EURO Journal on Computational Optimization 11 (2023) 100062

Contents lists available at ScienceDirect

EURO Journal on Computational Optimization

journal homepage: www.elsevier.com/locate/ejco

International migrant flows: Coalition formation among countries and social welfare

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ARTICLE INFO

Keywords:
Migration networks
Coalitions
Variational inequality
Equilibrium flows
Social welfare
Policies

ABSTRACT

In this paper, we consider policy interventions for international migrant flows and quantify their ramifications. In particular, we further develop a recent equilibrium model of international human migration in which some of the destination countries form coalitions to establish a common upper bound on the migratory flows that they agree to accept jointly. We also consider here a scenario where some countries can leave or join an initial coalition and investigate the problem of finding the coalitions that maximize the overall social welfare. Moreover, we compare the social welfare at equilibrium with the one that a supranational organization might suggest in an ideal scenario. This research adds to the literature on the development of mathematical models to address pressing

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https://doi.org/10.1016/j.ejco.2023.100062

issues associated with problems of human migration with insights for policy and decision-makers.

1. Introduction

Migration of people is a topic of great interest to researchers, government officials and policy makers and one that merits additional attention because of migratory flows across national boundaries due to climate change, conflicts, violence and wars, as well as poverty and the desire of certain migrants to attain greater economic opportunities and prosperity. For example, according to the latest available data from the United Nation's International Organization of Migration [7], there were approximately 281 million international migrants in the world in 2020. COVID-19 then impacted migratory flows, providing a major systemic shock. Now the major invasion of Ukraine by Russia, beginning on February 24, 2022, has resulted in over 7,800,000 Ukrainians as refugees across Europe, as of November 29, 2022, the largest such migration crisis since WWII [22]. And, on the other side of the globe, more than 6.1 million refugees and migrants have left Venezuela due to the economic and humanitarian crisis there, the International Organization of Migration reports [8]. In 2013, the United Nations [21] identified international migration as a global phenomenon that is increasing in scope, complexity and impact.

Governments that are being faced with challenges associated with migrant flows are increasingly turning to imposing regulations, as noted by Nagurney and Daniele [13]. Bertossi [3] emphasized that, in the past three decades, there have been immense efforts expended to control the migration of people across national borders. Helbling and Leblang [6], in turn, stated strongly that regulations associated with migration are essential to each nation since they impact the composition of the residents in the country. The United Nations [21] recognized that migration policies in origin as well as in destination countries play a pivotal role in affecting the flows of migrants. Furthermore, governments, in their management of migratory flows, typically, consider different classes of migrants, with examples of classes including: skilled workers, family members of migrant workers, refugees and asylum seekers.

Janning and Moller [9] noted that migration-related issues will be on top of European Union member states' agendas in forthcoming years. Furthermore, they stated in their report that, of the eighteen issues that the European Council on Foreign Relations asked policy makers and experts to rank in order of importance, common immigration and asylum policy rose to first place. Interestingly, in their study, they also emphasized that coalition-building is of increasing importance within European Union member nations. The NGO Refugees International [19] in 2021 issued an advocacy letter calling for a coalition of willing countries in the European Union to relocate refugees and asylum seekers.

Given the reality of human migration and the attention that it is receiving globally, it is imperative to have rigorous frameworks that can inform policy and decision-makers on this important topic.

We now provide some background on the mathematical modeling of international human migration flows and detail the contributions in this paper, in which an equilibrium model is developed that adds to the literature by advancing the inclusion of policies in the form of coalitions, a topic which has been only minimally explored quantitatively.

The first attempt to provide a mathematical model of migration was pioneered by the English geographer Ernest Ravenstein who formulated his seven laws of migration in 1885 [18], using for his investigation real data coming from population censuses. Being a complex phenomenon, the investigation of migration has been carried out by scholars of different fields, ranging from psychology to statistics and demography as well as economics and even operations research. From the mathematical modeling point of view, various approaches are possible, and our contribution is framed in microeconomic theory and, more precisely, within equilibrium analysis, where we consider multiple classes of migrants and multiple countries. With each country and class of migrant is associated a utility function that describes the potential benefits perceived by that class of migrants. A migration cost is also assigned to each pair of origin-destination countries and class of migrant in order to describe not only the economic cost of displacement, but also the social and psychological cost associated with the migration choice by each class of migrant.

The variational inequality approach to the analysis of equilibrium flows in migration networks was initiated by A. Nagurney (see, e.g., [11,12]). In a recent paper [13], Nagurney and Daniele constructed the first international network model of human migration in which some countries could bound the incoming flows of migrants from some other countries (see also [6]). Such bounds reflect imposed policies. They also applied variational inequality theory for the formulation, analysis, and numerical computations. The paper was then generalized in [16] through the introduction of multiple routes between origin and destination locations that the migrants could take, and these could consist of multiple links. Nagurney, Daniele, and Cappello [14], in turn, introduced a system-optimization framework for human migration with a focus on population distributions and proved that subsidies could guarantee that multiclass migrants, behaving selfishly in a user-optimizing manner, once the subsidies were imposed, would migrate in a manner that would guarantee a system optimum (see also [15]).

The model in [13] was also extended in the paper [17] by Passacantando and Raciti, who considered the possibility that a set of countries forms a coalition in order to impose a global upper bound on incoming flows. The introduction of coalitions gives rise to a new definition of equilibrium which was proven to be equivalent to a derived variational inequality.

In this paper, we take into account the fact that coalitions can change because some countries may wish to enter or leave an existing coalition. This gives rise to various scenarios, and a comparison among the potential sets of coalitions should then be performed with respect to some criterion. To provide a measure of "how good" a set of coalitions is, we, thus, define and compute the social welfare at equilibrium. Moreover, to measure how far the solution thus obtained is from an ideal scenario, we also compute the social welfare optimum, and the ratio between these two quantities.

The paper is structured as follows. In the following Section 2, we introduce the notation used and present the general international migration network equilibrium model on which we base our analysis. In Section 3, we consider the possibility of coalition formation, state the governing equilibrium conditions and establish their equivalent variational inequality formulation. We also provide a monotonicity lemma. Section 4 is then devoted to the investigation of the different scenarios associated with possible coalition changes. In this respect, in order to compare different sets of coalitions, we compute the social welfare at equilibrium for each of them; moreover, in order to assess how far from an ideal situation the best sets of coalitions are, we also compute the corresponding optimal social welfare, and, hence, a kind of *price of anarchy* associated with the individual choices of the international migrants. In Section 5, we illustrate our analysis through several numerical examples. In the concluding Section 6, we summarize our findings and outline some suggestions for future research.

2. The general international human migration network equilibrium model

In what follows, vectors in $\mathbb{R}^m$ are thought of as columns; when involved in matrix operations, $a^\top$ denotes the transpose of vector a and $a^\top b$ the canonical scalar product in $\mathbb{R}^m$. We consider a set $\mathcal{N} = \{1, \dots, N\}$ of N countries and assume that the population of migrants of each country can be divided into K different classes which constitute the set $\mathcal{K} = \{1, \dots, K\}$. The countries are thus considered as the nodes of a network where the arcs represent the migratory routes. Let b_i^k denote the initial population of migrants of class k in country i and p_i^k the current population of migrants of class k in country i after a migratory phase has occurred (no repeat or chain migration is considered in this model); populations can be grouped into a vector $p \in \mathbb{R}^{KN}$, such that

$$p = (p_1^1, \dots, p_N^1, \dots, p_1^K, \dots, p_N^K) = (p^1, \dots, p^k, \dots, p^K).$$

The migratory flow from country i to country j, with $i \neq j$, of the class k is denoted by f_{ij}^k and we group flows into a vector $f \in \mathbb{R}^{KN(N-1)}$ such that:

$$f = (f^1, \dots, f^k, \dots, f^K),$$

where each f^k is a subvector with the $N(N-1)$ components, f_{ij}^k, ordered in an arbitrarily prescribed manner. Each class of migrants chooses a destination country according to the perceived attractiveness of that country, which is embodied in a utility function u_i^k, $i \in \mathcal{N}, k \in \mathcal{K}$. The utilities are grouped into the vector

$$u = (u_1^1, \dots, u_N^1, \dots, u_1^K, \dots, u_N^K) = (u^1, \dots, u^k, \dots, u^K).$$

A common assumption is that the utility associated with each country and migrant class can depend on the whole population vector; hence, $u : p \mapsto u(p) \in \mathbb{R}^{KN}$, so as to take into account possible competition or saturation effects. Another factor that influences the migration choice is the cost c_{ij}^k faced by migrants of class k migrating from country i to country j. All the costs are grouped into a vector in the same manner as the flows.

For the sake of generality, the cost is assumed to depend on the whole network flow; hence, $c : f \mapsto c(f) \in \mathbb{R}^{KN(N-1)}$. As remarked in [12], c does not represent the mere economic cost of migration but also encompasses the social and psychological difficulties connected with the migration decision.

The current population after migration can be expressed, for each class and each country, in terms of the initial one and the net flow as:

$$p_i^k = b_i^k + \sum_{j \in \mathcal{N} \setminus \{i\}} f_{ji}^k - \sum_{j \in \mathcal{N} \setminus \{i\}} f_{ij}^k, \qquad \forall\, i \in \mathcal{N},\ \forall\, k \in \mathcal{K}, \tag{1}$$

where, for each class and each country, the outgoing flow cannot exceed the initial population:

$$\sum_{j \in \mathcal{N} \setminus \{i\}} f_{ij}^k \le b_i^k, \qquad \forall\, i \in \mathcal{N},\ \forall\, k \in \mathcal{K}, \tag{2}$$

and $f_{ij}^k \ge 0$.

In an initial model we also assume that some countries can decide to control the incoming flows of a certain type. We then introduce the additional constraints:

$$f_{ij}^k \le B_{ij}^k, \qquad \forall\, i \in \mathcal{N},\ j \in \mathcal{N} \setminus \{i\},\ k \in \mathcal{K}, \tag{3}$$

where it is understood that $B_{ij}^k = +\infty$ corresponds to the case where country j does not impose any limitation to flows of type k coming from country i. With this last condition, the set of feasible flows is substantially the one considered in [13], but we prefer to work in the space of flows only and get rid of the population variables. Thus, for later use, we observe that the utility function can be expressed as a function of the flows by using the conservation equation (1) and defining:

$$U(f) := u(b + Df), \tag{4}$$

where D is a block-diagonal matrix of dimension $KN \times KN(N-1)$ whose diagonal blocks are all equal to the node-arc incidence matrix E of the graph associated with the countries, and whose elements e_{va} are given by:

$$e_{va} = \begin{cases} -1 & \text{if } v \text{ is the origin of arc } a, \\ 1 & \text{if } v \text{ is the destination of arc } a, \\ 0 & \text{otherwise.} \end{cases}$$

3. Coalition formation and equilibrium conditions

We explain here the assumptions behind our model of coalition formation and illustrate our formal definition of a coalition by some examples. Let us assume that we can identify a group of destination countries, each of which is currently adopting the policy of bounding a certain class of migrant flow from a certain group of origin countries. Some of those destination countries can then decide to gather and establish a common global upper bound on the incoming flows under consideration. In the sequel, we make the following assumption:

The global bound imposed by the coalition is the sum of the individual bounds of the countries belonging to the coalition.

In the case where a country leaves a coalition, its contribution to the global upper bound is spun off. On the other hand, if a new country wishes to join a certain coalition, it is understood that it has already established an individual upper bound on the same class of migrants that the initial coalition is controlling, or it is willing to do so. Hence, this effective, or potential, upper bound has to be added.

To formalize our problem, let us denote by $\mathcal{R}$ the set of country coalitions and by r the generic coalition. Moreover, let:

$S_r^d =$ the set of countries in coalition r (destination countries);
$S_r^o =$ a subset of countries which do not belong to coalition r (origin countries);
$S_r^m =$ the set of migrant classes coming from countries belonging to S_r^o on which coalition r imposes an upper bound denoted by B_r.

Notice that, within this framework, each coalition is associated with a single constraint; hence, in the case where the same group of countries wants to impose additional constraints we formally need to introduce new coalitions, as illustrated in the following example.

Let us consider the following set of countries: $\mathcal{N} =$ {France, Germany, Italy, Spain, United Kingdom, Switzerland, Poland, Libya, Syria} and two classes of migrants: $\mathcal{K} =$ {skilled workers (class 1), without qualifications (class 2)}.

a) Assume that France, Germany, Italy and Spain join to impose a global bound on migrants without qualifications coming from Libya and Syria.

$S_r^d = \{\text{France}, \text{Germany}, \text{Italy}, \text{Spain}\}$;
$S_r^o = \{\text{Libya}, \text{Syria}\}$;
$S_r^m = \{\text{without qualifications (class 2)}\}$.

The corresponding coalitional constraint is thus:

$$\sum_{i \in S_r^o} \sum_{j \in S_r^d} f_{ij}^2 \leq B_r.$$

b) Assume that, in the previous case, France, Germany, Italy and Spain also wish to bound class 1 migrants coming from Poland. We then need to introduce two coalitions, r_1 and r_2, such that:

$S_{r_1}^d = \{\text{France, Germany, Italy, Spain}\};$
$S_{r_1}^o = \{\text{Libya, Syria}\};$
$S_{r_1}^m = \{\text{without qualifications (class 2)}\};$

$S_{r_2}^d = \{\text{France, Germany, Italy, Spain}\};$
$S_{r_2}^o = \{\text{Poland}\};$
$S_{r_2}^m = \{\text{skilled workers (class 1)}\}.$

The corresponding constraints now read as:

$$\sum_{i \in S_{r_1}^o} \sum_{j \in S_{r_1}^d} f_{ij}^2 \leq B_{r_1}, \qquad \sum_{i \in S_{r_2}^o} \sum_{j \in S_{r_2}^d} f_{ij}^1 \leq B_{r_2}.$$

c) In the case where France, Germany, Italy and Spain wish to impose two different upper bounds on the incoming flows of class 2 from Libya and Syria, we introduce two different coalitions analogous to the previous case:

$S_{r_1}^d = \{\text{France, Germany, Italy, Spain}\};$
$S_{r_1}^o = \{\text{Libya}\};$
$S_{r_1}^m = \{\text{without qualifications (class 2)}\};$

$S_{r_2}^d = \{\text{France, Germany, Italy, Spain}\};$
$S_{r_2}^o = \{\text{Syria}\};$
$S_{r_2}^m = \{\text{without qualifications (class 2)}\}.$

The corresponding constraints now read as:

$$\sum_{i \in S_{r_1}^o} \sum_{j \in S_{r_1}^d} f_{ij}^2 \leq B_{r_1}, \qquad \sum_{i \in S_{r_2}^o} \sum_{j \in S_{r_2}^d} f_{ij}^2 \leq B_{r_2}.$$

Within this framework, the feasible set of flows is given by:

$$C := \Big\{ f = (f_{ij}^k)_{k \in \mathcal{K}, i \in \mathcal{N}, j \in \mathcal{N} \setminus \{i\}} : \ f \geq 0, \sum_{j \in \mathcal{N} \setminus \{i\}} f_{ij}^k \leq b_i^k, \ \forall\, i \in \mathcal{N}, \ \forall\, k \in \mathcal{K},$$
$$\sum_{i \in S_r^o} \sum_{j \in S_r^d} \sum_{k \in S_r^m} f_{ij}^k \leq B_r, \ \forall\, r \in \mathcal{R} \Big\}.$$

We now recall the definition of equilibrium under joint regulations recently put forward in [17], along with its variational inequality formulation. For each $r \in \mathcal{R}$, we make use of an indicator function connected with the sets previously defined as follows:

$$I_{S_r^o \times S_r^d \times S_r^m}(i,j,k) = \begin{cases} 1 & \text{if } i \in S_r^o, j \in S_r^d, k \in S_r^m, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 1. (Equilibrium under joint regulations)
A flow $\bar{f} \in C$ is an equilibrium flow if, $\forall\, r \in \mathcal{R}, \forall\, i \in \mathcal{N}, \forall\, k \in \mathcal{K}$, there exist γ_r and β_i^k such that the following three conditions hold:

$$\gamma_r \begin{cases} = 0 & \text{if } \sum_{i \in S_r^o} \sum_{j \in S_r^d} \sum_{k \in S_r^m} \bar{f}_{ij}^k < B_r, \\ \geq 0 & \text{if } \sum_{i \in S_r^o} \sum_{j \in S_r^d} \sum_{k \in S_r^m} \bar{f}_{ij}^k = B_r, \end{cases} \tag{5}$$

$$\beta_i^k \begin{cases} = 0 & \text{if } \sum_{j \in \mathcal{N} \setminus \{i\}} \bar{f}_{ij}^k < b_i^k, \\ \geq 0 & \text{if } \sum_{j \in \mathcal{N} \setminus \{i\}} \bar{f}_{ij}^k = b_i^k, \end{cases} \tag{6}$$

$$U_j^k(\bar{f}) - U_i^k(\bar{f}) - c_{ij}^k(\bar{f}) \begin{cases} = \beta_i^k + \sum_{r \in \mathcal{R}} \gamma_r I_{S_r^o \times S_r^d \times S_r^m}(i,j,k) & \text{if } \bar{f}_{ij}^k > 0, \\ \leq \beta_i^k + \sum_{r \in \mathcal{R}} \gamma_r I_{S_r^o \times S_r^d \times S_r^m}(i,j,k) & \text{if } \bar{f}_{ij}^k = 0. \end{cases} \tag{7}$$

In order to illustrate the meaning of the conditions above, we can consider the particular case where $\mathcal{R}$ consists of only one coalition. The left-hand side of (7) represents the net gain that migrants of class k coming from country i perceive when moving towards country j. Let us notice that, when the upper bound B_r is not reached, then γ_r is equal to zero and the net gain of the migrants of the class only depends on the country of origin. Otherwise, the net gain can depend on both the country of origin and the destination, and is greater than in the previous case.

The following theorem provides a variational inequality formulation of the equilibrium defined above (see [17] for the proof).

Theorem 1. *Let $T : \mathbb{R}^{KN(N-1)} \to \mathbb{R}^{KN(N-1)}$ be the operator defined by: $T(f) := -D^\top U(f)$, where D is the matrix defined in (4). We have then that $\bar{f} \in C$ is an equilibrium flow according to Definition 1 if and only if $\bar{f}$ solves the variational inequality problem of finding $\bar{f} \in C$ such that*

$$T(\bar{f})^\top (f - \bar{f}) + c(\bar{f})^\top (f - \bar{f}) \geq 0 \qquad \forall\, f \in C. \tag{8}$$

Remark 1. Since the set C is compact, it is sufficient to assume the continuity of U and c to ensure that (8) admits solutions.

A sufficient condition for the uniqueness of the solution is the strict monotonicity property of the operator entering the variational inequality.

Definition 2. An operator $A : \mathbb{R}^n \to \mathbb{R}^n$ is said to be monotone iff

$$[A(x) - A(y)]^\top (x - y) \geq 0, \qquad \forall\, x, y \in \mathbb{R}^n.$$

A is said to be strictly monotone if in the definition above the equality holds only for $x = y$.

Lemma 2. *If the operator $-u : p \mapsto -u(p)$ is monotone, then T is monotone. If $-u : p \mapsto -u(p)$ is monotone and $c : f \mapsto c(f)$ is strictly monotone, then $T + c$ is strictly monotone.*

Proof. Let us consider two arbitrary flows $f, f' \in \mathbb{R}^{KN(N-1)}$ and set $p = b + Df$, $p' = b + Df'$. Then, it is easy to check that the following chain of equalities holds:

$$\begin{aligned}
[T(f) - T(f')]^\top (f - f') &= [-D^\top U(f) + D^\top U(f'))]^\top (f - f') \\
&= [D^\top (U(f') - U(f))]^\top (f - f') \\
&= [U(f') - U(f)]^\top D(f - f') \\
&= [-u(p) - (-u(p'))]^\top (p - p') \geq 0.
\end{aligned}$$

The second part of the lemma is straightforward because the sum of a monotone and a strictly monotone operator gives a strictly monotone operator. □

4. Coalition changes and social welfare: the best coalitions problem

We now consider a scenario where a given set $\mathcal{R}_0$ of coalitions has already been established but some new countries wish to enter some coalitions in $\mathcal{R}_0$, or some countries in $\mathcal{R}_0$ wish to change their coalition or leave a coalition that they belong to in order to establish an individual bound. Before a new agreement is formalized, we are then left with a certain number of potential sets of coalitions: $P = \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_L\}$ and the natural question arises of comparing them according to some criteria. To this end, we define the *social welfare* associated with our model as:

$$W(f) = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} \sum_{k \in \mathcal{K}} \left[U_j^k(f) - U_i^k(f) - c_{ij}^k(f) \right] f_{ij}^k, \tag{9}$$

Since we are dealing with an equilibrium model, the quantity of interest is the social welfare at equilibrium:

$$W^{eq} = W(\bar{f}).$$

Thus, denoting the social welfare at equilibrium corresponding to the set of coalitions $\mathcal{R}_l$ by W_l^{eq}, we seek to determine

$$\max_{l=1,\dots,L} W_l^{eq}, \tag{10}$$

and call a set of coalitions $\mathcal{R}_s$ that satisfies (10), a best set of coalitions at equilibrium. Furthermore, we can also consider the case where a supranational organization wishes to identify the flow patterns which would yield the highest social welfare. Thus, for a given set of coalitions, we introduce the quantity:

$$W^O = \max_{f \in C} W(f),$$

and look for the set of coalitions such that:

$$\max_{l=1,\dots,L} W_l^O. \tag{11}$$

In order to measure the difference between the social welfare associated with an ideal flow pattern suggested by the supranational authority and the social welfare at equilibrium, we then introduce, for each set of coalitions $\mathcal{R}_l$, the quantity:

$$\pi_l = \frac{W_l^{eq}}{W_l^O}, \tag{12}$$

which can be called the *price of anarchy* according to the terminology introduced in [20].

Remark 2. In [14], the authors constructed a multiclass migration network model under system-optimization, where the total utility associated with the multiclass population distribution at all the locations was maximized. Therein, a type of price of anarchy was also proposed consisting of the ratio of the total utility evaluated at the user-optimized solution versus that under the system-optimized solution. The model, however, did not have migration costs nor coalitions. Formulae for subsidies were provided. They can serve as powerful policy interventions in order to achieve a system-optimal population distribution although migrants can be expected to behave selfishly in a user-optimized fashion. Such subsidies serve a similar role in altering the behavior of migrants as tolls do in the context of travelers on congested urban transportation networks [10].

5. Numerical experiments

We now illustrate the concepts introduced in the previous sections with some numerical examples. We consider a problem with nine countries: $\mathcal{N} = \{$France (FR), Germany (DE), Italy (IT), Spain (ES), United Kingdom (UK), Switzerland (CH), Poland (PL), Libya (LY), Syria (SY)$\}$ and two classes of migrants: $\mathcal{K} = \{$skilled workers (class 1),

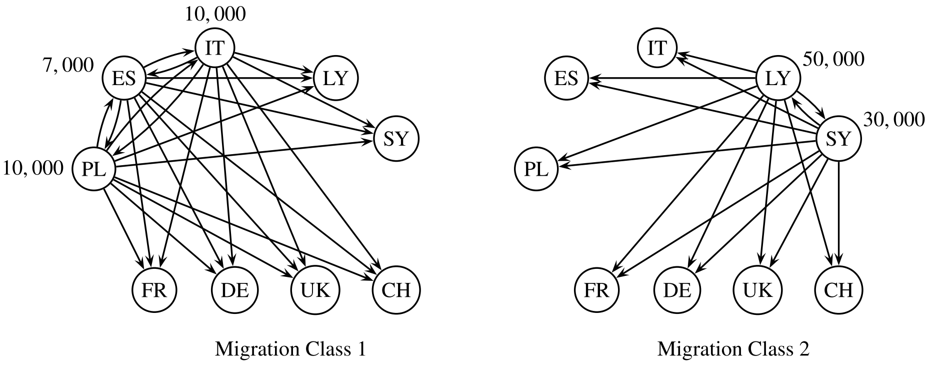

Fig. 1. Network structure of the problem considered in Section 5.

without qualifications (class 2)}. The initial populations of both classes of migrants are defined as follows:

$$\begin{array}{lllll} b_1^1=0 & b_2^1=0 & b_3^1=10,000 & b_4^1=7,000 & b_5^1=0 \\ b_6^1=0 & b_7^1=10,000 & b_8^1=0 & b_9^1=0 & \end{array} \quad \text{(Class 1)}$$

$$\begin{array}{lllll} b_1^2=0 & b_2^2=0 & b_3^2=0 & b_4^2=0 & b_5^2=0 \\ b_6^2=0 & b_7^2=0 & b_8^2=50,000 & b_9^2=30,000 & \end{array} \quad \text{(Class 2)}$$

Notice that the origin countries of migrants of class 1 are only Italy, Spain and Poland, while migrants of class 2 come only from Libya and Syria (see Fig. 1).

The utility functions associated with the countries are:

$$\text{Class 1:} \begin{cases} u_1^1(p) = -2p_1^1 - p_1^2 + 30,000 \\ u_2^1(p) = -2p_2^1 - p_2^2 + 40,000 \\ u_3^1(p) = -3p_3^1 - p_3^2 + 14,000 \\ u_4^1(p) = -3p_4^1 - p_4^2 + 14,000 \\ u_5^1(p) = -2p_5^1 - p_5^2 + 40,000 \\ u_6^1(p) = -4p_6^1 - 3p_6^2 + 40,000 \\ u_7^1(p) = -5p_7^1 - 2p_7^2 + 10,000 \\ u_8^1(p) = -10p_8^1 - 10p_8^2 + 100 \\ u_9^1(p) = -10p_9^1 - 10p_9^2 + 100 \end{cases} \quad \text{Class 2:} \begin{cases} u_1^2(p) = -p_1^1 - 2p_1^2 + 20,000 \\ u_2^2(p) = -p_2^1 - 2p_2^2 + 25,000 \\ u_3^2(p) = -2p_3^1 - 2p_3^2 + 12,000 \\ u_4^2(p) = -2p_4^1 - 2p_4^2 + 12,000 \\ u_5^2(p) = -2p_5^1 - 2p_5^2 + 25,000 \\ u_6^2(p) = -p_6^1 - 3p_6^2 + 25,000 \\ u_7^2(p) = -p_7^1 - 2p_7^2 + 9,000 \\ u_8^2(p) = -10p_8^1 - 10p_8^2 + 50 \\ u_9^2(p) = -10p_9^1 - 10p_9^2 + 50 \end{cases}$$

The migration cost functions for class 1 are defined as follows:

$$\begin{array}{llll} c_{31}^1(f) = f_{31}^1 + 20 & c_{32}^1(f) = 2f_{32}^1 + 30 & c_{34}^1(f) = f_{34}^1 + 20 & c_{35}^1(f) = 3f_{35}^1 + 40 \\ c_{36}^1(f) = 2f_{36}^1 + 20 & c_{37}^1(f) = 4f_{37}^1 + 90 & c_{38}^1(f) = 10f_{38}^1 + 1,000 & c_{39}^1(f) = 10f_{39}^1 + 1,000 \end{array}$$

$$c^1_{41}(f) = f^1_{41} + 10 \quad c^1_{42}(f) = 2f^1_{42} + 30 \quad c^1_{43}(f) = f^1_{43} + 20 \quad c^1_{45}(f) = 3f^1_{45} + 40$$
$$c^1_{46}(f) = 2f^1_{46} + 30 \quad c^1_{47}(f) = 4f^1_{47} + 90 \quad c^1_{48}(f) = 10f^1_{48} + 1{,}000 \quad c^1_{49}(f) = 10f^1_{49} + 1{,}000$$

$$c^1_{71}(f) = 3f^1_{71} + 30 \quad c^1_{72}(f) = 1f^1_{72} + 20 \quad c^1_{73}(f) = 2f^1_{73} + 40 \quad c^1_{74}(f) = 2f^1_{74} + 40$$
$$c^1_{75}(f) = 3f^1_{75} + 30 \quad c^1_{76}(f) = 4f^1_{76} + 40 \quad c^1_{78}(f) = 10f^1_{78} + 1{,}000 \quad c^1_{79}(f) = 10f^1_{79} + 1{,}000$$

$$c^1_{81}(f) = 2f^1_{81} + 20 \quad c^1_{82}(f) = 2f^1_{82} + 20 \quad c^1_{83}(f) = 3f^1_{83} + 30 \quad c^1_{84}(f) = 3f^1_{84} + 30$$
$$c^1_{85}(f) = 4f^1_{85} + 40 \quad c^1_{86}(f) = 4f^1_{86} + 40 \quad c^1_{87}(f) = 6f^1_{87} + 80 \quad c^1_{89}(f) = 10f^1_{89} + 10{,}000$$

$$c^1_{91}(f) = 2f^1_{91} + 20 \quad c^1_{92}(f) = 2f^1_{92} + 20 \quad c^1_{93}(f) = 3f^1_{93} + 30 \quad c^1_{94}(f) = 3f^1_{94} + 30$$
$$c^1_{95}(f) = 4f^1_{95} + 40 \quad c^1_{96}(f) = 4f^1_{96} + 40 \quad c^1_{97}(f) = 6f^1_{97} + 50 \quad c^1_{98}(f) = 10f^1_{98} + 10{,}000$$

The migration cost functions for class 2 are defined similarly to that for class 1: if $c^1_{ij}(f) = af^1_{ij} + a'$, then $c^2_{ij}(f) = af^2_{ij} + a'$ holds for any i, j. Here, we make such an assumption for simplicity.

The upper bounds on the incoming flows in some countries of each class of migrants are defined as follows, according to (3):

Class 1									
From\To	FR	DE	IT	ES	UK	CH	PL	LY	SY
IT	–	–	–	–	2,000	1,000	–	–	–
ES	–	–	–	–	1,000	500	–	–	–
PL	–	–	–	–	1,500	750	–	–	–
Class 2									
From\To	FR	DE	IT	ES	UK	CH	PL	LY	SY
LY	3,000	4,000	2,000	2,000	3,000	1,000	2,000	–	–
SY	3,000	4,000	2,000	2,000	3,000	1,000	2,000	–	–

We consider an initial scenario $\mathcal{R}_0$, containing seven coalitions, and eight potential scenarios defined as in Table 1.

It is easy to check that the map $-u$ is monotone and the map c is strongly monotone; thus, both the existence and uniqueness of the equilibrium flow for any scenario are guaranteed. Since the variational inequality (8) to be solved has a polyhedral feasible region and an affine and strongly monotone map, we reformulated it as an equivalent convex quadratic optimization problem (see [1]) and solved it by means of the MATLAB function `quadprog` from the optimization toolbox. Computations were implemented in MATLAB R2022 and tested on an Apple M1 Max with 64 GB of RAM, running under macOS 13.0.

The values of the social welfare at equilibrium W^{eq}_l, the best social welfare W^O_l and the price of anarchy π_l for each considered scenario $\mathcal{R}_l$, with $l = 0, \dots, 8$, are summarized in Table 2. Notice that scenario $\mathcal{R}_4$ gives the best set of coalitions with respect to both the social welfare at equilibrium and the price of anarchy, while scenario $\mathcal{R}_6$ is the best set of coalitions with respect to the best value of social welfare.

Table 1
Initial scenario $\mathcal{R}_0$ and potential scenarios.

Scenario	Coalition	S_r^d	S_r^o	S_r^m
$\mathcal{R}_0$	r_1	UK	IT, ES, PL	class 1
	r_2	CH	IT, ES, PL	class 1
	r_3	FR, DE	LY, SY	class 2
	r_4	IT, ES	LY, SY	class 2
	r_5	UK	LY, SY	class 2
	r_6	CH	LY, SY	class 2
	r_7	PL	LY, SY	class 2
$\mathcal{R}_1$	$r_1, r_2, r_4, r_5, r_6, r_7$			
	r_8	FR	LY, SY	class 2
	r_9	DE	LY, SY	class 2
$\mathcal{R}_2$	$r_1, r_2, r_3, r_5, r_6, r_7$			
	r_{10}	IT	LY, SY	class 2
	r_{11}	ES	LY, SY	class 2
$\mathcal{R}_3$	r_1, r_2, r_5, r_6, r_7			
	r_{12}	FR	LY, SY	class 2
	r_{13}	DE	LY, SY	class 2
	r_{14}	IT	LY, SY	class 2
	r_{15}	ES	LY, SY	class 2
$\mathcal{R}_4$	r_1, r_2, r_5, r_6, r_7			
	r_{16}	FR, DE, IT, ES	LY, SY	class 2
$\mathcal{R}_5$	r_1, r_2, r_4, r_5, r_6			
	r_{17}	FR, DE, PL	LY, SY	class 2
$\mathcal{R}_6$	r_1, r_2, r_5, r_6			
	r_{18}	FR, DE, IT, ES, PL	LY, SY	class 2
$\mathcal{R}_7$	r_1, r_2, r_4, r_6, r_7			
	r_{19}	FR, UK	LY, SY	class 2
	r_{20}	DE	LY, SY	class 2
$\mathcal{R}_8$	r_1, r_2, r_4, r_6, r_7			
	r_{21}	FR, DE, UK	LY, SY	class 2

Table 2
Social welfare at equilibrium, best social welfare and price of anarchy of the considered scenarios.

Scenario	Social welfare at equil. (10^9)	Best social welfare (10^9)	Price of anarchy
$\mathcal{R}_0$	7.7501	8.5286	0.9087
$\mathcal{R}_1$	7.7456	8.5244	0.9086
$\mathcal{R}_2$	7.7501	8.5285	0.9087
$\mathcal{R}_3$	7.7456	8.5243	0.9087
$\mathcal{R}_4$	**7.7592**	8.5331	**0.9093**
$\mathcal{R}_5$	7.7374	8.5288	0.9072
$\mathcal{R}_6$	7.7429	**8.5332**	0.9074
$\mathcal{R}_7$	7.7493	8.5292	0.9086
$\mathcal{R}_8$	7.7508	8.5305	0.9086

Table 3 and Table 4 report the equilibrium flow and the final populations of migrants in the scenario $\mathcal{R}_4$, respectively. It is interesting to notice that the formation of coalitions allows more flexibility in the control of incoming flows. For example, in scenario $\mathcal{R}_4$, France, Germany, Italy and Spain form a coalition to bound the overall flow of migrants

Table 3
Equilibrium flow in the scenario $\mathcal{R}_4$.

Class 1									
From \ To	FR	DE	IT	ES	UK	CH	PL	LY	SY
IT	3,301	2,704	–	501	1,524	992	0	0	0
ES	2,657	2,377	0	–	1,306	660	0	0	0
PL	1,239	5,845	194	454	1,670	598	–	0	0
Class 2									
From \ To	FR	DE	IT	ES	UK	CH	PL	LY	SY
LY	5,192	5,404	3,607	3,669	4,031	2,000	2,685	–	0
SY	1,069	1,281	858	920	1,969	0	1,315	0	–

Table 4
Final populations of migrants in the scenario $\mathcal{R}_4$.

	FR	DE	IT	ES	UK	CH	PL	LY	SY
Class 1	7,197	10,926	1,172	955	4,500	2,250	0	0	0
Class 2	6,261	6,685	4,465	4,589	6,000	2,000	4,000	23,412	22,588

without qualifications from Libya and Syria. The total flow sums up to 22,000, which is exactly the sum of the individual bounds that each country would impose on migrants of class 2 from Libya and from Syria. However, some individual bounds can be relaxed, thanks to the coalition. Italy receives 3,607 migrants of class 2 from Libya and 858 from Syria, which sum up to 4,465 which is greater than the 4,000 migrants of class 2 (from Libya and Syria) which it would have received based on its individual upper bounds. The same analysis applies to Spain which receives 3,669 migrants of class 2 from Libya and 920 from Syria; thus, relaxing its individual upper bound of 4,000 migrants. These examples are stylized but do provide insights as to the type of quantitative results that can be obtained, and which can also assist policy and decision-makers. Furthermore, they demonstrate the potential benefits of collaboration through coalition formation.

Table 5 and Table 6 report the utility at equilibrium of the migrants in the countries in scenario $\mathcal{R}_4$ and the migrants' net gain at equilibrium, respectively. In particular, Table 6 nicely illustrates the equilibrium conditions. For instance, not all the migrants of class 1, who are skilled workers, in Italy leave their country (only 9,022), and the net gain associated with France, Germany and Spain, which do not limit this flow, is zero at equilibrium. On the other hand, the net gain associated with United Kingdom and Switzerland, which do impose a bound on the flow of Italian migrants of class 1, is positive. Furthermore, there is no incoming flow of Italian migrants of class 1 towards Poland, Libya and Syria, and the corresponding net gain is, thus, negative. A similar analysis applies to migrants of class 1 from Spain or Poland, but, this time, all workers of class 1 leave their respective country of origin. As a result, the net gain associated with France, Germany, Italy and Spain, which do not limit this flow (and which actually receive a positive equilibrium flow) is positive. As to migrants of class two from Libya and Syria, it is worthwhile to notice that the net gain is the same (223,949) for the four countries in the coalition (France, Germany, Italy and Spain).

Table 5
Utility at equilibrium of the migrants in the countries in the scenario $\mathcal{R}_4$.

	FR	DE	IT	ES	UK	CH	PL	LY	SY
Class 1	9,344	11,462	6,023	6,545	25,000	25,000	2,000	$-234,023$	$-225,776$
Class 2	280	704	728	912	4,000	16,750	1,000	$-234,073$	$-225,826$

Table 6
Net gain at equilibrium for migrants in the scenario $\mathcal{R}_4$.

Class 1									
From\To	FR	DE	IT	ES	UK	CH	PL	LY	SY
IT	0	0	–	0	14,364	16,972	$-4,113$	$-241,047$	$-232,800$
ES	133	133	-541	–	14,497	17,105	$-4,635$	$-241,568$	$-233,321$
PL	3,596	3,596	3,596	3,596	17,960	20,569	–	$-237,023$	$-228,777$
Class 2									
From\To	FR	DE	IT	ES	UK	CH	PL	LY	SY
LY	223,949	223,949	223,949	223,949	221,910	242,783	218,885	–	$-1,753$
SY	223,949	223,949	223,949	223,949	221,910	242,537	218,885	$-18,247$	–

6. Conclusion and further research perspectives

In this paper, we refined several previous equilibrium models of international human migration with a focus on policies relevant to governments and with an investigation on the potential benefits of coalition formation.

In particular, in extending the analysis in [17], we further investigated the formation of coalitions of countries aimed at bounding the incoming flows of some specific classes of migrants. To measure how effective a coalition formation is, we computed the social welfare at equilibrium for each set of coalitions that is likely to be realized. This number was then compared with the optimal social welfare and, in our numerical examples, we found that the social welfare at equilibrium is approximately 10% less than the optimal social welfare.

To further extend the model, the possibility of uncertain data could be taken into account, along the same lines as in [4,5]. Furthermore, the distribution of migrants among countries could also be introduced via the modeling of countries as players of a, possibly cooperative, game and with migrants acting selfishly. In addition, allowing for multiple routes from origin countries to destination countries is another possible extension with limits on unsafe and illegal routes being imposed [2]. We leave such directions for future research.

Acknowledgements

The authors wish to thank the reviewers for helpful comments and suggestions on an earlier version of the paper.

The first two authors are members of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA - National Group for Mathematical

Analysis, Probability and their Applications) of the Instituto Nazionale di Alta Matematica (INdAM - National Institute of Higher Mathematics). This research was partially supported by the research project “Programma ricerca di ateneo UNICT 2020-22 linea 2-OMNIA” of the University of Catania. This support is gratefully acknowledged.

The third author acknowledges the organizers of the ODS conference, which took place in Florence, Italy, August 30 – September 2, 2022, during which the seeds of this collaboration were germinated.

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