}^2_g$	Interpretation	$W_{i,T}$
Mkt–RF	0.999	Strong	408,143.84
SMB	0.991	Strong	33,407.41
HML	0.968	Strong	7880.41
RMW	0.547	Moderate	303.48
CMA	0.816	Strong	1159.24

Notes: The first two columns of the table show the estimated signal-to-noise ratios ($\hat{R}^2_g$) for each of the five FF factors, along with their interpretation. A value close to one indicates that the variation in the factor is largely explained by latent components, implying a strong factor. Conversely, lower values suggest higher idiosyncratic noise. The last column then provides the Wald test statistics for the null hypotheses $H_{0,i}: \ \eta_{i1} = \eta_{i2} = ... = \eta_{i\hat{p}} = 0$. The 5% critical value corresponds to the 95% quantile of the chi-squared distribution with 6 degrees of freedom, equal to 12.5916.

However, several factors appear to be affected by measurement error. From Equation (2) it is possible to decompose the variance of the observable factors in the variance due to the exposure to the true risk factors and the residual variance which can be attributable to the measurement error and noise. In particular, defining $G = \xi + \eta V + Z$ the matrix of $d \times T$ observable factors, we get that

$$Var(G) = Var(\eta V) + Var(Z).$$

Therefore, in the case of tradable factors, rather than using the noisy version of observable factors, it is possible to use their denoised version, $\hat{G} = \hat{\eta}\hat{V}$.[5] As shown in Figure 6, the variance decomposition indicates that although most of the variation in the observable factors is captured by the latent components, some factors retain a non-negligible proportion of idiosyncratic noise. In particular, the HML, RMW, and CMA European factors exhibit measurement errors, with the noise component especially pronounced for RMW and CMA. This suggests that these factors are less tightly connected to the latent structure identified by the 3PEM. Similar results can be observed in Figures 7 and A3, where the actual and denoised factor observations are plotted against each other. Factors such as Mkt–RF, SMB, and display points that lie very close to the 45-degree line, indicating that their variation is largely captured by the latent factor structure and that measurement noise is limited. In contrast, the wider dispersion visible for RMW and, to a lesser extent, for CMA signals a weaker relationship with the latent factors. Their observations deviate more substantially from the 45-degree line and suggest a higher noise component and lower signal strength within the 3PEM framework. While this paper does not aim to identify the precise sources of this noise, one potential explanation for the higher noisiness of RMW and CMA in Europe is that they rely on firm-level accounting measures. Even among IFRS-reporting listed firms, cross-country differences in accounting practices persist (Eisenschmidt & Krasodomska, 2021). Sectoral differences further amplify this effect, as firms in different industries follow distinct accounting conventions and exhibit diverse profitability and investment patterns. Therefore, these factors could make RMW and CMA less stable and more dispersed compared to factors like the market, SMB, or HML, which are based on broader, aggregated market signals. Figures 4 and 5 also confirm that RMW is the noisiest factor in the set since it displays greater estimation uncertainty in its corresponding risk premium. Notably, when comparing the 2PEM and 3PEM estimates, the 3PEM produces a more stable, lower-variance estimate of the RMW risk premium (which is also statistically significant under 2PEM). This contrast indicates that the 3PEM is more effective at absorbing spurious variation generated by measurement error. From an economic perspective, these results suggest that factor pricing in Europe differs from the U.S. not only in magnitude but also in structure. While broad sources of risk such as the market and size factors remain well identified, factors linked to firm-level accounting characteristics appear more fragile and sensitive to institutional heterogeneity. This helps explain why standard factor models often deliver unstable or weak results in European applications and highlights the importance of explicitly accounting for measurement error when interpreting factor premia outside the U.S.

To further illustrate the ability of 3PEM to distinguish economically meaningful factors from irrelevant ones, we introduce a purely spurious factor generated as random noise and subject it to the same variance–decomposition and correlation analysis as the observable Fama–French factors. The results Table 5 and in Figure 8 clearly show that this artificial factor is almost entirely orthogonal to the latent factor space, with correlations that remain close to zero across all latent components. Consistent with this, the variance decomposition assigns nearly all of its variation to the idiosyncratic noise component. This behavior demonstrates that the 3PEM is able to correctly classify the spurious factor as non-informative since it is not aligned with any latent source of systematic risk. Therefore, these findings reinforce the suggestion to use the 3PEM to filter out irrelevant or noisy

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factors and confirm its theoretical advantage in settings where observable characteristics may include weak or non-priced components.

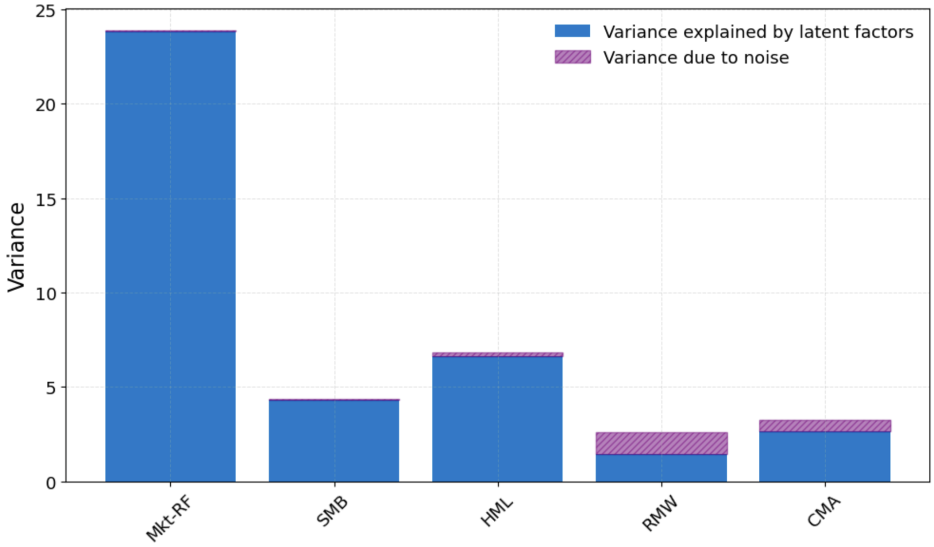

Figure 6. Variance Decomposition of each observable FF factor. *Notes:* The figure shows the decomposition of the variance of the five FF factors, distinguishing between the variance explained by the latent factor and that sourced by noise. The variance is reported as a percentage.

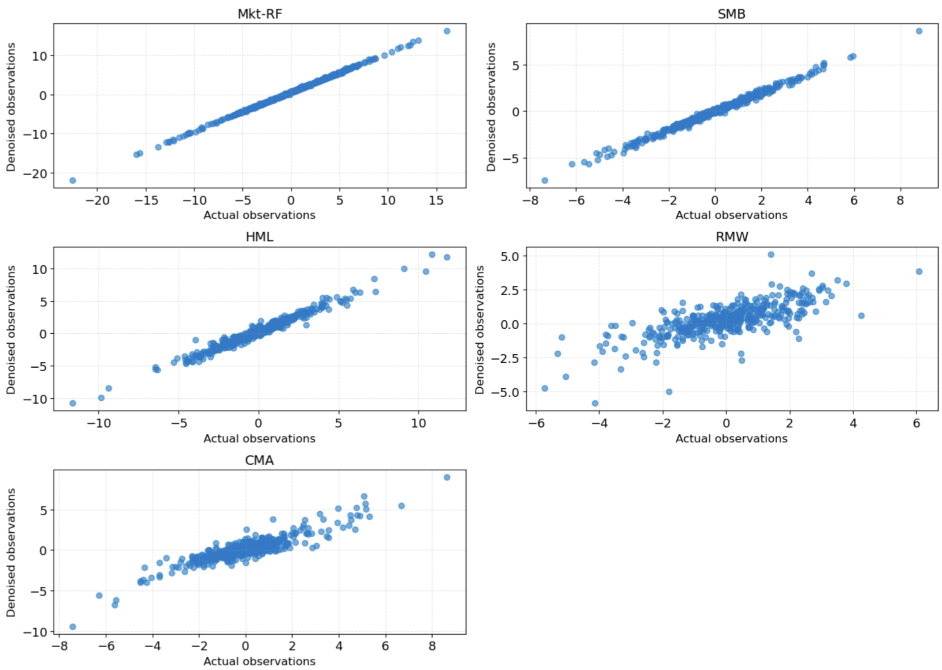

Figure 7. Actual versus denoised factor observations. *Notes:* The figure plots the scatter plots of the five denoised FF factors compared to observed ones.

Hence, in the next part of analysis, we compute the denoised version of the observable factors according to Equation (3), which allow us to separate the systematic component captured by the latent structure from the idiosyncratic noise associated with each factor.

Table 5. Correlation coefficients between latent factors and the spurious factor.

Latent Factor	Spurious
1st	−0.0489
2nd	−0.1318
3rd	−0.0062
4th	0.0893
5th	0.0581
6th	0.0882

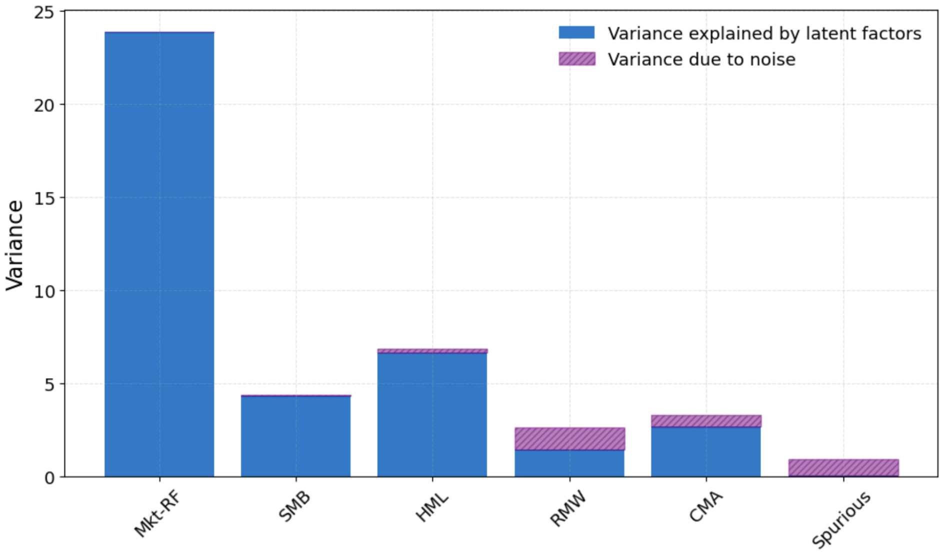

Figure 8. Detection of a spurious factor via variance decomposition.

3.4. Risk Exposures and Expected Values at Stock-Level

Building on prior results, the analysis now extends to individual stocks. In this section, the analysis applies the Fama–French five-factor (FF5) model, under the restricted zero-beta rate assumption, to estimate factor loadings and expected returns for 2327 European stocks. The study aims to compare results under two distinct setups: (i) Raw factors: Stock-level betas are estimated using the raw FF5 factors through time-series regressions, while the risk premia are those computed at the portfolio level using the 2PEM framework. (ii) Denoised factors: Stock-level betas are estimated using the denoised FF5 factors obtained via the 3PEM framework, together with risk premia computed at the portfolio level through the 3PEM. In both setups, stock-level betas are estimated using ordinary least squares (OLS) regressions corrected for heteroskedasticity and autocorrelation, with standard errors adjusted for up to six lags. Expected stock returns are subsequently derived by multiplying the estimated betas by the corresponding vector of risk premia. In the time-series regressions used to estimate stock-level factor loadings, employing denoised factors leads to a modest improvement in the model's explanatory power. As reported in Table A2, the mean and median adjusted R^2 remain largely unchanged when denoised factors are used, while the upper quartile of the distribution shifts upward, indicating a modest improvement in explanatory power for stocks with stronger factor exposures. Although the magnitude of the change is small, the upward shift in the upper part of the R^2 distribution suggests that denoising the factors reduces idiosyncratic noise in the time-series estimation of betas, leading to somewhat more stable factor exposures across stocks.

Figure 9 shows the distributions of stock-level betas estimated using raw and denoised factor observations. The results indicate that, for the market and size factors, the distributions remain nearly identical, suggesting that these factors are already well captured by their observable counterparts. In contrast, for HML, RMW, and CMA, the distributions widen after denoising, exhibiting greater cross-sectional dispersion in stock exposures. This increased dispersion highlights that the denoising process uncovers meaningful heterogeneity in how individual stocks load on these factors—heterogeneity that was previously masked by noise. The effect is particularly pronounced for the profitability and investment factors, confirming that denoising identifies the true underlying exposures rather than merely increasing estimation error. Overall, this evidence supports the conclusion that denoising enhances the precision and interpretability of factor loadings, especially for factors subject to measurement error.

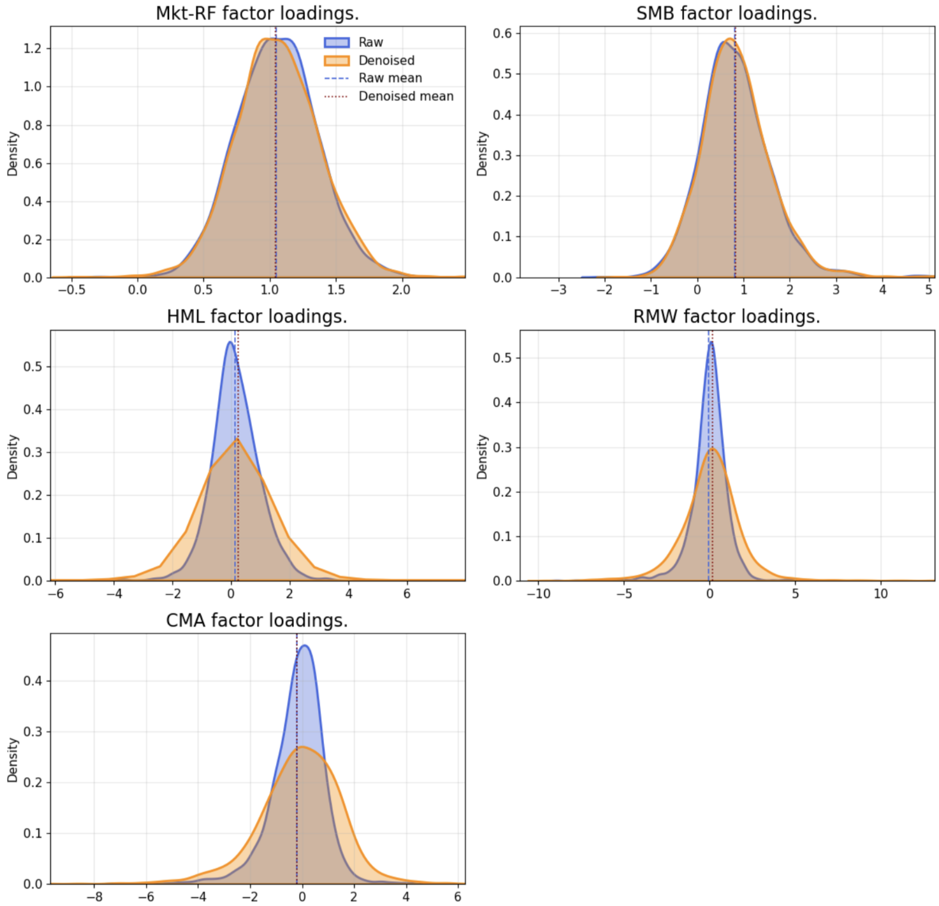

Figure 9. Cross-sectional distribution of the estimated raw and denoised factor loadings. *Notes:* The figure plots the empirical distributions of the estimated raw and denoised factor loadings. These factor loadings were estimated using the dataset of individual stocks. The mean values of the factor loadings are also shown (vertical lines). All estimates refer to the restricted zero-beta rate five-factor specification.

Figure 10 illustrates the differences in the distribution of expected monthly stock returns across estimation procedures. The 2PEM approach yields a wider dispersion with heavier tails, resulting in both overestimation and underestimation of expected returns compared to the 3PEM approach. These deviations are likely driven by noise contaminating the tradable factors, which raises spurious covariances for some stocks while reducing true exposures for others. Furthermore, the root mean squared difference between the vectors of expected returns estimated using the 2PEM approach and those obtained using denoised factors and 3PEM risk premia is approximately 1.18% across stocks. This measure captures the cross-sectional deviation between the expected return estimates implied by the two procedures rather than a simple average difference in monthly returns. It indicates that the two approaches generate non-negligible differences in the cross-sectional pattern of expected returns, which may affect portfolio allocation decisions and the interpretation of risk–return trade-offs.

Figure 10. Cross-sectional empirical distributions of the expected values of stock returns estimated using the 2PEM and 3PEM. *Notes:* The figure plots the empirical distributions of the estimated expected values of returns for the individual stocks. The expected values are computed by applying the 2PEM and the 3PEM. The mean values of the returns are also shown (vertical lines).

In certain cases, the discrepancies are substantially larger. For example, Figure A4 highlights individual stocks to illustrate how expected values change when denoised factors are used instead of raw factors. For stocks such as ATO–FR and IGR–GB, expected returns estimated with raw factors are biased and substantially underestimated, particularly when the raw specification yields large negative values. After denoising, expected returns adjust to more plausible levels, indicating that the 3PEM correction effectively mitigates distortions caused by measurement error. For other stocks, such as ONE–AU and HTRO–SE, the differences are smaller, reflecting that the impact of denoising depends on the extent to which each asset's exposure is affected by noise in the factor structure.

3.5. Out-of-Sample Validation of Denoising Procedure

While denoising materially alters the distribution of estimated betas and expected returns, a natural question is whether these changes reflect improved information about future returns or merely reweight existing noise. To address this question, we perform a simple out-of-sample validation exercise designed to assess the usefulness of denoised expected returns in explaining subsequent realized returns. In particular, we estimate the stock-level factor loadings using rolling windows of the preceding 60 months. Risk premia at the portfolio level are estimated using rolling windows of 300 months under both the 2PEM and 3PEM frameworks. This procedure yields 114 out-of-sample risk premium estimates for the period October 2015 to April 2025. Expected returns computed at month t are interpreted as predictions for month $t+1$ and are compared to realized simple stock returns over the period November 2015 to May 2025. All results are obtained under the restricted zero-beta rate assumption. Next, we compute estimation errors as the difference between realized and expected returns, and cross-sectional RMSE and MAE at both the monthly and yearly frequencies.

As shown in Figure 11, both RMSE and MAE for the 3PEM are consistently lower than those for the 2PEM. This indicates that the proposed denoising procedure and the 3PEM risk premia are able to reduce prediction errors, increase predictive power, and produce more accurate estimates of expected returns. Moreover, the cross-sectional RMSE remains relatively stable over an extended period up to November 2024. However, after this point, the FF5 factor model, regardless of the estimation method used, appears to lose predictive accuracy at cross-sectional level. This decline may reflect a structural shift in risk perception, whereby European market investors based their investment decisions less on company fundamentals or standard market factors and more on political risk. Notably, this period coincides with a change in the U.S. presidency and increased geopolitical uncertainty. Indeed, numerous studies have explored the impact of the U.S. election on global financial markets, supporting the notion that political events can significantly influence investor behavior and asset pricing (see, e.g., Ampountolas, 2025; Piserà et al., 2025).

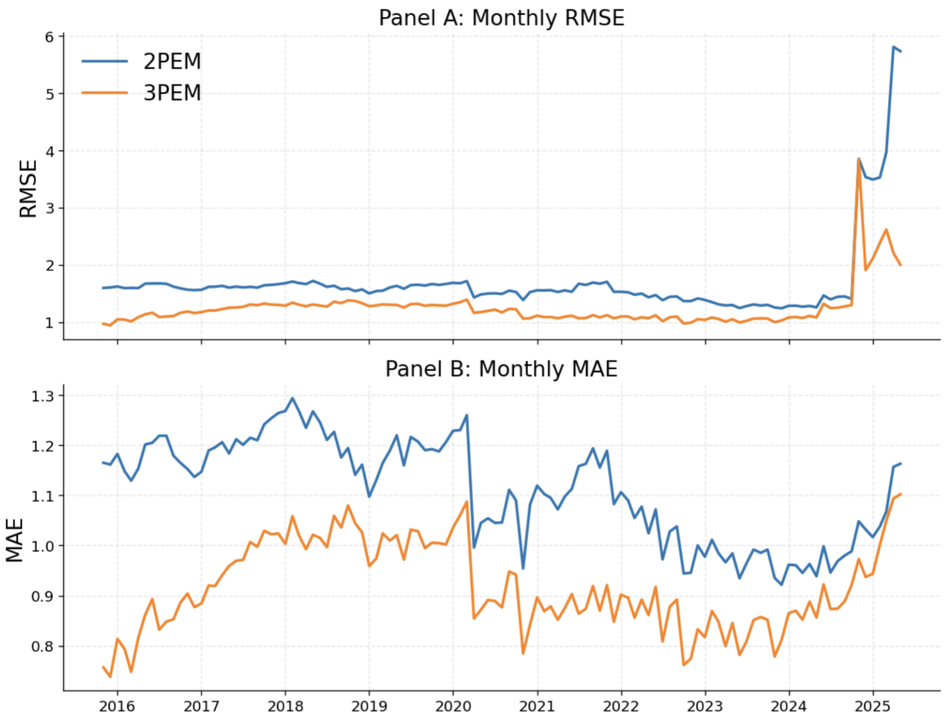

Figure 11. Monthly RMSE and MAE for the 2PEM and 3PEM over the individual stocks. *Notes:* The figure plots the RMSE (Panel **A**) and the MAE (Panel **B**) for the expected returns observed and estimated applying the 2PEM and the 3PEM. The results are obtained from an out-of-sample application from October 2015 to April 2025.

Additionally, Figure 12 presents the out-of-sample distribution of monthly absolute errors by year for the 2PEM and 3PEM models. It is evident that the error distribution under the 3PEM is narrower than that of the 2PEM, further confirming that the denoising procedure is able to improve predictive accuracy.

Importantly, this exercise is not intended as an investment backtest, but as a diagnostic check of whether denoising improves the economic content of model-implied expected returns. The results suggest that denoised factors yield expected return estimates that are more closely aligned with realized outcomes, supporting the practical relevance of the denoising step.

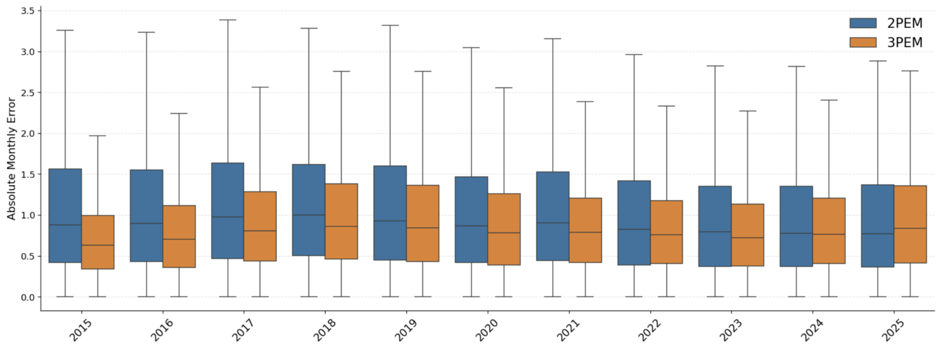

Figure 12. Cross-sectional annual distribution of monthly absolute errors for 3PEM and 2PEM. *Notes:* The figure provides annual boxplots of the monthly absolute errors for the expected returns observed and estimated applying the 2PEM and the 3PEM. The results are obtained from an out-of-sample application from October 2015 to April 2025.

4. Conclusions

This study provides a comprehensive evaluation of the Three-Pass Estimation Method (3PEM) within the European equity market, moving beyond a purely methodological application to address the documented challenges of factor noise and market heterogeneity.

In particular, we estimate factor risk premia in the presence of omitted factors and measurement error and obtain denoised versions of tradable factors that can be further used to compute expected returns at the stock level. Using a large panel of European portfolio and stock returns, and adopting the Fama–French five-factor model as a benchmark specification, we confirm that traditional two-pass procedures are highly sensitive to noise in observable factors and model misspecification. In contrast, the 3PEM delivers risk premium estimates that are both economically plausible and statistically robust, even when the observable factor set is incomplete or contaminated by measurement error.

Our empirical results highlight several key advantages of the 3PEM. First, the estimated risk premia are stable across alternative observable model specifications and align closely with the time-series averages of tradable factor returns, particularly for the market factor. This stability reflects the ability of the method to account for omitted-variable bias by conditioning on a latent factor space extracted from the cross-section of returns. Second, the 3PEM provides a natural diagnostic framework to assess factor relevance. Through signal-to-noise ratios and formal Wald tests, we show that all Fama–French factors are priced in the European market, while simultaneously identifying substantial measurement error in some of them—most notably in the profitability and investment factors.

Furthermore, as a main contribution of this paper, we demonstrate how the 3PEM can be employed as a practical tool for denoising tradable factors. By projecting observable factors onto the estimated latent space, we obtain purified factor series that more accurately capture systematic sources of risk. We demonstrate that the use of denoised factors modifies the cross-sectional distribution of the stock-level factor loadings. Notably, using denoised factors reduces the dispersion and instability observed in the traditional two-pass framework, revealing heterogeneity in factor exposures that would otherwise be obscured by noise. We also show that the denoising procedure improves the out-of-sample expected return estimates, leading to economically meaningful differences that might be relevant for portfolio construction and asset allocation.

Nevertheless, it is important to acknowledge that employing the 3PEM framework has several limitations. First, the method requires sufficiently large time-series and cross-sectional dimensions to fully exploit the information content of the underlying PCA. When the panel is short or sparse, the latent structure may be imprecisely estimated and not all relevant systematic components may be recovered. Second, although the results are generally robust to omitted factors and measurement errors, they remain sensitive to variations in the latent structure, and the method does not identify which factors are truly driving asset returns. Third, the 3PEM relies on the assumption of linear relationships between factors and returns, which may not hold in the presence of non-linear effects or regime shifts. Finally, out-of-sample performance can still be affected by structural breaks, extreme market events, or periods of increased uncertainty, limiting the generalizability of the estimates.

Overall, the 3PEM offers a consistent approach to factor selection, noise identification, and the development of cleaner factor inputs for subsequent applications. Although the analysis focuses on the Fama–French model and the European equity market, the methodology is entirely flexible and can be applied to different factor structures, asset classes, and markets.

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Appendix A. Estimation Approaches

The approach is based on a linear factor structure for r_t:

$$r_t = \beta f_t + u_t,$$

where f_t is a set of observable systematic factors. The two-pass cross-sectional regression approach is the most popular methodology to estimate risk premia in a linear multi-factor setting. Fama and MacBeth (1973) proposes to estimate risk premia in two steps. In the first step, the factor loadings β_i are estimated by time-series ordinary least squares (OLS) regression. In the second step, the risk premia vector is estimated by running a cross-sectional OLS regression at each time t, and taking the average of the cross-sectional regression coefficients.

Despite its simplicity, the two-pass approach suffers from the error-in-variable (EIV) problem: the risk premium estimator contains an estimation error through the estimated factor loadings. If we ignore the estimated errors, we understate the asymptotic variance of risk premia. Furthermore, if relevant factors are omitted in the linear structure, then an estimation bias could affect the estimated factor loadings and the risk premia.

The three-pass methodology in Giglio and Xiu (2021) aims to overcome these limitations. Let us write the model from Equations (1) and (2) in matrix notation as follows:

$$R = \beta\gamma\iota_T' + \beta V + U, \text{ and } G = \xi + \eta V + Z, \tag{A1}$$

where $R \in \mathbb{R}^{n\times T}$ is the matrix of excess returns, $\beta \in \mathbb{R}^{n\times p}$ is the matrix of factor loadings, ι_T is the T vector of ones, and risk premia and innovations are collected in $\gamma \in \mathbb{R}^{p\times 1}$

and $V \in \mathbb{R}^{p \times T}$, respectively. Furthermore, the matrix of observable factors is $G \in \mathbb{R}^{d \times T}$, $\xi \in \mathbb{R}^{d \times T}$ refers to the constant terms, and matrix $\eta \in \mathbb{R}^{d \times p}$ collects the loadings to the observed proxies factors. Finally, error terms are gathered in matrices $U \in \mathbb{R}^{n \times T}$ and $Z \in \mathbb{R}^{p \times T}$ with $\mathbb{E}[V] = \mathbb{E}[U] = \mathbb{E}[Z] = 0$. We also assume that $\mathbb{E}[UV'] = 0$ and $\mathbb{E}[ZV'] = 0$. Finally, the setting proposed in Equation (A1) can be easily extended to the unrestricted zero-beta rate as follows:

$$R = \gamma_0 \iota_n \iota_T' + \beta \gamma \iota_T' + \beta V + U, \tag{A2}$$

where $\iota_n \in \mathbb{R}^{n \times 1}$ is a vector of ones.

Based on this model setting, we provide a description of the three passes to estimate the risk premia vector of g_t.

1. *Estimation of V and β.* We apply PCA on $(nT)^{-1}\bar{R}'\bar{R}$, where $\bar{R}$ is the matrix of demeaned returns, such that the element $[\bar{r}_{i,t}]$ is defined as $\bar{r}_{i,t} = r_{i,t} - \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$. We then obtain the following eigenvalue decomposition:

$$(nT)^{-1}\bar{R}'\bar{R} = Q\Lambda Q',$$

where Q is an $T \times T$ matrix whose columns are the orthogonal eigenvectors of $(nT)^{-1}\bar{R}'\bar{R}$, and Λ is an $T \times T$ diagonal matrix whose diagonal elements are the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_T$. By decomposing $(nT)^{-1}\bar{R}'\bar{R}$ into its eigenvectors and eigenvalues, it is possible to identify the components that capture the most significant patterns in the asset returns.

In this setting, the true number p of latent factors is an unknown parameter. In line with Giglio and Xiu (2021), the following consistent estimator for p is introduced:

$$\hat{p} = \arg\min_{1 \le j \le p_{\max}} \Big((nT)^{-1}\lambda_j(\bar{R}'\bar{R}) + j \times \phi(n,T) \Big) - 1,$$

where the term $\lambda_j(\bar{R}'\bar{R})$ corresponds to the j-th eigenvalue of the matrix $\bar{R}'\bar{R}$, and $\phi(n,T)$ is a penalty term depending on the number of assets and the time periods considered.

Then, the following approximation holds:

$$(nT)^{-1}\bar{R}'\bar{R} \approx Q_{\hat{p}}\Lambda_{\hat{p}}Q_{\hat{p}}' = \sum_{i=1}^{\hat{p}} \lambda_i \xi_i \xi_i',$$

where $\Lambda_{\hat{p}}$ is the diagonal matrix containing the first $\hat{p}$ sorted eigenvalues, and $Q_{\hat{p}}$ is the matrix of their corresponding orthogonal eigenvectors. Finally, the estimated matrix of latent factors is defined as

$$\hat{V} = T^{1/2}(\xi_1 : \xi_2 : \ldots : \xi_{\hat{p}})',$$

and the corresponding estimated matrix of factor loadings:

$$\hat{\beta} = T^{-1}\bar{R}\hat{V}'.$$

2. *Estimation of γ.* In order to estimate the risk premia of latent factors, we regress the average returns of the n testing assets on the estimated factor loadings, $\hat{\beta}$:

$$\bar{r} = \hat{\beta}\gamma + u_t,$$

where $\bar{r} = \begin{bmatrix} \bar{r}_1 & \bar{r}_2 & \cdots & \bar{r}_n \end{bmatrix}'$, and the risk premia of estimated latent factors are given by

$$\hat{\gamma} = \left(\hat{\beta}'\hat{\beta}\right)^{-1}\hat{\beta}'\bar{r}.$$

3. *Estimation of* γ_t. From Equation (A1), we have

$$\bar{G} = G - E[G] = \xi + \eta V + Z - \epsilon = \eta V + Z,$$

where $\bar{G}$ is the matrix $(d \times T)$ of the demeaned observable factors. The true factors V are not directly observable and we can deduce a rotated version of them, $\hat{V}$ through PCA, and

$$\bar{G} = \eta\hat{V} + Z.$$

Thus, the time-series regression allows to project g_t onto the space spanned by the estimated factors $\hat{V}$ and we get

$$\hat{\eta} = \bar{G}\hat{V}'(\hat{V}\hat{V}')^{-1}.$$

Note that if the factors are tradable, then

$$G = \eta\hat{V} + Z, \tag{A3}$$

and $\hat{\eta} = G\hat{V}'(\hat{V}\hat{V}')^{-1}$. Finally, we estimate the risk premia of observable factors as follows:

$$\hat{\gamma}_g = \hat{\eta}\hat{\gamma}.$$

Moreover, we also get the denoised version of the observable factors as the fitted value of the model (A3):

$$\hat{G} = \hat{\eta}\hat{V}.$$

Finally, for the unrestricted zero-beta rate in model (A2), we get

$$\begin{aligned} \hat{\gamma}_0 &= \left(\iota_n'\mathbb{M}_{\hat{\beta}}\,\iota_n\right)^{-1}\iota_n'\mathbb{M}_{\hat{\beta}}\,\bar{r} \\ \hat{\gamma}_g &= \bar{G}\hat{V}'(\hat{V}\hat{V}')^{-1}\left(\hat{\beta}'\mathbb{M}_{\iota_n}\hat{\beta}\right)^{-1}\hat{\beta}'\mathbb{M}_{\iota_n}\,\bar{r}, \end{aligned}$$

where $\mathbb{M}_{\hat{\beta}} = I_n - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}'$ and $\mathbb{M}_{\iota_n} = I_n - \iota_n(\iota_n'\iota_n)^{-1}\iota_n'$.

Appendix B. Additional Tables and Figures

Table A1. 3PEM risk premia at changing the number of latent factors.

	$\hat{p}=1$			$\hat{p}=2$			$\hat{p}=3$			$\hat{p}=4$			$\hat{p}=7$			$\hat{p}=10$			$\hat{p}=20$		
Factor	**Est.**	**SE**	**Sig.**	**Est.**	**SE**	**Sig.**	**Est.**	**SE**	**Sig.**	**Est.**	**SE**	**Sig.**	**Est.**	**SE**	**Sig.**	**Est.**	**SE**	**Sig.**	**Est.**	**SE**	**Sig.**
Mkt–RF	0.542	0.274	**	0.526	0.258	**	0.528	0.259	**	0.570	0.259	**	0.573	0.261	**	0.581	0.262	**	0.577	0.262	**
SMB	0.033	0.021	*	0.065	0.104		0.064	0.104		0.013	0.105		0.010	0.102		0.021	0.102		0.019	0.102	
HML	0.062	0.043	*	0.059	0.040	*	0.045	0.156		0.275	0.170	*	0.269	0.165	*	0.273	0.165	**	0.303	0.171	**
RMW	−0.052	0.030		−0.050	0.029		−0.045	0.070		0.035	0.085		0.054	0.080		0.101	0.079		0.201	0.083	***
CMA	−0.054	0.025		−0.054	0.026		−0.062	0.087		0.195	0.111	**	0.189	0.124	*	0.188	0.125	*	0.149	0.123	

Notes: All estimates refer to the restricted zero-beta rate specification for the FF5 factor model. For the 3PEM, the latent structure is estimated for several values of p. All risk premia are expressed as percentages and computed using simple monthly returns. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

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Table A2. Distribution of first-pass adjusted R^2 across stocks.

Statistic	Raw Factors	Denoised Factors
Mean	0.3265	0.3290
Std. dev.	0.1487	0.1498
Minimum	−0.0457	−0.0583
25%	0.2247	0.2241
Median	0.3319	0.3353
75%	0.4293	0.4318
Maximum	0.7774	0.8117
Observations	2327	2327

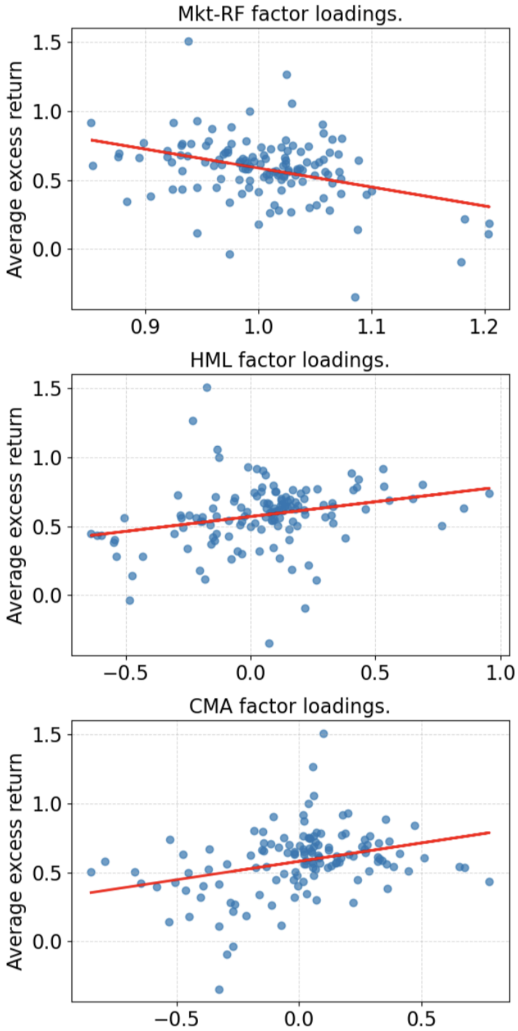

Figure A1. Scatter plots of average excess returns against first-pass betas for the five Fama–French factors within 2PEM framework. *Notes:* The figure plots the scatter plots of the estimated factor loadings compared to the average observed excess returns of individual stocks. The estimated loadings are obtained from the 2PEM time series step. All estimates refer to the restricted zero-beta rate five-factor specification.

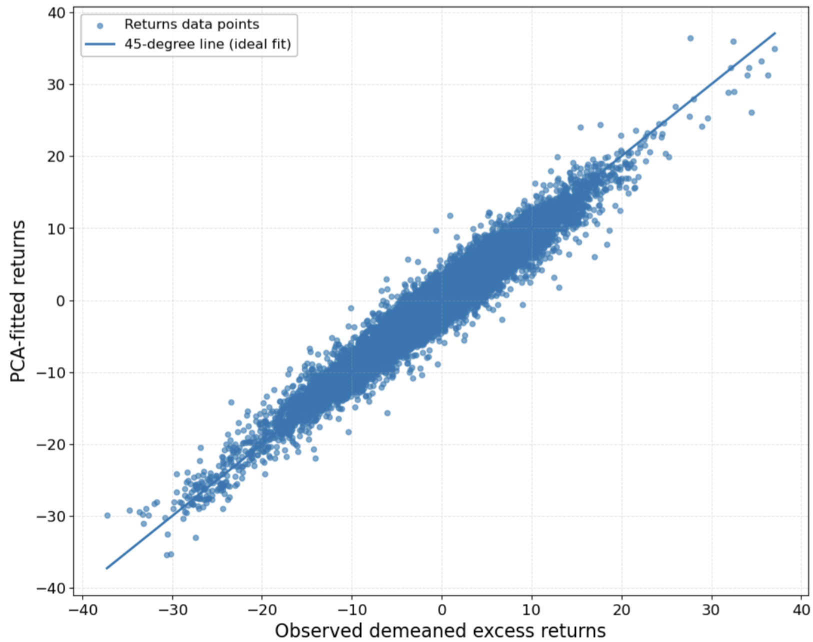

Figure A2. Reconstruction of demeaned excess returns using PCA-estimated latent factors. *Notes:* The figure shows a scatter plot based on the interaction between the observed excess returns of the 132 portfolios and their respective fitted values obtained using the PCA procedure.

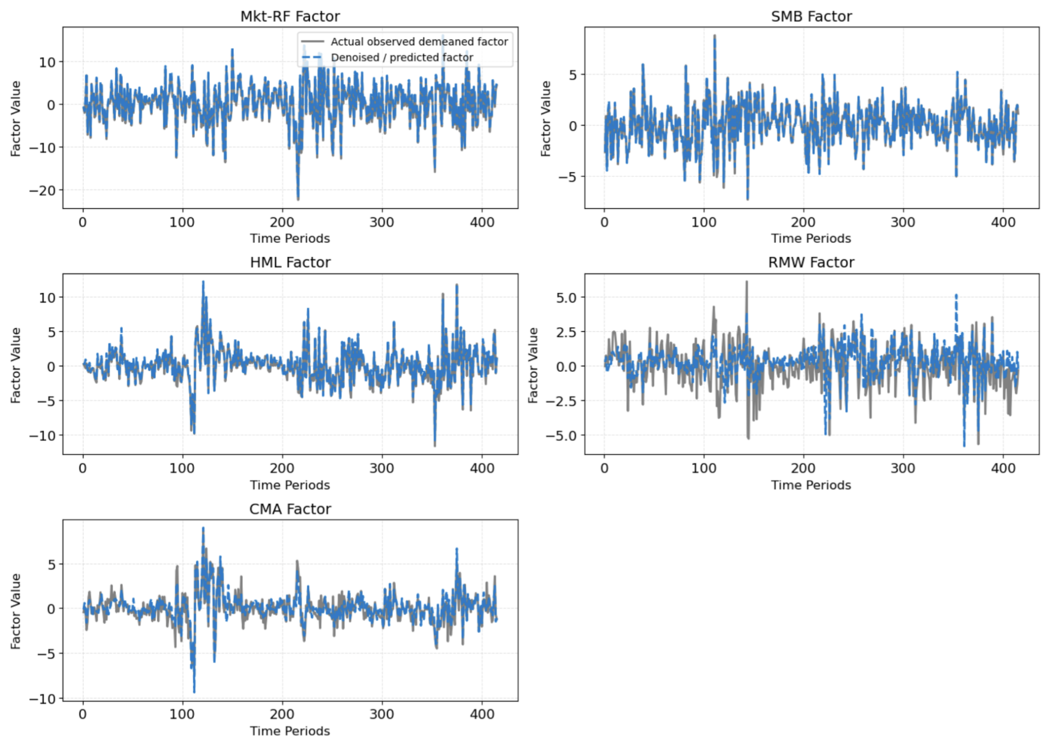

Figure A3. Time-series comparison between actual and denoised five FF factors. *Notes:* The figure plots the time-series observations for each FF factor. In particular, the dashed line corresponds to the denoised factors.

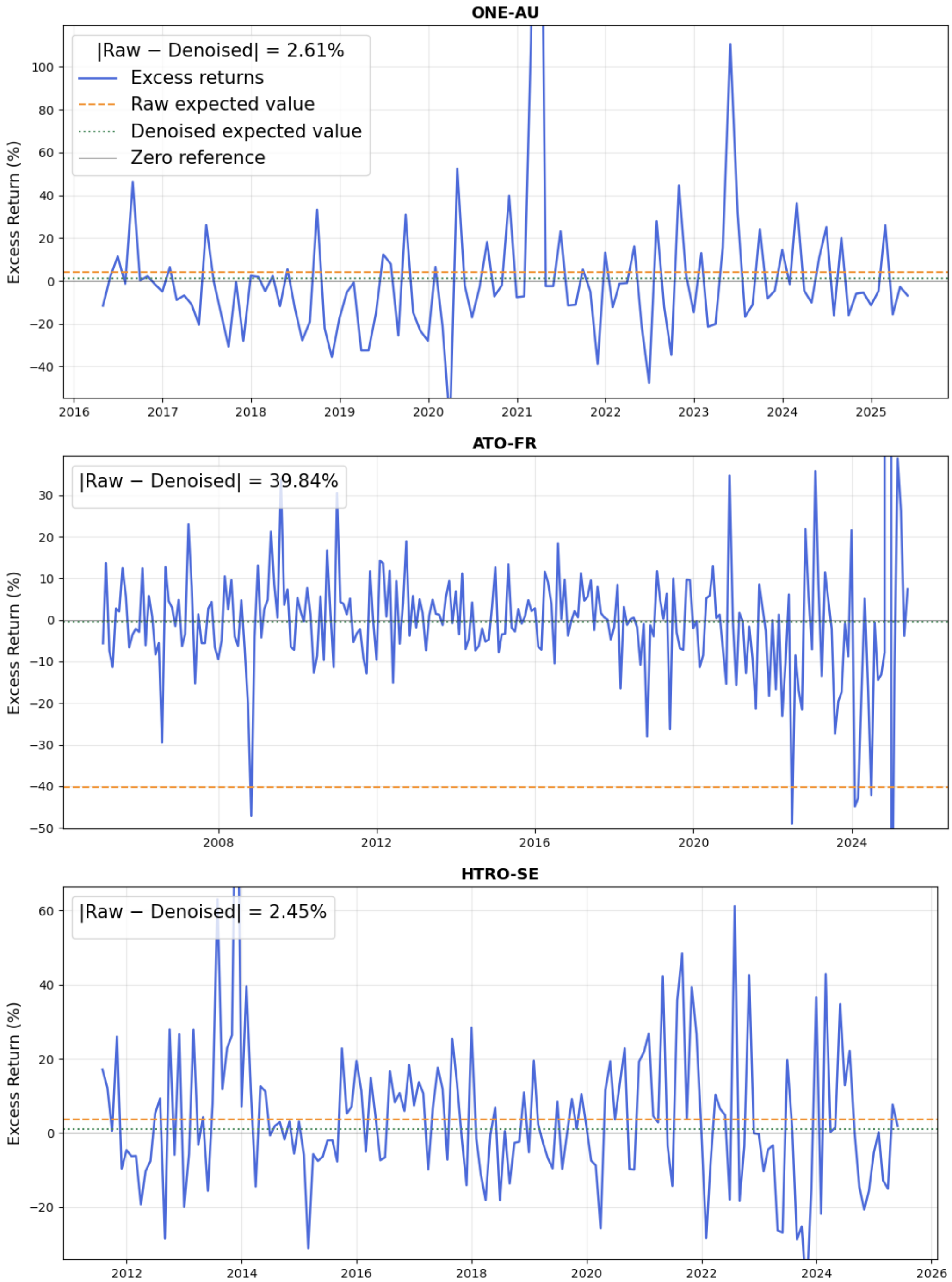

Figure A4. Time-series comparison between actual excess returns, raws, and denoised expected returns. *Notes:* The figure plots the time-series of excess returns for Oneview Healthcare PLC Chess Depository Interests repr 1 (ONE-AU), Atos SE (ATO-FR), and Hennes & Mauritz AB (HTRO-SE). Horizontal lines report the corresponding time-series average return for each asset.

Notes

1 For details on the estimation approach, see Appendix A.

2 Newly listed firms represent a distinct analytical category and are therefore excluded from the present analysis. We also allow for prices below one unit, as the analysis relies on simple rather than logarithmic returns. All prices are denominated in U.S. dollars to ensure consistency with the Fama–French factors, which are reported as simple returns based on dollar-denominated prices.

3 Based on the minimization approach in Equation (1), Bai and Ng (2002) proposed two classes of *PC* and *IC* criteria w.r.t the definition of the variance and penalty functions.

4 See Theorem 1 in Giglio and Xiu (2021).

[5] See Appendix A for details.

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