

Cosmology with the angular cross-correlation of gravitational-wave and galaxy catalogs: Forecasts for next-generation interferometers and the Euclid survey

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ABSTRACT

Context. The spatial clustering of galaxies has long been a key probe of cosmology. Gravitational-wave (GW) sources, which provide direct luminosity–distance measurements, have recently emerged as a complementary tracer of large-scale structures. The cross-correlation of GW and galaxy catalogs offers a novel way to test cosmic expansion.

Aims. We investigate the potential of tomographic GW–galaxy angular power spectra to constrain cosmological parameters, focusing on the Hubble constant and matter density, in the context of third-generation (3G) GW detectors combined with the Euclid survey.

Methods. We constructed our forecasts using realistic GW source populations and error models calibrated on recent detector designs. We adopted a Fisher–matrix approach, marginalized over nuisance parameters including tracer biases, primordial spectrum parameters, and baryon density, and compared different survey configurations, binning schemes, and GW detector networks.

Results. We find that tomographic cross-correlation can constrain H_0 at a percent or sub-percent precision, depending on the binning strategy, network configuration, and observing time. Combining galaxy autocorrelations with GW–galaxy cross-correlations improves constraints by up to a factor of ~ 10 relative to either probe alone. We further show that this performance requires multiple interferometers with accurate sky localization, and we discuss the added value of spectroscopic surveys and the detectability of GW clustering bias.

Conclusions. Our results demonstrate that this technique, applied to 3G GW detectors in synergy with large galaxy surveys, can deliver competitive measurements of cosmic expansion, even when marginalizing over a wide range of astrophysical and cosmological nuisance parameters.

Key words. gravitational waves – cosmological parameters

1. Introduction

The study of the three-dimensional large-scale distribution of galaxies is a cornerstone of modern cosmology. Its statistical properties probe structure formation, cosmology, and gravity (Kaiser 1987; Szalay et al. 1998; Yoo et al. 2009; Bonvin & Durrer 2011; Challinor & Lewis 2011), and are mapped by redshift surveys such as the Dark Energy Spectroscopic Instrument (DESI) (Aghamousa et al. 2016), Euclid (Laureijs et al. 2011; Mellier et al. 2025), the Legacy Survey of Space and Time (LSST) (Abell et al. 2009), and the Square Kilometre Array (SKA) (Braun et al. 2019; Bull 2016). The DESI survey recently released the latest clustering results (Adame et al. 2025), while Euclid will soon provide analyses of two-point correlations (de la Torre et al. 2025), the power spectrum (Pezzotta et al. 2024; Elkhatab et al. 2025), and lensing–clustering combinations (Paganin et al. 2024; Jelic-Cizmek et al. 2024; Tanidis et al. 2024; Tessore et al. 2025). Specifically, Euclid will deliver both a spectroscopic sample of $\sim 10^7$ galaxies at $0.9 < z < 1.8$ and a photometric sample of $\sim 10^9$ galaxies up to $z \sim 2$ (Mellier et al. 2025).

While galaxies are an established observational target for mapping the Universe, gravitational waves (GWs) have emerged

in recent years as a new, complementary tracer. Since their first direct detection in 2015 (Abbott et al. 2016), the LIGO–Virgo–Kagra (LVK) collaboration has reported 218 candidate detections from compact binary mergers (Abbott et al. 2019, 2021, 2023; Abac et al. 2025). These sources are complementary to galaxies because they provide a direct measurement of the luminosity distance to the source (Schutz 1986), but no direct redshift measurement (Schutz 1986; Finn & Chernoff 1993). This naturally motivates the study of GW clustering in distance space (Namikawa et al. 2016a,b; Zhang 2018; Namikawa 2021; Libanore et al. 2021; Garoffolo et al. 2021; Kumar 2023; Zheng et al. 2023; Vijaykumar et al. 2023; Zazzera et al. 2024; Begnoni et al. 2024; Dehghani et al. 2025), recently formulated in full generality by Fonseca et al. (2023) and shown to differ from its analog in redshift space (Bonvin & Durrer 2011). Because compact binaries form inside galaxies, both populations trace the same underlying dark matter field (with possible exceptions for primordial black holes¹). This enables the use of two-point GW–galaxy cross-correlations for both astrophysics (Raccanelli et al. 2016; Scelfo et al. 2018, 2020; Calore et al. 2020; Scelfo et al. 2022; Libanore et al. 2022;

¹ Even in this case, they remain correlated with the dark matter distribution, though with a different amplitude (Raccanelli et al. 2016; Scelfo et al. 2018).

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Balaudo et al. 2024; Gagnon et al. 2023; Zazzera et al. 2025) and cosmology.

The key cosmological feature is that the two tracers are observed in different spaces: galaxies in redshift space and GWs in distance space. Their cross-correlation in tomographic redshift and distance bins is therefore maximized only along the correct distance-redshift relation, allowing constraints on its parameters and, in particular, on the Hubble constant, H_0 . First suggested by Oguri (2016), Ferri et al. (2025) recently studied this in more detail. Other approaches based on two-point information in cosmology have been proposed (Camera & Nishizawa 2013; Nair et al. 2018; Mukherjee et al. 2020; Bera et al. 2020; Mukherjee et al. 2021a,b; Diaz & Mukherjee 2022; Mukherjee et al. 2024; Ghosh et al. 2025; Beltrame et al. 2024). These approaches are complementary to dark siren methods, which rely on prior information on the GW source population (see Pierra & Mastrogiovanni 2025, and references therein for a recent review). Importantly, cross-correlation techniques involve distinct systematics that remain to be fully explored, which makes them a complementary probe that can be used alongside other approaches.

Crucially, this program will reach its full statistical power with third-generation (3G) GW observatories: the Einstein Telescope (ET) in Europe (Abac et al. 2026) and the Cosmic Explorer (CE) in the US (Evans et al. 2023). These facilities are expected to detect millions of sources over ~ 10 years (Iacovelli et al. 2022a; Branchesi et al. 2023), comparable to galaxy survey catalogs, with subdegree angular resolution (Borhanian & Sathyaprakash 2024; Iacovelli et al. 2022a; Branchesi et al. 2023; Gupta et al. 2024). Third-generation (3G) detectors aim to achieve percent-level cosmological measurements of H_0 , which is critical to address the current tension between supernovae (SNe) and cosmic microwave background (CMB) (Riess et al. 2022; Aghanim et al. 2020), and to probe possible deviations from the standard cosmological model. These facilities are currently in a critical stage of planning. In particular, the ET collaboration is currently evaluating designs and sites (Branchesi et al. 2023; Abac et al. 2026). Synergies with CE are expected to be particularly powerful for cosmology and multimessenger science.

In this work, we assess the potential of 3G GW detectors to constrain the expansion history of the Universe through the GW-galaxy two-point correlation in tomographic redshift bins. We place particular emphasis on the Euclid survey. Cosmological studies based on cross-correlation techniques have been developed for LVK detectors, typically targeting H_0 (Mukherjee et al. 2020; Bera et al. 2020; Mukherjee et al. 2021a; Diaz & Mukherjee 2022; Mukherjee et al. 2024; Ghosh et al. 2025). In contrast, the 3G literature so far has focused mainly on astrophysical applications such as clustering bias measurements (Raccanelli et al. 2016; Scelfo et al. 2018, 2020; Calore et al. 2020; Scelfo et al. 2022; Libanore et al. 2022; Zazzera et al. 2025). Ferri et al. (2025) recently presented a dedicated investigation of constraints on cosmic expansion using the cross-correlation technique of Oguri (2016). Their analysis is based on simulations with fixed tracer bias and cosmological parameters, except for those governing the distance-redshift relation. In this work we take a complementary approach. Using a Fisher matrix formalism, we forecast the statistical power of 3G data, including nuisance parameters. Specifically, we adopt a fully agnostic treatment of tracer bias and marginalize over additional cosmological parameters. This allows us to explicitly quantify the information gain from combining galaxy autocorrelations with GW-galaxy

cross-correlations under realistic assumptions. Furthermore, we employ survey specifications and binning schemes calibrated to official Euclid requirements, and compare the constraining power of the photometric and spectroscopic samples. In the latter case, we also compare to a more optimistic design, namely the SKA Phase II spectroscopic survey (Bull 2016). We systematically investigate different binning strategies and quantify their impact on parameter inference. On the GW side, we adopt updated sensitivity curves for ET and CE and consider multiple 3G network configurations. We explicitly compare alternative ET geometries and their synergies with CE, with realistic observational errors on distance and sky localization. We also use updated compact-binary population models and revisit the detectability of the GW clustering bias under model-independent assumptions. Altogether, our forecasts provide a comprehensive assessment of how 3G detectors, in synergy with Euclid, can deliver competitive and robust constraints on the expansion history while accounting for astrophysical and cosmological uncertainties.

2. Methodology

2.1. Observable and cross-correlation signals

We considered the number counts of tracers X, Y of the underlying dark matter field (here, galaxies and GWs) as a function of sky position $\mathbf{n}$ and a tracer-dependent radial coordinate x (redshift z for galaxies, luminosity distance d_L for GWs). The formalism for cross-correlations is given in Scelfo et al. (2018) (see also Raccanelli et al. 2016; Scelfo et al. 2020, 2022, 2023; Bosi et al. 2023; Namikawa 2021; Fonseca et al. 2023; Libanore et al. 2021, 2022).

The fractional fluctuation of the tracer X at $(\mathbf{n}, x)$ is

$$\Delta^X(\mathbf{n}, x) = \frac{N^X(\mathbf{n}, x) - \langle N^X \rangle(x)}{\langle N^X \rangle(x)}. \quad (1)$$

This is a stochastic field with zero mean, and can be expanded in spherical harmonics, i.e.,

$$\Delta^X(\mathbf{n}, x) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} s_{\ell m}^X(x) Y_{\ell m}(\mathbf{n}). \quad (2)$$

The statistical properties of the field are encoded in the angular power spectrum in tomographic bins $C_{\ell}^{XY}(x_i, x_j)$, obtained by cross-correlating the harmonic coefficients of tracer X in bin i (centered at the radial coordinate x_i) with those of tracer Y in bin j (centered at the radial coordinate x_j):

$$\langle s_{\ell m}^X(x_i) s_{\ell' m'}^{Y*}(x_j) \rangle = \delta_{\ell \ell'} \delta_{m m'} C_{\ell}^{XY}(x_i, x_j). \quad (3)$$

The explicit expression of the power spectrum follows from the galaxy number density fluctuation in perturbation theory, binned into weighted redshift and distance intervals. This can be written as²

$$C_{\ell}^{XY}(x_i, x_j) = \frac{2}{\pi} \int dk k^2 P(k) \Delta_{\ell}^{X, x_i}(k) \Delta_{\ell}^{Y, x_j}(k), \quad (4)$$

where $P(k)$ is the primordial spectrum, and

$$\Delta_{\ell}^{X, x_i}(k) = \int_0^{\infty} dx w^X(x, x_i) \Delta_{\ell}^X(k, x) \quad (5)$$

² We adopted the `class` convention for the normalization of Fourier transforms, see e.g., Ref. Di Dio et al. (2013).

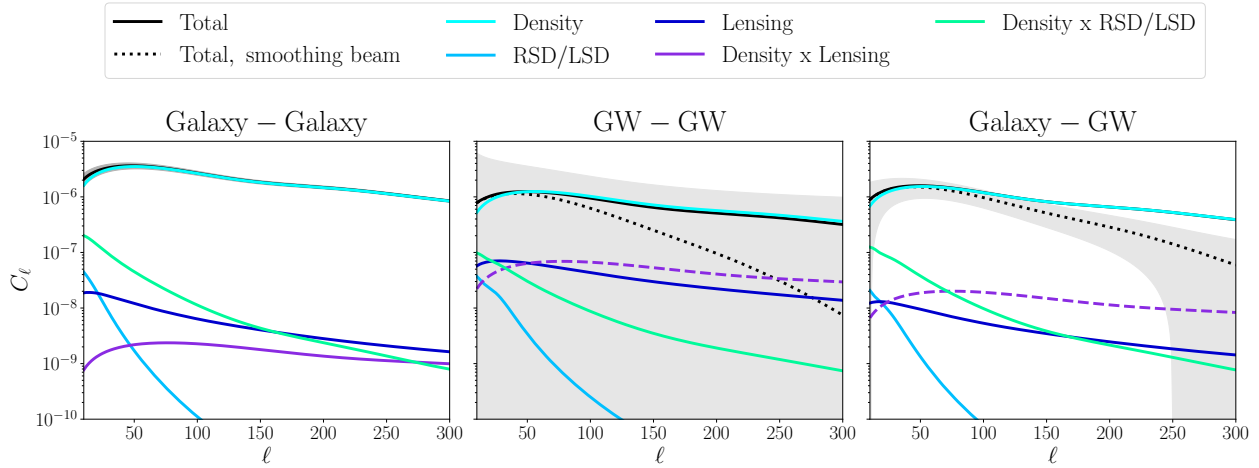


Fig. 1. Angular power spectra for the auto- and cross-correlation of the Euclid photometric sample with a GW catalog from five years of observations with two ET (L-shaped) and two CE detectors, shown for the redshift bin $z \in [1, 1.1]$. Colors indicate the different contributions to number-count fluctuations (see legend). Solid lines assume perfectly localized GW sources, while dotted black lines in the central and right panels include realistic smoothing due to their imperfect localization. Shaded areas show 1σ errors. Dashed lines indicate negative terms, which are plotted on the positive axis for comparison.

is the effective transfer function of the tracer X . The weight is a normalized window function centered at coordinate x_i . This can be expressed in terms of an unweighted window function $W^X(x, x_i)$ (encoding how the observed events are binned in either redshift or distance; more details are provided in Sect. 2.3) and the observed distribution of tracer X , dN_{obs}^X/dx , as follows:

$$w^X(x, x_i) = \frac{W^X(x, x_i) dN_{\text{obs}}^X/dx}{\int dx' W^X(x', x_i) dN_{\text{obs}}^X/dx'}, \quad (6)$$

The kernel $\Delta_\ell^X(k, x)$ encodes relativistic number-count contributions from perturbation theory. Bonvin & Durrer (2011) and Challinor & Lewis (2011) derived this expression in redshift space ($x \equiv z$); Zhang (2018) and Libanore et al. (2021) first studied it in the luminosity-distance space for GWs and SNe ($x \equiv d_L$); while Namikawa (2021) and Fonseca et al. (2023) provided a general derivation. The results differ from their counterparts in redshift space. Contributions are grouped into density, velocity (redshift-space distortion (RSD) + Doppler), lensing, and gravity terms. In luminosity-distance space, the RSD contribution becomes a “luminosity distance space distortion” (LSD) term (Zhang 2018; Libanore et al. 2021; Fonseca et al. 2023). This can be written schematically as

$$\Delta_\ell^X(k, x) = \Delta_\ell^{X,\text{den}}(k, x) + \Delta_\ell^{X,\text{len}}(k, x) + \Delta_\ell^{X,\text{RSD/LSD}}(k, x) + \Delta_\ell^{X,\text{dop}}(k, x) + \Delta_\ell^{X,\text{gr}}(k, x). \quad (7)$$

Here, we retain the dominant terms, namely density, lensing, and RSD/LSD (see Fonseca et al. (2023) for a full comparison)³. We computed Eq. (4) using the Limber approximation (Limber 1954; LoVerde & Afshordi 2008), restricting to scales where it is valid (Sect. 3.3). Appendix A details the explicit formulae.

With this formalism, we constructed the auto- and cross-correlation spectra of two given tracers. Specifically, we studied the galaxy-galaxy and GW-GW autocorrelation terms, C_ℓ^{g-g} and $C_\ell^{\text{GW-GW}}$, respectively, and their cross-correlation $C_\ell^{\text{GW-g}}$. Figure 1 shows these spectra for the Euclid photometric survey

and a GW detector network with two ET and two CE detectors in a redshift bin $z \in [1, 1.1]$ (survey details in Sect. 3). Colored lines indicate density, RSD, LSD, and lensing contributions, as well as their cross-terms⁴. Solid lines assume perfect GW localization, while dotted lines include smoothing due to limited sky localization (Sect. 2.3). The shaded bands give the expected 1σ errors.

2.2. Cosmological information and dependence on population properties and selection effects

Examination of Eq. (4) clarifies the source of cosmological information contained in the tomographic cross-spectrum $C_\ell^{\text{GW-g}}$. The dominant contribution is the density term, which relates the number-count fluctuations to the matter density through the tracer bias. In the Limber approximation, the expression is (see Appendix A):

$$C_\ell^{\text{GW-g}}(z_i, d_{L,j}) = \int dz w^g(z, z_i) w^{\text{GW}}[d_L(z, \lambda), d_{L,j}] \frac{dd_L}{dz}(z, \lambda) \times \frac{H(z)}{c r(z)^2} b_{\text{GW}}(z) b_g(z) P\left[\frac{\ell+1/2}{r(z, \lambda)}, z\right], \quad (8)$$

where the tracer biases are b_g and b_{GW} , the comoving distance is $r(z, \lambda) = d_L(z, \lambda)/(1+z)$, and λ denotes the parameters of the distance-redshift relation. Eq. (8) shows that if the galaxy window function w^g has non-zero support in a bin around redshift z_i , the GW window function w^{GW} will be non-vanishing in the (fixed) bin around $d_{L,j}$ only if $d_L(z, \lambda)$, for redshifts around z_i , falls inside the $d_{L,j}$ bin. That is, the cross-term is maximized when the GW and galaxy bins align along the correct distance-redshift relation (Oguri 2016). Figure 2 illustrates this using the signal-to-noise-ratio (S/N) of the cross-correlation between the Euclid photometric sample and a GW detector network composed of two ET and two CE detectors. We computed the S/N using Eqs. (23)–(25) below, following Scelfo et al. (2020). We divided the Euclid sample into 13 equally-populated redshift bins. We used a Planck 2018 cosmology to convert redshift bins

³ Although the signal is dominated by the density term, other contributions should be modeled since they affect inter-bin covariances and thus the forecasted errors, particularly the lensing term (Scelfo et al. 2018).

⁴ RSD/LSD-lensing cross-spectra are subdominant and omitted from the figure, but included in the analysis.

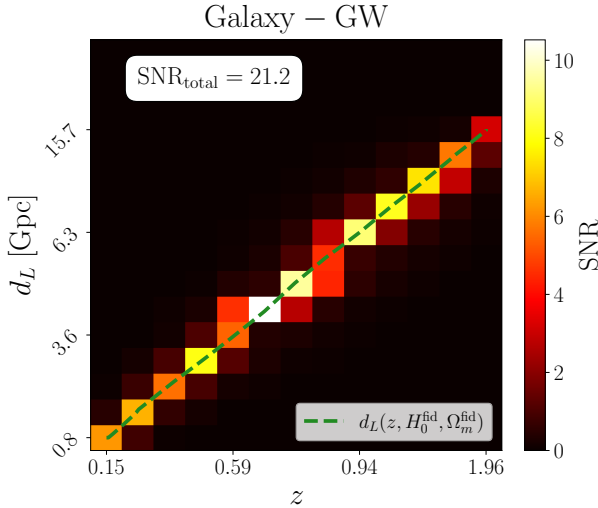


Fig. 2. Signal-to-noise ratio (S/N) of the cross-correlation between the Euclid photometric sample and a GW catalog from a network of two L-shaped ET detectors and two CE detectors. The S/N peaks along the fiducial distance–redshift relation (dashed green).

to distance, which defined the binning of the GW survey⁵. The S/N peaks along the true distance–redshift relation (dashed green line). This property allows to constrain its parameters (Oguri 2016).

Besides λ , the signal depends on cosmological parameters ($A_s, n_s, \Omega_b h^2$) that enter the power spectrum and on tracer properties. These must be marginalized over to avoid underestimating the uncertainty and bias. Four key terms encode the dependence on population properties (Scelfo et al. 2020):

- The observed redshift and distance distributions, dN_{obs}^X/dx . This sets the weighting factor in the window function (Eq. (6)). This can be determined directly from the observed sample. The cross-correlation then compares fluctuations in GW and galaxy counts, whose statistics are insensitive to the overall distribution or selection function. Different selection cuts or variations in the background source distribution alter the clustering of the remaining sources, i.e., their bias (next point). Hence, the assumed form of the binary black hole (BBH) merger distribution is likely not the dominant source of systematic uncertainty for this approach, although its impact has not yet been studied in detail and should be explicitly assessed in future work. By contrast, the clustering bias is certainly a possible source of systematic error and must therefore be modeled robustly, either by comprehensive forward modeling or by adopting a non-parametric approach, as we do in this work.
- Clustering bias, b_X . Source counts trace the dark matter density through a bias term (Appendix A). On the large scales considered here, the bias depends only on the redshift (Fosalba et al. 2015). Varying the bias linearly changes the signal amplitude. If the bias evolves with redshift, it can introduce correlations with the cosmological parameters. Here, we include a free bias parameter for each redshift bin and tracer. This enlarges the parameter space, but avoids strong assumptions on the functional form.
- Magnification bias and local count slope, s_X . This encodes lensing-induced changes in the number counts near the detection threshold and is defined as the derivative of the

comoving density with respect to the threshold. It affects lensing (Appendices A, B) and, more weakly, gravity and Doppler terms (Alonso et al. 2015; Maartens et al. 2021; Fonseca et al. 2023; Zazzerà et al. 2024).

- Evolution bias, $b_{e,X}$. This encodes variations in source counts from the formation of new objects at a fixed detection threshold. Similarly to magnification bias, it depends on both populations and survey specifications. In redshift space it enters the gravity term and, weakly, the velocity term, so we neglect it here. In distance space, it also affects lensing (Fonseca et al. 2023) (Appendix A).

Since the evolution and magnification bias enter only in subdominant terms, we fix them in this work.

2.3. Noise and measurement uncertainties

In addition to the angular power spectrum, we considered the effect of shot noise, which we assumed to be uncorrelated in different experiments and redshift and distance bins. This can also be decomposed into harmonic coefficients $n_{\ell m}(x_i)$. We obtain the harmonic coefficients of the observed signal by summing the signal and noise,

$$a_{\ell m}^X(x_i) = s_{\ell m}(x_i) + n_{\ell m}(x_i). \quad (9)$$

The expectation value of the noise is given by

$$\langle n_{\ell m}^X(x_i) n_{\ell' m'}^{Y*}(x_j) \rangle = \delta_{\ell \ell'} \delta_{m m'} \delta_{XY} \delta_{ij} \mathcal{N}^X(x_i), \quad (10)$$

where

$$\mathcal{N}^X(x_i) = \left[\int_0^\infty dx W^X(x, x_i) \frac{d^2 N_{\text{obs}}^X}{dx d\Omega} \right]^{-1} \quad (11)$$

is the inverse total number of observed sources per steradian in the i -th bin⁶. We also assumed that signal and noise are uncorrelated, i.e.,

$$\langle n_{\ell m}^X(z_i) s_{\ell' m'}^{Y*}(z_j) \rangle = 0. \quad (12)$$

Using these properties, the observed angular power spectrum $\tilde{C}_\ell^{XY}$ can be written as

$$\langle a_{\ell m}^X(x_i) a_{\ell' m'}^{Y*}(x_j) \rangle = \delta_{\ell \ell'} \delta_{m m'} \tilde{C}_\ell^{XY}(x_i, x_j),$$

$$\tilde{C}_\ell^{XY}(x_i, x_j) \equiv C_\ell^{XY}(x_i, x_j) + \delta_{XY} \delta_{ij} \mathcal{N}^X(x_i). \quad (13)$$

For GW sources, we accounted for the limited angular resolution of the detectors. The best achievable resolution sets a limiting scale in the angular power spectrum (Oguri 2016; Libanore et al. 2021; Scelfo et al. 2023; Bosi et al. 2023), given by $\ell_{\text{max}} = \pi/\Delta\Omega_{\text{min}}^{1/2}$, where $\Delta\Omega_{\text{min}}$ denotes the best 1σ sky-localization of the GW catalog. We imposed a hard cut on the signal at the limiting scale ℓ_{max} , and additionally modeled the suppression due to imperfect localization as an exponential damping. This follows from treating the localization error as a Gaussian beam and convolving it with the GW density contrast (Namikawa et al. 2016b; Calore et al. 2020; Libanore et al. 2021; Scelfo et al. 2023; Bosi et al. 2023). Thus, whenever one of the two tracers is a GW source, one has

$$C_\ell^{\text{GW-gal}}(d_{L,i}, z_j) \mapsto C_\ell^{\text{GW-gal}}(d_{L,i}, z_j) \times e^{-\ell(\ell+1)/\ell_{\text{damp}}^2}, \quad (14)$$

$$C_\ell^{\text{GW-GW}}(d_{L,i}, d_{L,j}) \mapsto C_\ell^{\text{GW-GW}}(d_{L,i}, d_{L,j}) \times e^{-2\ell(\ell+1)/\ell_{\text{damp}}^2}, \quad (15)$$

$$\ell_{\text{damp}}^2 = \frac{(2\pi)^{3/2}}{\Delta\Omega_{1\sigma}}, \quad (16)$$

⁵ We included three extra bins at the high-distance edge to fully cover the observed sample.

⁶ We note that $\mathcal{N}^X(x_i)$ depends on the overall source merger rate rather than just the shape of the distribution, as is the case for the signal C_ℓ .

where $\Delta\Omega_{1\sigma}$ is the solid angle corresponding to the 68% contour level of the angular error⁷.

Finally, the error on distance or redshift can be included in the unweighted window function by modeling it as the convolution between a “bare” window function and the likelihood of the distance or redshift profile (Oguri 2016; Calore et al. 2020). Therefore, for both GWs and galaxies, we modeled the unweighted window function $W^X(x, x_i)$ appearing in Eq. (6) as a step function $S^X(x, x_i)$ convoluted with a log-normal likelihood encoding the measurement uncertainty, denoted by $\mathcal{L}(x^{\text{obs}}|x)$. This yields

$$W^X(x, x_i) = \int_0^\infty dy S^X(y, x_i) \mathcal{L}(y|x) = \frac{1}{2} \left\{ \text{erf}[u(x^{i+1}, x)] - \text{erf}[u(x^i, x)] \right\},$$

$$u(y, x) = \frac{\ln y - \ln x}{\sqrt{2}\sigma_{\ln x}}, \quad (17)$$

where $\sigma_{\ln x}$ is the standard deviation of the log-normal, that is, the relative error on redshift or distance, and x^i, x^{i+1} denote the lower and upper edges of the i th bin in distance or redshift.

2.4. Fisher matrix and S/N

We estimated measurement errors using the Fisher matrix formalism. For a set of parameters $\{\theta_\alpha\}$, the Fisher matrix for the angular power spectrum assuming a Gaussian likelihood for the harmonic coefficients $a_{\ell m}$ is (Bellomo et al. 2020; Abramo et al. 2022)

$$F_{\alpha\beta} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_\alpha \partial \theta_\beta} \right\rangle = f_{\text{sky}} \sum_\ell \frac{2\ell + 1}{2} \text{Tr}[C_\ell^{-1} (\partial_\alpha C_\ell) C_\ell^{-1} (\partial_\beta C_\ell)], \quad (18)$$

where f_{sky} is the fraction of the sky observed by the experiment and C_ℓ is the covariance matrix. Specifically, we considered a galaxy survey binned into N redshift bins $z_1, \dots, z_N$, and a GW catalog binned into M luminosity distance bins $d_{L,1}, \dots, d_{L,M}$. The (symmetric) covariance matrix C_ℓ is expressed as (Scelfo et al. 2018)

$$C_\ell = \begin{pmatrix} \tilde{C}_\ell^{gg}(z_1, z_1) & \dots & \tilde{C}_\ell^{gg}(z_1, z_N) & \tilde{C}_\ell^{gGW}(z_1, d_{L,1}) & \dots & \tilde{C}_\ell^{gGW}(z_1, d_{L,M}) \\ & \ddots & \vdots & \vdots & \ddots & \vdots \\ & & \tilde{C}_\ell^{gg}(z_N, z_N) & \tilde{C}_\ell^{gGW}(z_N, d_{L,1}) & \dots & \tilde{C}_\ell^{gGW}(z_N, d_{L,M}) \\ & & & \tilde{C}_\ell^{GWGW}(d_{L,1}, d_{L,1}) & \dots & \tilde{C}_\ell^{GWGW}(d_{L,1}, d_{L,M}) \\ & & & & \ddots & \vdots \\ & & & & & \tilde{C}_\ell^{GWGW}(d_{L,M}, d_{L,M}) \end{pmatrix}, \quad (19)$$

where $\tilde{C}_\ell$ denotes the observed angular power spectrum as in Eq. (13). The matrix $\partial_\alpha C_\ell = \partial C_\ell / \partial \theta_\alpha$ contains the derivatives of the covariance-matrix elements with respect to a given cosmological parameter θ_α , while keeping all other parameters fixed. The forecasted marginal error on the parameter θ_α is given by the square root of the diagonal element of the inverse Fisher matrix, $\sigma_{\theta_\alpha} = \sqrt{F_{\alpha\alpha}^{-1}}$.

To assess the detectability of the cross-correlations, we computed their S/N, following the discussion of Scelfo et al. (2020).

⁷ Slightly different definitions of the damping factors appear in the literature, yielding different values of the numerical coefficient in the definition of ℓ_{damp}^2 . We are not aware of any established derivation of this numerical factor, so we provide one in Appendix C.

We organized the cross-correlation angular power spectra from different tracers and bin pairs in a vector $\mathbf{C}_\ell$ as follows:

$$\mathbf{C}_\ell = \left(C_\ell^{gg}(z_1, z_1), \dots, C_\ell^{gGW}(z_1, d_{L,1}), \dots, C_\ell^{GWGW}(d_{L,1}, d_{L,1}), \dots \right)^\top. \quad (20)$$

We associated a pair of indices $[I_1, I_2]$ to each element I of this array, corresponding to the tracer and the redshift bin of the angular power spectrum in each specific entry. With this notation, the related covariance matrix is

$$[\text{Cov}(\ell)]_{IJ} = \tilde{C}_\ell^{I_1 J_1} \tilde{C}_\ell^{I_2 J_2} + \tilde{C}_\ell^{I_1 J_2} \tilde{C}_\ell^{I_2 J_1}. \quad (21)$$

The diagonal entries of the covariance matrix are related to the forecasted 1σ error on the measured signal, (see e.g., Calore et al. (2020))

$$\delta C_\ell^{XY}(x_i, x_j) = \left\{ \frac{1}{(2\ell + 1) f_{\text{sky}}} \left[\left(C_\ell^{XY}(x_i, x_j) \right)^2 + \tilde{C}_\ell^{XX}(x_i, x_i) \times \tilde{C}_\ell^{YY}(x_j, x_j) \right] \right\}^{1/2}. \quad (22)$$

We used this expression to compute the shaded bands in Fig. 1. The S/N for each combination of redshift and distance bins at a given multipole ℓ can be expressed as

$$(\text{S/N})_{[I_1, I_2]}^2(\ell) = f_{\text{sky}} (2\ell + 1) \frac{\left(C_\ell^{[I_1, I_2]} \right)^2}{\left[\tilde{C}_\ell^{[I_1, I_1]} \tilde{C}_\ell^{[I_2, I_2]} + \left(\tilde{C}_\ell^{[I_1, I_2]} \right)^2 \right]}, \quad (23)$$

while the S/N of the full spectrum across all bins, taking their covariance into account, is given by

$$(\text{S/N})_{\text{total}}^2(\ell) = f_{\text{sky}} (2\ell + 1) \mathbf{C}_\ell^\top \cdot \text{Cov}^{-1} \cdot \mathbf{C}_\ell. \quad (24)$$

Finally, from Eq. (23) or Eq. (24), the cumulative S/N for $\ell < \ell_{\text{max}}$ is obtained from

$$(\text{S/N})^2(\ell < \ell_{\text{max}}) = \sum_{\ell'=\ell_{\text{min}}}^{\ell_{\text{max}}} (\text{S/N})^2(\ell'). \quad (25)$$

In Fig. 2 the color palette shows the cumulative S/N computed from Eq. (23), while each panel shows the cumulative total S/N obtained from Eq. (24).

3. Surveys, error forecasts, and implementation

This section provides details on the GW and galaxy catalog specifications, the error modeling, and the numerical implementation used.

3.1. Galaxy survey

For the galaxy survey, we considered the photometric sample of the Euclid satellite (Laureijs et al. 2011) as our fiducial scenario. This will have a sky coverage of 15 000 square degrees and deliver a total of $\sim 1.6 \times 10^9$ observed galaxies (Laureijs et al. 2011; Amendola et al. 2018). We adopted Euclid’s photometric rather than spectroscopic sample because its far larger source number reduces shot noise and its wider redshift coverage maximizes overlap with GW events. Although photometric redshifts are less precise, the GW angular resolution is the dominant limitation, making the higher number of

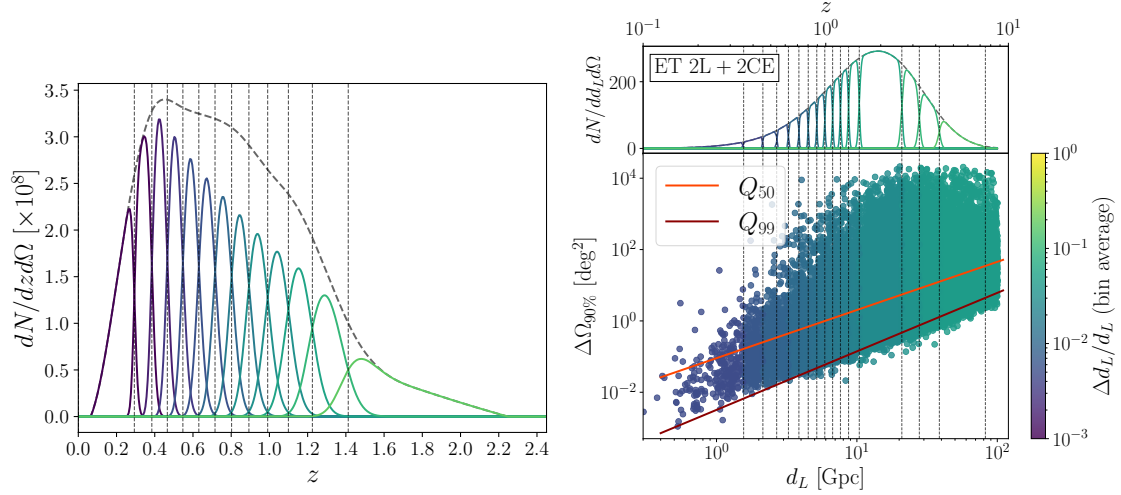


Fig. 3. Left: Redshift distribution of galaxies detected by the Euclid photometric survey in 13 equally populated bins, as in the Euclid preparation study (Pocino et al. 2021). Solid colored curves represent the weighted window function in each bin (see Eq. (6)), assuming a 5% relative error on the redshift. The dashed curve shows the overall event distribution. Vertical gray lines mark the bin edges. Right: Distribution of 90% sky localization areas of GW events detected in different distance bins. The color coding of the points corresponds to the relative 1σ distance uncertainty averaged over each bin. In the upper panel, the lines are the same as in the left panel, with the relative distance error modeled as described in the text. The redshift on the top axis is obtained by conversion of the distance with the fiducial cosmology.

photometric sources preferable for tomographic angular power spectra. We discuss the comparison with spectroscopic samples in Sect. 4.2. For modeling the photometric survey, we relied on public results from the Euclid flagship simulation (Pocino et al. 2021; Castander et al. 2025). We refer in particular to Table I and Appendix C of Lepori et al. (2022) which provide number density, galaxy bias, and magnification bias. We interpolated the values in Table I of Lepori et al. (2022) using a cubic spline to obtain the redshift profile. For the window function, we assumed a relative redshift uncertainty $\sigma_{\ln z} = 0.05$ (Lepori et al. 2022). As a reference case, we followed the Euclid preparation paper Pocino et al. (2021), and adopted the standard conservative choice of 13 equally populated redshift bins. Figure 3 (left panel) shows the overall redshift distribution together with the distribution in each bin, including the effect of redshift uncertainty. We discuss the effect of different binning choices in Sect. 4.1. For both the clustering and magnification bias, we assumed the cubic polynomial fit of the Euclid flagship simulation (see Appendix C of Lepori et al. (2022)). Values are reported in Appendix B.

3.2. Gravitational-wave survey

For GW sources, we considered resolved BBH mergers. As our default case, we considered a 3G GW detector network consisting of ET in its so-called two-L configuration (see Abac et al. (2026) for details). The two CE detectors consist of a 40 km detector and a 20 km arm detector. We denote this configuration as ET 2L + 2CE. We considered a five-year observing time, comparable to the duration of the Euclid survey. We analyze the dependence on the observing time in Sect. 4.3, considering observing times of one and ten years and different detector configurations.

Catalog. We used simulations of the expected population of astrophysical BBHs from the ET design study (Branchesi et al. 2023)⁸ (see also Mapelli et al. (2021, 2022) for details). The

catalog covers a data collection period of one year, with a total of $\sim 1.2 \times 10^5$ sources. We rescaled the amplitude of the observed source distribution to model longer observing periods. We forecast detectability and measurement errors on distance and sky localization using the Fisher-matrix code *gwfast* (Iacovelli et al. 2022b,a)⁹. We used the IMRPhenomXPHM waveform model (London et al. 2018) and a minimum frequency of 2 Hz with an 85% uncorrelated duty cycle in each detector in the network. We considered a source as detected if the optimal matched-filter S/N exceeded the threshold value of 12. We further restricted to events with a 90% credible level (CL) sky-localization area less than 4π steradians and with a relative error on the luminosity distance smaller than 100%¹⁰.

Source distribution, distance error, and binning We modeled the observed source distribution with the commonly used parametrization,

$$\frac{dN_{\text{obs}}^{\text{GW}}}{dd_L} = A \left(\frac{d_L}{d_{L,0}} \right)^\alpha e^{-(d_L/d_{L,0})^\beta}, \quad (26)$$

where we determined the values of $\{A, d_{L,0}, \alpha, \beta\}$ from the simulations. We report the result in Appendix B, together with the total number of detected sources after applying the distance cuts and sky-localization cuts described above. We obtained the relative error $\sigma_{\ln d_L}$ on the luminosity distance measurement in each bin, which enters the window function, as the median in the bin of σ_{d_L}/d_L , where σ_{d_L} is the 1σ error computed from the Fisher matrix.

⁹ The ET sensitivity curves are available at apps.et-gw.eu/tds/?content=3&r=18213. For CE–20 km and CE–40 km we used the curves available at dcc.cosmicexplorer.org/CE-T2000017/public. These are the same sensitivity curves used in Branchesi et al. (2023), Abac et al. (2026).

¹⁰ Larger localization volumes are unphysical and can appear in Fisher-matrix analyses for events for which a full Bayesian analysis would recover the prior distribution (Iacovelli et al. 2022a; Dupletsa et al. 2025).

⁸ Publicly available at apps.et-gw.eu/tds/?content=3&r=18321.

Figure 3 (right panel) shows the weighted window function in each bin for our reference binning scheme, together with the sky-localization errors. We obtained our distance bins by converting the edges of the galaxy survey redshift bins to distance using a fiducial cosmology. Because GW events can be detected at a larger distance than those obtained from Euclid’s maximum redshift, we added three equally populated distance bins at the high-distance edge of the distribution to avoid loss of information or bias.

Sky localization. As discussed in Sect. 2.3, we included the impact of limited angular resolution. We imposed a cut at the maximum angular scale $\ell_{\max} = \pi / \Delta\Omega_{\min}^{1/2}$, where $\Delta\Omega_{\min}$ is computed in each bin from the 99th percentile of the 90% CL localization error, $Q_{99}^{\Delta\Omega}(d_{L,i})$. This avoids outliers in the simulations that would yield overly optimistic predictions. The signal is further suppressed by the exponential damping term in Eq. (14), with $\ell_{\text{damp}}^2 = (2\pi)^{3/2} / \Delta\Omega_{1\sigma}$. Here $\Delta\Omega_{1\sigma}$ is the 50th percentile of the 1σ localization error, $Q_{50}^{\Delta\Omega}(d_{L,i})$. We provide tables with the resulting $\ell_{\max}$, ℓ_{damp} , and bin edges in Appendix C. Figure 3 (right panel) shows the localization area of the BBH sources together with the 50th and 99th percentiles as a function of distance. We find $\ell_{\text{damp}} \sim 1000$ for the closest bin ($z \in [0, 0.3]$), decreasing to ~ 600 at $z \in [0.3, 0.38]$, ~ 370 at $z \simeq 0.5$, ~ 200 at $z \simeq 1$, and ~ 100 at $z \simeq 2$. A further cut excludes the highly non-linear regime (Sect. 3.3), which additionally tightens the multipole range at low redshift.

Tracer bias. For the perturbation properties, we modeled the fiducial GW bias b_{GW} as in Peron et al. (2024), adopting the relation $b_{\text{GW}} = A_{\text{GW}}(1+z)^\gamma$. For the evolution and magnification bias, we followed Zazzer et al. (2024), which provides a third-order polynomial fit for the evolution and magnification bias for 3G detectors. We report the specific values in Appendix B.

3.3. Implementation, free parameters, and summary

We computed the Fisher matrix in Eq. (18) for the auto- and cross-tomographic angular power spectra of galaxy and GW surveys as in Eq. (4), considering the density, RSD, and lensing terms in the Limber approximation (see Appendix A). We used the publicly available python package COLIBRI¹¹, which we modified to handle tracers in different spaces and to include the number-count contributions considered in this work. The package computes the power spectrum $P(z, k)$ using CAMB (Lewis et al. 2000). We accounted for nonlinear scales using the HMcode-2020 scheme (Mead et al. 2021). To compute the derivatives, we employed the numdifftools library¹². We used the central-difference method and performed a series of robustness tests to validate the stability of the numerical derivatives and the accuracy of the Fisher-matrix inversion. More specifically, for each parameter we computed derivatives over a wide range of multipoles using different step sizes, identified a stability region where the derivatives are insensitive to this choice, and selected the step size accordingly. We performed this procedure independently for each parameter. To assess the stability of the matrix inversion, we computed the quantity $\epsilon = \|FF^{-1} - I\|$, where F is the Fisher matrix and I is the identity

matrix, and verified that it remained small in all the cases considered.

We assumed a flat Λ CDM cosmology, where the Hubble constant and the matter density, $\{H_0, \Omega_m\}$, determine the distance-redshift relation. To these, we added the amplitude and spectral index of the primordial power spectrum, $\{A_s, n_s\}$, and the physical baryon density $\Omega_b h^2$. We took fiducial values from Planck (2018) (Aghanim et al. 2020). We parametrized the galaxy and GW bias with one free parameter per bin for each tracer. We kept the evolution and magnification biases fixed, since they affect only the lensing term, which is subdominant, especially once the largest angular scales ($\ell \lesssim 10$) are excluded. A more general strategy would include large-scale effects and marginalize over these parameters as well (Zazzer et al. 2025). We leave this for future work. With the choices described above, our baseline Fisher analysis contained a total of $5 + N_{\text{bins}}^{\text{gal}} + N_{\text{bins}}^{\text{GW}} = 8 + 2 N_{\text{bins}}^{\text{gal}}$ parameters $\{\theta_\alpha\}$ (where $N_{\text{bins}}^{\text{gal}}$ and $N_{\text{bins}}^{\text{GW}}$ are the number of bins in the galaxy and GW surveys, respectively), namely

$$\theta = \{H_0, \Omega_m, \Omega_b h^2, A_s, n_s, b_{g,1}, \dots, b_{g,N_{\text{bins}}^{\text{gal}}}, b_{\text{GW},1}, \dots, b_{\text{GW},N_{\text{bins}}^{\text{GW}}}\}. \quad (27)$$

For the baseline configuration with 13 bins, this results in 34 parameters.

We restricted the multipole range to exclude highly non-linear scales. For bin i centered at z_i , we adopted a maximum multipole $\ell_{\max} = r(z_i) k_{i,\max}$, where $r(z_i)$ is the comoving distance at the bin center and the maximum wavenumber is defined as $k_{i,\max} \equiv \pi / (2R_i)$, where R_i is defined implicitly by requiring that the root mean square (rms) linear matter-density fluctuation smoothed with a spherical top-hat of radius R_i satisfies $\sigma^2(R_i, z_i) = 0.25$, which is a standard choice in galaxy surveys forecasts (Amendola et al. 2018). We computed this variance from the linear matter power spectrum $P_m(k, z)$ obtained from CAMB (as described above) as

$$\sigma^2(R, z) \equiv \langle \delta_R^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P_m(k, z) W^2(kR), \quad (28)$$

with $W(kR)$ the Fourier-space spherical top-hat window,

$$W(x) = \frac{3}{x^3} (\sin x - x \cos x). \quad (29)$$

We computed the integral numerically and solved for R_i using bisection root finding. This yields $k_{\max} \simeq 0.12 \text{ Mpc}^{-1}$ ($\ell_{\max} \simeq 130$) in the lowest redshift bin and $k_{\max} \simeq 0.21\text{--}0.24 \text{ Mpc}^{-1}$ ($\ell_{\max} \simeq 800\text{--}1000$) in the highest bins. For GW–galaxy and GW–GW correlations, $\ell_{\max}$ is the minimum between this nonlinearity cut and the angular scale allowed by GW localization (Sect. 3.2), with an additional exponential damping from finite sky resolution (Eq. (14)). Unlike the nonlinearity cut, the localization limit decreases with redshift, reflecting poorer constraints for distant GW sources. Overall, nonlinear scales dominate the cut at $z \lesssim 0.5$, while sky resolution dominates at $z \gtrsim 0.5$. A “sweet spot” occurs in the redshift bin $z \in [0.63, 0.72]$, where $\ell_{\text{damp}} = 345$, maximizing the S/N of the cross-correlation (Fig. 2). This multipole corresponds to the largest scale that effectively contributes to the GW-based constraints once all the discussed restrictions are accounted for¹³.

We set the minimum multipole to avoid inaccuracies of the Limber approximation. Following Tanidis & Camera (2019),

¹¹ github.com/GabrieleParimbelli/COLIBRI

¹² github.com/pbrod/numdifftools

¹³ Note that these values depend strongly on the assumed GW detector network (Sect. 4.3).

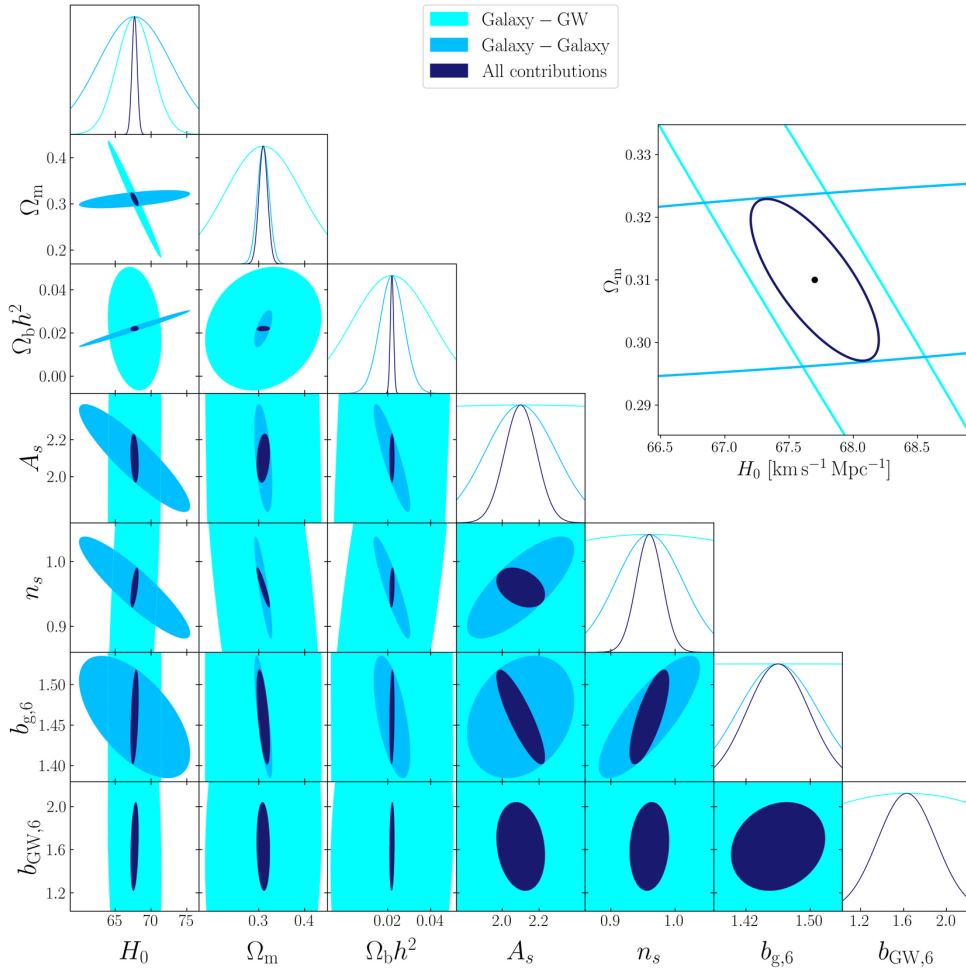


Fig. 4. Constraints at the 1σ on cosmological and galaxy (GW) bias parameter in a selected redshift (distance) bin from the Euclid photometric survey autocorrelation (blue), the cross-correlation between Euclid and ET 2L + 2CE (cyan), and their combination including the GW-GW autocorrelation term (navy blue). The inset shows the $H_0 - \Omega_m$ plane with a narrower range.

which assesses the accuracy of the Limber approximation in detail, we adopted $\ell_{\min} = 5$ for $z < 0.5$, $\ell_{\min} = 10$ for $0.5 < z < 0.75$, $\ell_{\min} = 15$ for $0.75 < z < 1.25$, and $\ell_{\min} = 20$ for $z > 1.25$.

4. Results and discussion

4.1. Euclid photometric sample combined with 3G detector networks

In this section we discuss results for the default configuration of this paper. As shown in Fig. 2, the cross-correlation is detected with a $S/N \sim 21$. We also find that the GW-GW autocorrelation gives a total S/N of 1.8, even though the signal in each individual multipole bin is not detectable (see Fig. 1).

Constraints on the cosmic expansion In Figure 4, we present the joint 1σ contours of H_0 , Ω_m , along with other cosmological parameters and the amplitude of the galaxy and GW bias in the sixth redshift (distance) bin, chosen so to maximize the S/N for the cross-correlation¹⁴. We show contours obtained from the galaxy-galaxy and GW-galaxy correlations separately, as well as from their combination. We report the constraints provided by

¹⁴ We include a free parameter quantifying the tracer bias for each bin, but showing a single one is sufficient to illustrate its effect.

each separate contribution, as well as by the combination of the probes, in Appendix D.

The cross-correlation between GW and galaxy data (cyan) provides substantial constraining power, driven by the effect discussed in Sect. 2.2, namely the presence of a peak in the cross-correlation signal along the correct distance-redshift relation. This results in the typical anti-correlation between H_0 and Ω_m . In particular, we find that the cross-correlation term alone provides a stronger constraint on H_0 than the galaxy autocorrelation. However, our constraints for the galaxy-galaxy autocorrelation term are based on a conservative cut on the maximum multipole, selected to exclude nonlinear scales. Regardless of this, the combination of the two probes leads to a substantial improvement in the measurement of H_0 , due to the different correlation directions in the $H_0 - \Omega_m$ plane. This results in an enhancement of a factor of ~ 6 in the precision on H_0 . Specifically, the constraint improves from $\sim 6\%$ when using the cross-correlation term alone (and $\sim 15\%$ with the galaxy-galaxy term alone) to a joint $\sim 1\%$ measurement (see Appendix D). This complementarity is illustrated in the inset of Fig. 4, where we show the constraint in the H_0, Ω_m plane on a narrower range. We note that, as a consequence, the constraints on other cosmological parameters ($\Omega_b h^2, A_s, n_s$) are also significantly improved. We forecast an overall precision of approximately 6% for the matter density parameter Ω_m . Overall, these constraints lead to a $\sim 0.7\%$ constraint on $H(z)$ at redshift $z_* \sim 0.25$, which minimizes the

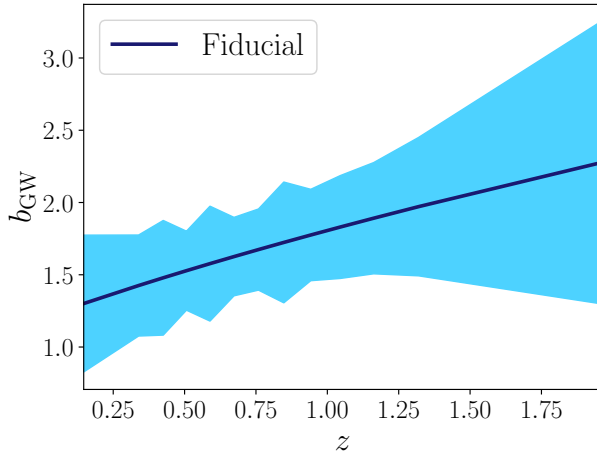


Fig. 5. Reconstruction of the GW clustering bias using an agnostic parametrization with one free parameter per redshift (distance) bin, using the Euclid photometric sample and five years of observations with the ET 2L + 2CE GW detector network.

relative uncertainty. We increased the sub-percent accuracy on H_0 and the percent-level accuracy on Ω_m by increasing the number of bins to at least 20 (see Sect. 4.1 below). Ferri et al. (2025) recently forecast a percent-level uncertainty on H_0 from simulations with this technique for a similar GW detector network. Our analysis yields a compatible precision, showing that such accuracy is achievable even when marginalizing over an extended set of nuisance parameters. For comparison, we obtain a fractional uncertainty of $\sim 0.6\%$ when fixing all nuisance parameters.

Astrophysical information While the main focus of this paper is the measurement of the cosmic expansion, we also present constraints on the GW clustering bias, shown in Fig. 5 as a function of redshift. With the ET 2L + 2CE configuration we find at best a $\sim 40\%$ constraint in $z \simeq [0.5, 1]$ after five years, which improves to $\sim 25\%$ after ten years (Appendix D). These bounds are less optimistic than in previous studies (Calore et al. 2020; Zazzera et al. 2025), mainly because we adopted a free bias parameter per redshift bin, as in real data analyses (Lepori et al. 2022), rather than parametric models matched to those used to obtain the fiducial values. While this widens the parameter space and weakens constraints, it is more robust against modeling systematics. Conversely, parametric assumptions can deliver tighter bounds, which may be useful for targeted astrophysical studies. For further comparison, Zazzera et al. (2025) assume perfect knowledge of galaxy bias (but include large-scale effects beyond Limber), while Calore et al. (2020) use a χ^2 analysis fixing all other parameters. Our results are closer to Vijaykumar et al. (2023), who find $\sim 20\%$ constraints from simulations with free bias parameters per bin, although using GW clustering alone in real space.

Dependence on the binning strategy We next examine how the choice of binning, both the number of bins and their definition in redshift and distance, affects the results. We tested four strategies:

- Fiducial cosmology, equally populated (default): sources are equally populated in redshift bins, then converted to distance with a fiducial cosmology. This would be “optimal” if the true cosmology were known.

- Wrong cosmology: as above, but adopting $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.32$, a model excluded by joint galaxy and GW correlations.
- Fiducial cosmology, equally spaced: as the default configuration, with the difference that the bins are equally spaced in redshift.
- Agnostic, equally spaced: bins equally spaced in both redshift and distance, maximizing robustness, but yielding less information since off-diagonal covariance terms are more important.

Figure 6 shows the relative uncertainty on H_0 (left) and Ω_m (center), and the figure of merit, $\text{FoM} = [\det(F^{-1})]^{-1/2}$ (Wang 2008) (left), as a function of the number of bins. Converting redshift bins to distance with a given cosmology extracts more information than agnostic binning, except at very high bin numbers where the two converge (30 bins). Using a wrong cosmology does not affect the statistical uncertainties, though a Fisher approach cannot capture systematic bias arising from such a choice.

Regarding the bin number, the uncertainty on H_0 converges to $\leq 1\%$ once more than 13 bins are used: $\sim 0.6\%$ for 20 bins and $\sim 0.5\%$ for 30, compared to $\sim 1.1\%$ for 13. The impact on Ω_m is stronger: with five years and 20 bins, we obtain $\sim 1.3\%$ precision versus $\sim 6\%$ with 13; with one year, we obtain $\sim 2.6\%$ on Ω_m and $\sim 1.3\%$ on H_0 . This suggests that increasing beyond the 13 bins used in Euclid forecasts would substantially benefit cross-correlation with GW surveys.

Crucially, bins must be equally populated: equally spaced schemes yield much weaker improvements since constraints on Ω_m come mainly from intermediate redshifts, and the equal number of sources keeps shot noise balanced across bins. The overall gain from 13 to 20 equally populated bins is also reflected in the FoM. The improvement comes from better disentangling the tracer bias from A_s and n_s . While the latter two are shared across bins, the bias is free in each bin. Thus, more bins provide independent slices where the bias can be constrained separately, reducing degeneracies with A_s and n_s as long as shot noise remains under control. In the limiting case of a single bin (no tomography), the bias and A_s would be fully degenerate, drastically weakening constraints on other parameters.

4.2. Spectroscopic versus photometric samples

In this section, we examine the impact of choosing a spectroscopic galaxy survey over a photometric one. Specifically, we compare the performance of the Euclid photometric sample to two spectroscopic surveys: the Euclid spectroscopic survey and the SKA phase II survey. In summary, we considered the following surveys:

- Euclid photometric survey (default): this is the default choice in this paper, described in Sect. 3.1.
- Euclid spectroscopic survey: to model this survey, we used results derived from the Euclid flagship simulation (Pocino et al. 2021; Castander et al. 2025), with fits provided in Jelic-Cizmek et al. (2024).

In particular, we refer to their Table 2 for the expected number density, which is shown in Fig. 7 as a dot-dashed line. This corresponds to a sample of $\sim 1.3 \times 10^7$ galaxies in the redshift range $z = 0.9$ and $z = 1.8$.

We assumed a relative redshift uncertainty of $\sigma_{\ln z} = 0.001$.

For modeling the bias, we used the cubic polynomial fits for the linear and magnification biases provided in Jelic-Cizmek et al. (2024).

- SKA Phase II: to assess the relevance of a larger spectroscopic survey, we considered the optimistic scenario of the SKA Phase II instrument. This survey covers 30 000 square

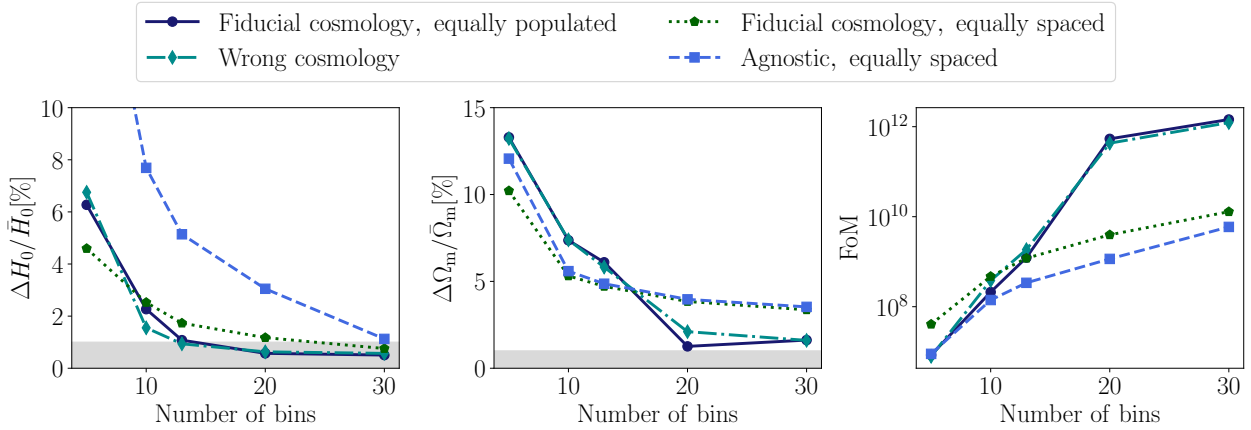


Fig. 6. Relative uncertainty on H_0 (left) and Ω_m (center), and $\text{FoM} = [\det(F^{-1})]^{-1/2}$ (right), as functions of the number of bins for different binning choices described in the text, using the Euclid photometric sample and five years of observations with the ET 2L + 2CE GW detector network. Shaded horizontal bands represent 1% constraints. A bar denotes the fiducial value.

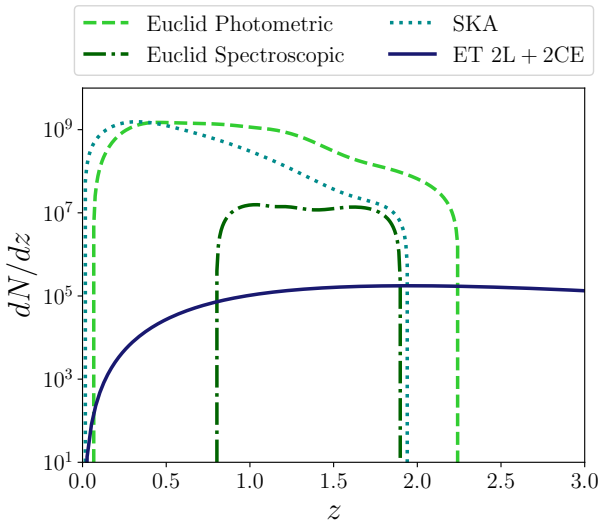


Fig. 7. Redshift coverage of the different surveys considered in Sect. 4.2, compared with the distribution of GW sources detected over five years of ET 2L + 2CE observations, converted from distance to redshift assuming the fiducial cosmology.

degrees and is expected to detect approximately 10^9 galaxies in the redshift range $0 \leq z \leq 2$. The survey is modeled following Bull (2016). We adopted the number density and magnification bias fitting formulae from Maartens et al. (2021), in particular their Eqs. (A19)–(A20).

Figure 7 shows the redshift distribution of the aforementioned surveys, compared to the GW catalog considered in this work. The redshift distribution of GW sources is obtained from the distance distribution assuming a fiducial cosmology. Figure 8 shows the relative uncertainty on H_0 (left) and Ω_m (center), and $\text{FoM} = [\det(F^{-1})]^{-1/2}$, as a function of the number of bins for the different surveys considered, combined with five years of observations of the ET 2L + 2CE configuration and the default binning scheme.

The Euclid spectroscopic sample provides constraints comparable to the Euclid photometric sample for a limited number of bins but shows saturation beyond ~ 12 bins. This saturation can be attributed to the larger shot noise in the galaxy autocorrelation term, as well as to the narrower redshift range spanned by the Euclid spectroscopic survey.

In the case of SKA, we instead find increased constraining power, with a promising $\sim 0.4\%$ constraint on H_0 and a $\sim 0.9\%$ constraint on Ω_m when combined with ET 2L + 2CE. This improvement is driven by the combination of a large number of sources, comparable to the Euclid photometric sample, and a superior redshift resolution. However, the gain does not continue indefinitely with increasing bin number: at high resolution, the constraining power is ultimately limited by the finite sky localization of GW sources and by the nonlinearity cut applied in the analysis. Adopting an equally spaced binning scheme might further improve the results.

We also note that in order to fully exploit the resolution of a spectroscopic survey, the correlation function may be a more promising summary statistic than the angular power spectrum. With the accuracy on distance estimates provided by 3G detectors, it would be interesting to compare the performance of different summary statistics in combination with a spectroscopic survey.

4.3. Comparison of different GW detector networks and geometries

In this section we compare the performance of different GW detector networks combined with the Euclid photometric sample, and examine the dependence on the observation time. We considered the following configurations, which are currently under active investigation (Abac et al. 2026):

- ET: ET alone, either as two L-shaped detectors with 15 km arms located at the Sardinia and Meuse-Rhine candidate sites (ET 2L), or as a single triangular detector in Sardinia with 10 km arms (ET Δ). The specific coordinates are given in Branchesi et al. (2023).
- ET + 1 CE: a network of 3G detectors made of ET (in either the 2L or triangular configuration) and a single CE detector in the US with 40 km arms.
- ET + 2 CE: a network of 3G detectors made of ET (in either the 2L or triangular configurations, with the former being our default choice) and two CE detectors in the US with 40 km and 20 km arms.

The CE detectors locations are given in Sect. 3.2.

Angular resolution The key difference between detector configurations is their angular resolution, which we modeled as described in Sect. 3.2, with detailed results reported in Appendix C. For ET alone, the median sky-localization error

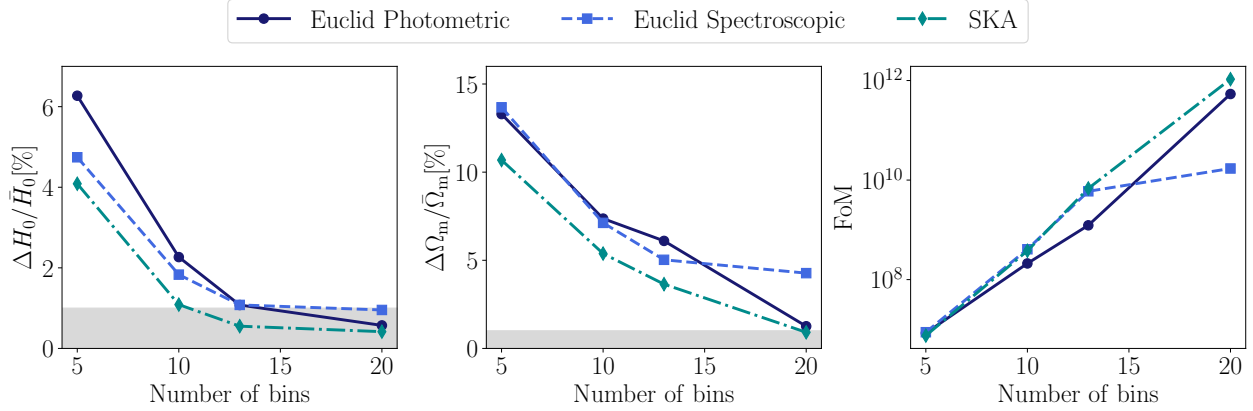


Fig. 8. Relative uncertainty on H_0 (left) and $\text{FoM} = [\det(F^{-1})]^{-1/2}$ (right) as a function of the number of bins for different photometric and spectroscopic surveys, using five years of observations with the ET 2L + 2CE GW detector network. Shaded horizontal bands represent 1% constraints. A bar denotes the fiducial value.

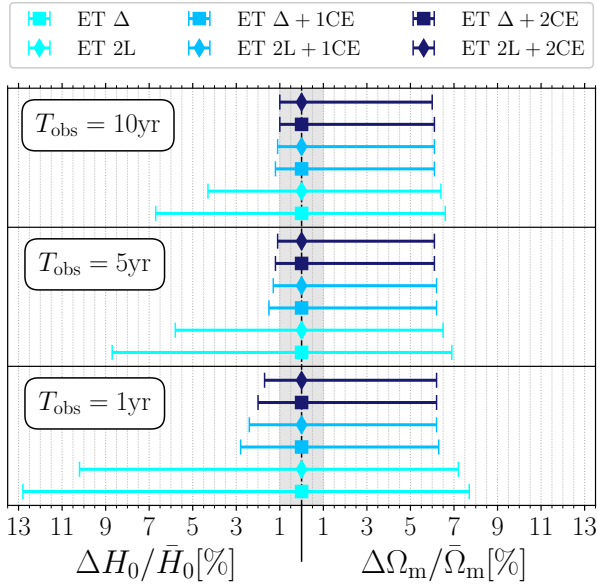


Fig. 9. Relative uncertainties on H_0 and Ω_m forecasted for the Euclid photometric sample combined with different possible GW detector networks, for different data-taking periods.

is substantially larger than in other setups, which strongly suppresses the signal and limits the usable multipole range. For ET Δ , the damping scale is $\ell_{\text{damp}} \lesssim 30$ in all but the first bin, making the angular power spectrum largely uninformative. The ET 2L configuration performs better, with $\ell_{\text{damp}} \sim 160$ in the first bin but < 80 in the others. A more stringent source-selection cut ($\Delta\Omega_{90\%} < 100, \text{deg}^2$) partially mitigates this. After removing the poorly localized events, which effectively lowers Q_{50} , the effective multipole range extends to $\ell_{\text{damp}} \sim 40\text{--}80$ for ET Δ and $\sim 50\text{--}100$ for ET 2L, with a corresponding improvement in distance uncertainties¹⁵. For ET-only networks, the effective multipole range is therefore determined by sky resolution rather than the nonlinearity cut. In contrast, for configurations including at least one CE detector, the transition between nonlinearity- and resolution-dominated regimes always occurs at $z \approx 0.5\text{--}0.7$.

¹⁵ For configurations with at least one CE, this additional selection cut has negligible impact on constraining power and is therefore not applied.

Detectability and constraints Across all configurations, we detect the cross-correlation after five years. The contrast between ET-only networks and those including CE is substantial: ET Δ reaches an S/N of ~ 1.3 and ET 2L an S/N of ~ 3.7 , whereas configurations with at least one CE achieve S/N values of $\sim 15\text{--}21$. The clustering of GW sources is not detected with $\text{S/N} > 1$ for ET-only configurations, nor with ET-CE networks after only one year of observations. Detailed values are reported in Appendix D.

Figure 9 summarizes the forecasted marginal uncertainties on H_0 and Ω_m for all configurations and observing times (one, five, and ten years). The corresponding numerical values are listed in Appendix D. With at least one CE, percent-level precision on H_0 becomes achievable in five years. When a single CE is added, the combination of auto- and cross-correlations yields a factor of ~ 10 improvement over either probe alone, compared to ~ 6 for ET 2L + 2 CE. Adding a second CE provides smaller gains than adding the first. With one year of observations, all CE-inclusive configurations already achieve 1.7% (ET 2L + 2 CE) to 3% precision (ET Δ + 1 CE).

For ET alone, the cross-correlation contributes relatively little information. The resulting constraints on H_0 are $\sim 70\%$ for ET Δ and $\sim 41\%$ for ET 2L, most of the information coming from the Euclid autocorrelation. Combining with the cross-term improves the constraint by factors of ~ 2 (ET Δ) and ~ 3 (ET 2L). The effectiveness of the cross-term clearly tracks the localization capabilities of the GW network: without triangulation from CE, the sky resolution of ET restricts the usable multipole range (Appendix C), especially for ET Δ . Tighter sky-localization cuts than those adopted here could be beneficial, at the cost of increased shot noise.

With ten years of observations and two CEs, the GW autocorrelation alone reaches an S/N of ~ 3.5 and constrains H_0 to $\sim 30\%$. In the same scenario, ET 2L alone achieves a precision of $\sim 4\%$ on H_0 .

5. Conclusions

Mapping the large-scale structure of the Universe with multiple tracers is becoming increasingly feasible. Ongoing galaxy surveys are expected to yield high-quality datasets over the coming years, containing measurements for up to billions of redshifts. Notably, the Euclid mission (Mellier et al. 2025) is currently in its data-taking phase, and exploring the potential of combining its datasets with other probes is of great interest. The

advent of 3G GW detectors promises to significantly enhance the detection rates of resolved sources, providing precise sky localizations and distance estimates. This progress opens the possibility of using GWs as tracers of the large-scale structure (LSS). GWs provide complementary tracers to galaxy surveys, as they probe the distance space rather than the redshift space. The conversion between distance and redshift is crucial for computing any summary statistics between the two, which causes the tomographic cross-correlation signal to peak along the distance-redshift relation (Fig. 2), thereby enabling constraints on cosmological parameters, as first suggested by Oguri (2016).

In this work, we investigated the potential of cross-correlating Euclid photometric data with 3G GW catalogs following this approach. Figure 4 illustrates the complementarity of these two probes. We show that their combination provides up to a tenfold improvement in cosmological constraints relative to each probe alone. We adopted the standard configuration from the Euclid preparation studies, e.g., Pocino et al. (2021), Lepori et al. (2022), Tanidis et al. (2024), which consists of 13 redshift bins. While this choice is conservative, we forecast a constraint of approximately 1% on H_0 after five years of observations (provided that at least one CE detector is operational alongside ET), marginalized over 33 nuisance parameters. Combining with constraints on Ω_m , we forecast a 0.7% constraint on the function $H(z)$ at the redshift $z_* = 0.25$, which minimizes the relative uncertainty. If we consider a more optimistic configuration of 20 redshift bins, the constraining power improves substantially. In this scenario, we can achieve a sub-percent ($\sim 0.6\%$) determination of H_0 and a $\sim 1.3\%$ constraint on Ω_m , the latter being an improvement of a factor of ~ 5 with respect to 13 bins. Such a precise measurement is only achieved with equally populated bins, which allows better suppression of shot noise. These results demonstrate that combining Euclid photometric data with GW measurements offers competitive constraints on the cosmic expansion history based on two datasets that trace the same portion of the Universe, without relying on external priors such as those from the cosmic microwave background, supernovae, or big bang nucleosynthesis.

From an astrophysical perspective, we find that the GW source clustering alone will be detectable by ET and CE detectors after five years of observations with a maximum S/N of ~ 1.8 . This would enable a measurement of the GW clustering bias, although our results are less optimistic than previous studies (Calore et al. 2020; Zazzera et al. 2025) due to our agnostic assumptions on its functional form. However, they do imply that the clustering bias can be measured with a fractional error of up to $\sim 40\%$ in redshift bins between $z \sim 0.5-1$ after five years (see Fig. 5), and up to $\sim 25\%$ after ten years, in a model-independent way.

Moreover, we discuss the possibility of using spectroscopic galaxy catalogs, in particular the Euclid spectroscopic sample and the SKA Phase II survey. The information from the Euclid spectroscopic survey saturates around 13 redshift bins, yielding constraints comparable to the Euclid photometric sample, while SKA Phase II could allow a sub-percent precision determination of the Hubble constant. However, other summary statistics, such as the correlation function, may be more appropriate to fully exploit the resolution of a spectroscopic survey. This would be an interesting topic to investigate in more detail.

Finally, we show that the full potential of the tomographic cross-correlation is realized only with a network of GW detectors capable of triangulating sources and achieving good source localization.

It is important to note that the constraints derived in this study can be considered optimistic, as they are based on a Fisher

matrix analysis. At the same time, we made several conservative assumptions: we excluded low multipole values consistently with the Limber approximation, although this restriction could be relaxed by including large-scale relativistic effects in the analysis; we also used a free parameter per bin for the clustering bias, without imposing priors such as continuity of the function. In addition, we restricted the analysis to linear or mildly nonlinear scales. However, these choices do not alter the conclusions regarding the complementarity between autocorrelation and cross-correlation terms. Looking ahead, real data applications will need to address systematic uncertainties, such as the dependence on GW population properties, the impact of selection cuts, and the proper inclusion of nuisance parameters and measurement errors beyond the approximations adopted here. These issues will be crucial for fully realizing the potential of combining GWs and galaxy surveys in cosmological studies.

Finally, throughout this work we assumed a Λ CDM framework, but testing extensions is equally important. In particular, when using GWs as probes, one must account for modified GW propagation, that is, a deviation of the GW luminosity distance from the standard relation in models with additional degrees of freedom (Belgacem et al. 2018a,b, 2019). As with H_0 , modified GW propagation can be constrained using two-point correlations (Mukherjee et al. 2021b; Cañas Herrera et al. 2021; Scelfo et al. 2023; Afroz & Mukherjee 2024). This should be accompanied by scalar-sector modifications probed through galaxy surveys, so that joint GW-galaxy correlations can simultaneously probe tensor and scalar perturbations (Scelfo et al. 2023).

Overall, two-point statistics provide a promising probe for determining the expansion history of the Universe and testing gravity at cosmological scales.

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Appendix A: Relativistic number counts

In this appendix, we provide details on the calculation of the signal in the Limber approximation.

We refer to Appendix A of [Scelfo et al. \(2018\)](#) for the full expressions of relativistic number counts in redshift space (which follows the notation of [Bonvin & Durrer \(2011\)](#), [Di Dio et al. \(2013\)](#)), and to Appendix B of [Fonseca et al. \(2023\)](#) for luminosity distance space. Note that [Fonseca et al. \(2023\)](#) report directly the transfer functions, see 5, using conformal time rather than redshift as integration variable. The expressions for the perturbations in luminosity distance space as functions of redshift can be read from those with the appropriate substitution of the jacobian from distance to conformal time with the one from distance to redshift. Explicitly, the density, RSD/LSD, and lensing contributions that we use in this work, read

$$\Delta_{\ell}^{X,\text{den}}(k, z) = b_X(z) \mathcal{T}_m(z, k) j_{\ell}(kr(z)), \quad (\text{A.1})$$

$$\Delta_{\ell}^{X,\text{len}}(k, z) = \frac{\ell(\ell+1)}{r(z)} \int_0^{r(z)} dr j_{\ell}(kr) \quad (\text{A.2})$$

$$\times A_L^X(r, r(z)) \mathcal{T}_{\Phi+\Psi}(z, k) \quad (\text{A.3})$$

$$\Delta_{\ell}^{X,\text{RSD/LSD}}(k, z) = A_{\text{LSD/RSD}}^X(z) k \mathcal{T}_V(z, k) j_{\ell}''(kr(z)) \quad (\text{A.4})$$

where $r(z) = c \int_0^z dz' H^{-1}(z')$ is the comoving distance. The lensing and RSD/LSD coefficients A_L^X , $A_{\text{RSD/LSD}}^X$ are given for the two tracers by

$$A_L^{\text{gal}}(r, \bar{r}) = \frac{1}{2}(5s^g - 2) \frac{\bar{r} - r}{r}, \quad (\text{A.5})$$

$$A_L^{\text{GW}}(r, \bar{r}) = \frac{1}{2} \left[\left(\frac{\bar{r} - r}{r} \right) (\beta - 2) + \frac{1}{1 + \bar{r}\mathcal{H}} \right], \quad (\text{A.6})$$

$$A_{\text{RSD}}^{\text{gal}} = \frac{1}{\mathcal{H}}, \quad (\text{A.7})$$

$$A_{\text{LSD}}^{\text{GW}} = \frac{2\gamma}{\mathcal{H}}, \quad (\text{A.8})$$

with

$$\gamma = \bar{r}\mathcal{H}/(1 + \bar{r}\mathcal{H}), \quad (\text{A.9})$$

$$\beta = 5s^{\text{GW}} - 1 + \gamma \left[\frac{2}{\bar{r}\mathcal{H}} + \gamma \left(\frac{\dot{H}}{H^2} + 1 - \frac{1}{\bar{r}\mathcal{H}} \right) - 1 - b_e^{\text{GW}} \right], \quad (\text{A.10})$$

where $\mathcal{H}(z) = H(z)/(1+z)$, b_e^{GW} and s^{GW} are respectively the evolution and magnification bias for GWs, and s^g is the magnification bias for galaxies. Finally, we denoted by $\mathcal{T}_{\Phi+\Psi}(z, k)$ and $\mathcal{T}_m(z, k)$ the transfer functions for the potentials Φ , Ψ and for matter respectively. Those are related to the initial curvature perturbation by

$$(\Phi + \Psi)(k, z) = \mathcal{T}_{\Phi+\Psi}(z, k) \Psi_{\text{in}}(k), \quad (\text{A.11})$$

$$\delta_m(k, z) = \mathcal{T}_m(z, k) \Psi_{\text{in}}(k) \quad (\text{A.12})$$

where $\Psi_{\text{in}}(k)$ has power spectrum $k^3 \langle \Psi_{\text{in}}(\mathbf{k}) \Psi_{\text{in}}^*(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \mathcal{P}(k)$ with $\mathcal{P}(k) = k^3 P(k) = A_s (k/k_*)^{n_s-1}$.

We use the above expressions to compute the power spectra in Eq. 4 using the Limber approximation ([Limber 1954](#)). In this case, one can evaluate integrals of the form 4 by making use of the asymptotic formula ([LoVerde & Afshordi 2008](#)):

$$\frac{2}{\pi} \int dk k^2 j_{\ell}(kr) j_{\ell}(kr') \simeq \frac{1}{r^2} \delta(r - r') + \mathcal{O}(\ell + 1/2)^{-2}, \quad (\text{A.13})$$

and evaluating the power spectrum at the wavenumber which projects on the angular scale ℓ , i.e.:

$$P(k, z) = P \left[k = \frac{\ell + 1/2}{r(z)}, z \right]. \quad (\text{A.14})$$

In GR one has $\Phi = \Psi$, and the potentials are related to the matter transfer function via the Poisson equation:

$$\mathcal{T}_{\Phi+\Psi} = -\frac{3H_0^2 \Omega_m}{k^2} (1+z) \mathcal{T}_m. \quad (\text{A.15})$$

For the density and lensing terms, we obtain (see also [Lepori et al. \(2022\)](#))

$$C_{\ell}^{\text{denX,denY}}(x_i, x_j) = \int_0^{\infty} \frac{cdz}{H(z)r^2(z)} \tilde{W}^X(z, x_i) \tilde{W}^Y(z, x_j) P \left(\frac{\ell + 1/2}{r(z)}, z \right), \quad (\text{A.16})$$

with $\tilde{W}^X(z, x_j) = J_X(z) b_X(z) w^X(z, x_i) H(z)/c$. Then,

$$\begin{aligned} C_{\ell}^{\text{lenX,lenY}}(x_i, x_j) &= \frac{\ell^2(\ell+1)^2}{(\ell+1/2)^4} \left(\frac{3H_0^2 \Omega_m}{c^2} \right)^2 \int_0^{\infty} \frac{dz_1}{r(z_1)} J_X(z_1) \\ &\times w^X(z_1, x_i) \int_0^{z_1} \frac{dz_2}{r(z_2)} J_Y(z_2) w^Y(z_2, x_j) \\ &\times \int_0^{\min(z_1, z_2)} \frac{cdx}{H(x)} r(x)^2 A_L^X(r(x), r(z_1)) \\ &\times A_L^Y(r(x), r(z_2)) (1+x)^2 P \left(\frac{\ell + 1/2}{r(x)}, x \right), \end{aligned} \quad (\text{A.17})$$

and

$$\begin{aligned} C_{\ell}^{\text{lenX,denY}}(x_i, x_j) &= -\frac{\ell(\ell+1)}{(\ell+1/2)^2} \frac{3H_0^2 \Omega_m}{c^2} \int_0^{\infty} \frac{dz_1}{r(z_1)} J_X(z_1) \\ &\times w^X(z_1, x_i) \int_0^{z_1} dz_2 J_Y(z_2) w^Y(z_2, x_j) b_X(z_2) \\ &\times (1+z_2) A_L^Y(r(z_2), r(z_1)) P \left(\frac{\ell + 1/2}{r(z_2)}, z_2 \right). \end{aligned} \quad (\text{A.18})$$

In the above equations, $J_X(z)$ denotes a jacobian factor that depends on whether the tracer X is in redshift (galaxies) or distance (GWs) space, with values $J_g(z) = 1$, $J_{\text{GW}}(z) = dd_L/dz$.

For the RSD/LSD term, the transfer function of the velocity is related to the matter one by the continuity equation

$$k \mathcal{T}_V(z, k) = -\frac{H(z)}{1+z} f(z) \mathcal{T}_m(z), \quad (\text{A.19})$$

with $f(z) \equiv d \log \mathcal{T}_m(z) / d \log a$ being the growth function. One can then use a recurrence relation for the spherical Bessel functions to express j_{ℓ}'' as a combination of j_{ℓ} which receives contributions from $\ell = -2, -1, 0, 1, 2$ ([Tanidis & Camera 2019](#); [Tanidis et al. 2024](#)). [Tanidis & Camera \(2019\)](#) showed that, for tracers in redshift space, the result can be expressed in the form of Eq. A.16, with a suitable definition of the window function, namely

$$\begin{aligned} \tilde{W}_{\ell, \text{RSD}}^X(z, z_j) &= \sum_{k=-1}^1 L_k(\ell) w^X \left[\frac{2\ell+1+4k}{2\ell+1} r(z), z_j \right] \\ &\times f \left[\frac{2\ell+1+4k}{2\ell+1} r(z) \right], \end{aligned} \quad (\text{A.20})$$

with

$$\begin{aligned} L_0(\ell) &= \frac{2\ell^2 + 2\ell - 1}{(2\ell - 1)(2\ell + 3)}, \\ L_1(\ell) &= \frac{(\ell + 2)(\ell + 1)}{(2\ell + 3)\sqrt{(2\ell + 1)(2\ell + 5)}}, \\ L_{-1}(\ell) &= -\frac{(\ell + 2)(\ell + 1)}{(2\ell - 1)\sqrt{(2\ell - 3)(2\ell + 1)}}. \end{aligned} \quad (\text{A.21})$$

For tracers in distance space, the only difference is that the LSD term has a different prefactor, cf. Eq. A.8, and carries an extra jacobian factor from distance to redshift space. As a result, the expression for LSD is

$$\begin{aligned} \tilde{W}_{\ell, \text{LSD}}^X(z, d_{L,j}) &= \sum_{k=-1}^1 L_k(\ell) w^X \\ &\times \left[\frac{2\ell + 1 + 4k}{2\ell + 1} r(z), d_{L,j} \right] \left(2f\gamma \frac{dd_L}{dz} \right) \Big|_{\frac{2\ell+1+4k}{2\ell+1} r(z)}, \end{aligned} \quad (\text{A.22})$$

with γ given in Eq. A.9. Finally, the growth rate is given in Λ CDM by $f(z) = \Omega_m(z)^{6/11}$ (Wang & Steinhardt 1998). The expressions for the RSD/LSD terms and their cross-terms with density and lensing contributions are then obtained by replacing the appropriate window functions in Eqs. A.16-A.18.

Appendix B: Details on population modeling

In this appendix we provide details on the modeling of the galaxy and gravitational-wave catalogs and the respective tracer biases.

Euclid photometric catalog bias. The linear and magnification bias for the Euclid photometric survey are:

$$\begin{aligned} b_g(z) &= b_{g,0} + b_{g,1}z + b_{g,2}z^2 + b_{g,3}z^3, \\ b_{g,0} &= 0.5125, \quad b_{g,1} = 1.377, \quad b_{g,2} = 0.222, \quad b_{g,3} = -0.249, \\ s_g(z) &= s_{g,0} + s_{g,1}z + s_{g,2}z^2 + s_{g,3}z^3, \\ s_{g,0} &= 0.0842, \quad s_{g,1} = 0.0532, \quad s_{g,2} = 0.298, \quad s_{g,3} = -0.0113. \end{aligned} \quad (\text{B.1})$$

Observed distributions of GW sources. The fits to the observed distribution of GW sources (see Eq. 26) is provided in Tab. B.1.

GW bias. The perturbation properties of GW sources are modeled as follows.

- Clustering bias b_{GW} :

$$b_{\text{GW}} = A_{\text{GW}}(1 + z)^\gamma, \quad (\text{B.2})$$

with fiducial values $A_{\text{GW}} = 1.20$, $\gamma = 0.59$ (Peron et al. 2024).

- Evolution and magnification bias (Zazzera et al. 2024):

$$\begin{aligned} b_{e,\text{GW}}(z) &= b_{e,\text{GW},0} + b_{e,\text{GW},1}z + b_{e,\text{GW},2}z^2 + b_{e,\text{GW},3}z^3, \\ s_{\text{GW}}(z) &= s_{\text{GW},0} + s_{\text{GW},1}z + s_{\text{GW},2}z^2 + s_{\text{GW},3}z^3. \end{aligned} \quad (\text{B.3})$$

The coefficients depend on the configuration, and are reported in Tab. B.2 for an S/N threshold of 12 for GW sources.

Appendix C: Effect of limited sky resolution

We detail here the impact of GW sky-localization accuracy on the estimation of the angular multipoles of galaxies.

We start from the probability distribution p for the reconstructed position of a GW event, assumed axis-symmetric around the true source and peaked at that location. We model it as a Gaussian in $\mu = \cos \theta$ (where θ is the polar angle):

$$p(\theta, \phi) = \frac{2}{(2\pi)^{3/2}\sigma_\mu} e^{-(\cos \theta - 1)^2/(2\sigma_\mu^2)}, \quad (\text{C.1})$$

with flat distribution in the azimuthal angle ϕ . Normalization ensures that the integral over θ and ϕ is unitary, assuming $5\sigma_\mu < 2$. The parameter σ_μ is related to the 1σ solid angle $\Delta\Omega_{1\sigma} = 2\pi\sigma_\mu$.

We stress that the modeling of the sky-localization kernel necessarily involves an approximation. In GW observations, one reconstructs a posterior distribution over directions from the strain data, mainly driven by triangulation. Within a Fisher-matrix treatment, this posterior is approximated as Gaussian in a chosen angular variable, which is not unique. One may equivalently adopt Gaussianity in θ or in $\mu = \cos \theta$. Here we adopt a Gaussian in μ , which is natural given the uniform distribution of sources on the sphere and allows for an analytic treatment of the angular integrals. We emphasize that alternative prescriptions would be equally legitimate: all such constructions rely on approximations and correspond to different choices of which variable is treated as Gaussian. Our choice aims to provide a transparent, consistent, and reproducible derivation of the damping effect.

The observed field $\tilde{\delta}(\theta, \phi)$ in the sky is the convolution of the true field with p , rotated to align with (θ, ϕ) :

$$\tilde{\delta}(\theta, \phi) = \int \delta(\theta', \phi' | \theta, \phi) p(\theta', \phi') d\Omega. \quad (\text{C.2})$$

Expanding both δ and p in spherical harmonics and using Wigner-D matrices yields

$$\tilde{\delta}(\theta, \phi) = \sum_{\ell m m_1} (-1)^m \delta_{\ell m_1} p_{\ell m} [D_{m_1 - m}^{(\ell)}]^*. \quad (\text{C.3})$$

The axial symmetry of p implies $p_{\ell m} = p_\ell \delta_{m,0}$, giving

$$\tilde{\delta}_{\ell m} = \tilde{p}_\ell \delta_{\ell m}, \quad \tilde{p}_\ell = \sqrt{\frac{4\pi}{2\ell + 1}} p_\ell. \quad (\text{C.4})$$

Thus $\tilde{p}_\ell$ acts as a damping function, which explicit form is given by

$$\tilde{p}_\ell = \frac{2}{\sqrt{2\pi}\sigma_\mu} \int_{-1}^1 L_\ell(\mu) e^{-(\mu-1)^2/(2\sigma_\mu^2)} d\mu, \quad (\text{C.5})$$

where L_ℓ are Legendre polynomials. Using their series expansion gives

$$\tilde{p}_\ell = \frac{2}{\sqrt{2\pi}\sigma_\mu} \sum_{k=0}^{\ell} \frac{(\ell + k)!}{(k!)^2(\ell - k)!} \frac{(-1)^k}{2^k} \sigma_\mu^{k+1} I_k, \quad (\text{C.6})$$

with $I_0 = \sqrt{\pi/2}$, $I_1 = 1$, and $I_k = I_0(k-1)!!$ for even k , $I_k = I_1(k-1)!!$ for odd k . For $\sigma_\mu \ll 1$, higher-order terms are suppressed, and at low ℓ :

$$\tilde{p}_\ell \simeq 1 - \ell(\ell + 1) \frac{\sigma_\mu}{\sqrt{2\pi}} \simeq e^{-\ell(\ell+1)/\ell_{\text{damp}}^2}, \quad (\text{C.7})$$

Table B.1. Parametric fit to the observed distribution of GW sources.

Detector network	$N_{\text{obs}}^{\text{GW}}$	Best-fit parameters			
		A [Gpc $^{-1}$]	$d_{L,0}$ [Gpc]	α	β
ET Δ 10 km	5.3×10^4	330.897	6.352	1.697	0.922
ET 2L 45° 15 km	6.1×10^4	155.232	3.466	2.152	0.765
ET Δ 10 km (localization cut)	1.05×10^4	99.0	6.89	1.25	0.97
ET 2L 45° 15 km (localization cut)	1.7×10^4	61.34	1.97	1.93	0.7
ET Δ 10 km + 1 CE	1.0×10^5	69.695	1.79	2.539	0.658
ET 2L 45° 15 km + 1 CE	1.03×10^5	626.25	1.533	2.619	0.658
ET Δ 10 km + 2 CE	1.1×10^5	40.143	1.364	2.693	0.625
ET 2L 45° 15 km + 2 CE	1.12×10^5	412.11	1.244	2.729	0.614

Notes. Coefficients of the parametric fit Eq. 26 a function of the luminosity distance, for the different observatories considered in this work, and one year of observation.

Table B.2. Evolution and magnification bias fits.

Detector network	$b_{\text{e,GW}}$				$s_{\text{GW}} \times 10^3$			
	$b_{\text{e,GW},0}$	$b_{\text{e,GW},1}$	$b_{\text{e,GW},2}$	$b_{\text{e,GW},3}$	$s_{\text{GW},0}$	$s_{\text{GW},1}$	$s_{\text{GW},2}$	$s_{\text{GW},3}$
ET alone	-1.45	-1.39	1.98	-3.63×10^{-1}	-8.3	45.4	13.6	-2.04
ET + CE	-1.45	-1.39	1.98	-3.63×10^{-1}	-5.59	29.2	3.44	2.58

Notes. Coefficients of the evolution and magnification bias fits in Eq. B.3 for GW sources, from Zazzera et al. (2024)

with

$$\ell_{\text{damp}}^2 = \frac{\sqrt{2\pi}}{\sigma_\mu} = \frac{(2\pi)^{3/2}}{\Delta\Omega_{1\sigma}}. \quad (\text{C.8})$$

The above equation shows that the damping multipole ℓ_{damp} is related to the 1-sigma error region in the localization of GW events.

Tables C.1–C.2 list the multipole and wavenumber cuts adopted in this work, including $\ell_{\text{max}}^{\text{NL}}$ from nonlinearity, $\ell_{\text{max}}^{\text{LOC}}$ from GW angular resolution, and ℓ_{damp} from finite sky localization.

Figure C.1 shows the localization areas, together with their 50th and 99th percentiles as function of distance, the source distributions and bins, for the different GW detector configurations considered in the paper.

Appendix D: Tables

We report here the values of the relative uncertainties obtained from various cases discussed in the main body of the paper.

Table D.1 contains values of the S/N of the GW auto-correlation term as well as of the cross-correlation of GW catalogs with the Euclid photometric sample. Table D.2 contains constraints corresponding to the main configuration studied in the paper, namely the Euclid photometric survey in combination with 5 years of observations of different GW detector networks (see table), with 13 equally populated bins. For comparison, we also report values for 1 and 10 years of observations for the configuration ET 2L + 2 CE in Tab. D.3. Finally, Tables D.4–D.6 contain the relative 1σ uncertainties obtained considering the auto and cross spectra of GWs and the Euclid photometric surveys separately.

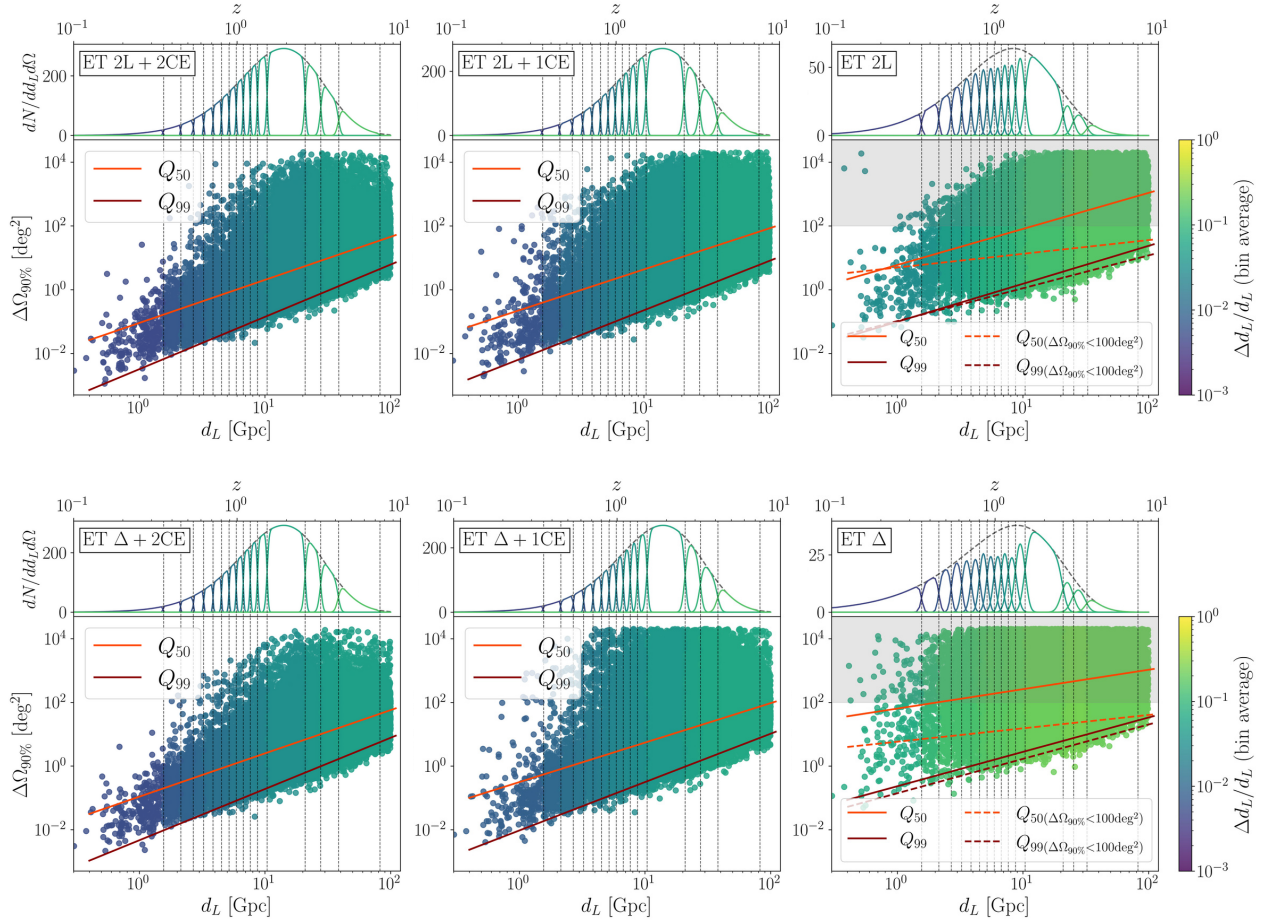


Fig. C.1. As in the right panel of Fig. 3, for different GW detector networks considered in this work. Shaded areas in the left column correspond to sources with 90% sky localization $\Delta\Omega_{90\%} < 100 \text{ deg}^2$, which we exclude from the analysis.

Table C.1. Bin edges for ET Δ .

Bin Edges	$k_{\text{max}}^{\text{NL}}$		ET Δ		ET Δ (loc. cut)		ET Δ + 1 CE		ET Δ + 2CE	
	$\ell_{\text{max}}^{\text{NL}}$	$\ell_{\text{max}}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\text{max}}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\text{max}}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\text{max}}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\text{max}}^{\text{LOC}}$
0 – 0.3	0.117	130	50	634	108	726	677	> 1000	1040	> 1000
0.3 – 0.38	0.122	171	31	294	85	304	401	> 1000	573	> 1000
0.38 – 0.47	0.128	214	20	259	65	305	306	950	416	> 1000
0.47 – 0.55	0.135	263	19	223	74	241	286	740	395	897
0.55 – 0.63	0.142	315	22	171	68	238	235	628	325	836
0.63 – 0.72	0.15	373	20	172	62	197	221	572	298	755
0.72 – 0.8	0.158	431	17	159	60	195	195	507	264	657
0.8 – 0.9	0.166	495	17	150	61	180	167	459	229	578
0.9 – 1	0.176	568	16	130	56	172	155	415	210	535
1 – 1.1	0.187	654	15	117	56	155	140	378	192	469
1.1 – 1.22	0.2	750	14	113	54	143	129	344	174	432
1.22 – 1.41	0.217	880	14	99	52	128	114	300	154	378
1.41 – 2.5	0.241	1068	12	80	48	107	83	229	110	285
2.5 – 3.2	/	/	10	69	46	98	62	161	80	195
3.2 – 4.2	/	/	10	62	46	86	51	131	66	158
4.2 – 8	/	/	8	49	43	74	38	102	49	122

Notes. Bin edges and corresponding values for the maximum wave number ($k_{\text{max}}^{\text{NL}}$, in Mpc) and multipole ($\ell_{\text{max}}^{\text{NL}}$) adopted, based on restricting the analysis to linear/mildly nonlinear scales, and values of the best resolved angular scale $\ell_{\text{max}}^{\text{LOC}}$ and damping scale ℓ_{damp} , coming from the finite sky resolution of GW events. This table refers to the ET Δ configuration.

Table C.2. Bin edges for the ET 2L configuration.

Bin Edges			ET 2L		ET 2L (loc. cut)		ET 2L + 1 CE		ET 2L + 2CE	
	$k_{\max}^{\text{NL}}$	$\ell_{\max}^{\text{NL}}$	ℓ_{damp}	$\ell_{\max}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\max}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\max}^{\text{LOC}}$	ℓ_{damp}	$\ell_{\max}^{\text{LOC}}$
0 – 0.3	0.117	130	165	718	179	732	877	> 1000	1200	> 1000
0.3 – 0.38	0.122	171	83	395	104	400	479	> 1000	634	> 1000
0.38 – 0.47	0.128	214	63	399	91	413	345	> 1000	471	> 1000
0.47 – 0.55	0.135	263	52	287	73	308	327	822	445	> 1000
0.55 – 0.63	0.142	315	51	207	76	224	270	751	374	935
0.63 – 0.72	0.15	373	51	224	73	229	256	697	345	861
0.72 – 0.8	0.158	431	40	241	68	255	223	625	299	756
0.8 – 0.9	0.166	495	39	196	63	209	192	551	268	679
0.9 – 1	0.176	568	37	184	57	215	172	487	239	612
1 – 1.1	0.187	654	33	161	57	182	159	462	219	546
1.1 – 1.22	0.2	750	29	141	53	162	149	414	197	497
1.22 – 1.41	0.217	880	26	135	53	169	127	355	175	441
1.41 – 2.5	0.241	1068	20	105	49	135	94	268	125	324
2.5 – 3.2	/	/	15	87	49	116	70	190	92	226
3.2 – 4.2	/	/	13	78	48	110	58	154	75	180
4.2 – 8	/	/	9	58	46	89	43	117	56	138

Notes. As in Tab. C.1, for the ET 2L configuration.

Table D.1. S/N.

Detector network	S/N GW-GW	S/N Cross-correlation
ET Δ	0.06	1.28
ET 2L	0.2	3.7
ET Δ + 1 CE	1.2	15.4
ET 2L + 1 CE	1.4	16.9
ET Δ + 2 CE	1.7	19.5
ET 2L + 2 CE ($T_{\text{obs}} = 1\text{yr}$)	0.4	9.6
ET 2L + 2 CE ($T_{\text{obs}} = 5\text{yr}$)	1.8	21.2
ET 2L + 2 CE ($T_{\text{obs}} = 10\text{yr}$)	3.5	29.6

Notes. S/N of the auto-correlation of GW sources and of their cross-correlation with the Euclid photometric sample, for different GW detector configurations.

Table D.2. Relative uncertainties.

Parameter	ET Δ	ET 2L	ET Δ + 1 CE	ET 2L + 1 CE	ET Δ + 2 CE	ET 2L + 2 CE
H_0	8.7	5.8	1.5	1.3	1.2	1.1
Ω_m	6.9	6.5	6.2	6.2	6.1	6.1
$\Omega_b h^2$	29.6	20.0	7.6	7.3	7.0	6.9
A_s	12.4	10.2	8.3	8.3	8.2	8.2
n_s	7.1	5.7	4.3	4.3	4.2	4.2
b_g bin 1-5	6.1	5.7	5.3	5.3	5.3	5.3
b_g bin 6-10	5.9	5.5	5.3	5.3	5.3	5.3
b_g bin 11-13	5.9	5.5	5.3	5.3	5.3	5.3
b_{GW} bin 1-5	> 100	> 100	60	60	40	40
b_{GW} bin 6-10	$\gg$ 100	$\gg$ 100	55	50	35	35
b_{GW} bin 11-16	$\gg$ 100	$\gg$ 100	> 100	> 100	> 100	90

Notes. Relative 1σ uncertainty on all the parameters of the analysis, for the Euclid photometric survey in combination with 5 years of observations of different GW detector networks with 13 equally populated bins.

Table D.3. Relative uncertainties with different observing times.

Parameter	$T_{\text{obs}} = 1 \text{ years}$				$T_{\text{obs}} = 10 \text{ years}$			
	Total	Galaxy	GW	Cross	Total	Galaxy	GW	Cross
H_0	1.7	15.2	> 100	16.4	0.9	15.2	27	5.1
Ω_m	6.2	8.2	$\gg 100$	> 100	6.0	8.2	> 100	44.6
$\Omega_b h^2$	8.3	23.3	$\gg 100$	$\gg 100$	6.7	23.3	$\gg 100$	> 100
A_s	8.3	18.1	$\gg 100$	$\gg 100$	8.2	18.1	$\gg 100$	> 100
n_s	4.3	10.8	$\gg 100$	> 100	4.2	10.8	$\gg 100$	43
b_g bin 1-5	5.4	6.8	/	$\gg 100$	5.3	6.8	/	> 100
b_g bin 6-10	5.4	6.9	/	$\gg 100$	5.3	6.9	/	90
b_g bin 11-13	5.4	6.9	/	$\gg 100$	5.3	6.9	/	> 100
b_{GW} bin 1-5	> 100	/	$\gg 100$	$\gg 100$	30	/	$\gg 100$	> 100
b_{GW} bin 6-10	80	/	$\gg 100$	$\gg 100$	25	/	$\gg 100$	80
b_{GW} bin 11-16	> 100	/	$\gg 100$	$\gg 100$	40	/	$\gg 100$	> 100

Notes. Relative 1σ uncertainty on all the parameters of the analysis, for the Euclid photometric survey in combination with 1 and 10 years of observations of ET 2L 15 km + 2 CE with 13 equally populated bins.

Table D.4. Relative uncertainties for ET alone.

Parameter	ET Δ				ET 2L			
	Total	Galaxy	GW	Cross	Total	Galaxy	GW	Cross
H_0	8.7	15.2	$\gg 100$	73	5.8	15.2	$\gg 100$	41
Ω_m	6.9	8.2	$\gg 100$	$\gg 100$	6.5	8.2	$\gg 100$	$\gg 100$
$\Omega_b h^2$	29.6	23.3	$\gg 100$	$\gg 100$	20.0	23.3	$\gg 100$	$\gg 100$
A_s	12.4	18.1	$\gg 100$	$\gg 100$	10.2	18.1	$\gg 100$	$\gg 100$
n_s	7.1	10.8	$\gg 100$	$\gg 100$	5.7	10.8	$\gg 100$	$\gg 100$
b_g bin 1-5	6.1	6.8	/	$\gg 100$	5.7	6.8	/	$\gg 100$
b_g bin 6-10	5.9	6.9	/	$\gg 100$	5.5	6.9	/	$\gg 100$
b_g bin 11-13	5.9	6.9	/	$\gg 100$	5.5	6.9	/	$\gg 100$
b_{GW} bin 1-5	> 100	/	$\gg 100$	$\gg 100$	> 100	/	$\gg 100$	$\gg 100$
b_{GW} bin 6-10	$\gg 100$	/	$\gg 100$	$\gg 100$	$\gg 100$	/	$\gg 100$	$\gg 100$
b_{GW} bin 11-16	$\gg 100$	/	$\gg 100$	$\gg 100$	$\gg 100$	/	$\gg 100$	$\gg 100$

Notes. Separate and total contributions of the galaxy-galaxy, GW-galaxy and GW-GW auto and cross-correlation terms to the constraints on cosmological and bias parameters, for the ET detectors alone in combination with the Euclid photometric survey, with 13 equally populated bins.

Table D.5. Relative uncertainties for ET in combination with one CE.

Parameter	ET Δ + 1 CE				ET 2L + 1 CE			
	Total	Galaxy	GW	Cross	Total	Galaxy	GW	Cross
H_0	1.5	15.2	95	12.1	1.3	15.2	85	11
Ω_m	6.2	8.2	$\gg 100$	91	6.2	8.2	$\gg 100$	83.2
$\Omega_b h^2$	7.6	23.3	$\gg 100$	> 100	7.3	23.3	$\gg 100$	> 100
A_s	8.3	18.1	$\gg 100$	> 100	8.3	18.1	$\gg 100$	> 100
n_s	4.3	10.8	$\gg 100$	95	4.3	10.8	$\gg 100$	87
b_g bin 1-5	5.3	6.8	/	$\gg 100$	5.3	6.8	/	> 100
b_g bin 6-10	5.3	6.9	/	$\gg 100$	5.3	6.9	/	> 100
b_g bin 11-13	5.3	6.9	/	$\gg 100$	5.3	6.9	/	> 100
b_{GW} bin 1-5	60	/	$\gg 100$	$\gg 100$	60	/	$\gg 100$	> 100
b_{GW} bin 6-10	55	/	$\gg 100$	$\gg 100$	50	/	$\gg 100$	> 100
b_{GW} bin 11-16	> 100	/	$\gg 100$	$\gg 100$	> 100	/	$\gg 100$	> 100

Notes. As in Tab. D.4, for ET in combination with one CE.

Table D.6. Relative uncertainties for ET in combination with two CE.

Parameter	ET Δ + 2 CE				ET 2L + 2 CE			
	Total	Galaxy	GW	Cross	Total	Galaxy	GW	Cross
H_0	1.2	15.2	64	8.2	1.1	15.2	54	7.2
Ω_m	6.1	8.2	$\gg 100$	64.5	6.1	8.2	$\gg 100$	54
$\Omega_b h^2$	7.0	23.3	$\gg 100$	> 100	6.9	23.3	$\gg 100$	> 100
A_s	8.2	18.1	$\gg 100$	> 100	8.2	18.1	$\gg 100$	> 100
n_s	4.2	10.8	$\gg 100$	72	4.2	10.8	$\gg 100$	64
b_g bin 1-5	5.3	6.8	/	> 100	5.3	6.8	/	> 100
b_g bin 6-10	5.3	6.9	/	> 100	5.3	6.9	/	> 100
b_g bin 11-13	5.3	6.9	/	> 100	5.3	6.9	/	> 100
b_{GW} bin 1-5	40	/	$\gg 100$	> 100	40	/	$\gg 100$	> 100
b_{GW} bin 6-10	35	/	$\gg 100$	> 100	35	/	$\gg 100$	> 100
b_{GW} bin 11-16	> 100	/	$\gg 100$	> 100	90	/	$\gg 100$	> 100

Notes. As in Tab. D.4, for ET in combination with two CE.