IIASA Workshop

Modeling of Environmental Dynamics

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Identification of Conductivity: Some Recent Results about a Composite Map (Identification for Control)

Giovanni Crosta

Universita‘ degli Studi di Milano

College of Sciences

Milan, Italy

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Identification for Control in 1 D

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Consider the composite *DATA – TO – RESULT* map

1 Provide some *uniqueness* conditions for

the *DATA – TO – PARAMETER* map

1.1 from *independent* data pairs =

= *regular* Cauchy pbm. w.r. to a in $(au')' =_{\text{a.e.}} f$

1.2 from a potential *stationary* at a point =

= *singular* Cauchy pbm. w.r. to a in $(au')' =_{\text{a.e.}} f$

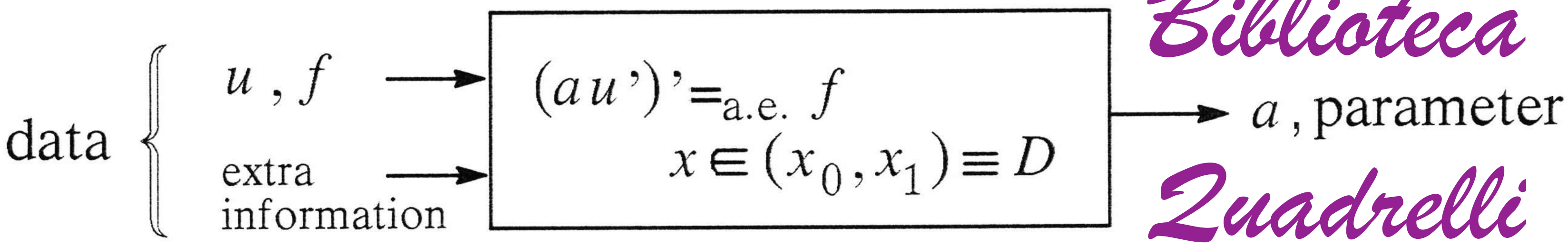
2 Provide the corresponding *stability* estimates for

the *DATA – TO – RESULT* map.

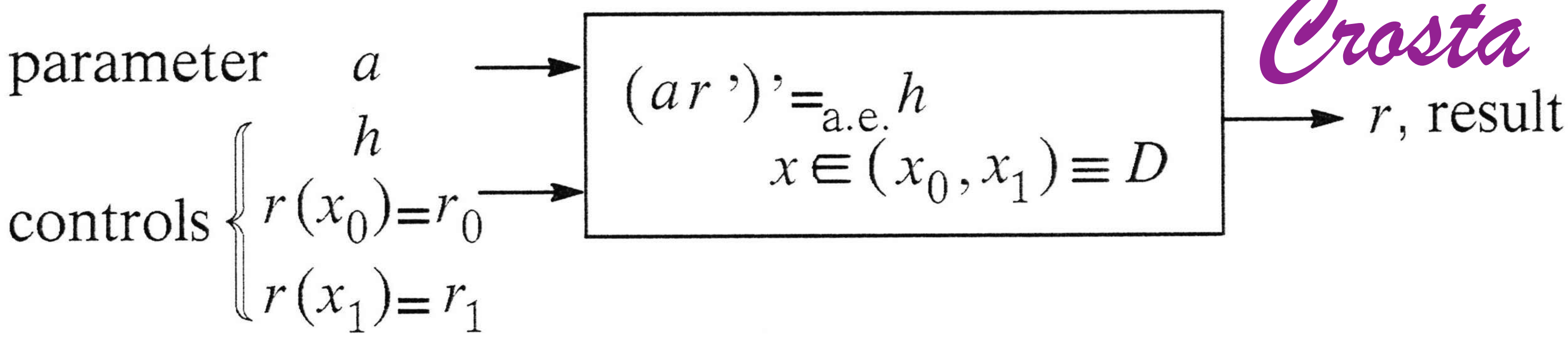
IDENTIFICATION FOR CONTROL IN 1 D

THE PARADIGM

INVERSE (*identification*) PROBLEM *ill – posed*



DIRECT (*control*) PROBLEM *well – posed*



"*The solution of the ill–posed problem is only an intermediate construct intervening between the available data and the intended application*"

(T. I. Seidman, 1990).

THE GENERAL CONTEXT: begin

(Seidman, 1990)

data		parameters		results
V	$\xleftarrow{\mathbf{P}}$	A	$\xrightarrow{\mathbf{Q}}$	Z
u	$\longleftarrow\!\!\mid$	a	$\longmapsto$	r

Thm. 0.1

Let A , V *Hausdorff;*

$\mathbf{P}_* : A \supset A_* \longrightarrow V$ *continuous injection;*

let $V_* := \mathbf{P}_*(A_*) \subset V$.

If A_* *is compact, then*

1) V_* *is compact*

2) $\mathbf{P}_*^{-1} : V_* \longrightarrow A_*$ *is continuous.*

Abstract Example: A not compact; $\mathbf{P} : A \longrightarrow V$;

define $\mathbf{E} : X \longrightarrow A$ (compact imbedding);

$B_M := \{\overline{a} \in X \mid \| \overline{a} \|_X \leq M \}$ (*prior knowledge*)

(*inverse problem*)

given $\overline{u} \in V$ *find* $\overline{a} \in X \cdot\!\ni\!\cdot \mathbf{P}(\overline{a}) = \overline{u}$.

(*key step*) *If* $A_* = \text{clos}[\mathbf{E}(B_M)]$ *then* A_* *compact.*

If $\mathbf{P}_* = \mathbf{P}\restriction_{A_*}$ *then Thm. 0.1 applies.*

Concrete Example

stability for the *INDEPENDENT* DATA PAIR – unique a .

(*strong hypotheses*)

If

$$f \in C^0(\bar{D}) \; ; \; |f(x)| \le c_s \, , \; \forall \, x \in \bar{D}$$

$$u \in C^2(\bar{D}) \; ; \; E_u = \emptyset \quad \text{and}$$

$$0 < c_M \le |\, u'(x) \,| \le c_P \, , \; \forall \, x \in \bar{D}$$

then

$$\{ \, a \mid a = \frac{F + c_1}{u'} \; ; \; a(x) \le a_H \, , \; \forall \, x \in \bar{D} \, \}$$

is uniformly bounded and equicontinuous hence (Arzelà's Thm.) compact.

(*relaxation*)

Detailed work at $\mathbf{Q} \cdot \mathbf{P}_*^{-1}$ (see 1.1, 2.1) weakens the continuity hypotheses on f, resp. u .

	becomes	
$f \in C^0(\bar{D})$		$f \in L^1(D)$
$\lvert f(x) \rvert \le c_s \, , \; \forall \, x \in \bar{D}$		$\lVert f \rVert_1 \le c_s$
$u \in C^2(\bar{D})$		$u \in C^1(\bar{D})$
$0 < c_M \le \lvert u'(x) \rvert \; \forall x \in \bar{D}$		$0 < c_M \le \lvert u_i' \rvert \, , \; i = 1, 2$
$\lvert u'(x) \rvert \le c_P \; \forall x \in \bar{D}$		$\lvert u_i' \rvert \le c_P \, , \; i = 1, 2$
	specific for the applicable	$\lvert v_2' u_1' - v_1' u_2' \rvert \ge \frac{1}{c_T}$
	uniqueness condition	$\lvert w_2' u_1' - w_1' u_2' \rvert \ge \frac{1}{c_T}$

1.1 – UNIQUENESS FROM *INDEPENDENT* DATA PAIRS
(a *Regular* Cauchy Problem)

$D := (x_0, x_1)$

$u\,,\, v \in C^1(\bar{D})\;;\; E_u := \{\, x \mid x \in \bar{D}\,,\; u'(x) = 0 \,\}\;;\; E_v :=$ analogous

$f\,,\, g \in L^1(D)\;;\; f\,,\, g \not\equiv 0;\; F(x) := \int_{x_0}^{x} f\,ds\;;\; G(x) := \int_{x_0}^{x} g\,ds$

$A_{ad} := \{\, a \mid a \in C^0(\bar{D}),\; 0 < a_L \le a(x)\,,\; \forall\, x \in \bar{D} \,\}$

Hp. $\exists\, a \in A_{ad} \;\cdot\!\ni\!\cdot\; \{\, (au')' =_{\text{a.e.}} f\,,\; (av')' =_{\text{a.e.}} g \,\}\;;\; E_u = E_v = \emptyset$

$$\exists\, y_1, y_2 \in \bar{D} \;\cdot\!\ni\!\cdot\; \frac{1}{v'(y_1)\,u'(y_2)} - \frac{1}{v'(y_2)\,u'(y_1)} \neq 0$$

Th. 1.1 $\exists!\; \hat{a} = \dfrac{F + c_1}{u'}$ *(reference solution)*

$$c_1 = \frac{(G_2 - G_1)\,u'_1\,u'_2 - F_2\,u'_1\,v'_2 + F_1\,u'_2\,v'_1}{u'_1 v'_2 - u'_2 v'_1}\,.$$

DATA – TO – *PARAMETER* map

1.2 – UNIQUENESS FROM A POTENTIAL *STATIONARY* AT A (*CRITICAL*) POINT (a *Singular* Cauchy Problem)

Hp. $\exists\ a \in A_{ad} \ \cdot\ni\cdot \ \{ (au')' =_{\text{a.e.}} f \}$

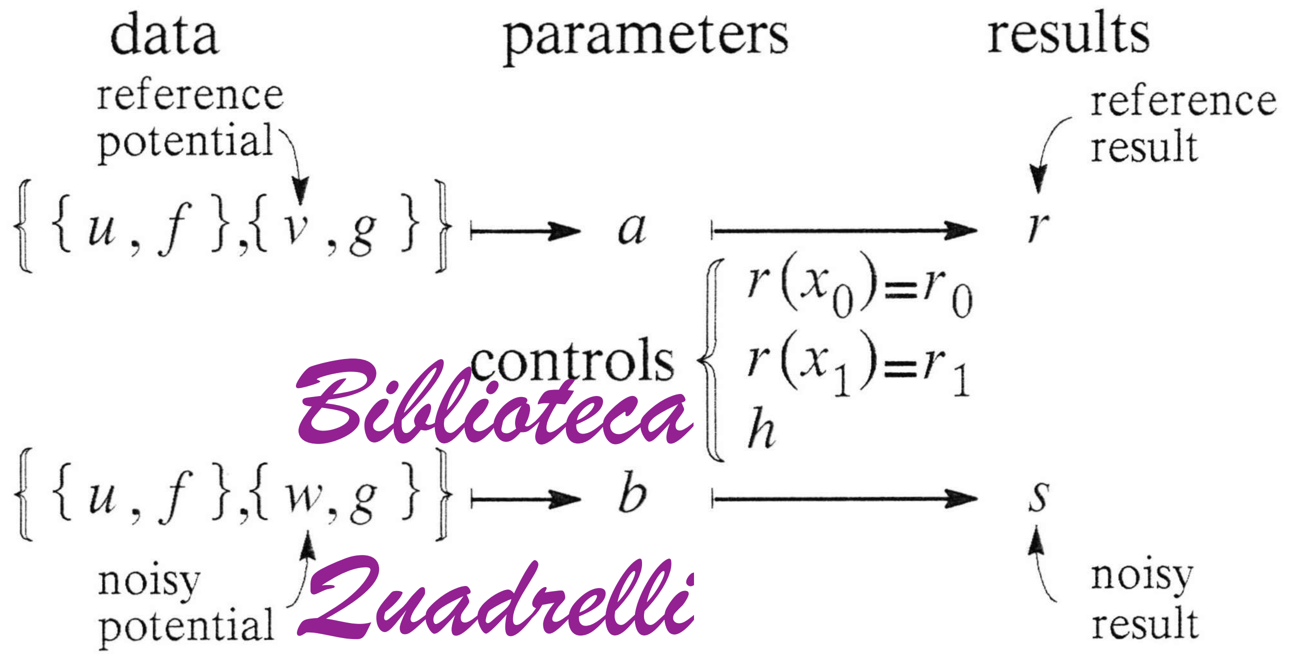
$E_u = \{ x_u \}$

Th. 1.2 $\exists!\ \hat{a} = \dfrac{F - F(x_u)}{u'}$ (*reference solution*)

Rem.: special case of Kitamura – Nakagiri (1977) uniqueness prop.

2.1 – STABILITY OF THE *DATA* – TO – *RESULT* MAP

Regular Cauchy Problem, begin



Target:

$$\| s - r \|_Z \leq c_I(D, f, g, h, u, r_0, r_1, a_L, a_H)\| w - v \|_V$$

2.1 – STABILITY OF THE *DATA* – TO – *RESULT* MAP
Regular Cauchy Problem, end

Hp. (additional, w.r. to uniqueness statement)

$\| f \|_1, \| g \|_1 \leq c_s$;

$| u_i' | , | v_i' | \leq c_P$, $i = 1, 2$ (noiseless potentials only)

$| u_i' | \geq c_M > 0$ ⟵ # uniqueness from *singular* Cauchy problem

$| v_2' u_1' - v_1' u_2' | , | w_2' u_1' - w_1' u_2' | \geq \frac{1}{c_T}$;

$a \leq a_H$ ⟵ No such constraint for b !

$\left| \frac{r_1 - r_0}{x_1 - x_0} \right| \leq c_r$; $\| h \|_1 \leq c_h$

Th. 2.1 *i*) $Z = W^{1,\infty}(D)$; $V = \{ w_i' - v_i' \mid i = 1, 2 \}$

ii) (uniform estimate)

$$\| s' - r' \|_\infty \leq C_0 (C_1 \Delta_{21} + C_2 \Delta_{21}^2)$$

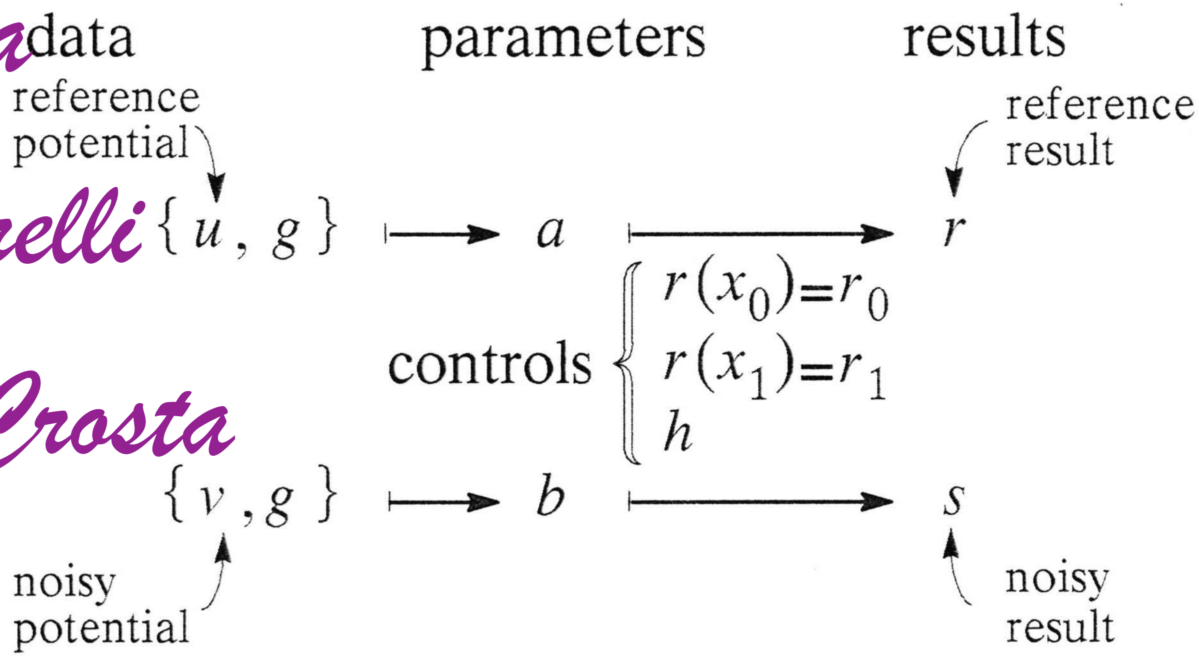
$$\Delta_i := | w_i' - v_i' | , i = 1, 2 \; ; \; \Delta_{21} = 2 c_P^3 c_s c_T^2 (\Delta_2 + \Delta_1)$$

$$C_0 = \frac{a_H}{a_L^2 c_M} \left(c_r + \frac{c_h}{a_L} \right); \; C_1 = 1 + \frac{a_H}{a_L} \; ; \; C_2 = \frac{1}{a_L c_M}$$

iii) ($W^{1,\infty}$ – norm estimate)

$$\| r - s \|_{1,\infty} \leq (1 + | x_1 - x_0 |) \| s' - r' \|_\infty$$

2.2 – STABILITY OF THE *DATA* – TO – *RESULT* MAP:
Singular Cauchy Problem, begin



2.2 – STABILITY OF THE *DATA* – TO – *RESULT* MAP
Singular Cauchy Problem, end

Hp. (additional, w.r. to uniqueness statement)

$\| \frac{1}{u'} \|_1 \, , \, \| \frac{1}{v'} \|_1 \leq c_v$ ⟵ major difference w.r. to *uniqueness* from *independence*)

Th. 2.2 *i*) $Z = W^{1,1}(D)$; $V = W^{1,\infty}(D)$

ii) (estimate, $E_u = E_v = \{ x_u \}$)

$$\| s' - r' \|_1 \leq 4 \frac{a_H^2}{a_L^2} c_v (c_r + \frac{c_h}{a_L}) \| v' - u' \|_\infty$$

c_r , c_h , a_H , a_L : usual meaning.

Rem.

Given the two point BV *control* pbm, $\| s' - r' \|_1$ is equivalent to the natural norm of $W^{1,1}$.

2.3 – STABILITY OF THE *DATA* – TO – *RESULT* MAP
Summary

Independent data =>

=> *regular* Cauchy problem for $a \cdot\exists\cdot (au')' =_{\text{a.e.}} f$ =>

=> L^{∞}– estimate for the *data* – to – *parameter* map.

Stationary potential e.g., $E_u = \{ x_u \}$ =>

=> *singular* Cauchy problem =>

=> L^1– estimate.

The *parameter* – to – *result* problem is well – posed.

The stability of the composite map inherits the stability of the *data* – to – *parameter* map.

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