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SOME PROPERTIES OF COMPOSED IDENTIFICATION–AND–CONTROL MAPS *

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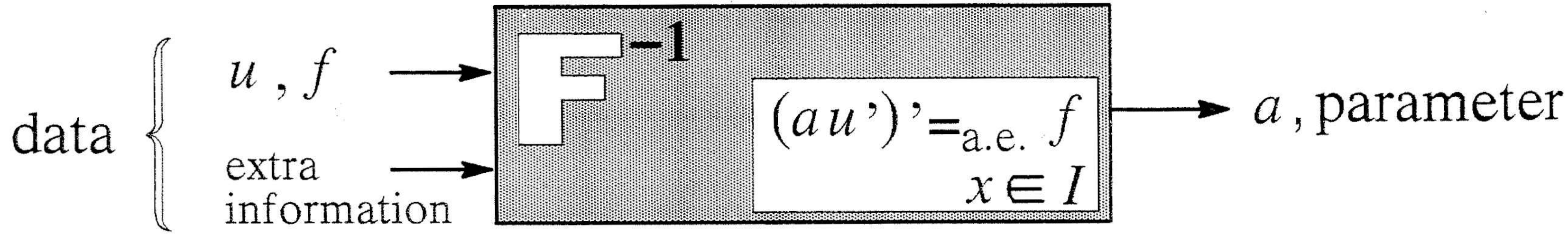
THE PARADIGM

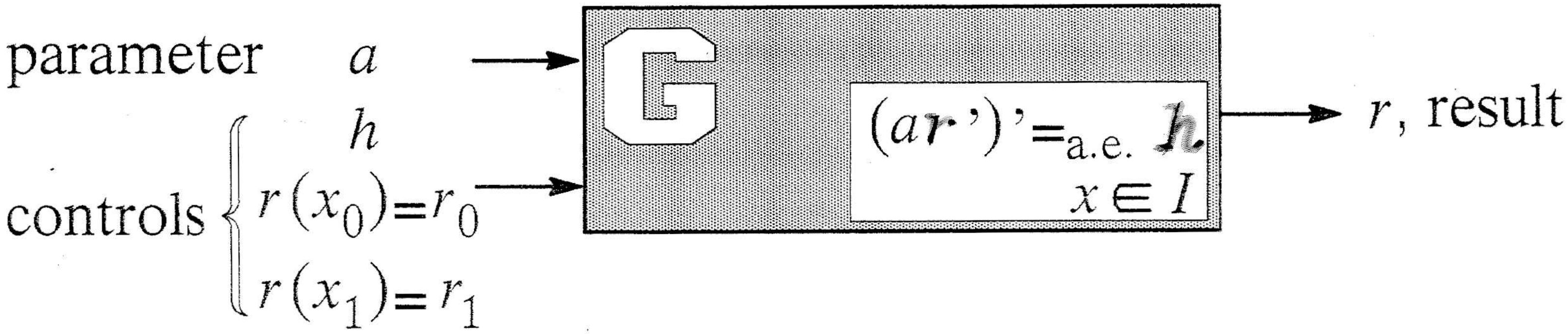
IDENTIFICATION FOR CONTROL

$$x \in (x_0, x_1) \equiv I$$

INVERSE (*identification*) PROBLEM *ill – posed*



DIRECT (*control*) PROBLEM *well – posed*



“*The solution of the ill–posed problem is only an intermediate construct intervening between the available data and the intended application*” (T I SEIDMAN, 1990).

THE GENERAL CONTEXT

(SEIDMAN, 1990)

data		parameters		results
$\mathbb{D}$	$\xleftarrow{\mathbb{F}}$	$\mathbb{P}_1$	$\xrightarrow{\mathbb{G}}$	$\mathbb{R}$
u	$\longleftarrow\!\!\shortmid$	a	$\longmapsto$	r

LEMMA 1 (a sufficient condition)

Let $\mathbb{P}_1$, $\mathbb{D}$ *Hausdorff subspaces;* $\mathbb{F}$ *a continuous injection;*
$\mathbb{D}_1 := \mathbb{F}(\mathbb{P}_1) \subset \mathbb{D}$. *If* $\mathbb{P}_1$ *is compact, then*

1) $\mathbb{D}_1$ *is compact*

2) $\mathbb{F}^{-1} : \mathbb{D}_1 \longrightarrow \mathbb{P}_1$ *is continuous.*

ABSTRACT EXAMPLE : $\mathbb{P}$ not compact; $\mathbb{F} : \mathbb{P} \longrightarrow \mathbb{D}$;

define $\mathbf{E} : \mathbb{X} \longrightarrow \mathbb{P}$ (compact imbedding);

$\mathbb{B}_M := \{\bar{a} \in \mathbb{X} \mid \| \bar{a} \|_{\mathbb{X}} \leq M \}$ *(prior knowledge)*

(inv. pbm) *given* $\bar{u} \in \mathbb{D}$ *find* $\bar{a} \in \mathbb{X} \cdot\!\ni\!\cdot \mathbb{F}(\bar{a}) = \bar{u}$.

(key step) *If* $\mathbb{D}_1 = \text{clos}[\, \mathbf{E}(\mathbb{B}_M)\,]$ *then* $\mathbb{D}_1$ *compact.*

If $\mathbb{F}_1 = \mathbb{F}\restriction_{\mathbb{P}_1}$ *then* LEMMA *1 applies.*

QUESTIONS

Question I : (topological conditions *vs.* state equations)
Since compactness is sufficient for the well posedness of the inverse problem, why not neglect system structure altogether and invoke some general topological conditions ?

Question II : (control problem *vs.* parameter identification)
Since the control problem is better posed than the inverse one, does the overall data–to–result map ($\mathbb{G}\circ\mathbb{F}^{-1}$) exhibit cancellation, *as transfer functions of interconnected control systems do ?*

THE SPECIAL CONTEXT

one dimensional heat equation in $I := (x_0, x_1)$

$f \in L^p(I)$; $a \in W^{1,p}(I)$, $0 < a_L \leq a \leq a_H$; $u \in W^{2,p}(I)$;

(identification) $(au')' =_{\text{a.e.}} f$ in I + extra information about a

(control) $(ar')' =_{\text{a.e.}} h$ in I ; $r(x_0) = r_0$, $r(x_1) = r_1$.

two dimensional heat equation in $D \subset \mathbb{R}^2$

$f \in L^p(D)$; $a \in W^{1,p}(D)$, $0 < a_L \leq a$; $u \in W^{2,p}(D)$;

(identification) $\nabla\cdot(a\nabla u) =_{\text{a.e.}} f$ in D + extra information about a

(control) $\nabla\cdot(a\nabla r) =_{\text{a.e.}} h$ in D ; $r|_{\partial D} = g$.

THM. 2 (RICHTER, 1981; preliminary and incomplete)

second result; reference result; second ("noisy") potential; reference potential; reference conductivity

$$\frac{\|\nabla(s-r)\|_{0,2}}{\|\nabla r\|_{0,2}} \leq C(a,D,f)\left\{\frac{\|\nabla a\|_{0,\infty}}{\min_D a}\|\nabla(V-u)\|_{0,\infty} + \frac{\|a\|_{0,\infty}}{\min_D a}\|\Delta(V-u)\|_{0,\infty}\right.$$

POSITIVE ANSWER TO QUESTION II

THM. 3 (Cauchy data for both inverse and control problems)

Let $F(x) := \int_{x_0}^{x} f\, ds$; $H(x) := \int_{x_0}^{x} h\, ds$; $(au')_0$ *and* $(ar')_0$ *fixed;*

$\min_{\bar{I}} |F + (au')_0| \geq k_N$; $|(ar')_0| \leq k_C$; $\| f \|_{0,p}$, $\| h \|_{0,p} \leq c_S$;

define $k_Q := \{|x_1 - x_0|^{\frac{1}{q}} c_S + k_C\} \frac{1}{k_N}$.

Then the data–to–result relation is $r' = \dfrac{H + (ar')_0}{F + (au')_0}\, u'$ *and satisfies*

$$\| s' - r' \|_{0,p} \leq k_Q \| v' - u' \|_{0,\infty} \text{ s.t., } k_Q > 1 ,$$

$$\| s'' - r'' \|_{0,p} \leq \frac{1 + k_Q}{k_N} c_S \| v' - u' \|_{0,\infty} + k_Q \| v'' - u'' \|_{0,p}$$

instead of e.g.,

$$\| s' - r' \|_{0,p} \leq k_Q^{2} \| v' - u' \|_{0,\infty}$$

as obtained from combining separate estimates.

ANOTHER POSITIVE ANSWER TO QUESTION II

Define $E_u := \{ x \mid x \in \bar{D} \,,\, u'(x) = 0 \}$; $E_V :=$ analogous.

THM. 4 (Uniqueness of a from critical point; Cauchy control pbm)

Let $E_u = E_V$; $\frac{1}{u'}$, $\frac{1}{V'} \in L^t(I)$, $t \geq 1$, *finite such that* $\frac{pt}{p+t} < 1$;

let $\| \frac{1}{V'} \|_{0,t} \leq c_V$, $|r'_0| \leq c_r$, $\| h \|_{0,p} \leq |x_1 - x_0|^{-\frac{1}{q}} c_h$. ↙ 2nd conductivity at x_0

Assume $\{ x_0 \notin E_u \wedge u'_0 = V'_0 \} \vee \{ x_0 \in E_u \wedge a_0 = b_0 \}$, *then*

$$\| s' - r' \|_{0,t} \leq (c_h + a_H c_r) \frac{c_V}{a_L} \| V' - u' \|_{0,\infty}$$

REMARKS

(estimating $\mathbb{G} \circ \mathbb{F}^{-1}$ *vs.* combining the estimates of $\mathbb{F}^{-1}$ and $\mathbb{G}$)

1) The structure of the estimate is preserved.

2) In a few (special) cases, cancellation occurs and constants from the composed map are consistently smaller.

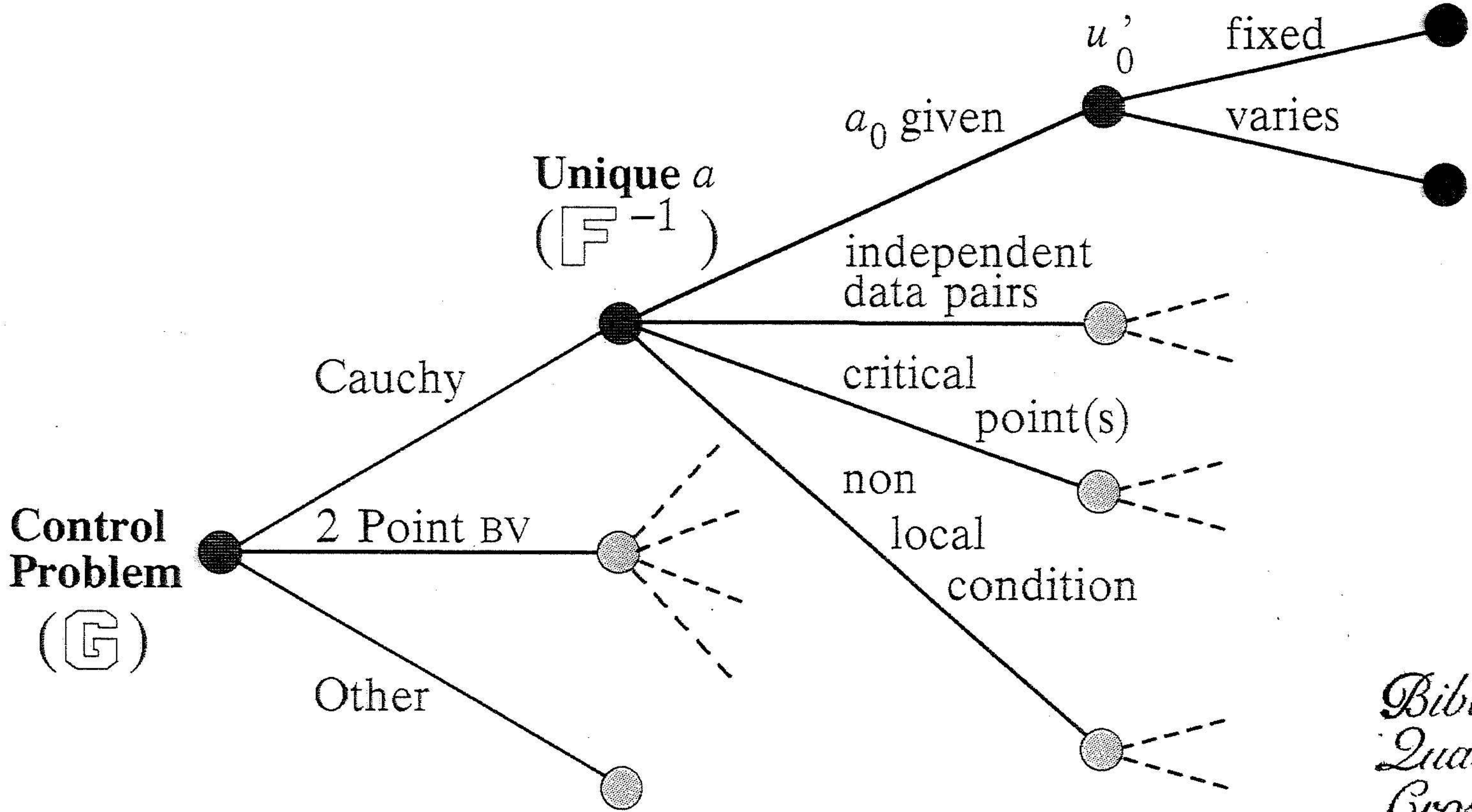
3) In general, constants depend on the estimation procedure and their values depend on the specific data.

Rubio and Quadrelli Crosta ;

EXAMPLE : uniqueness from independent data.

BY–PRODUCT

CLASSIFICATION
OF UNIQUENESS CONDITIONS
AND OF RELATED DATA – TO – RESULTS MAPS
(ONE DIMENSIONAL PROBLEMS)



ANSWER TO QUESTION I AND CONCLUSION

LEMMA 7: { *Relative compactness of* $G \sqsubset L^p(D, \alpha)$, $1 \leq p < \infty$ }

$$\Longleftrightarrow$$

{ [$\forall \varepsilon > 0$, $\exists\ \delta \ni \alpha(K) < \delta \Rightarrow \int_K |g|^p dK < \varepsilon\ \forall g$ *(absolute* α*–equi–continuity of integrals)*] $\wedge$ [*uniform* α*–measurability of* g *over* $\forall B \sqsubseteq D$]}.

APPLICATION : one dimensional problems.
Choose $a' \in G_a(I)$, $f \in G_f(I)$ and obtain $\mathbb{D}_1 = \{u |\ u'' \in G_u(I)\}$.

COUNTEREXAMPLE :
typical stability estimates from state equations require e.g.,

$$\| u'' \|_{0,p} \leq c_Q \ (\not\Rightarrow u'' \in G_u(I)) \text{ i.e.,}$$

the continuity of $\mathbb{F}^{-1}$ *is attained on larger data sets.*

REMARK : LEMMA 1 is just a sufficient condition.

SIDE BENEFIT : best estimation constants can be determined.

CONCLUSION : work at state equations is rewarding.