Game theory models of international agreements on adaptation to climate change

Mauro Passacantando

Abstract This note deals with the formation of coalitions of countries to jointly fight the adverse effects of climate change. We propose two game theory models of international agreements and compare them with the situation where each country individually develops new means to adapt to climate change.

Key words: International environmental agreement; Nash equilibrium; variational inequality; shared constraints

1 Introduction

The idea of establishing international agreements to reduce pollution due to industrial activities has its roots in the Kyoto Protocol that was adopted in December 1997 and entered into force on 16 February 2005, aiming at committing industrialized countries and economies in transition to reduce greenhouse gas (GHG) emissions in accordance with agreed individual targets. The failure of this first international agreement led to other attempts to coordinate the efforts of various countries in order to fight climate changes due to emissions. In particular, the Paris Agreement is a legally binding international treaty on climate change and was adopted by 196 Parties at the UN Climate Change Conference (COP21) in Paris on 12 December 2015. Although the full implementation of Paris agreement is considered a priority from the United Nations, it was soon recognized that the consequences of the environmental damage produced so far by GHG also requires joint efforts by the countries, to be effectively mitigated. In this respect, the word *adaptation* is used to denote the actions to be enforced in order to counterbalance the damage caused by climate change. For a survey on international environmental agreements with the objective

Mauro Passacantando
Department of Business and Law, University of Milan-Bicocca, Via degli Arcimboldi 8, 20126 Milan, Italy, e-mail: mauro.passacantando@unimib.it

of reducing emissions, we refer the reader to [8]. However, mathematical models of international agreements aiming at adaptation are quite recent. Our model is inspired by the game theory approach developed in [9, 10], where countries are considered as players whose utility functions depend on both emissions and investments and can decide whether or not to sign an international agreement to jointly invest in research and development (R&D) to find new means to adapt to climate change. The effect of the coalition on the overall emissions was then investigated. With respect to the above mentioned papers, our model differs in the following points: we consider the more realistic case of non-symmetrical countries, but consider the coalition as given, that is, we do not study the membership problem, and we do not consider spillover contributions. On the other hand, we provide two different coalition scenarios, and compare them with the one were each country invests alone. At last, in our variational inequality formulation of the games, the possibility of boundary solutions is allowed, while in [9, 10] the authors focused on the interior solution case.

The paper is organized as follows. In the following Section 2, we provide some background material and describe the three scenarios. In Section 3, we investigate the monotonicity properties of the three operators involved in the variational inequalities modeling the considered scenarios. Section 4 is devoted to illustrate our finding by means of a numerical example, while in the short concluding section we summarize our results and outline some future research perspectives.

2 Model

In what follows, vectors in $\mathbb{R}^m$ are thought of as columns, when involved in matrix operations, $a^\top$ denotes the transpose of vector a and $a^\top b$ the canonical scalar product in $\mathbb{R}^m$. The notation $x_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_m)$ and $x = (x_i, x_{-i}) = (x_1, \dots, x_m)$ will be used when we want to distinguish the role of x_i from all the other components of vector x.

We consider a set of M countries and assume, as is commonly done in the economic literature, that the industrial production P_i of country i generates a pollutant emission e_i and such emission is an increasing function of the production. Thus, the revenue of a country can be expressed as a function of its emissions. A standard functional form of the revenue R_i is

$$R_i(e_i) = \alpha_i\, e_i - \frac{1}{2} e_i^2, \qquad \alpha_i > 0.$$

The environmental impact D_i for each country i can depend from the emissions of all the countries. Specifically, we assume that, in absence of mitigation strategies, the impact is given by

$$D_i(e) = \beta \sum_{j=1}^{M} e_j, \qquad \beta > 0. \tag{1}$$

Let us now denote with k_i the effort in R&D of country i, which requires an investments given by

$$C_i(k_i) = \frac{c}{2}\,k_i^2, \qquad c > 0,$$

and assume that each country assigns to this activity a maximum budget of $\bar{k}_i$. The outcome of the research activity will allow a reduction of the environmental impact (1), which can be updated to

$$D_i(e, k_i) = (\beta - \theta\,k_i)\,\sum_{j=1}^{M} e_j, \tag{2}$$

where the parameter θ defines the adaptability (see [9]).

With these data, the utility function of country i, in absence of international agreements with other countries, is

$$u_i(e, k) = \alpha_i\,e_i - \frac{1}{2}e_i^2 - \frac{c}{2}\,k_i^2 - (\beta - \theta k_i)\,\sum_{j=1}^{M} e_j. \tag{3}$$

Furthermore, let $K_i := \min\{\bar{k}_i, \beta/\theta\}$ for each $i = 1, \ldots, M$.

We are now in position to consider a non-cooperative game, where each country aims at maximizing its welfare, and no international agreement is stipulated. We refer to this situation as *scenario 0*.

Scenario 0

Each country i controls the emission level e_i and the investment k_i in R&D with the aim of solving the following problem:

$$\max_{e_i, k_i}\; u_i(e, k) \tag{4}$$

$$\text{s. t. } 0 \le e_i \le E_i, \tag{5}$$

$$0 \le k_i \le K_i. \tag{6}$$

Constraint (5) fixes the upper bound on the emission, while constraint (6) expresses the fact that the investment in R&D of each country cannot exceed the corresponding budget and cannot change the environmental damage into a benefit.

For the subsequent development, it is useful to introduce the following notation. Let $z = (e_1, k_1, \ldots, e_M, k_M)$ and $z^1 = (e_1, k_1), \ldots, z^M = (e_M, k_M)$ and with a slight abuse of a notation write $u_i(e, k) = u_i(z)$. Moreover, let

$$G_i = [0, E_i] \times [0, K_i], \qquad G^0 = \prod_{i=1}^{M} G_i.$$

The optimization problems (4)–(6) then yield the following definition.

Definition 1 A point $z^* \in G^0$ is a Nash equilibrium for scenario 0 iff for each country $i \in \{1, \dots, M\}$ the following condition holds:

$$u_i(z^{*i}, z^{*-i}) \geq u_i(z^i, z^{*-i}), \qquad \forall\, z^i \in G_i. \tag{7}$$

It is well known that Nash equilibrium problems are equivalent to variational inequalities (see, e.g., [7, 13]). If we introduce the map $F^0 : \mathbb{R}^{2M} \to \mathbb{R}^{2M}$ made up with the partial gradients of u_i, defined as follows:

$$F^0(z) = -\left(\nabla_{z^1} u_1(z), \dots, \nabla_{z^M} u_M(z)\right), \tag{8}$$

then z^* is a Nash equilibrium according to (7) if and only if it solves the variational inequality problem $VI(F^0, G^0)$ of finding $z^* \in G^0$ such that:

$$F^0(z^*)^\top (z - z^*) \geq 0, \qquad \forall\, z \in G^0. \tag{9}$$

We recall now a useful definition and a classic existence and uniqueness theorem for variational inequalities.

Definition 2 An operator $T : \mathbb{R}^n \to \mathbb{R}^n$ is said to be monotone on a set $K \subset \mathbb{R}^n$ iff:

$$[T(x) - T(y)]^\top (x - y) \geq 0, \qquad \forall\, x, y \in K.$$

If the equality holds only when $x = y$, then T is said to be strictly monotone on K.

Theorem 1 (see, e.g., [3]) *If $K \subset \mathbb{R}^n$ is a compact convex set and $T : \mathbb{R}^n \to \mathbb{R}^n$ is continuous on K, then the variational inequality problem $VI(T, K)$ admits at least one solution. In the case that K is unbounded, existence of a solution may be established under the following coercivity condition:*

$$\lim_{\|x\| \to +\infty} \frac{[T(x) - T(x_0)]^\top (x - x_0)}{\|x - x_0\|} = +\infty,$$

for $x \in K$ and some $x_0 \in K$. Furthermore, the solution is unique if T is strictly monotone on K.

Scenario 1

We now consider the case where S out of the M countries decide to sign an international agreement in order to jointly develop some new technologies than can help moderate the impact of climate change. Let I be the set of these countries. The formation of this coalition affects both the utility functions and the constraint sets of its members. Specifically, each signatory country $i \in I$ aims to maximize the utility function

$$u_i^S(e, k) = \alpha_i e_i - \frac{1}{2} e_i^2 - \frac{c}{2} k_i^2 - \left(\beta - \theta \sum_{j \in I} k_j\right) \sum_{j=1}^{M} e_j, \tag{10}$$

with respect to variables e_i and k_i, subject to the constraints

$$0 \le e_i \le E_i, \tag{11}$$

$$k_i \ge 0, \tag{12}$$

$$\sum_{j \in I} k_j \le K_I, \tag{13}$$

where $K_I = \min \left\{ \beta/\theta, \sum_{j \in I} \bar{k}_j \right\}$. On the other hand, each non-signatory country $i \notin I$ aims to solve the problem

$$\max_{e_i, k_i} \; u_i(e,k) \tag{14}$$

$$\text{s. t. } \; 0 \le e_i \le E_i, \tag{15}$$

$$0 \le k_i \le K_i. \tag{16}$$

Thus, in this scenario, the countries that sign the agreement make a joint investment in R&D while maximizing their own utility function. We notice that constraint (13) yields to a generalized Nash equilibrium problem (GNEP). We mention here that GNEPs were introduced in [15] and called Nash equilibrium problems with shared constraints. Many decades later they were reformulated in the framework of variational and quasi-variational inequalities [2, 12]. A further extension to infinite dimension in due to [4, 11]. Let G^1 be the subset of $z \in \mathbb{R}^{2M}$ such that constraints (11)–(12) hold for any $i \in I$, (15)–(16) hold for any $i \notin I$ and the shared constraint (13) holds.

Definition 3 A point $z^* \in G^1$ is a generalized Nash equilibrium for scenario 1 iff

$$u_i^S(z^{*i}, z^{*-i}) \ge u_i^S(z^i, z^{*-i}), \quad \forall\, z^i \text{ such that } (z^i, z^{*-i}) \in G^1, \qquad \forall\, i \in I, \tag{17}$$

$$u_i(z^{*i}, z^{*-i}) \ge u_i(z^i, z^{*-i}), \quad \forall\, z^i \text{ such that } (z^i, z^{*-i}) \in G^1, \qquad \forall\, i \notin I. \tag{18}$$

It is well known that GNEPs may have infinite solutions, among which the so called *variational solution* is particular appealing for its socio-economic interpretation, and its stability properties. Specifically, variational solutions can be found by solving the variational inequality $VI(F^1, G^1)$, where

$$F^1(z) = -\left(\nabla_{z^1} u_1^S(z), \ldots, \nabla_{z^S} u_S^S(z), \nabla_{z^{S+1}} u_{S+1}(z), \ldots, \nabla_{z^M} u_M(z) \right), \tag{19}$$

and, for notational simplicity, we have assumed that the first S countries sign the international agreement.

Scenario 2

We now consider the case where the countries in the coalition I decide to maximize their overall utility and thus act as a single player who controls the variables

$z^I = (e_1, k_1, \dots, e_S, k_S)$ and whose utility function is

$$W^{\mathcal{S}}(e,k) := \sum_{i \in I} u_i^{\mathcal{S}}(e,k) = \sum_{i \in I} \left(\alpha_i e_i - \frac{1}{2} e_i^2 - \frac{c}{2} k_i^2 \right) - S \left(\beta - \theta \sum_{i \in I} k_i \right) \sum_{j=1}^{M} e_j .$$

In this scenario, the player representing the coalition I aims to solve the problem

$$\begin{aligned} \max_{(e_i,k_i)_{i \in I}} \quad & W^{\mathcal{S}}(e,k) && && (20) \\ \text{s. t.} \quad & 0 \le e_i \le E_i, && \forall\, i \in I, && (21) \\ & k_i \ge 0, && \forall\, i \in I, && (22) \\ & \sum_{i \in I} k_i \le K_I, && && (23) \end{aligned}$$

while each non-signatory country aims to solve problem (14)–(16). If we now denote with G_I the set defined by constraints (21)–(23) and

$$G^2 = \left\{ z \in \mathbb{R}^{2M} : \ \text{constraints (15), (16), (21), (22), (23) hold} \right\},$$

then scenario 2 yields to the following equilibrium definition.

Definition 4 A point $z^* \in G^2$ is a Nash equilibrium for scenario 2 iff:

$$W^{\mathcal{S}}(z^{*I}, z^{*-I}) \ge W^{\mathcal{S}}(z^I, z^{*-I}), \qquad \forall\, z^I \in G_I, \qquad (24)$$

$$u_i(z^{*i}, z^{*-i}) \ge u_i(z^i, z^{*-i}), \qquad \forall\, z^i \in G_i,\ \forall\, i \notin I. \qquad (25)$$

If we now define the map $F^2 : \mathbb{R}^{2M} \to \mathbb{R}^{2M}$ as follows:

$$F^2(z) = -\left(\nabla_{z^I} W^{\mathcal{S}}(z), \nabla_{z^{S+1}} u_{S+1}(z), \dots, \nabla_{z^M} u_M(z) \right), \qquad (26)$$

we then get that a point $z^* \in G^2$ is a Nash equilibrium for scenario 2 iff it solves $VI(F^2, G^2)$.

3 Monotonicity properties

In this section, we provide some sufficient conditions for the strict monotonicity of the operators F^0, F^1 and F^2 introduced in the previous section.

Proposition 1 *Let F^0 be the operator defined in* (8). *If $\theta(M+1)/2 < \min\{c, 1\}$, then F^0 is strictly monotone on $\mathbb{R}^{2M}$.*

Proof Since we have

$$\frac{\partial u_i}{\partial e_i} = -e_i + \theta k_i + \alpha_i - \beta, \qquad \forall\, i = 1, \dots, M,$$

$$\frac{\partial u_i}{\partial k_i} = -ck_i + \theta \sum_{i=1}^{M} e_i, \qquad \forall\, i = 1, \ldots, M,$$

the affine operator F^0 can be written as $F^0(z) = P^0 z + q^0$, where

$$P^0 = \left(\begin{array}{cc|cc|cc|cc} 1 & -\theta & 0 & 0 & \ldots & \ldots & 0 & 0 \\ -\theta & c & -\theta & 0 & \ldots & \ldots & -\theta & 0 \\ \hline 0 & 0 & 1 & -\theta & \ddots & & \vdots & \vdots \\ -\theta & 0 & -\theta & c & & \ddots & \vdots & \vdots \\ \hline \vdots & \vdots & \ddots & & \ddots & & 0 & 0 \\ \vdots & \vdots & & \ddots & & \ddots & -\theta & 0 \\ \hline 0 & 0 & \ldots & \ldots & 0 & 0 & 1 & -\theta \\ -\theta & 0 & \ldots & \ldots & -\theta & 0 & -\theta & c \end{array}\right), \qquad q^0 = \left(\begin{array}{c} \beta - \alpha_1 \\ 0 \\ \hline \vdots \\ \hline \beta - \alpha_M \\ 0 \end{array}\right).$$

The symmetric part of matrix P^0 is

$$\frac{P^0 + (P^0)^\top}{2} = \left(\begin{array}{cc|cc|cc|cc} 1 & -\theta & 0 & -\theta/2 & \ldots & \ldots & 0 & -\theta/2 \\ -\theta & c & -\theta/2 & 0 & \ldots & \ldots & -\theta/2 & 0 \\ \hline 0 & -\theta/2 & 1 & -\theta & \ddots & & \vdots & \vdots \\ -\theta/2 & 0 & -\theta & c & & \ddots & \vdots & \vdots \\ \hline \vdots & \vdots & \ddots & & \ddots & & 0 & -\theta/2 \\ \vdots & \vdots & & \ddots & & \ddots & -\theta/2 & 0 \\ \hline 0 & -\theta/2 & \ldots & \ldots & 0 & -\theta/2 & 1 & -\theta \\ -\theta/2 & 0 & \ldots & \ldots & -\theta/2 & 0 & -\theta & c \end{array}\right).$$

If $\theta(M+1)/2 < \min\{c, 1\}$, then

$$\theta + \frac{\theta}{2}(M-1) = \frac{\theta(M+1)}{2} < \min\{c, 1\},$$

thus the symmetric part of P^0 is diagonal dominant and positive definite. Hence, P^0 is positive definite as well and F^0 is strictly monotone on $\mathbb{R}^{2M}$. □

Proposition 2 *Let F^1 and F^2 be the operators defined in* (19) *and* (26)*, respectively. Then, we have:*

a) If $\theta(M+S)/2 < \min\{c, 1\}$, then F^1 is strictly monotone on $\mathbb{R}^{2M}$.
b) If $\theta S(M+S)/2 < c$ and $\theta S^2 + \theta(M-S)/2 < 1$, then F^2 is strictly monotone on $\mathbb{R}^{2M}$.

Proof It is similar to the proof of Proposition 1. □

4 Numerical experiments

In this section, we show a numerical example to illustrate the three scenarios described in Section 2.

We consider a set C of 20 countries divided into two subsets $C_1 = \{1, \dots, 10\}$ (developed countries) and $C_2 = \{11, \dots, 20\}$ (developing countries). We set parameters

$$\begin{array}{llll} \alpha_i = 10 & \forall\, i \in C_1, & \alpha_i = 2 & \forall\, i \in C_2, \\ E_i = 10 & \forall\, i \in C_1, & E_i = 2 & \forall\, i \in C_2, \\ \bar{k}_i = 4 & \forall\, i \in C_1, & \bar{k}_i = 0.5 & \forall\, i \in C_2, \end{array}$$

$\beta = 0.1$, $c = 0.2$ and $\theta = 0.001$.

We assume that in scenarios 1 and 2 the set of countries that sign the international agreement is $I = \{1, \dots, 15\}$, i.e., all the developed countries and half of developing countries. Table 1 shows the values of pollutant emission (e_i) and effort in R&D (k_i) of the 20 countries at the variational equilibrium in scenario 1 and at Nash equilibrium in scenarios 0 and 2.

	scenario 0		scenario 1		scenario 2	
Country	e_i	k_i	e_i	k_i	e_i	k_i
1	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
2	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
3	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
4	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
5	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
6	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
7	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
8	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
9	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
10	9.9006	0.5901	9.9089	0.5907	9.1375	2.8333
11	1.9005	0.5000	1.9089	0.5907	1.1375	2.8333
12	1.9005	0.5000	1.9089	0.5907	1.1375	2.8333
13	1.9005	0.5000	1.9089	0.5907	1.1375	2.8333
14	1.9005	0.5000	1.9089	0.5907	1.1375	2.8333
15	1.9005	0.5000	1.9089	0.5907	1.1375	2.8333
16	1.9005	0.5000	1.9005	0.5000	1.9005	0.5000
17	1.9005	0.5000	1.9005	0.5000	1.9005	0.5000
18	1.9005	0.5000	1.9005	0.5000	1.9005	0.5000
19	1.9005	0.5000	1.9005	0.5000	1.9005	0.5000
20	1.9005	0.5000	1.9005	0.5000	1.9005	0.5000

Table 1 Values of pollutant emission (e_i) and effort in R&D (k_i) of the 20 countries at the variational equilibrium in scenario 1 and at Nash equilibrium in scenarios 0 and 2, assuming that in scenarios 1 and 2 the countries in the set $I = \{1, \dots, 15\}$ sign the international agreement.

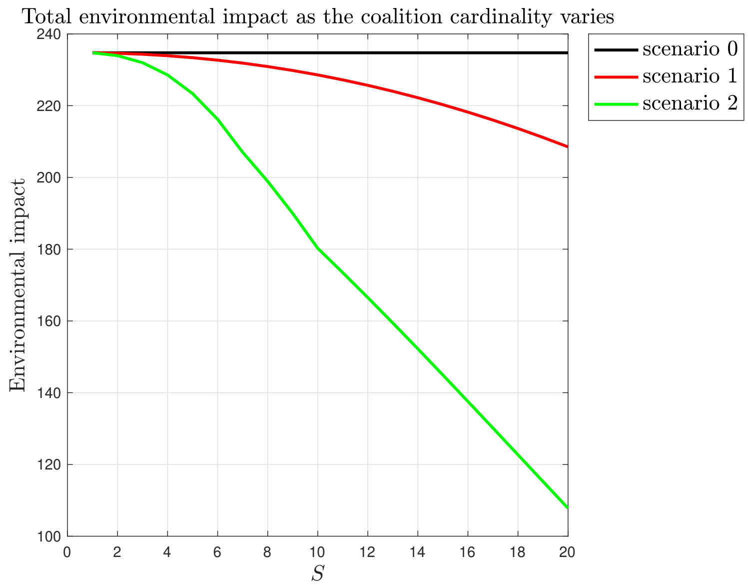

Fig. 1 Total environmental impact as the coalition cardinality varies.

Figure 1 shows the total environmental impact at equilibrium as a function of the coalition cardinality. The results suggest that in scenarios 1 and 2, in which countries may decide to sign agreements to jointly develop new technologies, the environmental impact at equilibrium is reduced with respect to scenario 0. Moreover, the more the number of countries in the coalition increases, the more environmental impact is reduced (especially in scenario 2).

5 Conclusion and further research perspectives

In this note we put forward a variational inequality approach to model international agreements among countries in order to fight the adverse effects of industrial pollution. Two kind of agreements are compared with the case where each country decides to individually invest in R&D, by means of a numerical example. Future research is needed to develop algorithms to effectively treat the case of large coalitions (100-200 countries). Another interesting research avenue is the modeling of uncertain parameters using the recent theory of random variational inequalities [5, 6, 14].

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