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Macroeconomic Perspectives on Sectoral Composition: Networks and Inequality

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Abstract

This dissertation investigates how the concept of elasticity of substitution – the degree to which factors of production can replace one another – and economic complementarity shapes sectoral interdependence, the propagation of shocks, and the distribution of labour income. Across two independent yet conceptually connected papers, I show that economic interdependencies act as the structural link between micro-level production organization and macroeconomic outcomes.

The first paper, *The Horizontal Geometry of Production Networks*, investigates structural interdependencies across sectors involved into a networked productive system. It develops a novel framework that extends beyond vertical supply chains the mechanism through which an idiosyncratic and independent shock propagates across networked economies. By introducing the concept of "horizontal geometry", capturing the network "economic" distances between sectors in terms of resembling Input-Output structures, the analysis demonstrates that shared upstream and downstream linkages generate horizontal complementarities, thereby reshaping patterns of sectoral comovement. Shock's transmission across the network emerges from the interaction between vertical supply chains and horizontal interdependencies: nearby sectors tend to move in opposite directions as horizontal transmission prevails, while distant sectors comove as vertical propagation dominates. At the aggregate level, horizontal propagation generates synchronized adjustments among sectors linked through common suppliers or customers, yielding persistent aggregate fluctuations even without vertical cascades or reliance on highly central sectors typically emphasised in vertical transmission mechanisms.

The second paper, *Industry Contribution to U.S. Wage Inequality*, investigates structural interdependencies across factors of production. It examines how evolving elasticities of substitution between technological capital, routine and non-routine workers drive the dynamics of real wage dispersion. Using a structural general equilibrium model estimated on U.S. industry data, I distinguish between a "quantity effect" – industry-level changes in factor endowments – and a "structural effect" – industry-level variations in substitution elasticities. The analysis shows that heterogeneous changes in elasticity parameters account for more than 90% of between-industry dispersion in wages, whereas variations in production input quantities own a fairly minor role. Rising labour market concentration is an amplifier of inequality. Overall, wage inequality primarily emerges from the asymmetric evolution of production technologies and sectoral complementarities rather than from factor accumulation alone.

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Introduction

The *fil rouge* connecting the two papers of this doctoral research lies in the study of how the concept underlying the *elasticity of substitution* generates economic interdependencies and governs both the diffusion of micro-level shocks and the distribution of wages in contemporary economies. The elasticity of substitution captures the degree to which production factors (such as intermediate inputs, capital types, or categories of labour) can replace one another when relative prices or technologies change. Far from being a purely technical parameter of production theory, it has evolved into one of the most powerful concepts linking microeconomic structures to macroeconomic outcomes: it represents a fundamental aspect through which the organization of production, the nature of technological change, or the propagation of shocks become connected with the aggregate behaviour of the economic system.

Throughout the thesis, sector-specific complementarities serve as the principal mechanism through which elasticities of substitution influence economic adjustment and interdependence. While I will return later in this introduction to present the two papers in detail – the first addressing production networks, the second focusing on labour market inequality – it is instructive to briefly highlight how substitution elasticities play a unifying role across both contributions.

Recent advances in the production network literature – where sectors rely on intermediate inputs purchased from other ones, creating a dense web of inter-sectoral trade linkages – have shown, most notably in Baqaee and Farhi (2019), that complementarities in intermediate inputs can generate counterintuitive propagation patterns. Specifically, when the production of a sector depends on inputs that are difficult to substitute, a positive, sector-specific shock can induce negative spillover effects on other sectors connected through vertical supply chains. In other words, independent and idiosyncratic shocks may produce asymmetric and negative comovements across the network, an effect that fundamentally relies on non-linearities in production, driven by the elasticity of substitution among intermediate inputs.

The first paper of this thesis theoretically discusses and empirically demonstrates that a similar negative comovement can emerge even within a linear production framework by exploiting what I term the *horizontal geometry* of an Input-Output economy. When sectors share similar sets of upstream suppliers or downstream buyers, their production activities become interdependent in two important ways. First, sectoral complementarities in production arise linearly from the observed similarities in network relationships. Second, these horizontal complementarities allow idiosyncratic shocks to propagate not only along traditional vertical supply chains, but also across sectors that interact with the same set of suppliers or buyers, also if completely disconnected one another. In this setting, competition for shared downstream customers or substitution among common upstream inputs can generate negative comovement, such that expansion in one sector coincides with contraction in another. This mechanism is formalized through measures of "network economic distance", which quantify the similarity of sectors' Input-Output linkages in terms of demand and supply. Analysis of these distances reveals a clear pattern: sectors that are "close" in the horizontal geometry of production tend to move in opposite directions, while more "distant" sectors comove as the standard vertical propagation dominates.

While the first paper endogenously and linearly captures sectoral complementarities across factors of production, the second paper on labour market outcomes in the United States (U.S.) demonstrates the importance of time-varying elasticities of substitution between technological capital and distinct types of labour (routine and non-routine job tasks) in shaping the evolution of real wage inequality. The structural estimation of the model I built allows to clearly separate between a "quantity effect" (industry-level changes in the composition of capital and labour types) and a "structural effect" (industry-level differences in how easily production inputs can be substituted) of technological change. Counterfactual analysis reveal that heterogeneous trends in substitution elasticities account for the bulk of observed between-industry wage variance, highlighting the critical role of technological structure – rather than merely factor accumulation – in driving wage inequality. In contrast, holding these elasticities constant reduces the explanatory power of the model to changes in input quantities alone, which account for only a minor fraction of wage dispersion. Henceforth, as the elasticities of substitution evolve heterogeneously across industries, they account for much of the observed inequality in wage dynamics, translating structural heterogeneity into distributional outcomes.

Both studies I propose reveal a unifying logic: economic interdependencies, whether across sectors or across factors of production, act as the fundamental structural force through which contemporary economies absorb, amplify, and redistribute technological and asymmetric shocks.

The conceptual roots of the elasticity of substitution traced back to the early Oxbridge's debates in capital theory during the 1930s. Its logic emerged from an intellectual and ideological exchange between John Richard Hicks (raised at University of Oxford) and Joan Robinson (raised at University of Cambridge), who sought to clarify how technological substitution shaped income distribution and the marginal productivity of capital. Robinson's *Economics of Imperfect Competition* (1933), and Hicks's *Theory of Wages* (1932) aimed to move beyond the fixed-coefficient production functions, implicit in classical theory. The debate centred on whether a change in relative factor prices – induced, for instance, by capital accumulation – necessarily altered the capital-labour ratio and, consequently, the distribution of income. Hicks introduced the elasticity of substitution as a quantitative measure of the responsiveness of production factors' proportions to changes in their relative prices, providing a formal mechanism through which technological substitution could explain variations in factor shares. Robinson, while sympathetic to the intuition, questioned the empirical relevance and the aggregation properties of the concept. This exchange established the elasticity of substitution as a central analytical bridge between technology and distribution, foreshadowing its later formalization in neoclassical production theory.

Historically, the role of substitution elasticities has evolved alongside macroeconomic thought. In early general equilibrium models, notably the Cobb-Douglas framework, substitution possibilities were assumed to be constant and uniform across factors of production, implying that aggregate outcomes depended primarily on quantities of capital and labour. The introduction of the *Constant Elasticity of Substitution (CES)* production function by Arrow, Chenery, Minhas, and Solow (1961) marked a turning point, allowing for non-linear heterogeneity in substitution patterns and opening the door to a richer understanding of technological change and factor shares. Subsequent research, especially within an economic growth perspective (*e.g.*, Uzawa 1962, de La Grandville 1989), further demonstrated how variations in elasticities could account for long-term growth differences and shifts in income distribution.

In the 1980s and 1990s, the concept of substitution possibilities became central to analyses of *Skill-Biased Technological Change (SBTC)* and wage inequality. Influential contributions by Katz and Murphy (1982), Autor, Katz, and Kearney (2006), and Acemoglu and Autor (2011) emphasized that technological progress alters the substitutability between skilled and unskilled workers, or between routine and non-routine tasks, reshaping the labour market by increasing the relative demand for skilled labour, hollowing out the middle-skilled class. Although still constant, these reinterpretations reframed the elasticity of substitution not merely as a static production parameter, but potentially as a structural force driving labour reallocation.

Recently, attention has turned to *networked economies*, where production is organized through complex webs of inter-sectoral trade linkages. Foundational contributions by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Baqaee and Farhi (2019) formalized their underlying macroeconomic logic. In these systems, elasticities of substitution determine how shocks spread across sectors along vertical supply chains (*i.e.*, a micro-originated shock transmits upstream or downstream): if intermediate inputs are easily substitutable, idiosyncratic shocks propagate evenly; if complementarities are strong, they generate negative comovement across sectors.

In Chapter 1, my first single-author research on production networks explores how the horizontal structure of inter-sectoral trade, extending beyond vertical supply chains, governs the propagation of shocks across the economy. By embedding complementarities in intermediate inputs within a tractable *horizontal geometry*, my work shows how shared (upstream and downstream) sectoral interconnections determine the direction of comovement between sectors in Input-Output economies. In such paper, serving as my Job Market Paper, titled ***The Horizontal Geometry of Production Networks***, I develop a novel analytical framework that advances the production networks literature by theoretically introducing and empirically validating the horizontal geometry of Input-Output economies. Unlike standard models that view inter-sectoral propagation as occurring only along vertical supply chains (*e.g.*, Long and Plosser 1983, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi 2012), my approach reveals that sectors are also connected by horizontal complementarities generated through similarities in their Input-Output structures. I formalize this by defining new measures of "network economic distances" between sectors based on shared upstream suppliers and downstream buyers relationships that complements the classical vertical propagation channel and yields richer predictions on how shocks diffuse across sectors. Demand-based distances overlap with traditional vertical linkages, dampening the classical amplification of shocks through shared input demand, while supply-based distances introduce competition effects among sectors selling to common downstream buyers, generating negative comovement through relative price effects and revenue reallocation. As for aggregate dynamics, horizontal propagation induces correlated adjustments across sectors sharing suppliers or buyers, producing persistent aggregate comovement even without vertical cascading effects and central sectors, crucial for standard network models. Importantly, horizontal mechanisms operate independently of direct trade intensity, uncovering a complementary dimension of shock transmission not captured in standard models. Patterns of sectoral comovement thus hinge on the network economic distance between sectors: nearby sectors move in opposite directions due to shared Input-Output relations, whereas distant sectors tend to comove as the standard (vertical) propagation prevails. These distance-based mechanisms operate independently of direct trade intensity, uncovering a new, parallel and complementary dimension of economic propagation of idiosyncratic shocks. Empirically, I validate these mechanisms using U.S. Input-Output and employment data, following the two-stage approach of Barattieri and Cacciatore (2023). I first isolate sector-specific structural shocks via panel Fixed Effects regressions, and then estimate their propagation across sectors using a Local Projections framework that distinguishes demand- and supply-based network distances. The results confirm the theoretical predictions: economically distant sectors exhibit positive comovement, while closely related ones often move in opposite directions determining negative comovement. By unifying vertical and horizontal propagation mechanisms within a multidimensional network framework, the paper provides a new analytical foundation for understanding sectoral comovement, and potentially the origins of aggregate fluctuations. More broadly, it linearly endogenizes intermediate-input complementarities and translates them into an empirically measurable form, offering a tractable framework that can be readily applied to study the Keynesian mechanism (*e.g.*, Guerrieri, Lorenzoni, Straub and Werning 2022), fiscal and trade policies transmission in networked economies.

In Chapter 2, my second single-author research on wage inequality due to sectoral composition examines how the interaction of labour-side task polarization and industry-specific production structures drives between-industry wage dispersion in the U.S. economy. The paper, titled ***Industry Contribution to U.S. Wage Inequality***, is the first in the literature to capture the role of industrial composition on widening wage inequality in U.S. I combine the employment polarization view (*e.g.*, Autor, Katz and Kearney 2006, Acemoglu and Autor 2011) – emphasizing technological change and the reallocation of workers across job tasks – with the industry dimension (*e.g.*, Haltiwanger, Hyatt and Spletzer 2024) – which highlights sector-specific production structures –, to argue that inequality stems not solely from differences in factor accumulation, long emphasized by the Skill-Biased Technological Change (SBTC) framework, but from evolving heterogeneous elasticities of substitution across sectors. Empirically, I document that industries exhibiting the largest wage growth are those with substantial ICT adoption and a rising share of non-routine workers relative to routine workers, while industries with smaller wage growth show slower technological uptake and more stable task composition. These patterns suggest that observed wage dispersion arises not solely from differences in factor accumulation, but from structural heterogeneity in industry-level technologies interacting with labour-side transformations. To formalize these mechanisms, I build a general equilibrium model of industries differing in capital-labour substitution elasticities and labour market concentration. Employment polarization is captured through substitution between routine and non-routine labour, while the industry dimension is modelled via cross-sector differences in the evolution of production technologies. Industry-specific elasticities determine how identical growth in capital and labour inputs translates into divergent wage outcomes, rendering observed inequality primarily structural rather than quantitative. I estimate the model on 3-digit U.S. NAICS industry data (2003-2022), combining ICT adoption, labour composition, and industry wage distributions. Counterfactual simulations disentangle the sources of wage inequality into two channels. The *structural effect* (industry-specific evolution of substitution elasticities and task reorganization) accounts for over 90% of observed between-industry wage dispersion, capturing how asymmetric technological change and labour reallocation shape wage outcomes. By contrast, the *quantity effect* (changes in ICT capital, routine and non-routine labour, and sectoral productivity) explains less than 15% of observed inequality. Task-based substitution emerges as the dominant transmission channel, while labour market concentration (workers' sorting and segregation across sectors) amplifies structural disparities between sectors. I further examine the potential role of wage markdowns (*monopsony power*) in shaping observed inequality: allowing firms to set wages below the marginal product of labour produces patterns of between-industry wage dispersion broadly similar to the competitive benchmark, indicating that monopsony effects are quantitatively minor relative to structural transformations (*e.g.*, Card, Rothstein and Yi 2024). The results demonstrate that while SBTC is necessary to rationalize the broad patterns of employment polarization, it is insufficient to explain the observed magnitude of wage dispersion. What ultimately drives contemporaneous wage inequality is the asymmetric evolution of sectoral production technologies and structural transformations in the labour market. Integrating both employment polarization and industry-level heterogeneity provides a unified framework for understanding cross-industry wage inequality in the U.S. economy.

Taken together, these two studies highlight how *elasticities of substitution are* *not static technological parameters but endogenous structural features* that determine how economies absorb and transmit shocks, and redistribute resources. By embedding complementarities within the horizontal geometry of production networks and by exploring industry-specific technologies, my thesis presents a framework for understanding how micro-level substitutability translates into macroeconomic propagation and inequality. The economic interdependence as a bridge between network, technology, and distribution forms the conceptual foundation of this doctoral thesis.

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Chapter 1

The Horizontal Geometry of Production Networks

by Valerio Dionisi*

Abstract

Modern economies are organized as dense production networks, yet sectoral comovement is typically understood only through vertical Input-Output linkages and cascade effects. This paper argues that this view is incomplete. I show that comovement fundamentally depends on horizontal complementarities: the extent to which sectors share common upstream suppliers or downstream buyers. These similar demand and supply network structures induce simultaneous responses to idiosyncratic shocks, shaping the sign of comovement. I develop a theoretical framework that disentangles vertical from horizontal transmission, offering a unified view of comovement rooted in the horizontal geometry of common economic interactions rather than their trade intensity. Unlike vertical propagation, horizontal transmission does not rely on cascading amplification or dominant sectors for aggregate fluctuations, but generates systematic comovement that prevents shocks from averaging out. Using U.S. Input-Output data, I show that sectoral employment responses vary systematically with demand and supply distances, with nearby sectors exhibiting dampened or opposing responses and distant sectors comoving positively.

Jel: C67, D57, E32, J21

Keywords: Input-Output economy, production networks, network distance, horizontal transmission, sectoral comovement, aggregate fluctuations

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1.1. INTRODUCTION

An hallmark of contemporary production systems is the comovement of economic activity across sectors. Recent advances in the production networks literature have demonstrated how independent, idiosyncratic shocks to a central sector can propagate through Input-Output linkages, generating sectoral adjustments in production and contributing to aggregate fluctuations. The presence of "(anti-)cascade effects" diffuses the shock along parallel supply chains via the tangle of Input-Output connections that tie sectors together.[1] In essence, a positive sectoral shock transmits positively and vertically to other network participants, thereby inducing economic variables across different sectors to move in response.[2]

One contribution of this paper is to theoretically assert that comovement depends crucially on the nature of *network economic distance* between sectors. Beyond vertical linkages, sectors can be additionally tied by demand and/or supply interdependencies, contingent upon their common structure of inter-sectoral trade. Specifically, sectors exhibits *factor input demand* network distance when buying intermediate inputs from a similar set of upstream suppliers, whereas *factor input supply* network distance materializes whenever sectors sell part of their production as an intermediate input to a similar set of downstream buyers (*e.g.*, Conley and Dupor 2003).

Under this perspective, an Input-Output structure not only reflects *vertical* complementarities in propagating shocks (*i.e.*, the intensity of the comovement between two sectors depends on their intensity of being interconnected; refer to Shea 2002), but also *horizontal* complementarities as sectors are additionally held together by structural demand and supply relations from having in common a resembling Input-Output geometry. While vertical propagation mechanisms are broadly utilized, this paper explores how horizontal interdependencies and network economic distances affect comovement across sectors participating in the production network, with theoretical insights tested on sectoral U.S. employment data.

On the empirical validation, a number of studies have documented the presence of sectoral comovement in employment levels (*e.g.*, Cooper and Haltiwanger 1990, Christiano and Fitzgerald 1998, Yedid-Levi 2016).[3] Yet, much of this evidence treats sectors as isolated units and gives limited attention to the role of inter-sectoral link-

[1] Central insight is that a positive shock to a sector enhances its productive performance, triggering: *(i)* a *downstream effect*, as other sectors have an incentive to expand their demand from that sector; *(ii)* an *upstream effect*, as higher production in the shocked sector raises its demand of intermediate inputs, benefiting its suppliers; and *(iii) network spillovers*, where the positive effects ripple beyond immediate trading partners, benefiting indirectly connected sectors. On the verticality of Input-Output economies refer to Pasinetti (1973).

[2] Yet, once complementarities in intermediate inputs constrain substitution possibilities, the transmission of shocks proves less straightforward, since non unitary elasticities of substitution limit the ability to adjust input combinations (*e.g.*, Corsetti et al. 2008, Atalay 2017, Baqaee and Farhi 2019).

[3] Sectoral activity and comovement are not separate phenomena but overlapping features. Various papers suggest how business cycles are driven by micro-level shocks rather than aggregate ones (*e.g.*, Long and Plosser 1987, Foerster et al. 2011, Gabaix 2011, Moro 2012, Garin et al. 2018), and that a synchronization between changes in output and employment, either at aggregate or sectoral level, is occurring (*e.g.*, Quah and Sargent 1993, Hornstein and Praschnik 1997, Stock and Watson 1999, Rebelo 2005, Barrot, Grassi, et al. 2021).

ages. Understanding how these Input-Output relationships shape sectoral comovement remains an open question, and there is a lack of empirical research specifically examining sectoral employment comovement within the context of a production network. Another contribution of this paper is to attempt at filling this empirical void.

From a network perspective, comovement is endogenous: sectors respond to one another because their production decisions are shaped by shared economic relationships. Aggregate fluctuations therefore reflect not only how shocks are amplified, but also how the structure of the production network shapes sectoral adjustments. When a shock occurs, it propagates vertically, cascading from suppliers to customers through input requirements, and generating amplification along supply chains. This vertical mechanism produces sequential effects that depend strongly on sectoral importance (*e.g.*, Acemoglu, Carvalho, et al. 2012, Carvalho 2014). At the same time, shocks also propagate through horizontal complementarities, as sectors sharing similar buyers or suppliers adjust simultaneously to common cost or supply conditions, even without direct Input-Output connections. These horizontal responses do not generate cascades, but instead induce systematic comovement across sectors. Even when individually small – so that no single sector is economically dominant –, such coordinated adjustments prevent shocks from averaging out, allowing aggregate fluctuations to emerge from the network's horizontal geometry itself.

Indeed, building a coherent theoretical framework that emphasizes the role of demand- and supply-driven network distances in shaping sectoral comovement within an Input-Output economy, and validating its main predictions using sector-level U.S. data, constitute the two central contributions of this paper.

Section 1.2 conceptually introduces the two complementary measures of demand and supply Input-Output economic distances. On the input side, sectors that buy from the same suppliers reveal simultaneous adjustments as they rebalance labour and intermediates. On the output side, sectors selling to the same buyers show coordinated relative price adjustments, and thus revenue shifts when downstream demand changes. These horizontal complementarities also connect disconnected nodes: as established by Theorem 1 even if sectors don't directly trade, the existence of a common neighbour sector makes them economically connected.

Section 1.3 interprets the horizontal transmission of asymmetric shocks. Under demand linkages, opposite comovement arises only when intermediate inputs in downstream buyers cannot be completely substituted away – a positive shock to one sector increases the price of a common supplier, and other downstream buyers with limited substitutability reduce demand. By contrast, in the supply-based case, opposite comovement is immediate and mechanical, as a sector capturing demand from a shared downstream buyer reduces activity among competing suppliers after a positive shock that effectively lowers its relative price. In this sense, demand linkages make comovement ambiguous and dependent on substitution patterns in downstream demand, while supply linkages make comovement transparent and directly tied to upstream competition for downstream markets.

In developing this perspective, a central theoretical result (Theorem 2) emerges:

measures of network "economic" distances are not themselves weighted by the Input-Output structure. In other words, sectoral distances in terms of common demand or supply relationships retain an independent force, distinct from the intensity of inter-sectoral trade flows. This separation strengthen the importance of horizontal complementarities between sectors. In addition, Theorem 3 formalizes how, for very large networks, horizontal effects from shared buyers and suppliers proliferate, making horizontal spillovers to dominate shock transmission over vertical chains.

Building on these effects, the analysis turns to employment comovement and its role on aggregate cycles. Because employment adjusts along both vertical and horizontal linkages, its comovement reflects how network distances determine both demand- and supply-side mechanisms in shaping the magnitude and sign of sectoral responses. Nearby sectors move in opposite directions due to shared Input-Output relations, causing opposing employment responses that dampen aggregate fluctuations, whereas distant sectors tend to comove as the vertical propagation prevails, generating broadly aligned employment movements. Since aggregate GDP emerge as the sum of many adjustments in sectoral employment, horizontal interactions govern whether shocks cumulate into economy-wide behaviours. Theorem 4 then formalizes that horizontal network distances alone can generate persistent aggregate fluctuations, even in the absence of dominant sectors or vertical propagation. With many sectors sharing upstream or downstream connections, shocks propagate along these common buyers and suppliers, inducing mutual responses across the network that accumulate at the aggregate level. Vertical mechanism is inherently amplifying: shocks transmit along production chains as input adjustments feed into upstream/-downstream sectors, generating cascading effects and aggregate responses. Differently, horizontal propagation induces comovement across sectors, but does not generate recursive feedback loops and cascading amplification.

As a validation of theoretical insights, the role of network distances on sectoral comovement is empirically tackled on U.S. employment and Input-Output data, discussed in Section 1.4. Section 1.5 implements the two-stage approach in Barattieri and Cacciatore (2023): first, sector-specific structural shocks are isolated by extracting residuals from panel Fixed Effects (FE) regressions; second, these identified shocks are used in a panel Local Projection (LP) analysis to estimate how employment changes in one sector affect others, distinguishing effects for both demand and supply network distances. The empirical results are threefold. *Firstly*, an increase in the set of intermediate inputs specific to a sector leads to a comparatively smaller rise in its employment with minimal, but alternate, effects from similar changes in more distant sectors. *Secondly*, sectors closer under factor input supply distance exhibit opposite employment comovement: an increase in employment in nearby sectors tends to reduce employment in the sector; for closely demand-linked sectors, the responses are often ambiguous (increases for some, reductions for others), but mostly pointing to a dampening effect on the vertical shock's transmission. *Thirdly*, positive comovement emerges among more distantly demand- and supply-related sectors, since sectoral employment rises in response to increases in more distant sectors.

To conclude, Section 1.6 examines the policy implications of the network's horizontal geometry and how incorporating it can improve the design of policy interventions.

Overall, these findings provide empirical support for the main theoretical predictions: horizontal complementarities in terms of similar demand and supply Input-Output relationships across sectors complement and balance the vertical transmission of sector-specific shocks, thereby revealing the multiple channels through which vertical and horizontal structures of sectoral interdependencies shape the distribution and the propagation of economic activity across networked economies.

Literature.– The objective of this paper is to advance both theoretically and empirically the understanding of how Input-Output economies work. On the theoretical side, it builds on the modern and rapidly expanding literature on production networks. In the wake of Gabaix (2011)'s "granular hypothesis", the seminal observation of Long and Plosser (1983) – that comovements across sectors are not dictated by a shared disturbance but rather by sectoral interdependencies –, has been modernized through a series of papers of the early 2010s (Acemoglu, Carvalho, et al. 2012, Carvalho and Gabaix 2013, Carvalho 2014, Barrot and Sauvagnat 2016). This renewed perspective gave rise to studies on production networks related to efficient economies (*e.g.*, Baqaee and Farhi 2019, vom Lehn and Winberry 2022, Liu and Tsyvinski 2024), monetary policy and nominal rigidities (*e.g.*, La'O and Tahbaz-Salehi 2022, Rubbo 2023, Ghassibe and Nakov 2025), inefficiencies (*e.g.*, Jones 2011, Grassi 2017, Baqaee 2018, Baqaee and Farhi 2020), policy-oriented issues (*e.g.*, Liu 2019, Grassi and Sauvagnat 2019, Lane 2025), endogenous network formation (*e.g.*, Oberfield 2018, Acemoglu and Azar 2020, Ghassibe 2024, Taschereau-Dumouchel 2025), economic growth (*e.g.*, Hausmann and Hidalgo 2011, Gualdi and Mandel 2019, McNerney et al. 2022), and international contexts (*e.g.*, Caliendo et al. 2022, Qiu et al. 2025, Huo et al. 2025). A common mechanism explored in these works emphasizes the vertical propagation of sectoral variations, where shocks originating in upstream sectors transmit downstream, amplifying their effects and contributing to aggregate outcomes.[4] My contribution broadens this perspective by introducing horizontal linkages between sectors, defined by their network "economic" distances as measured by common upstream or downstream Input-Output relationships. These novel interdependencies forge horizontal complementarities that bind even disconnected sectors, allowing asymmetric shocks to propagate across the network in ways beyond traditional vertical supply chains.

On the empirical side, relatively few studies examine the role of sectoral production networks on macroeconomic outcomes (*e.g.*, Acemoglu, Akcigit, et al. 2015, Ghassibe 2021, Barattieri and Cacciatore 2023, Barattieri, Cacciatore, and Traum

[4] As noted by Shea (2002), a (positive) sectoral shock has a price (decrease in its price level, and reduction in its nominal expenditure for intermediate inputs) and a quantity (its supply to other sectors increase, sector's production rises, thereby increasing its input demand) effects. These opposing forces affect upstream suppliers but, under Cobb-Douglas production function, they exactly offset each other and leaving intermediate inputs demand unchanged, thereby only affecting downstream suppliers. In general, only demand shocks can propagate upstream (*e.g.*, Acemoglu, Akcigit, et al. 2015, Barrot and Sauvagnat 2016, Ferrari 2024).

2023, Monti and Van Keirsbilck 2025). From this standpoint, the paper contributes novel empirical evidence on the role of Input-Output linkages in shaping sectoral dynamics, particularly in explaining the comovement of employment across sectors. Synchronized employment patterns are an important characteristic of business cycles, as most sectors tend to move together over time (*e.g.*, Christiano and Fitzgerald 1998). Several studies explore the nature and the sources of this sectoral comovement.[5] Cooper and Haltiwanger (1990) find that sectoral employment levels are positively correlated, in ways not fully explained by aggregate shocks. Cassou and Vázquez (2014) attribute high employment comovement to similar shock transmission channels. In Yedid-Levi (2016) comovement is stronger along the extensive margin (number of workers) than the intensive margin (hours per worker). Room for structural changes is in Kim (2020), where positive technology shocks in manufacturing raise employment in both manufacturing and services. While these studies provide valuable insights, they do not explicitly address how production networks shape employment comovement. By contrast, my paper focuses on short-run employment variations within a networked framework. Empirically, the paper contributes by *(i)* advancing the literature on the comprehension of Input-Output linkages and shocks' transmission via network "economic" distances, and *(ii)* providing unprecedented evidence on sectoral employment comovement from a production network perspective.

1.2. Foundational Theory

Preliminaries.– The economy is populated by a finite set of sectors, $\{s, s', s'', \dots, m\} \in \Phi(s)$, and denote by Φ_s the set of all sectors not including sector-s. Each produces a single good using either labour and a set of circulating intermediate inputs from other sectors. The whole economic system is thus represented by an $m \times 1$ vector of sectoral outputs, $\boldsymbol{Y} = [y(s) > 0]$, an $m \times 1$ vector of sectoral employment levels, $\boldsymbol{N} = [n(s) > 0]$, each associated to its output elasticity of labour $\alpha(s) \in (0, 1)$, and by an $m \times m$ square matrix, $\boldsymbol{X} = [x(s, s') \geq 0]$, indicative of the inter-sectoral trade of intermediate input quantities, with $x(s, s') = 0$ stating that sector-s does not make use of the good produced by sector-s' in producing its own good. Finally, an $m \times m$ square matrix, $\boldsymbol{H} = [\alpha(s, s') \geq 0]$, defines the intensity of the goods produced by sectors as intermediate inputs in other sectors. Denote its Leontief inverse transformation as $\boldsymbol{\mathcal{H}} = (\boldsymbol{I} - \boldsymbol{H})^{-1} = [\ell(s, s') \geq 0]$, with $\boldsymbol{I}$ being an $m \times m$ identity matrix.

Under these specifications, a perfectly competitive final good producer combines outputs from sectors, $Y = \prod_{s \in \Phi(s)} y(s)^{\beta(s)} \backsimeq \boldsymbol{\beta}' \boldsymbol{Y}$, where $\boldsymbol{\beta}$ is a $1 \times m$ vector of $\beta(s) \in (0, 1)$, a parameter governing the relative importance of each sectoral good in the definition of aggregate output Y. Total consumption is an aggregator over sectoral consumption: let $\boldsymbol{C} = [c(s)]$ be an $m \times 1$ vector of final consumption levels, with generic element $c(\cdot) > 0$, it holds that $C = \prod_{s \in \Phi(s)} c(s)^{\beta(s)} \backsimeq \boldsymbol{\beta}' \boldsymbol{C}$.

Within a unit mass, household-i is getting utility from consumption of all sectoral

[5] Theories beyond employment comovement are in Rogerson (1987) and Boldrin et al. (2001).

goods and leisure dis-utility from working in a given sector. Utility function is

$$\mathcal{U}_i := f^u\Big(\{c_i(s)\}_{\forall s\in\Phi(s)}\,,\,n_i(s)\Big)$$

with individual sectoral consumption and labour supplied characterized by a price, $p(s)$ and $w(s)$, respectively, the latter chosen as the numeraire.

A representative firm characterizes each sector, producing its own good, $y(s)$, using either a set of intermediate inputs bought from other sectors, $x(s,s')$, $\forall s'\in\Phi(s)$, and labour force, $n(s)$. Production function in sector-s is then

$$y(s) = z(s)\,f^y\Big(n(s)\ ,\ \{x(s,s')\}_{s'\in\Phi(s)}\Big)$$

where $z(s)$ is an exogenous Hicks-neutral sectoral productivity. Under a general perspective, assume the following regularity conditions hold.

ASSUMPTION 1 (Production technology requirements) *As main characteristics of a production function: (i) it has constant returns to scale in either* $x(s,s'\in\Phi(s))$ *and* $n(s)$*, so that production inputs shares sum to one,* $\alpha(s)+\sum_{s'}\alpha(s,s')=1$*; (ii) it is differentiable, continuous, strictly quasi-concave, homogeneous of degree one, and increasing in* $z(s)$*,* $x(s,s'\in\Phi(s))$*, and* $n(s)$*; (iii) the case* $f^y(0\ ,\ \cdot)$ *is ruled out as labour input is essential to production; (iv) at least two elements of* $x(s,s'\in\Phi(s))$ *have to be positive, thus* $f^y(\cdot\ ,\ 0)$ *cannot exist; consequentially, (v) part of the sectoral output is directly produced by the sector,* $x(s,s)>0$*,* $\forall s\in\Phi(s)$*, that is a production plan with a bundle of intermediate inputs as* $f^y(\cdot\ ,\ x(s,s'\in\Phi_s))$ *is precluded.*

The first two sub-assumptions are common in production technology;[6] the other ones ensure the production function to be consistent with the empirical exploration in Section 1.5. By way of *A1.iii*, workers cannot be fully substituted by any combination of intermediate inputs (so that sectoral output is finite), and changes in intermediates' quantity will determine subsequent variations in the labour force. The assumptions characterizing the intermediate inputs bundle, *A1.iv* and *A1.v*, ensure the sector not only to participate in the network, but also that part of the production process is roundabout: for sector-s delivering its intermediate good $y(s)$ requires to be at least vertically integrated (buying from one upstream sector), while imposing that a fraction of intermediate inputs must be directly produced within the sector.

Each representative firm is seeking to maximize profits in a perfectly competitive environment, and optimality conditions for both intermediate inputs and demanded labour are then continuous function as $x(s,s') = f^y_{x(s,s')}\big(y(s),p(s),p(s')\big)$ and $n(s) = f^y_{n(s)}\big(y(s),p(s),w(s)\big)$, so that their combination delivers

$$x(s,s') = f\Big(\alpha(s,s'),p(s')\ ;\ \alpha(s),n(s),w(s)\Big) \tag{1.1}$$

[6] Constant returns to scale, as well homogeneity of degree one, purely address the technical relation between inputs and output, ruling out any interference of external factors (*e.g.*, relative price changes associated to scalable production, non-homogeneity of degree one due to market dynamics). Continuity and differentiability guarantees well-defined (no "jumps") and smooth marginal products, while strict quasi-concavity ensures convex technologies and unique optimal input choice.

Sector-s's optimal demand for intermediate inputs of sector-$s' \in \Phi(s)$ is increasing in their inter-sectoral trade intensity, $\alpha(s, s')$, and in labour market variables, $w(s)$ and $n(s)$, while it is decreasing in $\alpha(s)$, and in its purchased intermediates' price, $p(s')$. Effectively, the ratio among factors of production depends on their relative intensity in production, and on their relative price.

Distances.– In a networked economy, comovement of production inputs is endogenously induced: each sector both relies on and supplies to others, so a change in one sector's optimal demand spreads through the network, thus reshaping the relation among factors of production within and between sectors. A rigorous comparison of two sectors can be conducted by examining their relation in the production network, specifically in terms of their trading partnerships with the same set of sectors.

Paraphrasing Conley and Dupor (2003), two types of network "economic" distances between sectors, sketched in Panel 1.1a of Figure 1.1, can be defined:

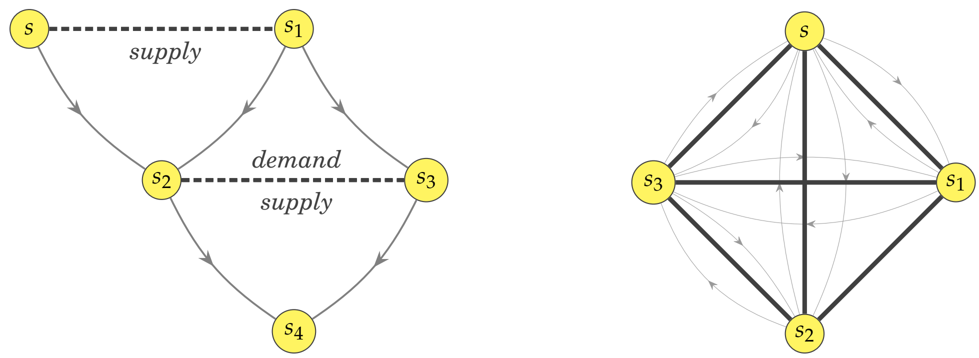
(a) *factor input demand*, when sectors are buying their intermediate inputs from similar upstream sectors;

(b) *factor input supply*, when sectors are selling their production as intermediate inputs to similar downstream sectors.

Under this perspective, connections among sectors extend beyond their flows of intermediate inputs. These additional interdependencies highlight the complexity of inter-sectoral dynamics and complement the *Leontief inverse*, a reminiscent result from Leontief (1936): it captures how sectors not directly trading among each others are indirectly related – *e.g.*, a sector buying from an upstream one introjects such sector's purchases as well the other of more upstream sectors –, thus mainly emphasizing the depth of vertical linkages through layers of Input-Output relationships. Network-based economic distances, by contrast, allow to consider how participants are mapped not only vertically, but horizontally as well: sectors can be also closely related because they are relying on the same suppliers or serving the same customers. These broader notions of sectoral linkages provide a richer understanding of the structure of production networks and help to explain *horizontal complementarities* in demand and supply that classical Input-Output analysis overlook. They reveal how seemingly unrelated sectors can experience synchronized fluctuations, and how the structure of common upstream or downstream connections shapes the propagation of independent and idiosyncratic shocks. In other words, the notion of sectoral interdependencies is not confined to "who trades with whom", but also emerges from "who depends on whom": horizontal complementarities thereby deliver a novel perspective on how sectoral shocks reverberate across a networked economy.

Panel 1.1b sketches a stylized economy associated to its distance-based network: distances effectively double Input-Output linkages and bound disconnected sectors.

EXAMPLE 1 (Distance in the network) *Consider an economy populated by four sectors,* $\{s, s_1, s_2, s_3\} \in \Phi(s)$*, where some trade with all others, while some do not. The resulting production network of Panel 1.1b displays an Input-Output matrix,* **H***, where some cells equal zero. Focus on the pair* $\{s_1, s_2\}$*: they trade with all other sectors*



(A) DISTANCE DEFINITION

(B) DISTANCE NETWORK

FIGURE 1.1: HORIZONTAL INPUT-OUTPUT GEOMETRIES

Note: the figure presents a stylized production network and its corresponding network of sectoral distances. Panel 1.1a illustrates both network distance definitions. Panel 1.1b highlights the horizontal dimension of the network, where solid thick lines connecting all vertexes indicate the distance relationships between sectors; production network linkages are depicted with oriented thin lines, and represent the inter-sectoral trade flows derived from the Input-Output matrix.

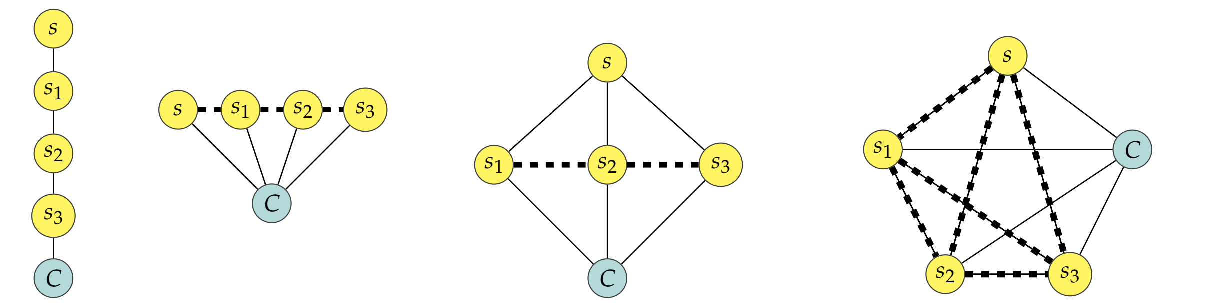
but not with each other, so $\alpha(s_1, s_2) = 0 = \alpha(s_2, s_1)$. *Nevertheless, their linkages with the remaining sectors generate a common structure of inter-sectoral trade. An Input-Output architecture induces a unique geometry of sectoral distances (both in demand and supply), as depicted in Panel 1.1b. Yet, while the underlying trade and distance structures are unique, the horizontal metrics between sectors vary under alternative specifications of network distance, as Examples 2-3 clarify.*

1.2.1. STYLIZED ECONOMIES AND HORIZONTAL GEOMETRY

Before turning to technicalities, I illustrate the economic relevance of the network's horizontal geometry through a set of highly stylized production structures, sketched in Figure 1.2. I consider three benchmark configurations: *(i) sequential economies*, where transmission occurs purely along a vertical chain (sector-by-sector towards the final consumer) or, alternatively, purely horizontally (distinct sectors supplying only the final consumer); *(ii) rhomboid economies*, where sectors depend on a common upstream supplier while simultaneously selling to the final consumer; and *(iii) star economies*, with all sectors mutually connected while serving the final consumer.

Sequential economies.– Consider the "snake" in the left graph of Figure 1.2a. Connections are purely vertical, and asymmetric shocks propagate sequentially along the supply chain, with no scope for horizontal interdependencies. By contrast, in the "spider" on the right graph, sectors sell directly to the final consumer without trading among themselves. This represents a purely horizontal structure, characteristic of standard multi-sector models with no inter-sectoral trade. Supply-based horizontal complementarities are central: even without explicit trade linkages, sectors are tied together through their shared exposure to final demand. A shock to one sector reshapes consumers' expenditure across all others, propagating horizontally through final demand reallocation and generating comovement *absent any network structure*.

Rhomboid economies.– A simple network in Figure 1.2b combines both sequential economies into a unique structure. This configuration blends vertical and hori-



(A) SEQUENTIAL (B) RHOMBOID (C) STAR

FIGURE 1.2: BENCHMARK ECONOMIES

Note: the figure illustrates three benchmark economic systems and their associated horizontal geometry (dashed) highlighting common Input-Output structures (solid). Sectors are in yellow, while the teal circles with C represent the final consumer.

zontal propagation, as shocks travel vertically (via sector-specific supply chain) and horizontally across sectors simultaneously exposed to common supplier and final demand. Rhomboid economies thus epitomize horizontal complementarities, as even sectors not directly trading with each other become tightly synchronized through their shared dependence on upstream and downstream markets.[7]

Star economies.– Figure 1.2c classifies three archetypes. In fully symmetric and balanced star networks, where all sectors buy and sell with identical intensity, network distances collapse to zero: propagation is only vertical, but the network structure becomes irrelevant as sectoral shocks average out (*e.g.*, Lucas 1977, Acemoglu, Carvalho, et al. 2012). In partially symmetric star networks, trade intensities in buying and selling are equal but sector-specific; vertical and horizontal dimensions coexist, though the two distance measures coincide. Finally, under asymmetric star networks (*i.e.*, different trade intensities across sectors) generate parallel vertical and horizontal transmission channels, with comovement emerging endogenously from network distances: horizontal complementarities in production and shared exposure to final demand weave a dense web of horizontal interdependencies, and sectors with differing upstream and downstream connections experience unrelated effects.

These illustrative examples represent extreme cases. In more realistic, highly interconnected systems, the boundaries between these benchmark economies blur: vertical supply chains coexist with horizontal interdependencies through shared buyers and suppliers, and sectors participate simultaneously in multiple propagation layers.

1.2.2. RECOVERING DEMAND AND SUPPLY DISTANCES

Consider the case in which two sectors, say $\{s, s'\}$, are buying intermediate inputs from the same sector, say s^*. In this scenario, by substituting out the combined optimality conditions in eq. (1.1) for such sectors purchasing from the same upstream seller then one obtains, after total *log*-differentiation, that

[7] This structure closely mirrors the configuration underlying the "Keynesian transmission mechanism" (*e.g.*, Guerrieri et al. 2022). The horizontal geometry offers a tractable stylization of this logic, as it formalizes how distance-based complementarities in production and final consumption create systemic interdependencies.

$$d\log\mathcal{B}_{s^*\rightarrow}\left(s'\right) = \frac{\alpha\left(s,s^*\right)}{\alpha\left(s',s^*\right)}\, d\log\mathcal{B}_{s^*\rightarrow}\left(s\right) \tag{1.2}$$

Given $\gamma^{\mathcal{B}}_{(\cdot)} = \frac{1}{\log\overline{x}(\cdot,s^*)}$ and $\delta^{\mathcal{B}}_{(\cdot)} = \frac{\log\overline{n}(\cdot)}{\log\overline{x}^2(\cdot,s^*)}$ being steady-state basis for production inputs, the differential elements $d\log\mathcal{B}_{s^*\rightarrow}\left(s\right) = \gamma^{\mathcal{B}}_{s}\, d\log n\left(s\right) - \delta^{\mathcal{B}}_{s}\, d\log x\left(s,s^*\right)$ and $d_{s^*\rightarrow}\log\mathcal{B}\left(s'\right) = \gamma^{\mathcal{B}}_{s'}\, d\log n\left(s'\right) - \delta^{\mathcal{B}}_{s'}\, d\log x\left(s',s^*\right)$ identify the *log*-difference of employment levels and intermediate inputs in the two sectors buying from the same sector. Subscript "$s^* \rightarrow$" reads as "purchasing from sector-s^*" by sectors in brackets.

Iteratively, the above condition can be rewritten accounting for all sectors to which the pair is buying from:

$$\boldsymbol{\mathcal{F}}\left[s,s'\right] = \left(\frac{\alpha\left(s,s\right)}{\alpha\left(s',s\right)}, \frac{\alpha\left(s,s'\right)}{\alpha\left(s',s'\right)}, \ldots, \frac{\alpha\left(s,m\right)}{\alpha\left(s',m\right)}\right) = \boldsymbol{d}^{fd}\left[s,s'\right]$$

where, given $\mathcal{F}\left[s,s'\right] = d\log\mathcal{B}_{s^*\rightarrow}\left(s'\right) / d\log\mathcal{B}_{s^*\rightarrow}\left(s\right)$, the indicator $\boldsymbol{\mathcal{F}}\left[s,s'\right]$ is a $1 \times m$ vector of changes in the ratio of any difference in production inputs within the pair of sectors, and $\boldsymbol{d}^{fd}\left[s,s'\right]$ is the associated $1 \times m$ vector of their relative intermediate usage intensity when sectors $\{s,s'\}$ are buying from each of the other sectors in the economy, yielding a unique value, $d^{fd}\left[s,s'\right]$. Stacking such condition across all sectors, then the following Lemma holds.

LEMMA 1 (Factor input demand) *Changes in the ratio of production inputs' quantities across any pair of sectors buying from the same sector(s) is driven by their ratio of intermediate intensities with whom the pair is purchasing from:* $\boldsymbol{\mathcal{F}} = \boldsymbol{\mathcal{D}}^{fd}$.

Proof in Appendix A.

In other words, the comovement of inputs of production for a pair of sectors is determined by their distance in the production network. In fact, in Lemma 1, each cell of the $m \times m$ matrix $\boldsymbol{\mathcal{F}}$ displays the differential variation across factors of production between the row-sector and the column-sector, while matrix $\boldsymbol{\mathcal{D}}^{fd}$ is given by

$$\underset{m\times m}{\boldsymbol{\mathcal{D}}^{fd}} = \begin{bmatrix} \left(\frac{\alpha(s,s)}{\alpha(s,s)}, \ldots, \frac{\alpha(s,m)}{\alpha(s,m)}\right) & \left(\frac{\alpha(s,s)}{\alpha(s',s)}, \ldots, \frac{\alpha(s,m)}{\alpha(s',m)}\right) & \cdots & \left(\frac{\alpha(s,s)}{\alpha(m,s)}, \ldots, \frac{\alpha(s,m)}{\alpha(m,m)}\right) \\ \left(\frac{\alpha(s',s)}{\alpha(s,s)}, \ldots, \frac{\alpha(s',m)}{\alpha(s,m)}\right) & \left(\frac{\alpha(s',s)}{\alpha(s',s)}, \ldots, \frac{\alpha(s',m)}{\alpha(s',m)}\right) & \cdots & \left(\frac{\alpha(s',s)}{\alpha(m,s)}, \ldots, \frac{\alpha(s',m)}{\alpha(m,m)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\alpha(m,s)}{\alpha(s,s)}, \ldots, \frac{\alpha(m,m)}{\alpha(s,m)}\right) & \left(\frac{\alpha(m,s)}{\alpha(s',s)}, \ldots, \frac{\alpha(m,m)}{\alpha(s',m)}\right) & \cdots & \left(\frac{\alpha(m,s)}{\alpha(m,s)}, \ldots, \frac{\alpha(m,m)}{\alpha(m,m)}\right) \end{bmatrix}$$

The greater the value of each cell, the closer the sectors are within the production network.[8] This concept of "network proximity" is foundational on the theory side. In

[8] A lower trade intensity ratio in eq. (1.2) implies that a shock to s transmits mildly to s'. For $\alpha\left(s,s^*\right) < \alpha\left(s',s^*\right)$, the resulting change in its relative quantities, $d\log\mathcal{B}_{s^*\rightarrow}\left(s\right)$, affects that of s', $d\log\mathcal{B}_{s^*\rightarrow}\left(s'\right)$, less than proportionally, signaling low horizontal complementarity. Conversely, if $\alpha\left(s,s^*\right) > \alpha\left(s',s^*\right)$, then $d_{s^*\rightarrow}\left[s,s'\right] > 1$, resulting in a more-than-proportional horizontal complementarity. The same logic applies to eq. (1.3).

the empirics, the interpretation is inverted: entries of matrix $\mathcal{D}^{fd}$ are the sum over a sequence of *Euclidean distances* across the elements of the intensity ratios,

$$d^{fd}\left[s,s'\right] = \sqrt{\left[\alpha\left(s,s\right)-\alpha\left(s',s\right)\right]^2 + \left[\alpha\left(s,s'\right)-\alpha\left(s',s'\right)\right]^2 + \cdots + \left[\alpha\left(s,m\right)-\alpha\left(s',m\right)\right]^2}$$

thereby representing a certain network "economic" distance between two sectors, with larger values denoting how dissimilar they are in terms of shared Input-Output structure when buying from upstream sellers. Henceforth, matrix $\mathcal{D}^{fd}$ is reflecting the proper distance in common demand mapping sectors, whose characterizing elements, $d^{fd}\left[s,s'\right] > 0$, $\forall s,s' \in \Phi\left(s\right)$, are symmetric outside the main diagonal (the distance from s to s' is the same as from s' to s), and $diag\left(\mathcal{D}^{fd}\right) = 0$.

EXAMPLE 2 (Factor input demand metrics) *Consider the pair of sectors* $\{s_1, s_2\}$ *in an Input-Output matrix mirroring Example 1, where the common sectors from which both are purchasing are* $\{s, s_3\}$*, and assume fictional trade intensities for sector-*s_1 *to be* $\alpha\left(s_1,s\right) = 0.2$ *and* $\alpha\left(s_1,s_3\right) = 0.1$*, while that of sector-*s_2 *are* $\alpha\left(s_2,s\right) = 0.1$ *and* $\alpha\left(s_2,s_3\right) = 0.2$*. Accordingly, the metrics expressing their factor input demand distance relation is composed of* $\boldsymbol{d}^{fd}\left[s_1,s_2\right] = (.2,.1),(.1,.2)$ *when buying from sector-*s *and sector-*s_3*, respectively. Demand-based distance value is positive even in the absence of direct trade linkage between considered sectors in the pair.*

Consider now the case in which two sectors, say $\{s,s'\}$, are selling their own good as an intermediate input to the same sector, say s^*. In this scenario, by substituting out the combined optimality conditions in eq. (1.1) for such sectors trading to the same downstream buyer, total *log*-differentiation leads to

$$d\log \mathcal{Q}_{\to s^*}\left[s,s'\right] = \frac{\alpha\left(s^*,s\right)}{\alpha\left(s^*,s'\right)}\, d\log \mathcal{P}_{\to s^*}\left[s',s\right] \tag{1.3}$$

Given $\gamma^{\mathcal{Q}}_{(\cdot)} = \frac{1}{\log \overline{x}(s^*,\cdot)}$, $\delta^{\mathcal{Q}}_{(\cdot)} = \frac{\log \overline{x}(s^*,\neq\cdot)}{\log \overline{x}^2(s^*,\cdot)}$, $\gamma^{\mathcal{P}}_{(\cdot)} = \frac{1}{\log \overline{p}(\cdot)}$ and $\delta^{\mathcal{P}}_{(\cdot)} = \frac{\log \overline{p}(\neq\cdot)}{\log \overline{p}^2(\cdot)}$ being steady-state components for intermediate inputs and their associated price levels, then differential elements $d\log \mathcal{Q}_{\to s^*}\left[s,s'\right] = \gamma^{\mathcal{Q}}_{s'}\, d\log x\left(s^*,s\right) - \delta^{\mathcal{Q}}_{s'}\, d\log x\left(s^*,s'\right)$ and $d\log \mathcal{P}_{\to s^*}\left[s',s\right] = \gamma^{\mathcal{P}}_{s}\, d\log p\left(s'\right) - \delta^{\mathcal{P}}_{s}\, d\log p\left(s\right)$ identify the *log*-difference of intermediate inputs and price levels between both sectors selling to the same downstream sector. Subscript "$\to s^*$" reads as "selling to sector-s^*" by sectors in brackets.

Following the previous logic scheme, then the following Lemma must hold.

LEMMA 2 (Factor input supply) *Changes in the ratio of intermediate input profits across any pair of sectors selling to the same sector(s) is driven by their ratio of intermediate intensities with whom the pair is trading to:* $\boldsymbol{\mathcal{R}} = \mathcal{D}^{fs}$.

Proof in Appendix A.

In other words, the comovement across (marginal) revenues of production for a pair of sectors is determined by their distance in the production network. When sectors are selling to the same downstream sector, the quantity sold of sector-s, given a change in those of any sector-s', is determined by the change in their relative prices and scaled by their network intensities in trade.

EXAMPLE 3 (Factor input supply metrics) *Consider the pair of sectors* $\{s_1, s_2\}$ *in an Input-Output matrix mirroring Example 1, where the common sectors to which both are selling are* $\{s, s_3\}$*, and assume fictional trade intensities for sector-*s_1 *to be* $\alpha(s, s_1) = 0.4$ *and* $\alpha(s_3, s_1) = 0.3$*, while that of sector-*s_2 *are* $\alpha(s, s_2) = 0.6$ *and* $\alpha(s_3, s_2) = 0.6$*. Accordingly, the metrics expressing their factor input demand distance relations is composed of* $\boldsymbol{d}^{fs}[s_1, s_2] = (.4, .3), (.6, .6)$ *when selling to sector-*s *and sector-*s_3*, respectively. Supply-based distance value is positive even in the absence of direct trade linkage between considered sectors in the pair.*

An important aspect to consider when introducing distances in the production network is that, under the same configuration of the inter-sectoral trade structure, sectors can be in a different "network economic distance relation" depending on whether they are buying from or selling to other sectors. Moreover, all sectors are simultaneously connected by network distances due to common demand and supply.

THEOREM 1 (On the configuration of network "economic" distances) *Let the set of sectors being* $\Phi(s)$*, and* $\alpha^j(\cdot)$*, for* $j = \{fd, fs,\}$*, the normalized Input-Output fixed coefficients. Then, demand- and supply-based distance matrices satisfy: (i) for any* $s, s' \in \Phi(s)$ *there exists a common neighbour-*s^**, so that both* $\mathcal{D}^{fd}$ *and* $\mathcal{D}^{fs}$ *are non-negative and full; (ii) a given Input-Output structure uniquely determines* $\mathcal{D}^{fd}$ *and* $\mathcal{D}^{fs}$*, which differ entrywise unless the corresponding bilateral trade with the common neighbour coincides; and (iii) if* $\alpha^j(\cdot)$*'s are non-symmetric, then all entries of* $\mathcal{D}^{fd}$ *and* $\mathcal{D}^{fs}$ *differ; conversely, if* $m \geq 4$*, then both share at least one identical entry whenever at least* $m-2$ *sectors trade symmetrically with a common sector and at least one additional symmetric trade relation elsewhere in the network exists.*

Proof in Appendix A.

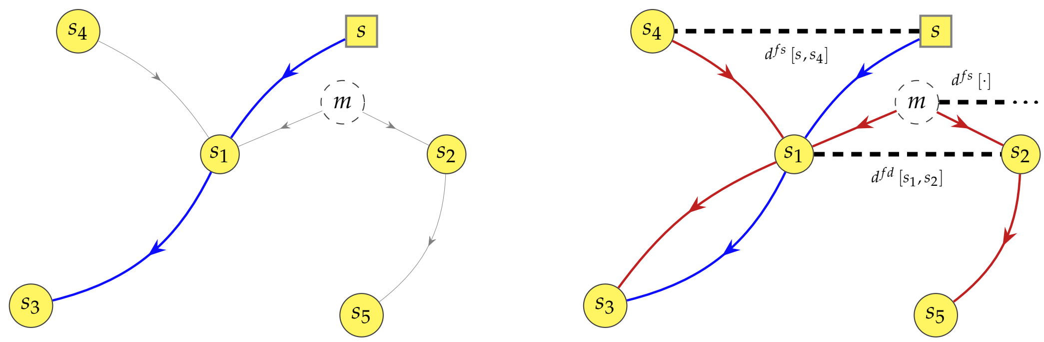
The first two clearly emerge in Examples 2-3 and Figure 1.1. The final property clarifies when the distance between two sectors is uniquely determined: if two sectors buy and sell the same relative quantity of intermediate inputs with a common third sector, then their factor input demand and factor input supply distances coincide. Their actual quantities may differ; what matters is that, for each sector individually, its purchases or sales with the common sector match in relative terms.[9]

COROLLARY 1 (Coinciding distances) *For* $m \geq 4$*, if a common sector trades symmetrically with* $S = m - 2$ *others, then* $\frac{S(S-1)}{2}$ *sectors exhibit symmetric distances.*

1.3. AN INSPECTION INTO THE HORIZONTAL TRANSMISSION

Outlined horizontal geometry introduces new implications for the propagation of idiosyncratic sectoral shocks across the economy. Crucially, the magnitude of such propagation depends not only on the size of (direct and Leontief) connections among

[9] A "normalized" network refers to an Input-Output matrix once rows or columns are scaled by their sum. Entries thus express relative and not absolute values, required to build $\mathcal{D}^{fd}$ and $\mathcal{D}^{fs}$; refer to Appendix B.



(A) VERTICALITY

(B) VERTICALITY AND DISTANCES

FIGURE 1.3: HORIZONTALITY OF A TRANSMISSION

Note: stylized example of how a sectoral shock propagates across a production network. Coloured and oriented bold-curves trace the propagation mechanism: vertical transmission is in blue, while additional transmission paths from the horizontal geometry are in red. Oriented thin-grey curves indicate further existing Input-Output (I-O) connections, and network economic distances through common nodes are represented by dashed lines. The underlying network structure is identical in both panels. Panel 1.3a shows a purely vertical propagation of a shock to sector-s, following standard sectoral supply-chain logics. In contrast, Panel 1.3b depicts a richer mechanism, where the shock also spreads horizontally through sectors sharing common upstream suppliers or downstream buyers: in this case, even sectors not directly connected through the network, or along the sector-s supply chain, are affected by shared demand or supply relationships.

sectors, but also on their network "economic" distance. For instance, as shown in Figure 1.3, a shock originating in a given sector propagates vertically through downstream industries along the supply chain, while concurrently transmitting horizontally across sectors. Notably, this horizontal transmission arises even between sector pairs that are neither directly nor indirectly linked, reflecting their shared demand and/or supply relationships with common upstream or downstream sectors.[10]

EXAMPLE 4 (Propagating behaviour under horizontal distances) *Consider an economy populated by* $\{s, s_1, s_2, s_3, s_4, s_5, m\} \in \Phi(s)$ *sectors, with* m *here representing the rest of the Input-Output network. Some trade with others, while some others do not:* s_1 *is buying from* $\{s, s_4, m\}$ *while selling to* s_3*;* s_2 *trades from* m *and to* s_5*. Suppose a supply shock to sector-*s *originates. In Panel 1.3a, the shock propagates downstream: it directly affects* s_1 *and indirectly (i.e., Leontief effect) reaches* s_3*, yielding a purely vertical transmission with others unaffected. Unknown remain the potential effects of induced adjustments in* s_1*, even though it shares linkages with others. Panel 1.3b illustrates both vertical and horizontal propagation: a shock to* s *not only affects its immediate trading connections but also alters its interaction with* s_4*: changes in* s_1 *triggered by the initial shock generate an horizontal effect that links* $\{s, s_4\}$ *through their common supply-based connection with* s_1*. By analogy, this reasoning falls under supply distances of* $\{s, s_4\}$ *with the rest of other sectors-*m *selling to* s_1*,* $d^{fs}[\cdot]$*. With due demand-based scheme, analogous is the logic underlying* $d^{fd}[s_1, s_2]$*, simultaneously buying from* m*. As a result, subsequent adjustments in* $\{s_4, m\}$ *feed back into*

[10] The example is presented as the more intuitive case of a sectoral supply shock propagating downstream, affecting sectors that depend on the initially shocked sector. Yet, the same underlying logic applies also in the case of a demand shock, where the direction of a transmission is reversed, with the sectoral shock propagating upstream as sectors increase demand for intermediate inputs from their suppliers. In the presence of an already-existing Input-Output linkage between distanced sectors, the transmission is progressively amplified.

s_1, compounding the Leontief propagation towards s_3, and into s_2: reverberating the initial shock on s, via $\{s_1, m\}$ to s_2, an additional vertical effect further downstream, from s_2 to s_5, is generated. All things considered, the total transmission is not unique but multiple. This extra "horizontal" geometry reinforces or mitigates the strength of vertical propagation, with horizontal complementarities depending on the nature of the demand / supply relationships each sector has with the shocked sector and related.

Example 4 illustrates the central message of the paper: incorporating factor input demand and factor input supply network economic distances, and thereby horizontal complementarities between sectors, fundamentally reshapes the vertical transmission of micro-level shocks. The new propagation stems from similar demand/supply trade relationships between sectors sharing a comparable Input-Output structure (*i.e.*, common upstream suppliers or downstream buyers), and this paper is mostly on the conceptualisation of the $d[\cdot]$-types of linkages. Stated differently, distance-based complementarities that emerge along the network's horizontal architecture alter the vertical transmission by either compounding or mitigating the propagating behaviours of a micro-originated shock.

Ultimately, along the vertical propagation of sectoral shocks, it is the horizontal geometry of the network structure that leads to positive or opposite comovement of economic variables when intermediate inputs are circulating between sectors.

1.3.1. Reduced form for distance-lead comovement

A shadowed implication of the framework developed so far is that intermediate inputs generally function as substitutes, allowing sectors to freely adjust their input mix in response to shocks. However, the horizontal geometry of a network can embed two aspects: *(i)* vertical propagation through the supply chain may remain the dominant channel, and what may appear as a horizontal transmission can in fact be the by-product of vertical propagation; and *(ii)* intermediate inputs complementarity in production may alter the transmission of shocks. This tension between horizontal complementarities at the network level and substitutability at the input level sets the stage for a more precise characterization of how sectoral comovement occurs.

In the context of factor input supply, alleviating the tension is straightforward since sectors' competition for common downstream buyers yields a more direct and unambiguous form of horizontal interdependence through relative prices. Conversely, the dual transmission in factor input demand makes the direction and the strength of induced comovement subtle, emerging the need for conditions that systematically identify when positive or opposite comovement arises under input substitutability.[11]

The structure of upstream connections, in fact, renders demand-based linkages ambiguous to interpret. Pivotal is the way in which upstream sector's price pass-

[11] Overlap of vertical and horizontal transmission under demand distances. A shock in sector-s that expands its demand for intermediate inputs alters the input supply composition of sector-s^*; subsequent adjustments in s^* then spill over to sector-s'; and the subsequent reactions of s' may again feed back into s^*, thereby completing a cycle of vertical propagation across interconnected sectors not directly operating through relative effects.

through translates shocks into input reallocation across sectors, potentially inducing opposite comovement (substitute away from more expensive suppliers) even when vertical links suggest positive propagation. Considering a single-input case (purchasing intermediate inputs from only one upstream supplier, with no substitution possibilities), and an extended multiple-input case (allowing substitutability as sectors source from multiple suppliers), and assuming a Constant Elasticity of Substitution (CES) among intermediate inputs, I provide the following conditions.

PROPOSITION 1 (Demand-driven comovement) *In the single-input case, comovement in demand implies $\delta_{s^*\rightarrow}[s,s'] \propto -\sigma\,\tau^{fd}$, determined by substitution elasticity σ and upstream pass-through τ^{fd}. In the multi-input CES case, then*

$$\delta_{s^*\rightarrow}\left[s,s'\right] \;\propto\; -\,\sigma\left[1-e\left(s,s^*\right)\right]\tau^{fd}$$

with relative intermediate inputs changes from common supplier, $\delta_{s^*\rightarrow}[s,s'] = \frac{\partial \log x(s,s^*)}{\partial \log x(s',s^*)}$, jointly determined by substitution elasticity, downstream sectoral expenditure share, $e(\cdot)$, and upstream pass-through. Opposite comovement occurs if $\tau^{fd} > 0$.

Proof in Appendix A.

Opposite comovement between downstream sectors arises through upstream adjustments in supply. Intuitively, when s' experiences a positive shock it might bids up the price of inputs from the common supplier-s^*, $\tau^{fd} = \frac{\partial \log p(s^*)}{\partial \log x(s',s^*)} > 0$, through increased demand. If this higher input cost makes production more expensive for s, its demand from s^* falls, generating opposite comovement. Whether this happens hinges on the complementarity between s^*'s input and the others purchased by s: when $\sigma > 0$, any price increase in s^* alters s's input mix; in the limit $\sigma \rightarrow \infty$, inputs become perfect substitutes and s fully shifts away from s^*, generating no comovement with s'. In the general multi-input CES case, the effect is further scaled by sector-s's expenditure share on s^*, $e(s,s^*)$, so that sectors more reliant on s^* are strongly affected. Thus, demand-driven comovement reflects not automatic crowding-out, but the interplay of upstream price pass-through (from vertical propagation), and downstream demand adjustments (indirectly generated by horizontal complementarities).

Considering instead supply-based distances, since shocks to a supplier directly affect its relative price against competitors for common downstream buyers, the induced reallocation of demand across suppliers creates a clear channel for comovement, with horizontal and vertical effects occurring in parallel.

PROPOSITION 2 (Supply-driven comovement) *In the single-input case, no condition exists for comovement in supply due to the absence of competition for downstream markets. Differently, in the multi-input CES case,*

$$\delta_{\rightarrow s^*}\left[s,s'\right] \;\propto\; \sigma\, e\left(s^*,s'\right)\,\tau^{fs}$$

with relative intermediate inputs changes to common buyer, $\delta_{\rightarrow s^*}[s,s'] = \frac{\partial \log x(s^*,s)}{\partial \log x(s^*,s')}$, jointly determined by substitution elasticity, σ, sectoral revenues for upstream supplier, $e(\cdot)$, and downstream pass-through. Opposite comovement occurs if $\tau^{fs} < 0$.

Proof in Appendix A.

Opposite comovement between upstream suppliers mechanically arises from competition for downstream buyers. Intuitively, when s' receives a positive shock it lowers its relative price, capturing a larger share of common buyer-s^*'s demand from adjustments in upstream price, $\tau^{fs} = \frac{\partial \log p(s')}{\partial \log x(s^*,s')} < 0$. This revenue reallocation necessarily comes at the expense of the competing supplier-s, generating opposite comovement even when $\sigma \to \infty$ (shifting away of s^*'s demand from s, increased towards s' due to relative price effect). The magnitude depends on two forces: the elasticity of substitution σ across sector-s^*'s inputs controls how easily s^* reallocates spending, while the expenditure share $e\,(s^*, s')$ scales the importance of s' for sector-s^*.

Discussion.– A networked economy inherently embeds complementarities in intermediate inputs – which matters in magnitude but not in the direction of comovement –, as sectors depend on overlapping sets of buyers and suppliers. In the demand-distance case, sectoral comovement is ambiguous: it reflects a combination of vertical propagation along the supply chain and horizontal complementarities from shared upstream suppliers, making it difficult to disentangle the two channels. In the supply-distance case, opposite comovement emerges when multiple upstream suppliers compete for the same downstream sectors, and horizontal transmission directly arises from the reallocation of inputs in response to adjustments in relative prices: comovement is transparent, and the mechanism governing the interaction between vertical and horizontal dimension is mechanically determined.[12]

Importantly, the discrepancy between the ambiguous effects in demand and the clear responses in supply is in accordance with non-linear network theories (*e.g.*, Atalay 2017, Baqaee and Farhi 2019), where downstream complementarities – and hence complementarities between purchased intermediate inputs – matter the most. In my framework, this dimension is embodied by supply-based distances, which create horizontal interdependencies among sectors selling to the same buyers. Delving into the downstream pass-through effect, when one supplier experiences a positive shock, downstream complementarities of common buyers cause other suppliers to adjust their production in response, and the propagation operates directly through the interdependence of upstream inputs used together in downstream production. Such amplification is largely absent under demand-based distances: the rationale beyond the upstream pass-through effect merely captures how easily a downstream buyer can substitute away from its upstream sellers, without generating true horizontal spillovers among buyers themselves.[13] In other words, supply-based distances generate genuine horizontal complementarities among sectors whose productions are complementary in downstream buyers, whereas demand-driven horizontal complementarities reflect only substitution patterns across common upstream suppliers.

[12] Baqaee and Farhi (2019) develop a "global" theory where complementarities can amplify or reverse the impact of idiosyncratic shocks. While their framework emphasizes non-linear amplification, mine linearizes these non-linearities: network "economic" distances naturally capture how vertical and horizontal interdependencies interact so that they summarize the same complementarity forces that drive non-linear dynamics.

[13] Indeed, considering complementarities between goods from an upstream (common) supplier and those of other horizontally related buyers would mechanically yield a supply-based network distance.

1.3.2. VERTICAL AND HORIZONTAL NON-DEPENDENCE

A key step in the analysis is to establish a dimensional separation.

THEOREM 2 (On the weighting of network "economic" distances) *In an Input-Output economy, the horizontal dimension of a network (defined by demand- and supply-based distance matrices) is not weighted by its vertical dimension (defined by the directed or Leontief inverse matrix).*

Proof in Appendix A.

The theorem postulates that, within an Input-Output economy, the horizontal geometry of the network – captured by demand- and supply-driven distances – remains unweighted and independent by its vertical trade intensities, encoded in the Input-Output matrix. This distinction is crucial for understanding the economics of production networks: while the vertical structure reflects the intensity of inter-sectoral trade linkages, the horizontal structure identifies demand/supply complementarities across sectors related through common upstream suppliers or downstream buyers. In other words, by separating the horizontal from the vertical geometry, the theorem clarifies that whatever distance matrix is not merely a transformation of the Input-Output matrix, but a complementary object that reveals otherwise hidden propagation channels within the production network.

Nevertheless, as linkages densify and the network expands, horizontal complementarities become increasingly important (with more sectors sharing common suppliers and buyers, interdependencies multiply), amplifying the complexity of shock transmission far beyond what purely vertical Input-Output structures would predict.

THEOREM 3 (On the importance of network "economic" distances) *Consider a network of size-m with Input-Output matrix $\mathbf{H}_m := \boldsymbol{\mathcal{V}}_m \cup \boldsymbol{\mathcal{D}}_m$, where $\boldsymbol{\mathcal{V}}_m$ encodes vertical buyer-supplier ties and $\boldsymbol{\mathcal{D}}_m$ encodes horizontal complementarities between sectors. Let $\boldsymbol{\rho}_m$ be a vector of shocks, and let $\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})}\boldsymbol{\rho}_m$ and $\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})}\boldsymbol{\rho}_m$ denote the associated matrices for propagation channels. Then, as the network size grows ($m \to \infty$), the vertical propagation vanishes: $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})}\boldsymbol{\rho}_m\right\|_{\triangle} \Big/ \left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_{\triangle} \to 0$.*

Proof in Appendix A.

In other words, for larger production networks (*e.g.*, firm-to-firm network), vertical buyer-supplier chains become too sparse and elongated to transmit shocks effectively, while horizontal complementarities grow denser and increasingly bind sectors together. As more network participants buy and sell, shared buyers/suppliers become more likely, and the more important horizontal complementarities become. In the limit where network size-$m \to \infty$, horizontal effects dominate: bigger networks transmit shocks not as cascading supply chains, but as clusters of horizontally-related sectors. This dominance persists even when vertical supply chains themselves expand endogenously as the network grows in size.

COROLLARY 2 (Endogenous expansion) *In growing networks, the horizontal geometry asymptotically dominates shock propagation even as vertical chains expand.*

1.3.3. SECTORAL COMOVEMENT AND AGGREGATE BEHAVIOUR

Equilibrium adjustments in production networks are central to understand the origins of aggregate business cycles. Within this perspective, an important question emerges: to what extent does employment comovement shape the dynamics of aggregate fluctuations in output? In a similar theoretical set-up vom Lehn and Winberry (2022) show that, in equilibrium, the impact of a given sector-specific shock on real Gross Domestic Product (GDP) – exploiting the *Divisia Index* – can be decomposed into its propagation on aggregate Total Factor Productivity (TFP) and its effect on aggregate employment. Fluctuations in aggregate real GDP growth are the result of

$$d\log Y = \sum_{s\in\Phi(s)} \Big(\lambda\,(s)\; d\log z\,(s) + \nu\,(s)\; d\log n\,(s)\Big) \tag{1.4}$$

where $\lambda\,(s) = \frac{p(s)y(s)}{PY}$ is the usual *Domar weight* of sector-s, and $\nu\,(s) = \frac{p^Y(s)y^Y(s)}{PY}$ its *value-added Domar weight*, with $p^Y\,(s)\,y^Y\,(s)$ being the sector-specific nominal value-added. These components identify the ratio of the gross nominal and value-added intermediate output of sector-s to GDP, respectively, and both define the importance of a given sector to aggregate fluctuations and business cycle dynamics.

Employment comovement thus constitutes a central phenomenon to investigate under inter-sectoral trade network: the cyclical behaviour of the economy origins from both changes in sectoral TFP, $d\log z\,(s)$, and in sectoral employment, $d\log n\,(s)$. Focusing on employment comovement offers distinct advantages. It provides a directly observable measure of sectoral activity, avoiding the interpretability (whether there is shock transmission or shocks correlation; see Huo et al. 2025) and the identification challenges associated with productivity.[14] As such, through the lens of eq. (1.4), different values of network distances govern the sign of the shock's transmission:

$$d\log Y = \sum_{s\in\Phi(s)} \lambda\,(s)\; d\log z\,(s) + \sum_{s\in\Phi(s)} \begin{cases} \nu\,(s)\;\Delta\,\text{comovement} & \textit{under major distance} \\ \nu\,(s)\;\Delta\,\text{reallocation} & \textit{under minor distance} \end{cases}$$

In fact, the model delivers the following insight into employment comovement.

PROPOSITION 3 (Propagation of sectoral variations) *The overall production network effects on sectoral employment is summarized by:*

$$d\,log\,\mathbf{N} = \Theta\Big\{d\,log\,C + d\,log\,\boldsymbol{\mathcal{S}} + d\,log\,\boldsymbol{\mathcal{V}} + d\,log\,\boldsymbol{\mathcal{D}}\,(j)\Big\}, \quad \textit{for} \quad j = \{fd, fs\}$$

where $\boldsymbol{\mathcal{V}}$ *and* $\boldsymbol{\mathcal{D}}\,(j)$ *represent the vertical and horizontal effects, respectively, with* $\boldsymbol{\mathcal{D}}\,(fd) = \boldsymbol{\mathcal{D}}^{fd}\Big[d\,log\,\mathbf{N} - \Big(\boldsymbol{\mathcal{L}}'_{\Phi(s)}\,d\,log\,\mathbf{N}\Big)\,\mathbf{1}\Big]$ *and* $\boldsymbol{\mathcal{D}}\,(fs) = \boldsymbol{\mathcal{D}}^{fs}\Big[d\,log\,\mathbf{P} - \Big(\boldsymbol{\mathcal{L}}'_{\Phi(s)}\,d\,log\,\mathbf{P}\Big)\,\mathbf{1}\Big]$.

Proof in Appendix A.

Mitigated by a bundle of structural parameters (Θ), changes in sectoral employ-

[14] Identifying the impact of network "economic" distances on comovement is challenging through productivity, because horizontal linkages propagate through production inputs, and TFP residually absorbs measurement errors, obscuring the precise antecedents of comovement. Sectoral employment, instead, is a directly observable input, offering a clearer test of how shared upstream and downstream connections transmit shocks.

ment are a function of aggregate consumption (C), both sector-specific productivity and consumption of its good ($\boldsymbol{\mathcal{S}}$), other sectors' productivities, employment levels, and intermediate inputs usage ($\boldsymbol{\mathcal{V}}$) whose magnitudes depend on the Input-Output matrix, and network "economic" distance effects ($\boldsymbol{\mathcal{D}}$) for both demand (through employment differentials) or supply (through relative price differentials) horizontal linkages. Square parentheses denote the difference between the change in all other sectors and the sector's own change. For example, considering sector-s employment (same for relative prices), $d\log \boldsymbol{N} - \left(\boldsymbol{\mathcal{L}}_s' \, d\log n\left(s\right)\right)\mathbf{1} = \left[d\log n\left(s\right) - d\log n\left(s\right), \ldots, d\log n\left(m\right) - d\log n\left(s\right)\right]$ where $\boldsymbol{\mathcal{L}}_s$ is a $m \times 1$ selection vector and $\mathbf{1}$ an $m \times 1$ vector of ones, both replicating the reference sector's change across all sectors, ensuring that the horizontal propagation term captures relative deviations. For all sectors:

$$\boldsymbol{\mathcal{D}}^{fd}\left[d\log \boldsymbol{N} - \left(\boldsymbol{\mathcal{L}}_{\Phi(s)}' \, d\log \boldsymbol{N}\right)\mathbf{1}\right] = \begin{bmatrix} \sum_{s'\in\Phi(s)} d^{fd}\left[s,s'\right] \cdot \left[d\log n\left(s'\right) - d\log n\left(s\right)\right] \\ \vdots \\ \sum_{s\in\Phi(s)} d^{fd}\left[m,s\right] \cdot \left[d\log n\left(s\right) - d\log n\left(m\right)\right] \end{bmatrix}$$

The interpretation of Proposition 3 follows Subsection 1.3.1: sectors sharing upstream suppliers exhibit ambiguous comovement, as employment changes in other sectors enter in both vertical and horizontal components; sectors sharing downstream buyers undergo relative-price reallocation of supply, producing opposite comovement. Moreover, vertical propagation prevails under low proximity (high distance, low theoretical $\boldsymbol{\mathcal{D}}^j$), leading to positive sectoral comovement. For low distances, horizontal forces dominate, thereby inducing opposite comovement. Both demand ambiguity and employment-based horizontal effects are tested empirically in Section 1.5.

Finally, it is worth assessing the extent to which the mere horizontal geometry of networked economies contributes to aggregate fluctuations.

THEOREM 4 (On aggregate cycles) *Consider an Input-Output economy of m sectors. Let $\lambda\left(s\right)$ denote sector-s's Domar weight, and assume no sector is dominant, in the sense that $\lambda\left(s\right) \asymp \frac{1}{m}$, $\forall s \in \Phi\left(s\right)$. Beside vertical propagation, if sectors share upstream suppliers or downstream buyers, then independent, idiosyncratic sectoral shocks generate non-vanishing aggregate GDP fluctuations:* $\liminf_{m\to\infty}\ var\left(Y\right) > 0$. *As many horizontally-related sectors move in response, an idiosyncratic shock does not average out, and aggregate volatility survives even as the economy grows.*

Proof in Appendix A.

Aggregate cycles need not originate from economically dominant sectors nor from vertical production chains. Even when sectors are small, idiosyncratic shocks generate persistent aggregate volatility under horizontal linkages: fluctuations from sectoral responses are induced by shared Input-Output structures rather than through usual economic centrality or vertical propagation (more in Subsection 1.4.2). Horizontal interdependences thus prevent shocks from averaging out in aggregate.

COROLLARY 3 (Granular sectoral weights) *Economically large sectors amplify aggregate fluctuations induced by horizontal network propagation.*

Allowing for heterogeneous Domar weights – so that some sectors represent a larger fraction of aggregate GDP – does not alter the horizontal transmission mechanism identified in Theorem 4. Instead, it strengthens its aggregate impact: when a highly horizontally connected sector is also economically large, its shock propagates across the horizontal cluster and carries greater weight in aggregate output, thereby amplifying GDP fluctuations. Horizontal transmission and sectoral importance operate as complementary channels: horizontal geometry ensures persistence, while uneven granularity magnifies the magnitude of aggregate volatility.

Vertical and horizontal transmissions thus reflect distinct mechanisms. Vertical propagation operates through the Input-Output matrix: shocks cascade along production chains via observed linkages, generating supply chain amplifications that scale up mechanically with sectoral size, as measured by Domar weights. Granularity therefore magnifies aggregate responses by strengthening vertical cascades. Horizontal propagation, instead, acts on comovement: sectors sharing many suppliers or buyers respond jointly, without inducing cascades, but generating systematic correlation that survives aggregation. Granularity amplifies horizontal spillovers only when large sectors belong to dense horizontal clusters; absent such similarity, dominant but isolated sectors do not generate aggregate effects. Aggregate fluctuations thus reflect an interaction: vertical linkages determine the magnitude of cascades, while horizontal geometry governs whether shocks cumulate or average out.

1.4. Data and Network Anatomy

From the United States' Bureau of Economic Analysis (BEA), available data display information, for 3-digit U.S. 2017 North American Industrial Classification System (NAICS) sectors, on value-added, employment, net exports, and inter-sectoral linkages exploiting Input-Output (I-O) matrices. The (balanced) panel data built is made of 65 private sectors over the period 1998-2022. For additional details and an extended discussion of the data refer to Appendix B.

Distance measures are derived from the *directed* network, $\boldsymbol{H}$, and not from its *Leontief inverse*, $\boldsymbol{\mathcal{H}}$. Whereas it incorporates indirect sectoral exposures as well, and thus amplifies the magnitude of interconnections beyond direct relationships, the directed network isolates the immediate Input-Output intensities, offering a more parsimonious and transparent representation of demand/supply linkages. Employing the directed configuration to characterize matrices $\boldsymbol{\mathcal{D}}^{fd}$ and $\boldsymbol{\mathcal{D}}^{fs}$ avoids overstating distance values between sectors, preventing the inflation of horizontal complementarities and instead preserving the strictly observed interdependencies.[15]

[15] While *sparsity* is preserved under both representations, the directed network prevents the exaggeration of distance values that would emerge under the Leontief inverse system. From the perspective of Example 4, the directed structure ensures distances to stem from immediate network ties, consistently with the absence of an indirect link: the additional propagation from s_1 to s_3 – *i.e.*, the Leontief inverse mechanism of indirect network effects – would not be artificially introduced to compute network distances, rather based on common upstream and downstream sectors. Put differently, the network distance between s and s_4, conditional on adjustments in s_1, would remain contained by overlooking the additional Leontief-based linkage towards s_3.

1.4.1. DISTANCE NETWORKS

Each cell $d^j[s,s'] \in [0,1]$ of matrix $\mathcal{D}^j$, for $j = \{fd, fs\}$ and $\forall s, s' \in \Phi(s)$, identifies the Euclidean distance (square root of the sum for squared differences of network intensities, see Section 1.2) between dyads of sectors in demand or supply relative to common sectors.[16] The empirical estimation will consider network distances at their *extensive margin* for classification purposes (reminiscent from Theorem 2). Such matrices are labelled as $\mathcal{D}^j_{ext}$, whose generic entry is $d^j_{ext}[s,s'] = \{1, 2, \ldots, d_{max}\}$. Differently from the theory, values closer to one correspond to shorter horizontal distances.

As for their characterization, network distance-d among any two sectors $\{s,s'\}$ is computed as a *shortest path problem* from *intensive margin* distance matrices, $\mathcal{D}^j$, implementing an algebraic version (*i.e.*, using matrix multiplication) of the traditional Dijkstra (1959)'s or Roy (1959)-Floyd (1962)-Warshall (1962)'s types of algorithms, since it is *(i)* suitable for discrete-value adjacency matrices – as the ones for intensive margin distances –, and *(ii)* fairly efficient for small to medium-sized graphs – as a sector-level production network. Appendix B reports the technical version of the implemented algorithm which, in plain words, reads as follows.

(Shortest path algorithm) *Central idea is to find, for each single node, all the other nodes that can be reached in one step, then in two steps, and so on. Each time a node is reached for the first time, it records the number of steps it took to get there – this is the shortest path approach. In other words, find the shortest number of steps it takes to get from every node to every other node in a network. In particular:*

(a) identify the direct connections between any pair of nodes (those that take exactly one step to be connected);

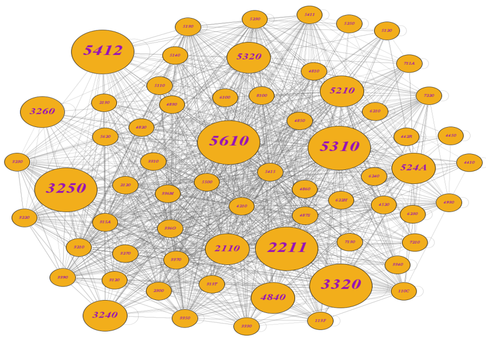
(b) for not-connected pairs, increase the path length by 1 in each round (two steps, then three steps, etc.). Multiply the graph by itself to discover which new pairs of nodes are now connected through longer paths;

(c) if a pair of nodes becomes connected (i.e., only if it has not be already found a shorter path), record the current length as the shortest distance between them;

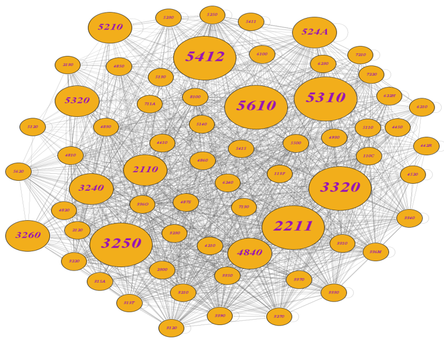
(d) repeat this process until any new reachable pairs of nodes is found. For any pair of nodes that are still unreachable (i.e., the distance is still zero and they are not linked in the network), set their distance to infinity.

To illustrate the constructed distance matrices Figure 1.4 displays, for each extensive margin value, the network distance relationship among sectors based on both factor input demand and factor input supply horizontal linkages. Importantly, each connection does not necessarily represent inter-sectoral trade between two sectors, but rather it reflects an horizontal linkage based on demand- or supply-driven network distance relationships. A few concatenated points are then worth noting.

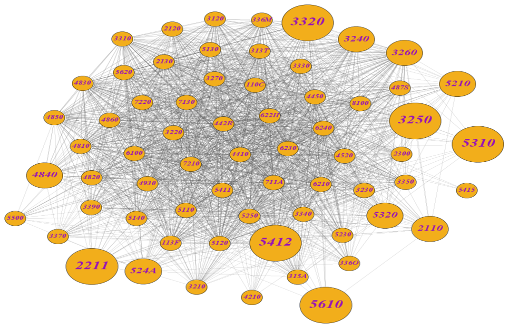
[16] Small transactions under 1% of total purchases (or sales) are disregarded (*e.g.*, Conley and Dupor 2003, Acemoglu, Carvalho, et al. 2012, Carvalho 2014). This approach enables to compute a finite set of network distances at the extensive margin, as it removes any bias stemming from meaningless sectoral interconnections.



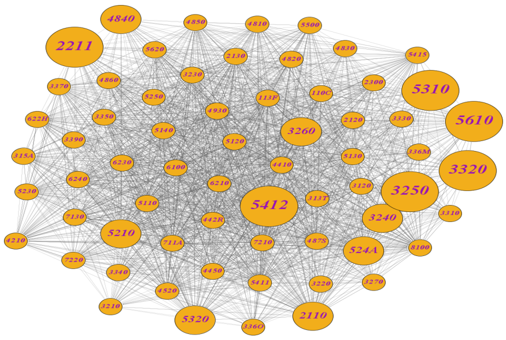
(A) DEMAND, $d = 1$



(B) SUPPLY, $d = 1$



(C) DEMAND, $d = 2$



(D) SUPPLY, $d = 2$

FIGURE 1.4: VISUALIZATION OF PRODUCTION NETWORK DISTANCES

Note: each panel of the figure represents the distance-based inter-sectoral production network corresponding to the (*commodity-by-commodity total direct requirements*) U.S. Input-Output matrix in the year 2007. *Factor input demand* and *factor input supply* network distances are computed implementing the *shortest path algorithm* on distance matrices whose values are at the intensive margin, as well derived from the *directed* configuration of the production network. Panels 1.4a and 1.4c correspond to the demand-based network distances, with $d = 1$ for low distance (closest sectors) and $d = 2$ for major distance (further sectors), among any pair of sectors; analogously it is for the supply-based network distances in Panels 1.4b and 1.4d. There are 65 3-digit U.S. 2017 NAICS private sectors, and each orange node corresponds to one of them. Two sectors connected under $d = 1$ cannot be linked under $d = 2$, and otherwise; constructed distance matrices are full (Theorem 1). Biggest nodes correspond to top 10% of mostly connected sectors (*i.e.*, major number of inter-sectoral connections), while intermediate ones the additional top 20%. Figure drawn with the software package *Gephi*, version 0.10, exploiting the *ForceAtlas2* layout algorithm. *Source*: BEA and own calculations.

First, the distribution of horizontal complementarities is uneven across distance levels. At very short range ($d = 1$), the demand-based network appears slightly denser than the supply-based one, suggesting that sectors are more frequently linked through common upstream suppliers than through shared downstream buyers. However, this asymmetry vanishes as the distance increases ($d = 2$), where both types of horizontal ties spread more diffusely across the network.

Second, sectors with the highest number of Input-Output linkages tend to concentrate at the very core of the production system when considering only close connections ($d = 1$). As shown in Panels 1.4a and 1.4b, both the largest hubs (top 10% by connectivity) and the mid-sized hubs (top 20%) are similarly central, occupying short distances from the network's nucleus.

Third, and in contrast with the previous point, these same highly connected sectors tend to become relatively more peripheral once longer distances ($d = 2$) are considered. Panels 1.4c and 1.4d show that, at this range, their positions are more evenly distributed across the network, reflecting the fact that their trade structures – both upstream and downstream – are highly similar to those of other sectors. In other words, the most connected sectors are less unique at longer distances, as commonalities in demand and supply linkages become more widespread.

Altogether, these observations highlight that the structure and role of horizontal complementarities depend crucially on the notion of distance. At shorter distances, horizontal linkages cluster around the same set of key sectors, whereas at longer distances, such complementarities are more evenly distributed across the production network. This pattern confirms that sectors typically share both upstream suppliers and downstream buyers at smaller but not at a greater distance, and the same vertical production system generates distinct horizontal geometries (Theorem 1).

1.4.2. Aggregate dynamics

To illustrate the implications of horizontal geometry for business cycles, I simulate aggregate GDP responses to sectoral shocks exploiting the network's vertical (Input-Output chains) and horizontal (shared buyers/suppliers) structures, relying on observed Input-Output and intensive margin distance matrices from the 2007 BEA tables. Both dimensions remain unweighted, as dictated by Theorem 2.

Sectoral outputs evolve as $\boldsymbol{y}_t = \boldsymbol{\mathcal{P}}^{(\mathcal{V},\mathcal{D})}\,\boldsymbol{\varepsilon}_t$, where matrix $\boldsymbol{\mathcal{P}}^{(\cdot)}$ encodes how sector-specific shocks in $\boldsymbol{\varepsilon}$ propagate, following a mean-reverting AR(1) process with persistence ρ. Aggregate Impulse Response Functions (IRFs) trace the path $\{Y_t\}_{t=0}^{T}$ following an initial shock in $t = 0$, and evolve over time according to

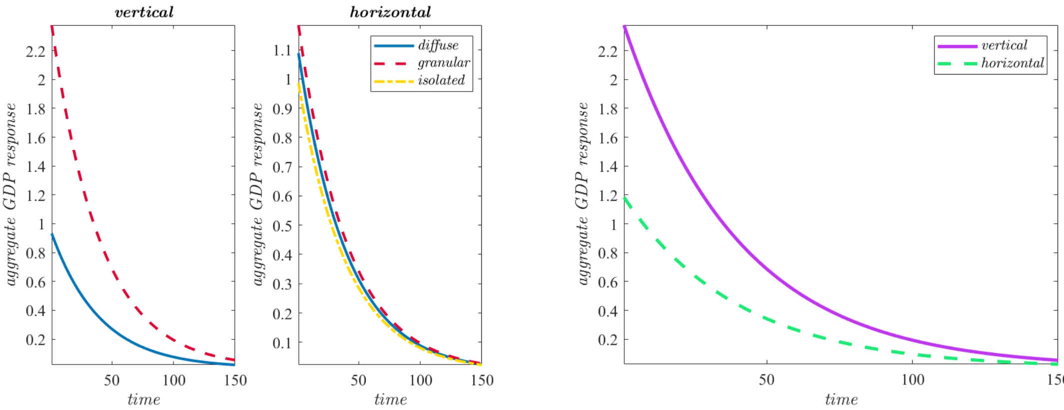
$$Y_t = \boldsymbol{\lambda}'\boldsymbol{y}_t \quad \longleftrightarrow \quad Y_t = \boldsymbol{\lambda}'\boldsymbol{\mathcal{P}}^{(\mathcal{V},\mathcal{D})}\left(\rho_t\,\boldsymbol{\varepsilon}_0\right)$$

where $\boldsymbol{\lambda}$ bundles Domar weights (*i.e.*, relative sectoral contribution to GDP).

Figure 1.5 shows the outcome. Panel 1.5a mirrors the logic of Theorem 4 and Corollary 3. Vertical propagation, governed by the Leontief inverse, amplifies shocks primarily through parallel supplier-customer chains:[17] granularity mechanically increases aggregate volatility because economically large sectors transmit shocks proportionally more strongly. Horizontal propagation, by contrast, does not rely on Input-Output flows but depends on the geometry of production network similarities. Idiosyncratic shocks diffuse strongly across sectors with common network linkages, even in the absence of direct trade, and aggregate effects depend on the distribution of economic proximity. Granularity amplifies horizontal spillovers only if large sectors are embedded in dense horizontal clusters; when such sectors are isolated in the similarity space, horizontal spillovers remain weaker despite Domar weights' concentration. Indeed, this transmission mechanism is not driven by size alone but by the alignment between sectoral dominance and network's horizontal geometry.

Additionally, vertical propagation generates larger aggregate responses than hor-

[17] Sectoral shocks average out in aggregate for "balanced" – *i.e.*, almost symmetric – Input-Output networks (*e.g.*, Acemoglu, Carvalho, et al. 2012), and not necessarily for equal Domar weights.



(A) TRANSMISSION

(B) GRANULAR WEIGHTS

FIGURE 1.5: AGGREGATE FLUCTUATIONS

Note: under "diffuse", sectoral Domar weights are evenly distributed; if concentrated, the economy is "granular"; dominant yet weakly horizontally connected sector is "isolated". Shock transmission is "vertical" if inducing cascades via network linkages, "horizontal" if generating sectoral comovement via shared Input-Output structure. *Source*: BEA and own calculations.

izontal propagation (Panel 1.5b). This outcome is not a numerical artifact, but a direct consequence of the economic mechanisms embedded in: vertical transmission is stronger because it compounds along production chains, while horizontal transmission increases covariance across sectoral variables and creates comovement, not amplification. Such distinction explains why granularity is always amplifying under vertical linkages, but only conditionally amplifying under horizontal ones.[18]

The simulation illustrates that aggregate cycles depend not only on the presence of large sectors, but on the interaction of sectoral adjustments with network similarities. Key is that the horizontal geometry alone is sufficient to prevent sectoral fluctuations from vanishing in aggregate, even in the absence of relevant sectors.

1.5. EMPIRICS

In this empirical section sector-level data for the United States (U.S.) economy are exploited to test the predictions about sectoral comovement in Section 1.3. Estimation relies on the two-stage procedure outlined by Barattieri and Cacciatore (2023): first, I perform a series of sectoral panel Fixed-Effect (FE) regressions to isolate the exogenous dynamics in a given input of production not due to other (sector-specific, other sectors-specific and aggregate) factors; then, each estimated residual is exploited as an identified structural shock into a Local Projection (LP) analysis, consisting in a battery of (predictive) panel regressions on separated employment series, in order to asses the network-based short-run cumulative variations in sectoral employment.[19]

[18] Figure 1.5 represents a sectoral network. In highly sparse sparse networks, however, amplification from vertical effects vanishes (Theorem 3), and horizontal transmission alone potentially drives aggregate volatility.

[19] Indeed, the identified shock is the residual (exogenous) change in a sector after controlling for its own production inputs and output dynamics and fixed effects. It does not correspond to a pure demand- or supply-side disturbance; instead, it may arise from either (or both) and captures unexpected adjustments within the sector, thereby tracing how shocks propagate across the production network.

1.5.1. INTERMEDIATE INPUTS AND SECTORAL EMPLOYMENT

Sections 1.2 and 1.3 rest on the premise that adjustments in the stock of circulating intermediate inputs lead to synchronized sectoral employment responses. Considering variations within a sector and across its production network neighbours, the first step isolates changes in input sourcing plausibly orthogonal to employment dynamics. The following panel fixed-effect regression over $s \in \Phi(s)$ sectors is estimated:

$$\begin{aligned} x_t(s) = \beta_n\, \dot{n}_t(s) \;+\; \sum_d \beta_{n(d)}\, \dot{n}_t(\Phi_s, d) \;+\; \sum_k \beta_{n(k)}\, \dot{n}_t(\Phi_s, k) \\ +\; \beta_{z(s)}\, \mathcal{Z}_t(s) \;+\; \beta_{\dot{Z}(s)}\, \dot{\mathcal{Z}}_t(s) \;+\; \beta_z\, \mathcal{Z}_t \;+\; \beta_{\dot{Z}}\, \dot{\mathcal{Z}}_t \;+\; \psi(s) \;+\; u_t^x(s) \end{aligned} \tag{1.5}$$

where $x_t(s)$ identifies the set of intermediate inputs of sector-s (as a share of its output), $\dot{n}_t(s)$, $\dot{n}_t(\Phi_s, d)$, and $\dot{n}_t(\Phi_s, k)$ represent its employment growth, that of its economically closer and further sectors, and that of its upstream and downstream sectors, respectively. The latter two measures are defined as

$$\begin{aligned} \dot{n}_t(\Phi_s, d) &= \sum_{s' \neq s \in \Phi_s} \Delta\, n(s', d) \qquad , \quad \forall d \in \boldsymbol{\mathcal{D}}^j_{ext} \quad and \quad j = \{fd, fs\} \\ \dot{n}_t(\Phi_s, k) &= \sum_{s' \neq s \in \Phi_s} \ell(s, s')\; \Delta\, n(s', k) \quad , \quad \forall k = \{up, dw\} \end{aligned}$$

where $\Delta n(\cdot) = n_t(\cdot) - n_{t-1}(\cdot)$. As for vertical propagation, each change in employment is weighted by the Leontief inverse relation, $\ell(s, s') = \left[1 - \alpha(s, s')\right]^{-1}$, between sector-$s$ and each of the other sectors whenever these are located upstream or downstream to sector-s, thereby capturing direct and indirect network exposure. Differently, changes in employment under extensive margin network demand or supply distances seize all the horizontal relationships across sectors which, in line with Theorem 2, are not leveraged by the Leontief inverse weights.

A set of control variables characterizes the second line of eq. (1.5). $\mathcal{Z}_t(s)$ and $\dot{\mathcal{Z}}_t(s)$, comprise the labour force size of the sector, both its level and growth rate of value-added, and its net exports that control for changes in demand/supply not due to internal factors. The set of aggregate controls, $\mathcal{Z}_t$ and $\dot{\mathcal{Z}}_t$, is made of aggregate value-added and aggregate employment in size as well as in growth terms. Finally, while β's are coefficients to be estimated, $\psi(s)$ imposes fixed-effects to control for unobserved heterogeneity across sectors. Standard errors are clustered at sector-level. For each sector, the estimated residual $\widehat{u}_t^x(s)$ identifies the exogenous variation in the share of production of sector-s that is bought from Input-Output connections.

Impulse Response Functions (IRFs) of sectoral employment to changes in the sector-specific intermediate inputs rely on the estimation of Local Projections, consisting in performing predictive panel regression techniques of the identified structural shock, $\widehat{u}_t^x(s)$, on the cumulative difference in sectoral employment levels.[20]

[20] Cumulative variables are likely to be highly correlated with the error term because they embed past values – bequeathing past errors – of the variable of interest, leading to biased and inconsistent estimates. Differencing cumulative measures soothes this endogeneity concern.

Given $\hbar = \{1, \ldots, h\}$ time-horizons, and for all sectors $\{s, s', \ldots, m\} \in \Phi(s)$, a battery of panel FE predictive regressions of the form

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\widehat{u}^x,\hbar}\, \widehat{u}^x_t(s) \;+\; \psi(t+\hbar) \;+\; \psi_\hbar(s) \;+\; \nu^x_{t+\hbar}(s) \tag{1.6}$$

is performed, with $\dot{n}^{cum}_{t+\hbar}(s) = n_{t+\hbar}(s) - n_{t-1}(s)$ denoting the cumulative change in sector-s employment at each horizon. Estimated coefficient $\beta_{\widehat{u}^x,\hbar}$ is the response of the variation in the cumulative difference of sectoral employment at time $t+\hbar$ given the identified shock hitting in period-t. Prediction error term $\nu^x_{t+\hbar}(s)$ is horizon-specific, while standard errors and bootstrapped Confidence Interval (CI) of $\beta_{\widehat{u}^x,\hbar}$ are clustered by sectors. To avoid measurement errors due to unobserved heterogeneity in response, sectoral fixed-effects $\psi_\hbar(s)$, and horizon fixed-effects $\psi(t+\hbar)$ – that remove common trends in the comovement of sectoral employment –, are imposed.[21]

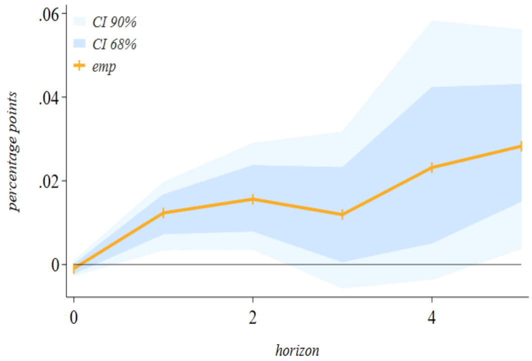
Outcomes from eq. (1.6) estimate the cumulative responses of sectoral employment to changes in their set of intermediate inputs. Analogous estimation is performed to variations in nearby sectors, considering the d^{th}-distance between sectors:

$$\dot{n}^{cum}_{t+\hbar}(s) = \beta_{\widehat{u}^x(d),\hbar} \sum_{s' \neq s} \widehat{u}^x_t(s', d) \;+\; \psi(t+\hbar) \;+\; \psi_\hbar(s) \;+\; \nu^x_{t+\hbar}(s, d) \tag{1.7}$$

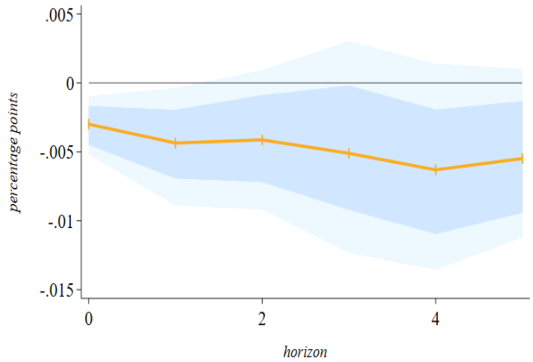
Performed estimation of eqs. (1.6) and (1.7), under *Leontief inverse* network structure and *factor input supply* distances, delivers Figure 1.6. The cumulative (short-term) response of sector-specific employment levels to a 1% change in its output share of intermediate input is depicted in the top panel: averaging across sectors, the increase induces a statistically-significant positive change in sectoral employment, in line with Proposition 3, where an increase in intermediates will drive up total production of a given sector, and thus its employment. Whether this effect can be attributed to a complementarity between production factors is an open question; however, the magnitude of the change on the y-axis may ratifies the Cobb-Douglas specification: an increase in the set of intermediate inputs by 1% induces a positive shift in employment by less than 1% upon impact. For a given increase in one unit of intermediates it is required less than one worker so that factors of production may be at least substitute, as the Cobb-Douglas production function instructs.

The same conclusions do not apply to changes in intermediate inputs of other linked sectors in the production network. Results are almost always not statistically significant at 90% except when the shock hits, and the magnitude of the variation is negligible. However besides statistical significance (which may be due to external market dynamics factors), it is interesting to note the different response between Panels 1.6b and 1.6c: for intermediate inputs changes in closer sectors there is an average negative effect, while changes in further sectors are positively related with variations in sectoral employment. Under *Leontief inverse* and *factor input demand*

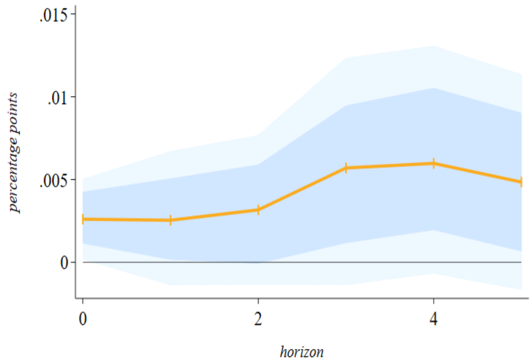
[21] Differently from the first-stage, where I control common trends over time using aggregate variables, $\psi_\hbar(t+\hbar)$ removes the possibility of common trend in the *response* of sector-specific employment to idiosyncratic shocks; the possibility that employment responses of sector-s and sector-s' to sector-specific shocks are influenced by a common trend between the two is ruled out.



(A) SECTOR-SPECIFIC



(B) $d = 1$



(C) $d = 2$

FIGURE 1.6: SECTORAL EMPLOYMENT RESPONSE TO INTERMEDIATE INPUTS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the sector-specific set of intermediate inputs as a share of its value-added for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. In particular, Panel 1.6a represents the sector-level employment response to changes in its intermediate inputs, while Panel 1.6b-1.6c to changes in the set of intermediates in closer (distance equal to 1) and further (distance equal to 2) sectors. The solid-orange line corresponds to the average response of employment across sectors, while shadow-blue and shadow-light blue areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eqs. (1.6)-(1.7). *Source*: BEA and own calculations.

network distances, opposite results are obtained (increase for closer sectors, decrease for further ones).[22] Such observations might suggest that certain types of intermediate inputs are more essential than others in production (*e.g.*, Carvalho and Voigtländer 2015), but it is actually the type of economic (demand or supply) distance among sectors that determines their importance both in the production of sectoral output and in their network centrality of being larger buyers or suppliers.

1.5.2. SECTORAL COMOVEMENT OF EMPLOYMENT

I now analyse how sectoral employment adjusts in response to employment changes in other sectors, depending on their network economic distance. The previous identi-

[22] In Appendix C, Panel C.1a of Figure C.1, I show how that the same impact of Panel 1.6a still holds when network distances in eqs. (1.6)-(1.7) are identified under *factor input demand* relations. Results for distances' effects (Panels C.1b and C.1c) identify stronger but opposite results.

fication and estimation scheme is developed, but now purging the dynamics of sector-specific employment with changes in its and other sectors' value-added, since employment is not exogenous to growth rates of sectoral output.[23]

Identification.– To identify exogenous variations in sectoral employment, $n_t(s)$, a panel fixed-effects regression over each sector $s \in \Phi(s)$ is estimated:

$$\begin{aligned} n_t(s) = \beta_y \, \dot{y}_t(s) \; + \; \sum_d \beta_{y(d)} \, \dot{y}_t(\Phi_s, d) \; + \; \sum_k \beta_{y(k)} \, \dot{y}_t(\Phi_s, k) \\ + \; \beta_{z(s)} \, \mathcal{Z}_t(s) \; + \; \beta_{\dot{\mathcal{Z}}(s)} \, \dot{\mathcal{Z}}_t(s) \; + \; \beta_z \, \mathcal{Z}_t \; + \; \beta_{\dot{\mathcal{Z}}} \, \dot{\mathcal{Z}}_t \; + \; \psi(s) \; + \; u_t^n(s) \end{aligned} \quad (1.8)$$

where $\dot{y}_t(s)$ is the value-added growth for sector-s, $\dot{y}_t(\Phi_s, d)$ identifies the value-added growth of its related sectors according to network distances, and $\dot{y}_t(\Phi_s, k)$ represents the value-added growth of its upstream and downstream sectors. Each measure is defined as that of eq. (1.5) but using $y(s)$ instead of employment. Moreover, for this specification, sector-level controls, $\mathcal{Z}_t(s)$ and $\dot{\mathcal{Z}}_t(s)$, comprise the labour force size of the sector, its value-added level and net exports, while aggregate controls, $\mathcal{Z}_t$ and $\dot{\mathcal{Z}}_t$, are analogous to eq. (1.5). Estimated coefficients are the β's, given sectoral fixed-effects, $\psi(s)$, and sectoral standard errors.[24] Sector-specific estimated residual $\widehat{u}_t^n(s)$ from eq. (1.8) identifies the exogenous variation in sectoral employment.

Local Projection analysis.– Sectoral comovement is defined as the response of employment in a given sector to changes in employment of other sectors. This can be addressed using Local Projections of the following form:

$$\dot{n}_{t+\hbar}^{cum}(s) = \beta_{\widehat{n}(d),\hbar} \sum_{s' \neq s} \widehat{u}_t^n(s', d) \; + \; \psi(t+\hbar) \; + \; \psi_\hbar(s) \; + \; \nu_{t+\hbar}^n(s, d) \quad (1.9)$$

with $\dot{n}_{t+\hbar}^{cum}(s) = n_{t+\hbar}(s) - n_{t-1}(s)$ denoting the cumulative change in sector-s employment at each horizon given the identified employment shock in closer or further sectors at the d^{th}-distance. Coefficient $\beta_{\widehat{n}(d),\hbar}$ is the $\hbar$-step-ahead response of sector-specific employment in cumulative difference due to the identified shock $\widehat{u}_t^n(s', d)$. Still, sector and horizon fixed effects, $\psi_\hbar(s)$ and $\psi(t+\hbar)$ respectively, are imposed to remove unobserved heterogeneity and common trends across sectors in the comovement of sectoral employment. Prediction error term $\nu_{t+\hbar}^n(s, d)$ is horizon- and distance-specific in each predictive panel regression, with standard errors and boot-

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[23] Hornstein and Praschnik (1997) highlight positive comovement of output and employment across sectors (as strong cross-sectoral employment correlations support the observed comovement in value-added), driven by intermediate inputs in durable goods production. Similarly, Acemoglu, Akcigit, et al. (2015) examine how sectoral shocks propagate through production networks while resulting in output and employment comovement, and Sandqvist (2017) demonstrates that the strength of these linkages on comovement varies over time and intensifies in downturns. More recently, Barattieri, Cacciatore, and Traum (2023) find that targeted government spending benefits recipient and upstream sectors but reduces output and employment downstream.

[24] In both identifications I exclude time fixed-effect while controlling for variations in aggregate variables. Comovement between aggregate and sectoral variables is well established in the literature. Stock and Watson (1999) use quarterly data over 1953:I-1996:IV period showing the positive association of the cyclical component of real GDP with aggregate employment and hours worked, and sectoral employment. Strong pro-cyclicality characterizes sectoral hours worked and employment, positively correlated with aggregate output in Huffman and Wynne (1999). As from Rebelo (2005), the correlation between aggregate and sectoral hours worked is .68 to .80. DiCecio (2009) emphasizes the comovement to be driven by sticky wages.

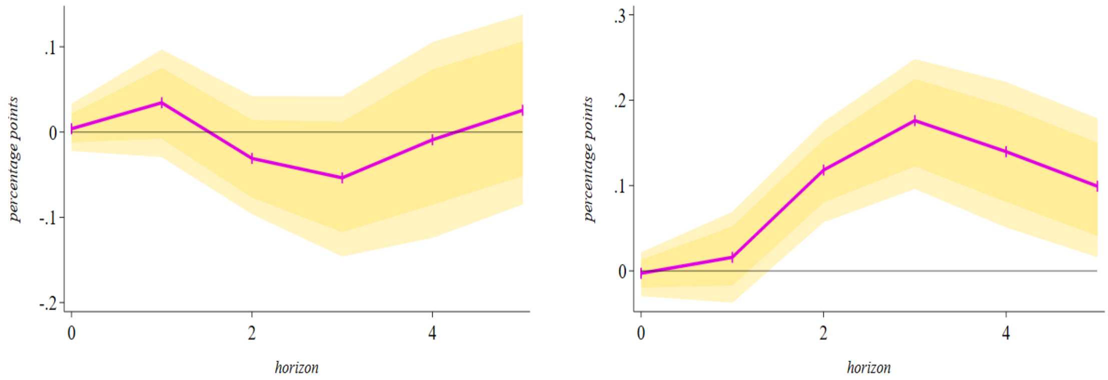
strapped Confidence Interval (CI) of $\beta_{\hat{u}^x,\hbar}$ clustered by sectors.

Impulse responses from eq. (1.9) estimate employment comovement as the average cumulative response of sectoral employment to changes in employment in other sectors, distinguishing between both types of *factor input demand* and *factor input supply* distances. As a preview, the theoretical insights of Sections 1.2 and 1.3 find empirical support: sectoral employment shocks propagate differently depending on the types of "economic" distance in the production network (Proposition 3). Sectors sourcing intermediate inputs from common upstream suppliers do not necessarily display positive employment comovement: sectors tend to comove when far apart, though responses for closely demand-linked sectors remain muted or ambiguous (rising in some cases and falling in others) due to the overlapping interplay of vertical and horizontal propagation (Proposition 1). Consistent patterns also emerge when distances are defined on the supply side. Sectors selling intermediate inputs to the same downstream buyers exhibit opposing employment comovements at short network distances, effectively counteracting the standard vertical mechanism and illustrating how competition for shared downstream buyers can dampen or reverse the expected positive transmission, while positive comovement arises as supply distance increases. In other words, the downstream demand pass-through is accordingly generating opposite comovement (Proposition 2): the factor input supply distance majorly disrupts vertical supply chain effects. Overall, these findings underscore how, at short network distances, horizontal propagation takes over, offsetting any positive comovement effect, and eclipsing the standard vertical transmission of idiosyncratic shocks. Finally, the empirics ratify the notion that as sectors are increasingly connected, horizontal complementarities matter the most (Theorem 3).

Results for factor input demand distances

Considering sectors whose distance is defined in terms of buying from the same set of upstream suppliers, estimating eq. (1.9) yields Figure 1.7. As already emphasized in Subsection 1.3.1, demand-based complementarities are inherently ambiguous to interpret, since they depend on the joint action of vertical propagation and horizontal spillovers. The conditions derived in Proposition 1 try to clarify this ambiguity: positive comovement in downstream sectors arises only when the combined effect of upstream price pass-through (transmitted through the common supplier) and downstream substitutability across intermediate inputs works in the same direction. When buyers can easily substitute inputs, an increase in employment in one sector strengthens demand for the shared supplier without crowding out the other sector, producing positive comovement. By contrast, when inputs are strong complements, a positive shock in one sector raises the supplier's price, eroding the competitiveness of the other sector and dampening comovement. Where these forces conflict or remain weak, observed correlations are muted or ambiguous.

These mechanisms echo in observed responses. Panels 1.7a and 1.7b show that a 1% increase in employment in closely linked sectors ($d = 1$) produces an ambiguous (statistically not significant) response, whereas more distant sectors ($d = 2$) consis-



(A) $d = 1$ (B) $d = 2$

FIGURE 1.7: EMPLOYMENT COMOVEMENT UNDER DEMAND LINKAGES

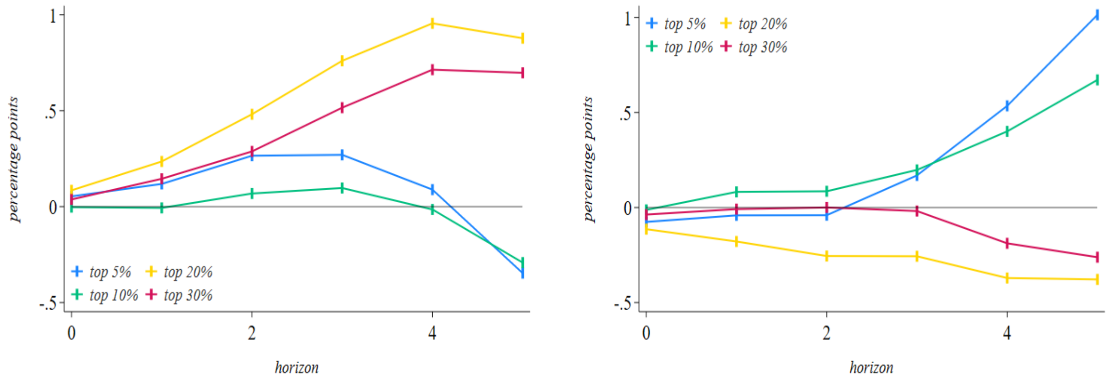
Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 1.7a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 1.7b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.

tently display positive comovement. However, sectoral shocks to employment levels shown in Panel 1.7a do not produce any statistically significant effects, as the estimated responses fail to reach significance at both the 90% and 68% confidence levels.

The lack of a significant effect may be driven by including all sectors indiscriminately, as sectors with fewer inter-sectoral linkages might introduce noise or bias to the estimation, masking and diluting the true magnitude of measured comovement in sectoral employment: restricting the sample to sectors with most Input-Output linkages could reveal more meaningful propagation effects, in line with Theorem 3. From these considerations the performed analysis is conducted focusing exclusively on highly interlinked sectors – *i.e.*, those sectors with the highest number of linkages within the considered production network –, to better capture the dynamics of employment propagation where inter-sectoral connections are strongest, and thus where network effects are more pronounced.

Nevertheless, ambiguous patterns emerge when partitioning sectors into finer subsets by the number of Input-Output linkages (Figure 1.8): comovement depends on the specific cluster considered. This confirm that demand-driven horizontal complementarities and vertical propagation operate jointly, often obscuring one another. Such mixed evidence ultimately reflects the inherent ambiguity of demand-based distances discussed in Subsection 1.3.1: because they capture substitution patterns across upstream intermediate goods rather than genuine horizontal complementarities, horizontal propagation remains weak and easily masked by vertical effects.

Observed demand-driven comovement broadly aligns with the theory developed in Sections 1.2 and 1.3. Positive comovement is most likely to emerge when sectors are “economically” distant, since vertical effects dominate and the conflicting role of horizontal spillovers is attenuated. For mostly connected sectors, however, the balance



(A) $d = 1$ (B) $d = 2$

FIGURE 1.8: DEMAND-BASED COMOVEMENT IN HIGHLY INTERLINKED SECTORS

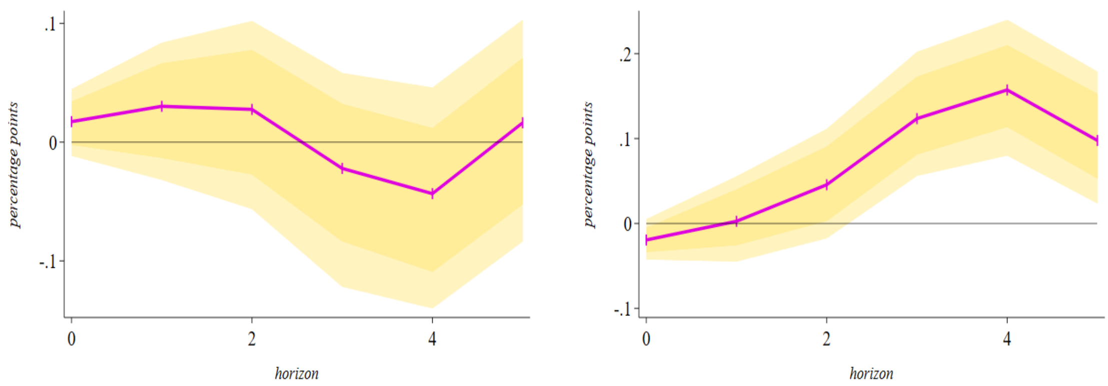
Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 1.8a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 1.8b plots the response to employment changes in further (distance equal to 2) ones. Each line corresponds to the average response of employment across sectors with different numbers of linkages, robust to 68% and 90% significance levels of bootstrapped Confidence Interval (CI) computed from eq. (1.9). If a plot is not appearing, it means there is no response for the specified distance value. *Source*: BEA and own calculations.

between vertical and horizontal channels is more delicate, producing the mixed responses observed in the data. This suggests that observed employment comovement mirrors an interplay of vertical and horizontal propagation, thereby highlighting the inherent complexity in interpreting demand-based complementarities. In this sense, factor input demand distances reveal how horizontal geometries can either reinforce or challenge the standard vertical logic of propagation, thereby adding an essential layer of complexity to the interpretation of comovement in production networks.

Results for factor input supply distances

Considering now sectors whose distance is defined in terms of selling to the same set of downstream buyers, Figure 1.9 plots the responses from the estimation of eq. (1.9). Interpreting the effect of further ($d = 2$) sectors' variations in employment yields the same results of the factor input demand case: from Panel 1.9b, a 1% increase in their employment increases sectoral employment as well, thus determining positive comovement and a positive transmission of the sector-idiosyncratic shock. Different is the case of closer ($d = 1$) sectors, where it appears a positive and then a negative cumulative effect on sector-specific employment with a small magnitude.

The lack of significance in Panel 1.9a might have two complementary interpretations: *(i)* changes in employment in nearby sectors have no measurable impact on others when the sectors are linked through a common set of downstream buyers; and *(ii)* the factor input supply network distance seems to dampen the vertical propagation of shocks across the production network, thereby weakening the expected comovement in production inputs across sectors. From the model's optimality conditions, this attenuation effect arises due to a sectoral output effect, $y(s)$. Specifically, when sectors



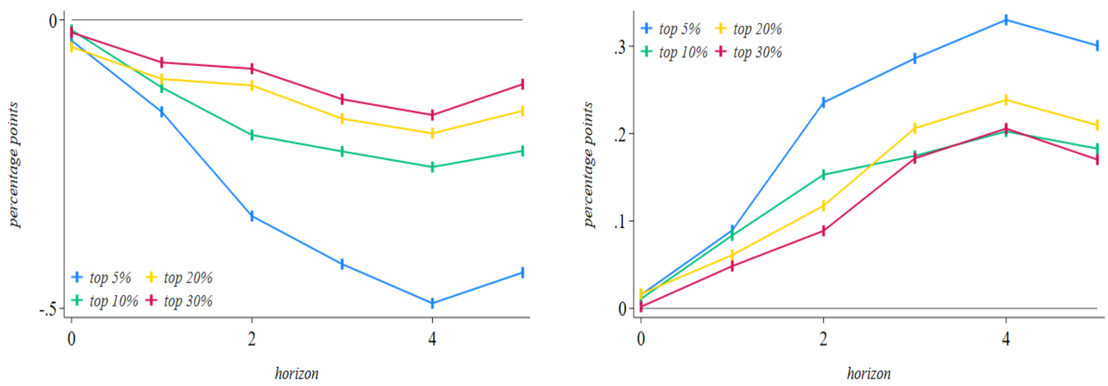
(A) $d = 1$ (B) $d = 2$

FIGURE 1.9: EMPLOYMENT COMOVEMENT UNDER SUPPLY LINKAGES

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 1.9a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 1.9b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.

share the same downstream buyers, a change in the relative revenues received from the common buyer leads to opposite changes in their relative supply of intermediate inputs (Lemma 2). This generates an inverse comovement in sectoral production levels and, consequently, an opposite comovement in sectoral employment levels (Subsection 1.5.1). In the context of Panel 1.9a, these opposing forces roughly cancel each other out when averaged across all sectors, resulting in the non-significant response. Accordingly, in analogy with the demand distances, clearer effects might be correctly identified when considering highly interlinked sectors.

Impulse responses for the clustered analysis are presented in Figure 1.10. The results for major network distances, shown in Panel 1.10b, largely mirror those found using the full sample, but with a notably higher magnitude in the point estimates responding to the structural employment shock: shocks in highly interconnected sectors generate stronger propagation effects through the production network, amplifying the employment response among these sectors. More strikingly, the results under minor supply distances, depicted in Panel 1.10a, fully confirm Proposition 3: any positive increase in employment levels within closely linked sectors induces a negative transmission on employment in other nearby sectors, resulting in an opposite comovement in sectoral employment levels. This inverse relationship highlights the counter-intuitive dynamics of factor input supply distance networks, where shocks do not propagate in a uniform manner but generate divergent employment responses depending on network proximity and the structure of interdependencies. Importantly, this opposite comovement effect grows stronger as the analysis narrows to an even smaller group of sectors with increasingly dense linkages: the closer the sectors are in the production network when supplying downstream, the more pronounced these opposing employment responses become. In sum, restricting the analysis to highly



(A) $d = 1$ (B) $d = 2$

FIGURE 1.10: SUPPLY-BASED COMOVEMENT IN HIGHLY INTERLINKED SECTORS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel 1.10a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel 1.10b plots the response to employment changes in further (distance equal to 2) ones. Each line corresponds to the average response of employment across sectors with different numbers of linkages, robust to 68% and 90% significance levels of bootstrapped Confidence Interval (CI) computed from eq. (1.9). *Source*: BEA and own calculations.

interlinked sectors (Theorem 3) provides a clearer picture of how employment shocks travel through the network, revealing that the Input-Output structure and its horizontal geometry fundamentally condition either the direction and the strength of sectoral comovement in employment levels.

Interpreting these results through Proposition 2 clarifies when opposite comovement arises: a positive shock to a sector contracts production in other closely supply-linked sectors, thereby reflecting the take over of supply-driven horizontal interdependencies (adjustments in upstream relative prices) on the vertical propagation (increase in downstream demand). Yet, downstream pass-through is weaker when sectors have dissimilar Input-Output structure in supply. All these theoretical and empirical results are in line with the non-linear theories in Atalay (2017) and Baqaee and Farhi (2019): downstream complementarities in intermediate inputs are crucial to generate opposite responses from an idiosyncratic shock (thus pivotal is the downstream pass-through effect).

1.5.3. ROBUSTNESS

Results on comovement hold along different "economically-based" robustness checks, sequentially presented in Appendix C. Initially, I shift the focus towards peripheral sectors within the production network. Much of the existing literature holds on the role of central sectors, typically measured using Bonacich-Katz centrality (*e.g.*, Carvalho 2014). The underlying premise is that a positive shock in a central sector transmits outward, potentially inducing broader economic expansions – *i.e.*, microeconomic shocks can scale up to macroeconomic consequences through network centrality. From a comovement perspective, periods of economic expansion (contraction) are expected only when a positive (negative) shock in a central sector effectively propagates to more peripheral sectors: fluctuating aggregate activity is not merely a direct realization of shocks to central nodes, but rather it stems from the manner in which such asymmetric shocks ripple across the network to other, more peripheral sectors.[25] If network distances were irrelevant, comovement would be expected universally. However, should demand- and supply-side linkages prove significant, the comovement patterns ought to remain consistent with those documented thus far.

I then test the robustness of the presented findings by employing alternative base years for the U.S. production network. My main analysis is anchored at the 2007 Input-Output structure, which conveniently divides the sample period into two equal sub-periods. Theoretically, the results should remain valid provided that the production network structure exhibits limited evolution over time.[26] Empirically, I replicate the full set of analyses using network data from other benchmark years.

Centrality scores.– Figures C.6 and C.7 present the impulse responses of employment to factor input demand and supply distances, respectively, considering only the peripheral sectors of the U.S. production network. Results are broadly consistent with those discussed in earlier sections: opposite comovement under supply distances, and positive comovement for shocks originated in further sectors. The only noteworthy exception concerns the response of employment to changes in closer sectors under demand-based distances: rather than exhibiting the ambiguity observed for all sectors (overlapping of vertical and horizontal dimensions), inspecting horizontal complementarities in demand for more peripheral sectors generate effects that resemble those observed under supply-based distances, with opposite comovement occurring across sectors sharing the same network structure in terms of buying from common suppliers. This refines the observed pattern under direct demand-based measures of Figure 1.7, suggesting that negative employment comovement among sectors purchasing from similar upstream suppliers largely occurs in sectors not playing a key role in the propagation of shocks.[27] These outcomes create a bridge between eq. (1.4) and the insights of Theorem 4: positive shocks in central sectors transmit unevenly to peripheral ones according to different network economic distance values, thereby inducing different types of aggregate behaviours.

Different base years.– All the documented results, derived using the 2007 U.S. tables, remain robust if replicated for alternative benchmark years (2002 and 2012), as depicted in Figures C.8, C.9, and C.10. This temporal consistency suggests that the structural features of the U.S. production network have remained remarkably

[25] Stated differently, it is not the initial shock itself that generates business cycle fluctuations, but the subsequent consequences it produces as it transmits across the network to less central nodes.

[26] A condition broadly supported in the empirical literature. For instance, Carvalho (2014) and Acemoglu, Ozdaglar, et al. (2016) document relative stability of U.S. sectoral production network and its role on shock propagation. Classical Input-Output analysis (*e.g.*, Leontief 1986, Miller and Blair 2009) also emphasises the persistence of production linkages in the short to medium run.

[27] In line with the discussion in Baqaee and Farhi (2019): within a sector, complementarities ("horizontal" in my framework) across intermediate inputs attenuate the aggregate effects of a positive idiosyncratic shock.

stable over time, in line with the academic debate. Such outcomes document the enduring nature of Input-Output linkages and the slow-moving evolution of production networks, thus lending further credibility to the proposed horizontal mechanisms.

1.6. Future Policy Perspectives

The Input-Output structure of the economy shapes how shocks propagate and contributes to business cycle dynamics. This paper has explored how the structure of production networks shapes sectoral comovement, moving beyond the traditional focus on vertical transmission to emphasize the role of horizontal economic relationships. By analysing both the demand- and supply-driven dimensions of sectoral interconnections, the analysis reveals that shocks do not propagate solely along upstream or downstream chains. Instead, sectors that share common suppliers or buyers experience intertwined vertical and horizontal effects. Overall, sectoral comovement arises not only from direct or indirect vertical production network linkages, but also from horizontal distances defined by shared inter-sectoral trade structure. By highlighting this additional dimension, the paper offers a new perspective on how idiosyncratic shocks propagate in networked economies: it is not merely the existence of Input-Output connections, but rather the demand and supply geometry of these linkages that fundamentally governs the transmission of micro-originated shocks and the resulting macroeconomic comovement patterns. Several are the contributions that horizontal complementarities can provide to the ongoing literature exploiting a production network perspective to form and deliver policy prescriptions.[28]

Fiscal policy.– A positive government-spending shock works through an increase in its demand of goods and services; affected sectors thus face an increase in their supply, stimulating an expansion of sectoral output. Enlarged production that meets the increased demand from the government then spread through Input-Output linkages as sectors rely on intermediate inputs sourced from their suppliers. In this regard, a micro-originated shock generates upstream effects (*e.g.*, Barattieri, Cacciatore, and Traum 2023). It is thus essential the horizontal dimension of a network since changes in demand and supply relationships – neglected in standard analysis – are at the core of mechanism triggered by an increased government demand. Moreover, horizontal effects play a critical role when considering the specific composition of public expenditure since certain sectors may be disproportionately exposed to changes in demand:[29] while the vertical transmission mechanism only depends on

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[28] On the theory side, horizontal complementarities can provide valuable insights into: *(i)* the functioning of global production networks (*e.g.*, Caliendo et al. 2022, Huo et al. 2025), extending beyond merely vertical supply chains; *(ii)* the complexity of firm-to-firm production networks (*e.g.*, di Giovanni et al. 2018, Boehm et al. 2019), as demand- and supply-based distance networks help to reveal relationships between nodes that would otherwise appear unconnected; *(iii)* the endogenous formation of production networks, since "economically" closer sectors tend to share similar Input-Output structures and might rely on common intermediate inputs that are more essential than others (*e.g.*, Carvalho and Voigtländer 2015); and *(iv)* the contribution of sectoral shocks to aggregate outcomes (*e.g.*, Acemoglu, Carvalho, et al. 2012, Atalay 2017, Baqaee and Farhi 2019).

[29] For example, a large-scale infrastructure program would boost demand for construction, cement, and

the intensity of inter-sectoral trade, incorporating measures of demand- and supply-driven network distances allows for a more accurate weighting in the propagation of sectoral shocks. Tariff policies operate through similar network mechanisms, not by raising demand directly, but rather altering relative prices (*e.g.*, Barattieri and Cacciatore 2023, Antonova et al. 2025, Clausing and Obstfeld 2025), effectively acting as revenue reallocation shocks (*i.e.*, my supply-based distances), lending further perspectives from network's horizontal geometries.

Industrial policy.– Two contemporaneous papers analyse how production networks amplify the impact of industrial policies supporting strategic sectors to promote sustained economic growth: essentials are the notions of "distortionary effects" (market imperfections accumulate through backward demand linkages; Liu 2019) and "downstream spillovers" (positive effects on buyers of targeted sectors; Lane 2025), and both perspectives exploit the verticality of sectoral connections.[30] Yet, how do industrial policies affect untargeted sectors exhibiting similar Input-Output structures to the targeted ones? This is where the horizontal dimension matters.

Keynesian transmission mechanism.– Complementarities (either in consumption or production) might turn an asymmetric supply shock, affecting a subset of sectors and reducing their demand for other sectors, into a demand-like shock at the aggregate level (the "Keynesian supply shock"; see Guerrieri et al. 2022). Production network is a natural field to study this mechanism (*e.g.*, Cesa-Bianchi and Ferrero 2021) and, reminiscent from Subsection 1.2.1 (Figure 1.2b), horizontal complementarities among sectors can help to clearly, easily and linearly identify demand and supply forces within the network that guide the Keynesian transmission. This extended mechanism is important for interventions aimed at stabilizing business cycles, acting as a suitable extension of the production network framework for monetary policy (*e.g.*, Pasten et al. 2020, Kalemli-Özcan et al. 2025).

Finally, by embedding sectoral similarities through common structures of Input-Output linkages, the horizontal dimension transcends the confines of production networks rendering it applicable to a broader class of economic interdependencies, including studies analysing investment (*e.g.*, vom Lehn and Winberry 2022) or financial (*e.g.*, Acemoglu, Ozdaglar, et al. 2015, Huremovic et al. 2025) networks.

It is along these avenues that I intend to orient my future research agenda.

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steel sectors, in turn strongly engaging with closely connected suppliers (such as mining, heavy machinery manufacturing, and architectural or engineering services). By contrast, weakly connected sectors with core infrastructure-related sectors (such as apparel manufacturing, publishing, residential cleaning services, or artisanal food production) would be only marginally affected.

[30] Explanations related to cross-country income differences, Input-Output economies and industrialization are presented in Ciccone (2002) and Fadinger et al. (2022). Conley and Ligon (2002) study how cross-country dependence and GDP growth rates are shaped by "economic distance".

1.7. Concluding Remarks

Initial inquiries into production networks chiefly centred on how disaggregation induced by inter-sectoral trade generates comovement of macroeconomic variables. Reviving this foundational perspective, this paper moves beyond traditional vertical linkages by theoretically introducing and empirically validating the horizontal geometry of Input-Output economies, measured through network "economic" distances reflecting similarities between sectors in terms of common upstream suppliers or downstream buyers. Constitutive premise is that sectors are densely interconnected not only by their direct trade flows but also through shared inter-sectoral relationships, providing a richer understanding of the complex architecture underlying any capitalist economy. Demand-based distances promote an overlap of horizontal and vertical propagation, as shocks transmitted through shared upstream suppliers interact with parallel supply chain effects. By contrast, supply-based distances directly threaten the design of vertical transmission, as revenue reallocation among sectors selling to the same downstream buyers counteracts the standard propagation of shocks. These mechanisms operate independently of inter-sectoral trade intensities, offering a powerful refinement to prevailing network analysis. As for aggregate fluctuations, horizontal propagation induces correlated adjustments across sectors sharing suppliers or buyers, generating persistent aggregate comovement even without vertical cascading effects and central sectors. By conceptualising these horizontal complementarities, the paper enriches the understanding of how independent and idiosyncratic shocks permeate the production system. Ultimately, it reveals that the demand/supply architecture of an Input-Output structure and its horizontal geometry critically shape the transmission of shocks and comovement, offering novel insights into the networked nature of economic activity.

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Appendix

(Outline) *In this appendix I report all the material complementary to the main text; it is made of additional tables and figures, and of discussions on further analytical results. Section A deepens further all the theoretical findings, where the subsections consequentially derive all the salient elements of the partial equilibrium results on production network distances of Section 1.2, prove the theorems in Section 1.3, and derive and prove all the characterizing features of the outlined Input-Output general equilibrium model. Section B consolidates and further discusses the data presented in Section 1.4. Finally, Section C complements and enriches the exposition of the empirical findings in Section 1.5.*

A. Theoretical results and model derivation

Proofs for theoretical results on network economic distances

(Proof of Lemma 1) *Consider a Cobb-Douglas economy as stated in Section 1.3. The combination of optimal demand for both labour and intermediate inputs is*

$$x\left(s,s'\right) = \alpha\left(s,s'\right) w\left(s\right) n\left(s\right)\ \alpha\left(s\right)^{-1} \frac{1}{p\left(s'\right)}$$

Then, consider the case in which two sectors $\{s,s'\}$ *are buying their own intermediate inputs from the same sector-*s^**. Associated system of equations is*

$$\begin{cases} x\left(s,s^*\right) = \alpha\left(s,s^*\right) w\left(s\right) n\left(s\right)\ \alpha\left(s\right)^{-1} \dfrac{1}{p\left(s^*\right)} \\ x\left(s',s^*\right) = \alpha\left(s',s^*\right) w\left(s'\right) n\left(s'\right)\ \alpha\left(s'\right)^{-1} \dfrac{1}{p\left(s^*\right)} \end{cases}$$

$$\longrightarrow \quad \begin{cases} x\left(s,s^*\right) = \alpha\left(s,s^*\right) \left[x\left(s',s^*\right) \dfrac{1}{\alpha\left(s',s^*\right) n\left(s'\right)} \alpha\, p\left(s^*\right)\right] n\left(s\right)\ \alpha^{-1} \dfrac{1}{p\left(s^*\right)} \\ w = x\left(s',s^*\right) \dfrac{1}{\alpha\left(s',s^*\right) n\left(s'\right)} \alpha\, p\left(s^*\right) \end{cases}$$

where I exploit the fact that wage is the numeraire of the economy, so that $w\left(s\right) = w$*,* $\forall s \in \Phi\left(s\right)$*, and, without any loss of generality (to simplify notation and build better intuition),* $\alpha\left(s\right) = \alpha$*,* $\forall s \in \Phi\left(s\right)$ *as well. The above substituted equation will result in*

$$\frac{x(s,s^*)}{x(s',s^*)} = \frac{\alpha(s,s^*)}{\alpha(s',s^*)}\frac{n(s)}{n(s')} \quad \equiv \quad \frac{n(s')}{x(s',s^*)} = \frac{\alpha(s,s^*)}{\alpha(s',s^*)}\frac{n(s)}{x(s,s^*)} \tag{A.1}$$

Now, note that solving the representative firm's problem with log-quantities will report exactly the same optimality conditions. Henceforth, it is not necessary to take logs in total log-differentiation. In other words, consider the following expression, in which both $n(\cdot)$ and $x(\cdot)$ are already expressed in logarithmic form:

$$\frac{\log n(s')}{\log x(s',s^*)} = \frac{\alpha(s,s^*)}{\alpha(s',s^*)}\frac{\log n(s)}{\log x(s,s^*)}$$

Let me define the following for notational convenience: $N(s) := \log n(s)$, $X(s,s^*) := \log x(s,s^*)$, and $d_{s^*\to}[s,s'] := \frac{\alpha(s,s^*)}{\alpha(s',s^*)}$. The expression becomes

$$\frac{N(s')}{X(s',s^*)} = d_{s^*\to}[s,s']\,\frac{N(s)}{X(s,s^*)}$$

I proceed by performing a first-order Taylor expansion around the steady state. Let $\overline{N}(s)$ and $\overline{X}(s,s^*)$ denote the steady-state values of $N(s)$ and $X(s,s^*)$, respectively, and let $dN(s)$ and $dX(s,s^*)$ denote their log-deviations. Using the total differential of a ratio, it is possible to obtain

$$d\left(\frac{N(s)}{X(s,s^*)}\right) = \frac{1}{\overline{X}(s,s^*)}\,dN(s) - \frac{\overline{N}(s)}{\overline{X}^2(s,s^*)}\,dX(s,s^*)$$

and similarly for sector-s', $d\left(\frac{N(s')}{X(s',s^*)}\right) = \frac{1}{\overline{X}(s',s^*)}\,dN(s') - \frac{\overline{N}(s')}{\overline{X}^2(s',s^*)}\,dX(s',s^*)$. Since $d_{s^*\to}[s,s']$ is a constant parameter, it is possible to differentiate both sides of the original equation, $\frac{N(s')}{X(s',s^*)} = d_{s^*\to}[s,s']\,\frac{N(s)}{X(s,s^*)}$:

$$d\left(\frac{N(s')}{X(s',s^*)}\right) = d_{s^*\to}[s,s']\,d\left(\frac{N(s)}{X(s,s^*)}\right)$$

Inserting differentials, I derive the following first-order log-linearized expression

$$\frac{1}{\overline{X}(s',s^*)}dN(s') - \frac{\overline{N}(s')}{\overline{X}^2(s',s^*)}dX(s',s^*) = d_{s^*\to}[s,s']\left(\frac{1}{\overline{X}(s,s^*)}dN(s) - \frac{\overline{N}(s)}{\overline{X}^2(s,s^*)}dX(s,s^*)\right)$$

which, given $d_{s^*\to}[s,s'] = \frac{\alpha(s,s^*)}{\alpha(s',s^*)}$, will deliver the two-sector condition in Subsection 1.2.2. This result illustrates that the log-linearized response of the ratio between logged employment and logged intermediate inputs in sector-s' is proportional to that of sector-s, with the scaling factor given by their relative intensity of intermediate input usage, $\alpha(s,s^*)\,/\,\alpha(s',s^*)$. Steady-state values $\overline{N}(\cdot)$ and $\overline{X}(\cdot,s^*)$ cannot be avoided in this log-linearization due to the non-linear nature of the ratio between logarithms.

Stacked across all sectors, the above condition yields Lemma 1. Note that the same would have been obtained under Constant Elasticity of Substitution (CES) technology between $n(s)$ and $x(\cdot)$, and for any production function satisfying Assumption 1.

□

(Proof of Lemma 2) *Consider a Cobb-Douglas economy as stated in Section 1.3. The combination of optimal demand for both labour and intermediate inputs is*

$$x\left(s,s'\right) = \alpha\left(s,s'\right) w\left(s\right) n\left(s\right)\ \alpha\left(s\right)^{-1} \frac{1}{p\left(s'\right)}$$

Then, consider the case in which two sectors $\{s,s'\}$ *are selling their own intermediate inputs to the same sector-*s^**. Associated system of equations is*

$$\begin{cases} x\left(s^*,s\right) = \alpha\left(s^*,s\right) w\left(s^*\right) n\left(s^*\right)\ \alpha\left(s^*\right)^{-1} \dfrac{1}{p\left(s\right)} \\ x\left(s^*,s'\right) = \alpha\left(s^*,s'\right) w\left(s^*\right) n\left(s^*\right)\ \alpha\left(s^*\right)^{-1} \dfrac{1}{p\left(s'\right)} \end{cases}$$

$$\longrightarrow \quad \begin{cases} x\left(s^*,s\right) = \alpha\left(s^*,s\right)\left[x\left(s^*,s'\right) \dfrac{1}{\alpha\left(s^*,s'\right) n\left(s^*\right)} \alpha\left(s^*\right)\ p\left(s'\right)\right] n\left(s^*\right)\ \alpha\left(s^*\right)^{-1} \dfrac{1}{p\left(s\right)} \\ w\left(s^*\right) = x\left(s^*,s'\right) \dfrac{1}{\alpha\left(s^*,s'\right) n\left(s^*\right)} \alpha\left(s^*\right)\ p\left(s'\right) \end{cases}$$

The above substituted equation will result in

$$\frac{x\left(s^*,s\right)}{x\left(s^*,s'\right)} = \frac{\alpha\left(s^*,s\right)}{\alpha\left(s^*,s'\right)} \frac{p\left(s'\right)}{p\left(s\right)} \tag{A.2}$$

which, if log-linearized around its steady state (note that solving the representative firm's problem with log-quantities will report exactly the same optimality conditions; thus, not necessary to take logs in total log-differentiation), closely following the proof for Lemma 1, will deliver the two-sector condition in Subsection 1.2.2 which, stacked across all sectors, delivers the result in Lemma 2. Note that the same result would have been obtained under Constant Elasticity of Substitution (CES) technology between $n\left(s\right)$ *and* $x\left(\cdot\right)$*, and for any production function satisfying Assumption 1.*

For completeness, consider the following expression

$$\frac{\log x\left(s^*,s\right)}{\log x\left(s^*,s'\right)} = \frac{\alpha\left(s^*,s\right)}{\alpha\left(s^*,s'\right)} \frac{\log p\left(s'\right)}{\log p\left(s\right)}$$

and define the following for notational convenience: $P\left(s\right) := \log p\left(s\right)$*,* $X\left(s^*,s\right) := \log x\left(s^*,s\right)$*, and* $d_{\rightarrow s^*}\left[s,s'\right] := \frac{\alpha\left(s^*,s\right)}{\alpha\left(s^*,s'\right)}$*. Thus, the expression becomes*

$$\frac{X\left(s^*,s\right)}{X\left(s^*,s'\right)} = d_{\rightarrow s^*}\left[s,s'\right] \frac{P\left(s'\right)}{P\left(s\right)}$$

Proceed by performing a first-order Taylor expansion around the steady state. Let $\overline{X}\left(s^*,s\right)$ *and* $\overline{P}\left(s\right)$ *denote the steady-state values of* $X\left(s^*,s\right)$ *and* $P\left(s\right)$*, respectively, and let* $d\,X\left(s\right)$ *and* $d\,P\left(s\right)$ *to denote their log-deviations. Using the total differential of a ratio, then*

$$d\left(\frac{X\left(s^*,s\right)}{X\left(s^*,s'\right)}\right) = \frac{1}{\overline{X}\left(s^*,s'\right)}\, d\,X\left(s^*,s\right) - \frac{\overline{X}\left(s^*,s\right)}{\overline{X}\left(s^*,s'\right)^2}\, d\,X\left(s^*,s'\right)$$

In the same manner the right-hand side can be manipulated, $d\left(d\left(s^* \mid s, s'\right) \frac{P(s')}{P(s)}\right) = d\left(s^* \mid s, s'\right) \left(\frac{1}{\overline{P}(s)} d\,P\left(s'\right) - \frac{\overline{P}(s')}{\overline{P}^2(s)} d\,P\left(s\right)\right)$, *since* $d_{\to s^*}\left[s, s'\right]$ *is not changing. Differentiating both sides of the original equation,* $\frac{X(s^*,s)}{X(s^*,s')} = d_{\to s^*}\left[s, s'\right] \frac{P(s')}{P(s)}$, *one obtains*

$$\frac{1}{\overline{X}\left(s^*, s'\right)} d\,X\left(s^*, s\right) - \frac{\overline{X}\left(s^*, s\right)}{\overline{X}\left(s^*, s'\right)^2} d\,X\left(s^*, s'\right) = d_{\to s^*}\left[s, s'\right] \left(\frac{1}{\overline{P}\left(s\right)} d\,P\left(s'\right) - \frac{\overline{P}\left(s'\right)}{\overline{P}\left(s\right)^2} d\,P\left(s\right)\right)$$

This result demonstrates how the log-linearized relationship between price and quantity ratios in sectors $\{s, s'\}$ *depends on their relative intensity* $d_{\to s^*}\left[s, s'\right] = \frac{\alpha(s^*,s)}{\alpha(s^*,s')}$. *Similar to the previous proof, the steady-state values* $\overline{X}\left(s^*, \cdot\right)$ *and* $\overline{P}\left(\cdot\right)$ *appear due to the non-linear nature (i.e., the ratio of logarithms) of the initial expression.*

□

(Proof of Theorem 1) *Let* $\Phi\left(s\right)$ *denote the finite set of sectors in the economy. Each sector can buy intermediate inputs from other sectors and, simultaneously, can sell its output to other sectors. Define this Input-Output bilateral trade intensity as*

$$\alpha\left(s, s'\right) : \Phi\left(s\right) \times \Phi\left(s\right) \to \left[0, \infty\right), \quad \text{the amount of inputs purchased by } s \text{ from } s'$$

Express then each fixed coefficient $\alpha\left(\cdot\right)$*'s in terms of share of sectoral total purchases or sales – i.e.,* $\alpha^{fd}\left(s, s'\right) = \frac{\alpha(s,s')}{\sum_{s' \in row} \alpha(s,s')}$ *and* $\alpha^{fs}\left(s, s'\right) = \frac{\alpha(s,s')}{\sum_{s' \in column} \alpha(s,s')}$;[31] *these values allow to know not only "who trades with whom" but how much in relative terms.*

Define the upstream (buying intermediate input from) and downstream (selling output to) sets of neighbours of sector-s^* *as*

$$\mathcal{N}^{up}\left(s^*\right) := \left\{s \in \Phi\left(s\right) : \alpha^{fs}\left(s^*, s\right) > 0\right\} \quad \text{and} \quad \mathcal{N}^{dw}\left(s^*\right) := \left\{s \in \Phi\left(s\right) : \alpha^{fd}\left(s, s^*\right) > 0\right\}$$

Simplify temporarily with $\Phi\left(s\right) = \{s, s', s^*\}$*: the upstream set points for the share of purchases of the common sector-*s^* *from sectors* $\{s, s'\}$*, and the downstream set represents the share of sales of the common sector-*s^* *to sectors* $\{s, s'\}$*. These are the same information in the Input-Output matrix, but now fully abstract. For any pair of sectors* $\{s, s'\} \in \Phi\left(s\right)$*, let the set of common neighbours be*

$$\mathcal{S}_{s,s'} := \left\{s^* \in \Phi\left(s\right) : s^* \in \mathcal{N}^{up}\left(s\right) \cap \mathcal{N}^{up}\left(s'\right) \quad \text{or} \quad s^* \in \mathcal{N}^{dw}\left(s\right) \cap \mathcal{N}^{dw}\left(s'\right)\right\}$$

for any common sector-s^**. These common neighbours are the sectors that can horizontally connect two already-connected or otherwise unconnected sectors vertically. Indeed, define the horizontal linkage (or network "economic" distance) metrics*

$$d_{s^* \to}\left[s, s'\right] := f\left(\alpha^{fd}\left(s, s^*\right), \alpha^{fd}\left(s', s^*\right)\right) \quad \text{and} \quad d_{\to s^*}\left[s, s'\right] := f\left(\alpha^{fs}\left(s^*, s\right), \alpha^{fs}\left(s^*, s'\right)\right)$$

[31] Row-normalized values show each sector's sales share to other sectors, highlighting how similarly sectors source from common upstream suppliers; analogously, column-normalized values show each sector's input share from other sectors, highlighting how similarly sectors sell to common downstream buyers. For further references, refer to Appendix B.

for $s^* \in \mathcal{S}_{s,s'}$. Relative to sectors in brackets, subscript "$s^* \rightarrow$" reads as "purchasing from sector-s^*", while subscript "$\rightarrow s^*$" reads as "selling to sector-s^*". Define further the vectors of factor input demand and supply distances for the pair $\{s, s'\}$ as

$$d^{fd}\left[s,s'\right] := \left\{d_{s^*\rightarrow}\left[s,s'\right] : s^* \in \mathcal{S}_{s,s'}\right\} \quad \textit{and} \quad d^{fs}\left[s,s'\right] := \left\{d_{\rightarrow s^*}\left[s,s'\right] : s^* \in \mathcal{S}_{s,s'}\right\}$$

Part 1: horizontal linkage exists even if two sectors are not connected. *Assume $\alpha^j(s,s') = 0$ and $\alpha^j(s',s) = 0$, for $j = \{fd, fs\}$. If there exists $s^* \in \Phi(s)$ such that $s^* \in \mathcal{N}^{up}(s) \cap \mathcal{N}^{up}(s')$ or $s^* \in \mathcal{N}^{dw}(s) \cap \mathcal{N}^{dw}(s')$, then*

$$d^{fd}\left[s,s'\right] > 0 \ \textit{and} \ d^{fs}\left[s,s'\right] > 0$$

quantify the horizontal linkages between $\{s, s'\}$ in common upstream demand or downstream supply, respectively. Therefore, these sectors are horizontally connected through a common neighbour sector-s^*: even though $\{s, s'\}$ don't trade directly within the production network, the fact that they both interact with s^* makes them economically connected; this is the "horizontal complementarity" concept.

Part 2: factor input demand and supply distances are generally different. *Let $\{s, s'\} \in \Phi(s)$ and let $s^* \in \mathcal{S}_{s,s'}$ be a common neighbour. By definition, the network "economic" distance metrics are $d_{s^*\rightarrow}\left[s,s'\right]$ and $d_{\rightarrow s^*}\left[s,s'\right]$. Unless the trade is symmetric (next part of the proof), that is when $\alpha^{fd}(s,s^*) = \alpha^{fs}(s^*,s) \ \forall s \in \Phi(s)$, then $d_{s^*\rightarrow}\left[s,s'\right] \neq d_{\rightarrow s^*}\left[s,s'\right]$. Extending to all common neighbours $s^* \in \mathcal{S}_{s,s'}$, it follows that*

$$d^{fd}\left[s,s'\right] := \left\{d_{s^*\rightarrow}\left[s,s'\right] : s^* \in \mathcal{S}_{s,s'}\right\} \quad \neq \quad d^{fs}\left[s,s'\right] := \left\{d_{\rightarrow s^*}\left[s,s'\right] : s^* \in \mathcal{S}_{s,s'}\right\}$$

The network "economic" distance between two sectors depends on whether one looks at it from the perspective of demand or supply between one sector and the common one, and they usually don't match; these two sets of numbers are different because buying and selling patterns are usually not symmetric. Hence, the same Input-Output structure always generates two distinct factor input demand and factor input supply distance metrics unless trade linkages with common sectors are perfectly symmetric.

Part 3: symmetric interactions imply coinciding distances for at least one common neighbour. *Suppose that for some common sector-s^*, the trade is perfectly symmetric: s buys the same relative amount from s^* as it sells to s^*, and same for s'. If it exists a common $s^* \in \mathcal{S}_{s,s'}$ such that $\alpha^{fd}(s,s^*) = \alpha^{fs}(s^*,s)$ and $\alpha^{fd}(s',s^*) = \alpha^{fs}(s^*,s')$, with these being of different values, then*

$$d_{s^*\rightarrow}\left[s,s'\right] := f\Big(\alpha^{fd}(s,s^*), \alpha^{fd}(s',s^*)\Big) \quad = \quad f\Big(\alpha^{fs}(s^*,s), \alpha^{fs}(s^*,s')\Big) =: d_{\rightarrow s^*}\left[s,s'\right]$$

and therefore $d^{fd}\left[s,s'\right] \cap d^{fs}\left[s,s'\right] \neq \emptyset$.[32] *In this case, the distance metrics are*

[32] In other words, if sectors $\{s, s'\}$ buy and sell symmetrically with s^* but at different Input-Output intensities, then the demand and supply network "economic" distances between $\{s, s'\}$ are the same.

exactly equal: demand-based distance is the same of the supply-based distance for that neighbour – or distance vectors $\forall s^* \in \Phi(s)$. If two sectors trade symmetrically with a common sector, then their economic distance looks the same whether it is measured by demand or supply. The building intuition is that symmetry in relative trade "aligns" the two perspectives of network "economic" distance, which would otherwise differ.

Finally, it is left to prove the minimum number of sectors required for such coinciding distances to exist. Suppose the economy has m sectors, $\Phi(s) = \{s, s', s^*, \ldots, m\}$, and consider a sector-$s^*$ acting as the common neighbour. To construct a pair of sectors $\{s, s'\}$ whose factor input demand and supply distances coincide via s^*, it is required: (i) the common neighbour s^* itself, and (ii) at least two distinct sectors s and s' that trade with s^* symmetrically. Now, compare two network sizes.

***Three-sector network**. Let the sectors be $\{s, s', s^*\} \in \Phi(s)$ with $s^* \in \mathcal{S}_{s,s'}$, so that the minimum number of sectors is $m = 2 + 1 = 3$. Under trade symmetry, I have already claimed that the two distance metrics coincide: $d_{s^* \to}[s, s'] = d_{\to s^*}[s, s']$. But a general condition also asks for an additional symmetric entry between another pair of sectors (distinct from the pair linked by s^*). With only three sectors there is no distinct third pair available that is disjoint from $\{s, s'\}$ and uses a different sector: the only other possible symmetric entries would involve the same sectors $\{s, s', s^*\}$ as well. Therefore, a case with $m = 3$ does not provide an independent additional symmetric entry: any symmetric pair must involve these three sectors and cannot be both an extra independent symmetric pair and keep the structure non-degenerate. Concretely, with $m = 3$ one may obtain a single coinciding distance metric via a common neighbour, but it is not general enough to satisfy the hypothesis requiring a separate additional symmetric entry independent of the same three nodes. Thus, a three-sector network is insufficient to guarantee a general statement of the theorem as formulated.*

***Bigger network**. To allow the existence of an additional symmetric entry between another pair of sectors, at least one more sector is required. Let $\{s, s', s^*, \ldots, t\} \in \Phi(s)$ with $s^* \in \mathcal{S}_{s,s'}$, and with at least one extra sector-t. Therefore, the absolute minimum network size to guarantee that there exists at least one coinciding distance vector is $m \geq 4$. Usual trade symmetry delivers $d_{s^* \to}[s, s'] = d_{\to s^*}[s, s']$. Because there is at least one additional sector-t, it is possible also to impose an additional symmetric entry between some other pair: this additional symmetric entry can be chosen so it is not the same symmetry used at $\{s, s'\}$ and s^* – i.e., it provides a distinct symmetric pair in the network. Thus, both ingredients required by the general theorem can be satisfied in a non-degenerate way when $m \geq 4$: a common neighbour symmetric with (at least) two sectors, and at least one independent symmetric pair elsewhere. In such a way, the network has enough nodes to realize the general hypothesis (common neighbour symmetric with $m - 2$ sectors and a separate symmetric pair), thereby guaranteeing the existence of at least one distance vector that coincides in the constructions of demand and supply distances.*

***General formulation**. Let $\Phi(s)$ be the set of sectors, with $|\Phi(s)| = m$, and let the set of common neighbours of s and s' to be*

$$\mathcal{S}_{s,s'} := \left\{ s^* \in \Phi(s) \setminus \{s, s'\} : s^* \textit{ is a common neighbour of } s \textit{ and } s' \right\}$$

Define the demand- and supply-based network "economic" distance multi sets as $d^{fd}[s,s'] := \left\{ f\left(\alpha^{fd}(s,s^*), \alpha^{fd}(s',s^*)\right) : s^* \in \mathcal{S}_{s,s'} \right\}$ *and, analogously,* $d^{fs}[s,s'] := \left\{ f\left(\alpha^{fs}(s^*,s), \alpha^{fs}(s^*,s')\right) : s^* \in \mathcal{S}_{s,s'} \right\}$. *Assume it exists a subset* $\mathcal{T}_{s,s'} \subseteq \mathcal{S}_{s,s'}$ *such that all sectors in* $\mathcal{T}_{s,s'}$ *satisfy bilateral symmetry with both* $\{s,s'\}$: $\alpha^{fd}(s,s^*) = \alpha^{fs}(s^*,s)$ *and* $\alpha^{fd}(s',s^*) = \alpha^{fs}(s^*,s')$, $\forall s^* \in \mathcal{T}_{s,s'}$. *Then, the set of coinciding distances is*

$$\mathcal{C}_{s,s'} := d^{fd}\left[s,s'\right] \cap d^{fs}\left[s,s'\right] = \left\{ f\left(\alpha^{fd}(s,s^*), \alpha^{fd}(s',s^*)\right) : s^* \in \mathcal{T}_{s,s'} \right\}$$

Thus, $\mathcal{C}_{s,s'} \neq \emptyset$ *if* $\mathcal{T}_{s,s'} \neq \emptyset$: *the demand-based and supply-based distance between two sectors* $\{s,s'\}$ *coincide if and only if there is at least one sector* $s^* \in \mathcal{T}_{s,s'}$ *that both trade with symmetrically. For the general statement of the theorem: if* $|\mathcal{T}_{s,s'}| \geq S-2$ *and there exists at least one additional symmetric pair* $\{s,t\} \in \Phi^2(s)$*, then*

$$\exists\, s,s' \in \Phi(s) : \mathcal{C}_{s,s'} \neq \emptyset$$

□

(Proof of Corollary 1 for Theorem 1) *The task is to determine how many coinciding distances should appear when* $m \geq 4$. *Let*

$$\mathcal{S}_{|s^*} = \left\{s, s', \ldots, S\right\} \subset \Phi(s) \setminus \{s^*\}, \qquad S = m-2$$

denote the set of sectors that trade symmetrically with the common sector s^*. *Symmetry means that for every* $s \in \mathcal{S}_{|s^*}$, $\alpha^{fd}(s,s^*) = \alpha^{fs}(s^*,s)$, *and similarly for any other directional equality required by the construction of* $d^{fd}[\cdot]$ *and* $d^{fs}[\cdot]$. *Recall that these equalities must hold after Input-Output matrix normalization (divide by rows for demand distance, by columns for supply distance). Which distances coincide and how many? For any unordered pair* $\{s,s'\} \subset \mathcal{S}_{|s^*}$, *the distances defined through the common neighbour* s^* *satisfy, as usual,* $d_{s^*\to}\left[s,s'\right] = d_{\to s^*}\left[s,s'\right]$. *Thus each unordered pair inside* $\mathcal{S}_{|s^*}$ *generates one coinciding demand- and supply-based network "economic" distance. Now, the idea is to count all pairs of sectors within the set of symmetric neighbours* $\mathcal{S}_{|s^*}$ *that share the common sector-*s^*. *Suppose that* $\mathcal{S}_{|s^*}$ *contains* $S = m-2$ *sectors that trade symmetrically with* s^*. *To find the coinciding distances, I consider unordered pairs* $\{s,s^*\} \in \mathcal{S}_{|s^*}$ *because each pair corresponds to one entry in the distance matrix (which is symmetric). The number of ways to pick two sectors from among the* m*-sectors is given by the binomial coefficient* $\binom{S}{2} = \frac{S(S-1)}{2}$. *It counts all distinct pairs of symmetric neighbours: each pair "shares" the same symmetric relation with the common sector, so that their demand- and supply-based network "economic" distances coincide.*[33] *Intuitively, for a common sector symmetric with* $m-2$ *neighbours,*

[33] If one sector s^* is symmetric with exactly $S = m-2$ other sectors, then, for $m = 5$ one gets $\binom{3}{2} = 3$ distinct unordered sector-pairs whose demand- and supply-based distances coincide – equivalently, 6 equal off-diagonal distance matrix entries, since each pair corresponds to (s,s') and (s',s). In fact, as $S = m-2 = 3$, then $\binom{S}{2} = \frac{3(3-1)}{2} = \frac{3\times 2}{2} = 3$.

every unordered pair of those neighbours has matching demand- and supply-based distances, giving $\binom{m-2}{2}$ *coinciding distance values; the formula comes from counting all distinct pairs among the* $m-2$ *symmetric sectors.*

□

Proofs for theoretical results on distance-based propagation

(Proof of Proposition 1) *Consider a buyer sector-*s *that demands an intermediate input supplied by an upstream sector-*s^**. The buyer's technology follows a CES aggregator with elasticity of substitution* $\sigma > 0$*. From cost minimization, the log-demand elasticity of the intermediate with respect to its own price is given by* $\frac{\partial \log x(s,s^*)}{\partial \log p(s^*)} = -\sigma$*. Suppose a shock in sector-*s' *increases demand for the input produced by* s^**. Let the upstream price respond according to* $\frac{\partial \log p(s^*)}{\partial \log x(s',s^*)} = \tau^{fd}$*. By the chain rule, the effect of the demand shock on sector-*s*'s input demand is then*

$$\frac{\partial \log x\,(s,s^*)}{\partial \log x\,(s',s^*)} = \frac{\partial \log x\,(s,s^*)}{\partial \log p\,(s^*)} \cdot \frac{\partial \log p\,(s^*)}{\partial \log x\,(s',s^*)} = -\left(\sigma \cdot \tau^{fd}\right)$$

Thus, the sign of the comovement depends on the product $\sigma \cdot \tau^{fd}$*. If* $\sigma \cdot \tau^{fd} > 0$*, the comovement is negative. The mechanism operates through two key channels: (i) the price channel, where a shock to sector-*s' *increases demand for inputs from* s^* *and, if upstream supply is inelastic, the price* $p\,(s^*)$ *rises, so that* $\tau^{fd} > 0$*; and (ii) the substitution channel, where the buyer sector-*s *responds by substituting away from the now more expensive input, and the magnitude of this effect depends on the elasticity of substitution* σ*. When both effects are active (positive* τ^{fd} *and sizeable* σ*), sector-*s *reduces demand, creating opposite comovement with the shocked sector.*

Consider instead a downstream sector-s *that buys a finite set of intermediate inputs supplied by upstream sectors. Let the composite intermediate input be a CES aggregator as* $x\,(s) = \left(\sum_{s^* \in \Phi(s)} \alpha\,(s,s^*)^{\frac{1}{\sigma}}\, x\,(s,s^*)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$*, where* $x\,(s,s^*)$ *is the quantity of the intermediate supplied by upstream sector-*s^* *to downstream buyer-*s*, while* $\alpha\,(s,s^*) \in (0,1)$ *are network intensity parameters, and* $\sigma > 0$ *is the (constant) elasticity of substitution across intermediate inputs. Define its associated (unit) price index for its composite intermediate by the usual CES formula,* $p_s = \left(\sum_{s^* \in \Phi(s)} \alpha\,(s,s^*)\, p\,(s^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$*, where* $p\,(s^*)$ *is the price of the intermediate supplied by upstream sector-*s^**.*

Under cost minimization, for a given level of composite intermediate $X\,(s)$*, the conditional demand for intermediate input* s^* *is given by the standard CES demand system. Define the expenditure share of variety* s^* *in the composite intermediate by* $e\,(s,s^*) = \frac{\alpha(s,s^*)p(s^*)^{1-\sigma}}{\sum_{s^*\in\Phi(s)} \alpha(s,s^*)p(s^*)^{1-\sigma}}$*, with* $\sum_{s^* \in \Phi(s)} e\,(s,s^*) = 1$*. A well-known property of the CES demand system is the (log) price elasticities for conditional demands:*

$$\frac{\partial \log x\,(s,s^*)}{\partial \log p\,(s^*)} = -\sigma\Big[1 - e\,(s,s^*)\Big] \quad , \quad \frac{\partial \log x\,(s,s^*)}{\partial \log p\,(s^{**})} = \sigma\, e\,(s,s^{**}) \quad \forall s^{**} \neq s^*$$

The former, own-price elasticity depends on the substitution parameter and on the expenditure share, while the latter, cross-price elasticity is proportional to the share of *the other intermediate input considered.*

Focus on a particular upstream sector-s^* that supplies an intermediate used by several downstream buyers. Consider an idiosyncratic positive demand shock in some sector-s' that increases aggregate demand for the good produced by s^*. Let $x(s', s^*)$ denote the demand shifter (or demand level) associated with the shocked sector-s' from sector-s^*. The upstream price of the good produced by s^* responds to the shock according to a (log) pass-through parameter $\tau^{fd} = \frac{\partial \log p(s^*)}{\partial \log x(s', s^*)}$ of demand changes into prices. By the chain rule, and noting that only $p(s^*)$ changes in response to the shock (holding other upstream prices fixed in this partial equilibrium exercise), one obtains

$$\frac{\partial \log x(s, s^*)}{\partial \log x(s', s^*)} = \frac{\partial \log x(s, s^*)}{\partial \log p(s^*)} \cdot \frac{\partial \log p(s^*)}{\partial \log x(s', s^*)} = -\sigma \left[1 - e(s, s^*)\right] \cdot \tau^{fd}$$

Three points are worth nothing: (i) the elasticity of substitution, σ, magnifies the response – the larger σ, the stronger buyer s substitutes away from an input whose price rises; (ii) the expenditure share, $e(s, s^*)$, attenuates the own-price effect – if buyer-s spends only a small fraction on the input from s^*, then $\left[1 - e(s, s^*)\right] \approx 1$ and the full $-\sigma$ factor operates, while if the share is large, the own-price sensitivity is mechanically smaller in magnitude; and (iii) the upstream pass-through, τ^{fd}, transmits the original downstream demand shock into a price change at the supplier – without such pass-through ($\tau^{fd} = 0$) the substitution channel vanishes.

Hence, the sign of the log-response is the sign of $-\sigma\left[1 - e(s, s^*)\right]\tau^{fd}$. Because $\sigma > 0$ and $1 - e(s, s^*) > 0$ (for an interior share), the sign is governed by τ^{fd}: if $\tau^{fd} > 0$ (upstream price rises) and $\sigma > 0$, buyer s reduces its demand for s^*'s input, thereby delivering negative comovement; by contrast, if $\tau^{fd} \approx 0$ (perfectly elastic upstream supply), the price channel is absent and do not expect opposite comovement via this substitution mechanism. Its logic is shown in the upper panel of Figure A.1.

□

(Proof of Proposition 2) *Consider an upstream supplier-s that sells a finite set of intermediate inputs to a downstream sector-s^*. Let the composite intermediate input be a CES aggregator $x(s^*) = \left(\sum_{s \in \Phi(s)} \alpha(s^*, s)^{\frac{1}{\sigma}}\ x(s^*, s)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, where $x(s^*, s)$ is the quantity of the intermediate supplied by upstream sector-s to downstream buyer-s^*, while $\alpha(s, s^*) \in (0, 1)$ are network intensity parameters, and $\sigma > 0$ is the (constant) elasticity of substitution across intermediate inputs. Associated is the (unit) price index from the usual CES formula, $p_{s^*} = \left(\sum_{s \in \Phi(s)} \alpha(s^*, s)\ p(s)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, with $p(s)$ denoting the price of input from upstream supplier-s.*

Under cost minimization, for a given level of composite intermediate $X(s^*)$, the conditional demand for intermediate input s is given by the standard CES demand system. Define the expenditure share of variety s in the composite intermediate by $e(s^*, s) = \frac{\alpha(s^*, s)\, p(s^*)^{1-\sigma}}{\sum_{s \in \Phi(s)} \alpha(s^*, s)\, p(s)^{1-\sigma}}$, with $\sum_{s \in \Phi(s)} e(s^*, s) = 1$. A well-known property of the CES demand system is the (log) price elasticities for conditional demands:

$$\frac{\partial \log x(s^*, s)}{\partial \log p(s)} = -\sigma\left[1 - e(s^*, s)\right] \quad , \quad \frac{\partial \log x(s^*, s)}{\partial \log p(s')} = \sigma\, e(s^*, s') \quad \forall s' \neq s$$

The former, own-factor demand elasticity is decreasing in own price, scaled by substitution elasticity and expenditure share, while in the latter, cross-price elasticities, demand for s *rises if another input-*s' *becomes more expensive relative to* s'*'s share.*

Focus on a particular downstream sector-s^* *that buys an intermediate used by several upstream sellers. Consider an idiosyncratic positive demand shock in some sector-*s' *that increases aggregate supply for the good purchased by* s^**. Let* $x(s^*, s')$ *denote the supply shifter (or supply level) associated with the shocked sector-*s' *to sector-*s^**. The upstream price of the good produced by* s' *responds to the shock according to a (log) pass-through parameter* $\tau^{fs} = \frac{\partial \log p(s')}{\partial \log x(s^*, s')}$ *of supply changes into prices.*

By the chain rule, and noting that only $p(s)$ *changes in response to the shock (holding other upstream prices fixed in this partial equilibrium exercise), obtaining*

$$\frac{\partial \log x(s^*, s)}{\partial \log x(s^*, s')} = \frac{\partial \log x(s^*, s)}{\partial \log p(s')} \frac{\partial \log p(s')}{\partial \log x(s^*, s')} = \sigma\, e(s^*, s') \cdot \tau^{fs}$$

Three transparent points: (i) the elasticity of substitution, σ*, magnifies the response – a larger* σ *amplifies the substitution away from* s *and toward* s' *when* $p(s')$ *falls; (ii) the expenditure share of the common downstream sector,* $e(s^*, s')$*, attenuates the price effect – the cheaper input* s' *represents a bigger share of* s^**'s bundle; and (iii) the downstream pass-through,* τ^{fs}*, transmits the original upstream supply shock into a price change at the buyer – without such pass-through (*$\tau^{fs} = 0$*) the substitution channel vanishes. If the downstream sector* s^* *uses only a single input from an upstream supplier (i.e. no possibility of substitutability across inputs), the elasticity of substitution is effectively zero (*$\sigma = 0$*). In this case, there is no scope for reallocation of expenditures across inputs: a price change in* s' *does not alter the demand for* s*:* $\frac{\partial \log x(s^*, s)}{\partial \log p(s')} = 0$*. Therefore, unlike the demand-driven case, the single-input supply case does not yield a sufficient statistic, as there is no mechanism generating comovement.*

Hence, the sign of the log-response is the sign of $\sigma\, e(s^*, s')\, \tau^{fs}$*. Because* $\sigma > 0$ *and* $e(s^*, s') > 0$ *(for an interior share), the sign is governed by* τ^{fs}*: if* $\tau^{fs} < 0$*, the product* $\sigma\, e(s^*, s')\, \tau^{fs}$ *is negative, implying negative comovement between* s *and* s' *through their common buyer; by contrast, if* $\tau^{fs} \approx 0$ *(perfectly elastic downstream demand), the price channel is absent and do not expect opposite comovement via this substitution mechanism. Its logic is shown in the lower panel of Figure A.1.*

□

(Proof of Theorem 2) *This part provides a purely intuition-driven mathematical proof of Theorem 2, as it relies on a structural distinction between horizontal and vertical operators on production In this sense, the theorem is proved as a "structural impossibility result". I then provide some elements for the proof.*

Economic intuition as a mathematical principle*. The economic intuition underlying Theorem 2 can be summarized as follows: horizontal interactions capture network similarity across peers (shared upstream suppliers and downstream buyers), whereas vertical interactions capture production inputs flows along idiosyncratic supply chains. Economically, horizontal effects operate <u>within</u> a layer of the production network, while vertical effects operate <u>across</u> layers. Mathematically, this implies that* *"horizontal effects depend on similarity of local neighbourhoods, whereas vertical effects depend on paths of arbitrary length".*

Theorem 2 is therefore a structural statement: operators that measure similarity of neighbourhoods cannot be weighted by operators that aggregate along paths.

***Minimal graph-theoretic setup**. Let $G = (m, E)$ be a finite directed weighted graph, with indicator-m representing the number of nodes in the network, and $E = \{(i,j) \in m \times m : \alpha_{ij} > 0\}$ is the set of directed edges representing non-zero linkages between nodes. Let $\boldsymbol{H} = [\alpha_{ij}] \in \mathbb{R}^{m\times m}$ denote the adjacency (Input-Output) matrix of G, with α_{ij} representing the inputs that node-i sources from node-j. When the spectral radius $\rho(\boldsymbol{H}) < 1$, the Leontief inverse $\boldsymbol{\mathcal{H}} := (\boldsymbol{I} - \boldsymbol{H})^{-1}$ exists. Production is sequential at the k-th round of input requirements, $\boldsymbol{\mathcal{H}} \equiv \sum_{k=0}^{\infty} \boldsymbol{H}^k$.*

Then, the need is to define what horizontal and vertical operators are. Matrix $\boldsymbol{\mathcal{D}} = [d_{ij}] \in \mathbb{R}^{m\times m}$ is called an "horizontal (distance) operator" if $d_{ij} = f^d(\alpha_{\cdot i}, \alpha_{\cdot j})$, where $\alpha_{\cdot i}$ denotes the immediate neighbourhood of node-i (common upstream suppliers or downstream buyers, that is the entire input shares/sales of node-i), and $f^d(\cdot)$ is a function that depends only on the overlap or similarity of these neighbourhoods (i.e., a fixed similarity or distance functional): horizontal effects are thus defined as finite-profile (column/row) dependence.[34] Differently, vertical effects are fundamentally different from profile comparison and involve repeated application of $\boldsymbol{V} = \{\boldsymbol{H}, \boldsymbol{\mathcal{H}}\}$. Indeed, an operator is called "vertical" if it aggregates information along paths of length greater than or equal to one, as it depends on recursive compositions of the Input-Output matrix – i.e., if it can be written as $\sum_{k\geq 1} \boldsymbol{V}^k A$ for some non-negative matrix A. The network matrix indicator $\boldsymbol{V}$ is the canonical vertical operator: it sums contributions from all (directed and indirected) paths in the network, thereby aggregating the effects of shocks along all production chains of arbitrary length: vertical effects are thus defined as recursive composition of production technologies.

***Locality versus propagation**. Here is the mathematical statement. In particular, through a series of lemmas, I am going to define the nature of both vertical and horizontal anatomies, and then prove the theorem by contradiction.*

LEMMA *3 (Locality of horizontal operators)* *If a certain horizontal entry is $d_{ij} = f^d(\alpha_{\cdot i}, \alpha_{\cdot j})$, then horizontal matrix $\boldsymbol{\mathcal{D}}$ depends only on a finite-dimensional representation of $\boldsymbol{V}$ and does not involve recursive application of $\boldsymbol{V}$.*

Proof: each entry d_{ij} is constructed from the vectors $\alpha_{\cdot i}$ and $\alpha_{\cdot j}$ through a fixed functional $f^d(\cdot)$. No term involves powers $\boldsymbol{V}^k$ with $k \geq 2$, nor any infinite series.

LEMMA *4 (Non-locality of the Input-Output weights)* *Any vertical operator involving $\boldsymbol{V}$ depends on all powers $\boldsymbol{V}^k$, for $k \geq 0$.*

Proof: by definition, $\boldsymbol{V} \equiv \sum_{k=0}^{\infty} \boldsymbol{V}^k$. Each power $\boldsymbol{V}^k$ represents k-fold composition of technologies. Hence, any object involving $\boldsymbol{V}$ depends on recursively propagated Input-Output relations.

[34] Horizontal distances therefore depend on *finite-dimensional technological profiles*. They compare how two sectors source inputs (or sell outputs), but they do not involve repeated application of the Input-Output matrix.

LEMMA 5 *(Incompatibility)* *No local operator contains a global path operator.*

Proof: assume by contradiction that a local operator $\mathcal{D}$ could be written as an equilibrium object $B = V'\widetilde{\mathcal{D}}$ for some $\widetilde{\mathcal{D}}$, so that horizontal effects enter the equilibrium multiplied by the Input-Output weights.[35] *By contrast, given the non-locality of V, the object B depends on arbitrarily long production paths on all recursive compositions V^k, besides the local neighbourhood similarity of $\mathcal{D}$. Then, B would inherit the non-local dependence of V, contradicting the locality established above: by the incompatibility lemma, such a representation is impossible.*

A re-written version of the incompatibility lemma delivers Theorem 2: horizontal distance operators cannot be multiplicatively weighted by the Input-Output weights.

Characterizing intuition. *Turning to a conceptual part of the proof, it is essential to recall that the horizontal dimension of the network, as captured by its network "economic" distances, is not confined to sectors already connected – directly or indirectly – through inter-sectoral trade. Rather, it also encompasses sectors without such direct or indirect linkages, which are nonetheless related through similar demand or supply relationships arising from common upstream suppliers or downstream buyers. This reasoning implies that certain sectors may exhibit a strictly positive distance linkage despite the absence of any Input-Output connection. In other words, considers two sectors, say $\{s, s'\}$, with zero (direct and Leontief inverse) inter-sectoral trade intensity – $\alpha(s, s') = \alpha(s', s) = 0$ and $\ell(s, s') = \ell(s', s) = 0$ –, held together by their demand/supply relationship with a common sector, say s^*, so that $d_{s^*\rightarrow}[s', s''] = d_{\rightarrow s^*}[s', s''] > 0$. In this case, the Input-Output matrix and the distance matrix would be given by $\boldsymbol{H} \equiv \boldsymbol{\mathcal{H}} = \begin{bmatrix} \cdot & 0 \\ 0 & \cdot \end{bmatrix}$ and $\boldsymbol{\mathcal{D}}^j = \begin{bmatrix} \cdot & > 0 \\ > 0 & \cdot \end{bmatrix}$, for $j = \{fd, fs\}$. Accordingly, multiplying the distance matrix by the production network matrix would effectively neutralise the role of distances. This rationale underpins Theorem 2.*

□

(Corollary to Theorem 2) *To make the result transparent to a broader theory, I now reformulate the "structural impossibility" argument to prove Theorem 2 using: (i) the language of Spectral Graph theory; and (ii) Green's functions/resolvent operators.*

Spectral Graph theory interpretation. *In Spectral Graph theory, powers of the adjacency matrix V^k encode walks of length-$k \in [0, \infty)$ on the network, and is therefore a global spectral object sensitive to the full eigenvalue distribution of V. Horizontal distance operators, by contrast, are kernel operators defined on the space of columns (or rows) of V. They depend only on first-order technological profiles and do not involve eigenvalue amplification or long-run diffusion. Theorem 2 therefore states that a kernel on the column/row space of V cannot be composed multiplicatively with the resolvent of V without changing its mathematical nature.*

---

[35] By construction, each entry of $\mathcal{D}$ depends only on finite-dimensional technological profiles $\{\alpha_{\cdot i}, \alpha_{\cdot j}\}$. Therefore, $V'\widetilde{\mathcal{D}}$ depends on recursive compositions of production technologies, while $\widetilde{\mathcal{D}}$ itself depends only on static comparisons of production technologies. However, horizontal effects arise from substitutability across technologies, not from iterated production chains: there is no economic or algebraic mechanism that generates higher-order compositions of V inside $f^d(\cdot)$. As a result, $V'\widetilde{\mathcal{D}}$ cannot arise from the equilibrium derivation.

Green's functions and resolvents. *The Input-Output indicator* $\mathbf{V}$ *is the Green's function (or resolvent) associated to a linear operator. It solves propagation problems of the form* $\mathbf{V}'y = x$*, by transmitting shocks through all production chains. An horizontal operator* $\mathcal{D}$ *is not a Green's function. They do not solve propagation problems and do not correspond to inverses of linear operators. Instead, they define a metric or similarity structure on the space of technologies. From this perspective, Theorem 2 asserts that a metric kernel on technologies (horizontal geometry) cannot appear inside a Green's function propagator (i.e., a vertical production dynamics). The two objects live in distinct mathematical categories: one is a similarity kernel, the other a resolvent of a linear operator.*

□

(Proof of Theorem 3) *Let* $G = (m, E)$ *be a finite directed weighted graph, with indicator-*m *representing the number of nodes in the network, and* $E = \{ (i,j) \in m \times m : \alpha_{ij} > 0 \}$ *is the set of directed edges representing non-zero linkages between nodes. Let* $\boldsymbol{H} = [\alpha_{ij}] \in \mathbb{R}^{m \times m}$ *denote the adjacency (Input-Output) matrix of* G*, with* α_{ij} *representing the inputs that node-*i *sources from node-*j*. Decompose as* $\mathbf{H}_m = \mathcal{V}_m + \mathcal{D}_m$*, where* $\mathcal{V}_m$ *encodes vertical Input-Output relations, that is the vertical supplier-buyer linkages, and* $\mathcal{D}_m$ *encodes horizontal complementarities, meaning pairwise similarity between sectors based on shared suppliers or buyers, or shared upstream/downstream network structures. The decomposition isolates the vertical and horizontal components.*

To characterize the asymptotic structure of the production network and analyse large networks, I embed the decomposed Input-Output matrix – $\mathbf{H}_m$*,* $\mathcal{V}_m$*, and* $\mathcal{D}_m$ *– into graphon spaces –* $\mathcal{W}_{\mathbf{H}_m}$*,* $\mathcal{W}_{\mathcal{V}_m}$*, and* $\mathcal{W}_{\mathcal{D}_m}$ *–, representing the global geometry of a large graph, where networks are represented by functions rather than finite matrices. For each* $m \times m$ *matrix* $\mathbf{H}_m$ *its associated graphon* $\mathcal{W}_{\mathbf{H}_m}$ *is a step-function on a measurable space* $U : [0,1]^2$*. Graphon convergence is measured in the "cut norm"* $\|U\|_\square$*, identified with subscript-*$\square$*, a way to capture global structural differences between graphs (i.e., convergence metric: how the network structure behaves in the limit). In fact, graphons allow to capture the "limiting geometry" of very large networks: if* $\mathcal{W}_{\mathbf{H}_m} \to \mathcal{W}$*, then the global structure stabilizes to a continuous object* $\mathcal{W}$*. The cut norm controls spectral norm (i.e., propagation metric: how big the shock propagation can be), identified with subscript-*$\triangle$*, up to a universal constant (i.e., the Lovász-Szegedy inequality):* $\|\boldsymbol{H}_m\|_\triangle \leq \sigma \|\mathcal{W}_{\mathbf{H}_m}\|_\square$ *for any specified constant* σ*: if a network is sparse in structure, then it cannot amplify shocks much.*

Assume the following. (1) vertical sparsity: supply chains do not scale with network size-m*, and each node has a certain number of suppliers/buyers; the vertical graphon becomes negligible in the limit,* $\|\mathcal{W}_{\mathcal{V}_m}\|_\square \to 0$*, thus implying* $\|\mathcal{V}_m\|_\triangle \to 0$*; (2) convergence of horizontal complementarities: network "economic" distances converge to a non-degenerate (non-zero) graphon, as they remain dense and globally relevant in the limit; in particular, there exists a symmetric graphon* $\mathcal{W} : [0,1]^2 \to [0,1]$ *such that* $\|\mathcal{W}_{\mathcal{D}_m} - \mathcal{W}\|_\square \to 0$ *and* $\|\mathcal{W}\|_\square > 0$*, which implies* $\liminf_m \|\mathcal{D}_m\|_\triangle > 0$*; this is the dominant network structure in the limit, as it creates the nontrivial eigenvalues* *responsible for persistent (and large) shock propagation; and (3) vector of bounded shocks: $\|\boldsymbol{\rho}_m\|_\infty \in [0,\infty)$, which ensures the consistency in scaling the results.*

Finally, define the shock's propagation operators. Given the $m \times m$ Input-Output matrix $\mathbf{H}_m$, define $\boldsymbol{\mathcal{P}}_m = f^{\mathcal{P}}(\mathbf{H}_m)$ as the operator that gives the total propagation of an initial idiosyncratic, independent and asymmetric shock to a certain node transmitting to its (direct and indirect) network connections. Similarly, by separating vertical and horizontal channels of propagation, define $\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})} = f^{\mathcal{P}}(\boldsymbol{\mathcal{V}}_m)$ and $\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})} = f^{\mathcal{P}}(\boldsymbol{\mathcal{D}}_m)$: the former measures propagation due solely to vertical buyer-supplier network ties; the latter transmits shocks through horizontal complementarities. $f^{\mathcal{P}}(\cdot)$ is a linear operator, continuous in the norm of its argument: for $f^{\mathcal{P}}(\cdot) : \mathbb{R}^{m\times m} \to \mathbb{R}^{m\times m}$, then $\|a-b\|_\triangle \to 0$ implies $\left\|f^{\mathcal{P}}(a) - f^{\mathcal{P}}(b)\right\|_\triangle \to 0$. This assumption ensures that whenever $\mathbf{H}_m, \boldsymbol{\mathcal{V}}_m, \boldsymbol{\mathcal{D}}_m \to 0$ in spectral norm, the corresponding propagator also vanishes.

***Step 1: Convergence of the full network geometry**.– Given the coexistence of vertical and horizontal geometries, the graphon for the total Input-Output matrix $\mathbf{H}_m$ is $\boldsymbol{\mathcal{W}}_{A_m} = \boldsymbol{\mathcal{W}}_{V_m} + \boldsymbol{\mathcal{W}}_{H_m}$. It follows that*

$$\|\boldsymbol{\mathcal{W}}_{\mathbf{H}_m} - \boldsymbol{\mathcal{W}}\|_\Box \leq \|\boldsymbol{\mathcal{W}}_{\boldsymbol{\mathcal{V}}_m}\|_\Box + \|\boldsymbol{\mathcal{W}}_{\boldsymbol{\mathcal{D}}_m} - \boldsymbol{\mathcal{W}}\|_\Box$$

By assumptions vertical part disappears, $\|\boldsymbol{\mathcal{W}}_{\boldsymbol{\mathcal{V}}_m}\|_\Box \to 0$, and the horizontal part converges, $\|\boldsymbol{\mathcal{W}}_{\boldsymbol{\mathcal{D}}_m} - \boldsymbol{\mathcal{W}}\|_\Box \to 0$. Thus, the whole network converges, $\boldsymbol{\mathcal{W}}_{\mathbf{H}_m} \to \boldsymbol{\mathcal{W}}$: The large-scale geometry of production networks is asymptotically determined entirely by horizontal complementarities.

***Step 2: Vertical propagation vanishes**.– As vertical sparsity implies $\|\boldsymbol{\mathcal{V}}_m\|_\triangle \to 0$ for $m \to \infty$, and since the transmission operator is continuous in its norm, $\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})} = f^{\mathcal{P}}(\boldsymbol{\mathcal{V}}_m)$, then $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})}\right\|_\triangle = \|f^{\mathcal{P}}(\boldsymbol{\mathcal{V}}_m)\|_\triangle \to \|f^{\mathcal{P}}(0)\|_\triangle = 0$. Since $\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})} = f^{\mathcal{P}}(\boldsymbol{\mathcal{V}}_m) \to f^{\mathcal{P}}(0) = 0$, continuity implies $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})}\boldsymbol{\rho}_m\right\|_\triangle \to 0$. Vertical spillovers die out because vertical links are too sparse to amplify anything when n is large enough, that is when vertical connections are too sparse to support long-range propagation.*

***Step 3: Horizontal propagation persists**.– As the graphon $\boldsymbol{\mathcal{W}}$ has positive mass: (i) the horizontal network is asymptotically connected; (ii) the expected weighted degree converges to a strictly positive constant; and (iii) spectral norm is bounded away from zero, $\liminf_m \|\boldsymbol{\mathcal{D}}_m\|_\triangle > 0$. Since $\liminf_m \|\boldsymbol{\mathcal{D}}_m\|_\triangle > 0$, and due to operator's continuity, then $\liminf_m \left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_\triangle > 0$. Thus, the horizontal geometry amplifies the shock as the network enlarges, so that $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_\triangle > 0$. Horizontal complementarities form a dense backbone through which node-specific shocks propagate as $m \to \infty$.*

***Step 4: Ratio dominance**.– Since $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})}\boldsymbol{\rho}_m\right\|_\triangle \to 0$ and $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_\triangle > 0$, then*

$\frac{\left\|\mathcal{P}_m^{(\mathcal{V})}\boldsymbol{\rho}_m\right\|_\triangle}{\left\|\mathcal{P}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_\triangle} \to 0$. *Vertical spillovers disappear relative to horizontal spillovers.*

Characterizing intuition.– *In large economies with static production networks, the number of direct upstream suppliers or downstream buyers per nodes does not scale with the network size-m. Thus, vertical propagation channel become negligible in the limit and vertical shock propagation cannot scale: vertical effects remain local and thin, thereby explaining the vanishing vertical transmission mechanism. In contrast, network participants tend to share connections with many others, especially when the number of nodes grows (i.e., that is for larger and larger networks). Shared buyers/suppliers become more likely, horizontal similarity becomes dense, and horizontal complementarities increase with network size-m. Thus, horizontal effects remain global and strong, thereby resulting in a global structure that persists in the limit. As a result, in very large production networks, horizontal effects tend to dominate the transmission of shocks, whereas vertical chain effects become negligible. While vertical propagation fades away, horizontal effects persist: large Input-Output economies are dominated by network similarity effects, not supply chain effects.*

□

(Proof of Corollary 2 for Theorem 3) *Let $\mathcal{W}_{\mathcal{V}_m}$ and $\mathcal{W}_{\mathcal{D}_m}$ denote the graphons associated with the vertical and horizontal components of the network, respectively. Shock propagation along these two is given by the linear operators $\mathcal{P}_m^{(\mathcal{V})}$ and $\mathcal{P}_m^{(\mathcal{D})}$.*

By assumption, $\|\mathcal{W}_{\mathcal{V}_m}\|_\square$ is non-decreasing in network size-m, and horizontal complementarities must satisfy $\|\mathcal{W}_{\mathcal{D}_m}\|_\square \geq a \ \|\mathcal{W}_{\mathcal{V}_m}\|_\square$ far a given $a > 0$. Even if vertical linkages increase with network size-m, the horizontal complementarities induced by shared suppliers or buyers grow faster, because each new vertical relation creates new horizontal connections among nodes that share Input-Output partners. Henceforth:

$$\|\mathcal{W}_{\mathcal{V}_m}\|_\square \Big/ \|\mathcal{W}_{\mathcal{D}_m}\|_\square \leq \frac{1}{a} \tag{T3.1}$$

The Lovász-Szegedy inequality provides universal constants $0 < b \leq c < \infty$, that is $\left\|\mathcal{P}_m^{(\mathcal{V})}\right\|_\triangle \leq c\,\|\mathcal{W}_{\mathcal{V}_m}\|_\square$ and $\left\|\mathcal{P}_m^{(\mathcal{D})}\right\|_\triangle \geq b\,\|\mathcal{W}_{\mathcal{D}_m}\|_\square$. When dividing these inequalities, $\frac{\left\|\mathcal{P}_m^{(\mathcal{V})}\right\|_\triangle}{\left\|\mathcal{P}_m^{(\mathcal{D})}\right\|_\triangle} \leq \frac{c}{b}\frac{\|\mathcal{W}_{\mathcal{V}_m}\|_\square}{\|\mathcal{W}_{\mathcal{D}_m}\|_\square}$. Plugging in eq. (T3.1) yields the uniform bound

$$\left\|\mathcal{P}_m^{(\mathcal{V})}\right\|_\triangle \Big/ \left\|\mathcal{P}_m^{(\mathcal{D})}\right\|_\triangle \leq \frac{c}{b}\frac{1}{a} \tag{T3.2}$$

Let the shock vector $\boldsymbol{\rho}_m \in \mathbb{R}^n$ be uniformly bounded, so that $\|\boldsymbol{\rho}_m\|_\triangle \leq a_\rho$ for some constant $a_\rho > 0$ independent of network size-m. By submultiplicativity of the operator norm, $\left\|\mathcal{P}_m^{(\mathcal{V})}\boldsymbol{\rho}_m\right\|_\triangle \leq \left\|\mathcal{P}_m^{(\mathcal{V})}\right\|_\triangle \times \|\boldsymbol{\rho}_m\|_\triangle \leq a_\rho \left\|\mathcal{P}_m^{(\mathcal{V})}\right\|_\triangle$. Similarly, $\left\|\mathcal{P}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_\triangle \leq a_\rho \left\|\mathcal{P}_m^{(\mathcal{D})}\right\|_\triangle$. Hence, for all network size-m, $\frac{\left\|\mathcal{P}_m^{(\mathcal{V})}\boldsymbol{\rho}_m\right\|_\triangle}{\left\|\mathcal{P}_m^{(\mathcal{D})}\boldsymbol{\rho}_m\right\|_\triangle} \leq \frac{\left\|\mathcal{P}_m^{(\mathcal{V})}\right\|_\triangle}{\left\|\mathcal{P}_m^{(\mathcal{D})}\right\|_\triangle}$. By construction, as a network with size-m grows, the horizontal component becomes strictly denser than the vertical one, so the denominator grows faster than the numerator. If combined *with eq. (T3.2) I obtain a uniform, n-independent, strictly finite upper bound. Hence* $\left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{V})} \boldsymbol{\rho}_m\right\|_{\triangle} \Big/ \left\|\boldsymbol{\mathcal{P}}_m^{(\mathcal{D})} \boldsymbol{\rho}_m\right\|_{\triangle} \xrightarrow[m\to\infty]{} 0$. *This establishes that, even under endogenous expansion of the vertical component, the horizontal dimension remains asymptotically dominant in determining the total propagation of shocks. So, in large networks, a growth in vertical edges produces a related growth in horizontal complementarities: the dominance of horizontal transmission survives when vertical linkages grow endogenously. In other words, even if network participants form more vertical linkages as the economy grows, the combinatorial explosion of shared upstream suppliers and downstream buyers ensures that horizontal complementarities still become the overwhelmingly dominant channel of shock propagation.*

□

(Proof of Theorem 4) *I want to formalize the role of horizontal propagation in generating aggregate output fluctuations in the absence of central or dominant sectors.*

Let $G = (m, E)$ *be a finite directed weighted graph, with indicator-m representing the number of nodes in the network, and* $E = \left\{ (i,j) \in m \times m : \alpha_{ij} > 0 \right\}$ *is the set of directed edges representing non-zero linkages between nodes. Let* $\boldsymbol{H} = \left[\alpha_{ij}\right] \in \mathbb{R}^{m\times m}$ *denote the adjacency (Input-Output) matrix of G, with* α_{ij} *representing the inputs that node-i sources from node-j. When the spectral radius* $\rho\left(\boldsymbol{H}\right) < 1$, *the Leontief inverse* $\boldsymbol{\mathcal{H}} := \left(\boldsymbol{I} - \boldsymbol{H}\right)^{-1}$ *exists. Production is sequential at the k-th round of input requirements,* $\boldsymbol{\mathcal{H}} \equiv \sum_{k=0}^{\infty} \boldsymbol{H}^k$.

Let each sector potentially experiences a an asymmetric, idiosyncratic and independent productivity shock. Let $\mathbb{P}$ *be a probability space. Define the productivity-related random variable* $\left\{\varepsilon_1, \ldots, \varepsilon_i, \varepsilon_j \ldots, \varepsilon_m\right\} : \mathbb{P} \to \mathbb{R}$ *such that*

$$\mathbb{E}\left[\varepsilon_i\right] = 0 \quad , \quad \mathbb{E}\left[\varepsilon_i^2\right] = \sigma^2 < \infty \quad , \quad \mathbb{E}\left[\varepsilon_i\, \varepsilon_j\right] = 0 \quad \textit{for} \quad i \neq j$$

The vector of micro-level shocks be $\boldsymbol{\varepsilon} := \left(\varepsilon_1, \ldots, \varepsilon_m\right)'$. *The covariance matrix of shocks is therefore* $\sigma^2 \boldsymbol{I}$. *In this sense, all correlations in equilibrium outcomes must come from the network, not from correlated shocks.*

Let $\boldsymbol{\lambda} = \left(\lambda_1, \ldots, \lambda_m\right)' \in \mathbb{R}^m$ *be the sectoral Domar weights, satisfying* $\lambda_i > 0$ *with* $\sum_{i=1}^{m} \lambda_i = 1$. *Define aggregate log-output as* $Y := \boldsymbol{\lambda}'\, \boldsymbol{\mathcal{H}}\, \boldsymbol{\varepsilon}$ *or, equivalently,*

$$Y = \sum_{i=1}^{m} v_i\, \varepsilon_i \qquad \boldsymbol{v} := \boldsymbol{\mathcal{H}}'\, \boldsymbol{\lambda}$$

where v_i *measures the general equilibrium importance of sector-i. This is the standard Acemoglu et al. (2012) representation: aggregate output is a weighted sum of sectoral outputs after vertical network propagation system.*

Given this configuration, it is then possible to define the variance of aggregate output from micro-level determinants under classical supply chain amplification: independent and idiosyncratic shocks generate correlated sectoral outputs through Leontief inverse-based vertical chains.

Lemma 6 ***(Vertical variance of aggregate output)*** *In an Input-Output economy, aggregate volatility depends on how concentrated the network-adjusted weights are*

$$var\,(Y) = \sigma^2\, \boldsymbol{\lambda}'\, \boldsymbol{\mathcal{H}}\, \boldsymbol{\mathcal{H}}'\, \boldsymbol{\lambda}$$

Proof: central idea is to compute the baseline aggregate output variance, ignoring horizontal linkages (i.e., network "economic" distances) between vertexes, to show the contribution of vertical chains alone. By definition, $Y = \sum_{i=1}^{m} v_i \varepsilon_i$*, with variance* $var\,(Y) = \mathbb{E}\left[\left(\sum_{i=1}^{m} v_i\, \varepsilon_i\right)^2\right]$*. Expanding the square,*

$$\begin{aligned} var\,(Y) &= \mathbb{E}\left[\sum_{i=1}^{m} v_i^2 \varepsilon_i^2 + \sum_{i \neq j} v_i v_j \varepsilon_i \varepsilon_j\right] && \text{where, using independence, then} \\ &= \sum_{i=1}^{m} v_i^2 \mathbb{E}\left[\varepsilon_i^2\right] + \sum_{i \neq j} v_i v_j \mathbb{E}\left[\varepsilon_i \varepsilon_j\right] && \text{Since the cross-terms vanish, then} \\ &= \sigma^2 \sum_{i=1}^{m} v_i^2 \end{aligned}$$

Since $\boldsymbol{v} = \boldsymbol{\mathcal{H}}'\, \boldsymbol{\lambda}$*, then* $\sum_{i=1}^{m} v_i^2 = \|\boldsymbol{v}\|^2 = \boldsymbol{\lambda}'\, \boldsymbol{\mathcal{H}}\, \boldsymbol{\mathcal{H}}'\, \boldsymbol{\lambda}$*. Aggregate volatility is the quadratic form induced by the propagation of micro-level shocks along vertical supply chains.*

It is now required to formalize the horizontal propagation channel. Let, for each sector-i, the upstream set (i.e., the suppliers of sector-i) and the "upstream (demand-based) similarity" be defined by

$$\mathcal{N}^{fd}\,(i) = \{m : \alpha_{im} > 0\} \quad , \quad d_{ij}^{fd} = \frac{\left|\mathcal{N}^{fd}\,(i) \cap \mathcal{N}^{fd}\,(j)\,\right|}{\left|\mathcal{N}^{fd}\,(i) \cup \mathcal{N}^{fd}\,(j)\,\right|}$$

If $\{i, j\}$ *buy from the same upstream sectors, then* $d_{ij}^{fd} > 0$*. This measures shared supplier exposure. Analogously let, for each sector-i, the downstream set (i.e., the buyers of sector-i) and the "downstream (supply-based) similarity" be defined by*

$$\mathcal{N}^{fs}\,(i) = \{m : \alpha_{mi} > 0\} \quad , \quad d_{ij}^{fs} = \frac{\left|\mathcal{N}^{fs}\,(i) \cap \mathcal{N}^{fs}\,(j)\,\right|}{\left|\mathcal{N}^{fs}\,(i) \cup \mathcal{N}^{fs}\,(j)\,\right|}$$

If $\{i, j\}$ *sell to the same downstream sectors, then* $d_{ij}^{fs} > 0$*. This measures shared buyer exposure. Combined horizontal similarity is then* $\boldsymbol{\mathcal{D}} = \boldsymbol{\mathcal{D}}^{fd} \cup \boldsymbol{\mathcal{D}}^{fs}$*. This matrix captures the full horizontal propagation logic. Recall that, by Theorem 1, matrix* $\boldsymbol{\mathcal{D}}^{j}$*, for* $j = \{fd, fs\}$*, is symmetric by construction, so is* $\boldsymbol{\mathcal{D}}$*. Horizontal linkages generate correlation between sectors sharing suppliers or buyers, independently of vertical propagation. Entry* $d_{ij} > 0$ *means that shocks hitting sector-i partially affect sector-j through common upstream suppliers or downstream buyers, independently of vertical propagation mechanism. The stronger the overlap, the stronger the induced correlation. This implies that even small sectors can transmit shocks horizontally if they share key upstream or downstream partners.*

Let the total propagation operator to be defined by

$$\boldsymbol{\mathcal{P}} := \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}' \cup \boldsymbol{\mathcal{D}}$$

which is the total covariance matrix reflecting the union of vertical and horizontal effects. Given the vertical propagation, $\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}'$*, and the horizontal propagation,* $\boldsymbol{\mathcal{D}}$*, the union operator* $\cup$ *implies that each channel contributes independently to aggre-*

gate variance, with no weighting or multiplicative interaction.[36] *separated structure makes the proof sharper, because horizontal propagation does not need vertical propagation to generate aggregate fluctuations (no dependence of horizontal propagation on vertical chains). In terms of economic intuition, aggregate fluctuations are driven by the superposition of two independent channels: shocks propagate both vertically along chains and horizontally via shared Input-Output connections. Horizontal propagation alone can generate aggregate fluctuations, even if vertical propagation is small.*

LEMMA 7 *(Variance of aggregate output)* *In an Input-Output economy, aggregate volatility depends on the combination of vertical and horizontal propagation*

$$var(Y) = \sigma^2 \boldsymbol{\lambda}' \boldsymbol{\mathcal{P}} \boldsymbol{\lambda}$$

Proof: substitute the total amplification operator into Lemma 6.

Let me, finally, impose some assumptions.

ASSUMPTION 2 *(Bounded vertical amplification)* *There exists $\chi > 0$ such that for all network size-m and all $\boldsymbol{x} \in \mathbb{R}^m$, $\boldsymbol{x}'\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}'\boldsymbol{x} \geq \chi \|\boldsymbol{x}\|^2$.*

It guarantees that vertical propagation is finite and non-degenerate. Element $\boldsymbol{x}$ is an arbitrary real vector of dimension equal to the number of sectors (think of it as a hypothetical pattern of sectoral exposure or weights, that is a subset of sectors, a weighted combination of sector outputs, a direction along which shocks are aggregated, ...).[37] *No matter how to aggregate sectors, vertical propagation does not collapse that aggregation to zero. In other words, the Input-Output network is not degenerate: there are no directions in which shocks completely die out, and vertical propagation has full rank and strength. This assumption is not required for the horizontal-only result and is included for completeness.*

ASSUMPTION 3 *(Diffuse Domar weights)* *There exist constants $q, r > 0$ such that, for all network size-m and every sector-i, $\frac{q}{m} \leq \lambda_i \leq \frac{r}{m}$.*

It ensures that no sector dominates fluctuations in aggregate GDP, thereby sharpening the analysis of aggregate fluctuations in large economies with no superstar sectors. In particular, it rules out granularity since $\lambda_i \sim \frac{1}{m}$. If aggregate volatility survives, it is not because one sector/node is big or dominates; rather, it is purely a network phenomenon.

ASSUMPTION 4 *(Macroscopic horizontal cluster)* *There exist sets $\mathcal{S} \subset \{1, \ldots, m\}$ such that, for the fraction $\mathcal{S}(\eta)$ of sectors belonging to the horizontal cluster:*

[36] By Theorem 2, horizontal propagation is not weighted by vertical Input-Output weights. Henceforth, an operator of the form $\boldsymbol{\mathcal{P}} := \boldsymbol{\mathcal{D}} \circ (\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}')$, denoting the Hadamard (entry-wise) product, is not consistent. Moreover, this would imply that strong (weak) vertical propagation generates stronger (weaker) horizontal covariance, thereby pushing towards the same direction.

[37] Later in the proof I will impose that $x_i = \lambda_i$ is the GDP-weight of sector-i, representing the aggregate exposure of its horizontal linkages.

1. $\frac{|\mathcal{S}|}{m} \to \mathcal{S}(\eta) > 0$;
2. $d_{ij} \geq \underline{d} > 0, \forall\, i, j \in \mathcal{S}$.

It ensures that a positive fraction of sectors are strongly connected horizontally, sufficient to generate non-vanishing variance. The first condition imposes that the cluster of network participants connected horizontally does not vanish as the economy grows (by Theorem 3). If $\mathcal{S}(\eta) = 0$, then the cluster becomes negligible, its contribution to aggregate output vanishes, and the horizontal propagation cannot generate aggregate fluctuations. Henceforth, $\mathcal{S}(\eta) > 0$ ensures macroeconomic relevance, and determines the order of magnitude of the horizontal cluster's aggregate weight (more later on). Moreover, $\mathcal{S}(\eta)$ is conceptually different from "large or important sectors" (Acemoglu et al. 2012). The latter own a large λ_i, generating a concentration of GDP. Differently, the horizontal clusters may imply small λ_i but a large number of sectors. Aggregate effect comes from extensive margin, not intensive margin. The second condition imposes positive horizontal connections (by Theorem 1).

Step 0: Aggregate variance through vertical and horizontal propagation*.– Recall that, in Lemma 6, I compute the baseline variance ignoring horizontal links, thereby determining the contribution of vertical supply chains alone to aggregate output, $var(Y) = \sigma^2 \boldsymbol{\lambda}' \boldsymbol{\mathcal{H}} \boldsymbol{\mathcal{H}}' \boldsymbol{\lambda}$: classical supply chain amplification, where independent shocks generate correlated sectoral outputs through Leontief chains. In Lemma 7, I introduce additional variance from sectors sharing upstream suppliers or downstream buyers, thereby capturing the key mechanism of horizontal propagation independent of vertical chains, $var(Y) = \sigma^2 \boldsymbol{\lambda}' \boldsymbol{\mathcal{P}} \boldsymbol{\lambda}$. These two independent channels coexist, and horizontal linkages generate aggregate variance regardless of vertical amplification.*

Step 1: Focus on horizontal cluster*.– Isolate network participants with strong horizontal linkages (that is that substantially connected horizontally with the others):*

$$x_i = \begin{cases} \lambda_i, & i \in \mathcal{S} \\ 0, & i \notin \mathcal{S} \end{cases} \quad \textit{thus implying} \quad \boldsymbol{\lambda}' \boldsymbol{\mathcal{P}} \boldsymbol{\lambda} \geq \boldsymbol{x}' \boldsymbol{\mathcal{P}} \boldsymbol{x}$$

Only nodes in the cluster contribute substantially to aggregate variance; other sectors' contributions are smaller. This allows a lower bound on variance.

Step 2: Bound contribution of the cluster*.– In what follows I am going to focus only on the horizontal propagation, as dropping the component $\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}'$ in $\boldsymbol{\mathcal{P}}$ gives a lower bound to fluctuations in aggregate output (i.e., considering only the sources of aggregate cycles from the horizontal geometry of a network; even if vertical chains were shut down, shared suppliers/buyers still induce correlation). From this, and expanding the previous step, then*

$$\boldsymbol{x}' \boldsymbol{\mathcal{D}} \boldsymbol{x} = \sum_{i,j \in \mathcal{S}} \lambda_i \lambda_j d_{ij}$$

which, by the cluster assumption ($d_{ij} \geq \underline{d}$),

$$\sum_{i,j \in \mathcal{S}} \lambda_i \lambda_j d_{ij} \geq \underline{d} \sum_{i,j \in \mathcal{S}} \lambda_i \lambda_j \quad \textit{with the latter being} \quad \underline{d} \left(\sum_{i \in \mathcal{S}} \lambda_i \right)^2$$

Strong horizontal linkages induce correlated shocks within the cluster, thereby ensuring non-vanishing variance. Each pair of sectors co-moves, and aggregate correlation scales with the square of total weight.

***Step 3: Bound cluster weight**.– I now bound λ_i, $\forall i \in \mathcal{S}$, using the diffuse weights assumption, $\lambda_i \geq \frac{q}{m}$. So, $\sum_{i \in \mathcal{S}} \lambda_i \geq |S| \times \frac{q}{m}$. From the cluster assumption,*

$$\left(\sum_{i \in \mathcal{S}} \lambda_i \right)^2 \geq \left(|\mathcal{S}| \frac{q}{m} \right)^2 \rightarrow \left(\mathcal{S}(\eta)\, q \right)^2 > 0$$

A positive fraction of the economy is in the horizontal cluster: aggregate contribution from horizontal linkages persists as $m \to \infty$. In fact, even if each sector is small, together they matter since the fact that many of them are horizontally connected induces substantial horizontal effect.

***Step 4: Combine bounds**.– When combining everything, it holds that*

$$var(Y) \geq \sigma^2 \underline{d} \left(\mathcal{S}(\eta)\, q \right)^2$$
$$\implies \quad \liminf_{m \to \infty} var(Y) > 0$$

Horizontal propagation alone generates persistent aggregate fluctuations even if no sector is dominant. Vertical propagation is not required for this effect. The aggregate variance from horizontal effects is strictly positive, independent of the network size-m, does not rely on large sectors, and does not rely on vertical propagation.

What the theorem states is that horizontal linkages (i.e., network "economic" distances) creates systematic correlation across many vertexes. Aggregate volatility potentially emerges without shocks to most central sectors, and the horizontal network geometry alone is sufficient. This is a new origin of aggregate fluctuations, distinct from: granular shocks to certain nodes, vertical chains, and superstar sectors/firms.

□

(Remark to Theorem 4) *Clarification on the definition of the covariance operator and the role of horizontal propagation. When proving Theorem 4, Appendix A, I refer to a covariance operator $\boldsymbol{\mathcal{P}}$ defined as the "union" of vertical and horizontal propagation mechanisms, denoted $\boldsymbol{\mathcal{P}} := \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}' \cup \boldsymbol{\mathcal{D}}$. Since the operator $\cup$ is not standard in linear algebra, I clarify its precise meaning and its role in the proof.*

The purpose of introducing $\boldsymbol{\mathcal{P}}$ is not to impose a specific functional form for how vertical and horizontal propagation interact, but rather to formalize the coexistence of two distinct channels (by Theorem 2) through which sectoral shocks may generate aggregate comovement. In particular, my results rely exclusively on lower-bound arguments for aggregate variance. Accordingly, it is sufficient to define $\boldsymbol{\mathcal{P}}$ as any symmetric *positive semidefinite (PSD) matrix satisfying the partial order conditions $\mathcal{P} \succeq \mathcal{D}$ and $\mathcal{P} \succeq \mathcal{H}\mathcal{H}'$, where $\succeq$ denotes the Loewner order on symmetric matrices.*

Equivalently, for any vector of Domar weights $\boldsymbol{\lambda}$, this definition ensures $\boldsymbol{\lambda}'\mathcal{P}\boldsymbol{\lambda} \geq \boldsymbol{\lambda}'\mathcal{D}\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'\mathcal{P}\boldsymbol{\lambda} \geq \boldsymbol{\lambda}'\mathcal{H}\mathcal{H}'\boldsymbol{\lambda}$. This definition formalizes the notion that vertical and horizontal propagation channels coexist without one mechanically weighting or scaling the other, and it is sufficient for establishing lower bounds on aggregate volatility. Importantly, no assumption is made that the two channels combine additively; the argument only requires that the contribution of horizontal propagation is not attenuated by the vertical structure of production.

For expositional simplicity, one may alternatively define $\mathcal{P} := \mathcal{H}\mathcal{H}' + \mathcal{D}$, which corresponds to the special case in which the two channels contribute additively to output covariance. This formulation is mathematically standard and yields identical lower-bound results. However, the more general definition above is preferred in order to remain consistent with the earlier result that vertical and horizontal propagation coexist but do not weight each other.

Finally, the proof makes use of the inequality $\boldsymbol{x}'\mathcal{D}\boldsymbol{x} \geq 0$ for a vector $\boldsymbol{x}$ supported on a horizontally connected cluster of sectors. While similarity matrices such as those based on shared suppliers or buyers (e.g., Jaccard-type measures) are not positive semidefinite in general, positive semi-definiteness is not required globally. The argument only relies on the restriction of $\mathcal{D}$ to a subset $\mathcal{S}$ of sectors forming a horizontal cluster, on which $\mathcal{D}$ is symmetric and element-wise positive. Since the quadratic form is evaluated on vectors supported on $\mathcal{S}$, non-negativity follows directly. This restricted use of $\mathcal{D}$ is sufficient to establish the lower bound on aggregate volatility driven by horizontal propagation.

□

(Proof of Corollary 3 for Theorem 4) *Closely following the logic of Theorem 4, allowing for heterogeneous Domar weights does not alter its conclusion.*

Step 0: Horizontal cluster and variance representation.– *From the proof of Theorem 4, the variance of aggregate output can be bounded from below using the quadratic form restricted to the horizontally connected cluster $\mathcal{S}$:*

$$var\,(Y) \geq \sigma^2\,\boldsymbol{x}'\mathcal{D}\boldsymbol{x}, \qquad with \quad x_i = \begin{cases} \lambda_i, & i \in \mathcal{S} \\ 0, & i \notin \mathcal{S} \end{cases}$$

Here, $\mathcal{S}$ satisfies $|\mathcal{S}|\,/m \to \mathcal{S}\,(\eta) > 0$ and $d_{ij} \geq \underline{d} > 0$ for all $i,j \in \mathcal{S}$. This step isolates the horizontal propagation, ignoring vertical chains.

Step 1: Lower bound on horizontal contribution.– *By construction of $\boldsymbol{x}$ and the properties of $\mathcal{D}$ on $\mathcal{S}$,*

$$\boldsymbol{x}'\mathcal{D}\boldsymbol{x} = \sum_{i,j\in\mathcal{S}} \lambda_i\lambda_j d_{ij} \;\geq\; \underline{d} \sum_{i,j\in\mathcal{S}} \lambda_i\lambda_j = \underline{d}\left(\sum_{i\in\mathcal{S}} \lambda_i\right)^2$$

In Theorem 4, Domar weights are not allowed to be heterogeneous and may include *large sectors, $\lambda_i \approx 1/m$, so $\sum_{i \in \mathcal{S}} \lambda_i \sim |\mathcal{S}|/m$. When allowing for granular Domar weights, some λ_i would be larger than others. If a shock hits one of these sectors, then $\sum_{i \in \mathcal{S}} \lambda_i \geq \bar{\lambda} \gg |\mathcal{S}|/m$. Horizontal propagation is amplified proportionally to the total weight in the cluster: large sectors make the quadratic form bigger, thereby increasing aggregate variance.*

Step 2: Combine bounds. *– Combining the previous inequalities yields*

$$var(Y) \geq \sigma^2 \boldsymbol{x}' \boldsymbol{\mathcal{D}} \boldsymbol{x} \geq \sigma^2 \underline{d} \bar{\lambda}^2 > 0$$

Compared to the diffuse Domar weight case (i.e., the original theorem), the presence of large sectors increases $\bar{\lambda}$, hence amplifying the aggregate effect of the horizontal geometry. Horizontal propagation generates non-vanishing aggregate fluctuations in all cases, with granularity magnifying this effect. In other words, horizontal propagation generates non-vanishing aggregate output fluctuations even when Domar weights are heterogeneous and some sectors are large.

□

Model derivation

(Households problem) *The household-i's utility problem is to maximize utility function subject to its budget constraint:*

$$\max_{c_i(s),\, n_i(s)} \quad \mathcal{U}_i\left(\left\{c_i(s)\right\}_{\forall s \in \Phi(s)}; n_i(s)\right) := \prod_{s \in \Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi}$$

$$s.t. \qquad \sum_{s \in \Phi(s)} p(s)\, c_i(s) = w(s)\, n_i(s) + \sum_{s \in \Phi(s)} D_i(s)$$

Sectoral consumption and labour supplied are $c_i(s)$ and $n_i(s)$, respectively, and each is determined by a price, $p(s)$ and $w(s)$. Parameter $\beta(s) \in (0,1)$ identifies the weight of each sectoral good in household i's consumption basket and $\sum_{s \in \Phi(s)} \beta(s) = 1$, ϕ is the inverse Frisch labour supply elasticity, measuring the elasticity of hours worked to the wage rate under a constant marginal utility of income. Finally, $D_i(s)$ is the constant share of sector-specific profits flowing from sector-s to household-i.

Utility maximization implies the Lagrangian function to be

$$\mathcal{L}_{c_i(s),\, n_i(s)} := \prod_{s \in \Phi(s)} c_i(s)^{\beta(s)} - \frac{n_i(s)^{1+\phi}}{1+\phi} + \\ - \psi^{\mathcal{L}} \left[\sum_{s \in \Phi(s)} p(s)\, c_i(s) - \left(w(s)\, n_i(s) + \sum_{s \in \Phi(s)} D_i(s) \right) \right]$$

with $\psi^{\mathcal{L}}$ being the penalty multiplier. Optimality conditions are in order

$$\frac{p(s)\,c_i(s)}{\beta(s)} = \frac{p(s')\,c_i(s')}{\beta(s')}$$

$$w(s) = n_i(s)^{\phi}\, p(s)\, c_i(s)\left[\beta(s)\;\; c_i\right]^{-1}$$

which, once aggregated across all households, $\frac{p(s)\int_i c_i(s)\mathrm{d}\,i}{\beta(s)} = \frac{p(s')\int_i c_i(s')\mathrm{d}\,i}{\beta(s')}$ *and* $w(s) = \int_i n_i(s)^{\phi}\,\mathrm{d}\,i\; p(s)\int_i c_i(s)\,\mathrm{d}\,i\left[\beta(s)\;\;\int_i c_i\,\mathrm{d}\,i\right]^{-1}$*, would report the sector-level optimality conditions for consumption and labour supply. The first condition for consumption defines how the total expenditure from households for sectoral goods gives rise to the relative importance of each good in total consumption, C. This is an aggregator over sectoral consumption levels:*

$$C = f^{c}\Big(c(s), c(s'), c(s''), \ldots, c(m)\;;\;\{\beta(s)\}_{s=1}^{m}\Big)$$

ASSUMPTION *5 (Final consumption requirements)* *As main characteristics of a final consumption aggregator: (i) it is strictly quasi-concave, non-decreasing and homogeneous of degree one in each of the sectoral consumption levels; (ii) consumption goods are normal, so that their demand increases with households' income; and (iii) the weight of sectoral goods in total consumption is driven by* $\beta(s)$*, which is the relative weight that households give in the consumption of the good produced by sector-*s*.*

Moreover, denote by $\mathbf{C} = \left[c(s)\right]$ *the* $m \times 1$ *vector of consumption levels over sectoral goods, with generic element defined as* $c(\cdot) > 0$*. Under this specification and the regularities in Assumption 5, it holds that* $C = \prod_{s\in\Phi(s)} c(s)^{\beta(s)} \propto \boldsymbol{\beta}'\mathbf{C}$*.*

□

(Sectoral optimization) *The production side of the economy is made of a finite set of sectors,* $\{s, s', \ldots, m\} \in \Phi(s)$*, populated by a representative firm whose output is the result of a Cobb-Douglas technology satisfying Assumption 1:*

$$y(s) = z(s)\Big(n(s)\Big)^{\alpha(s)} \prod_{s'\in\Phi(s)} \Big(x(s,s')\Big)^{\alpha(s,s')} \varkappa_j(s) \qquad \textbf{(A.3)}$$

where $\varkappa_j(s)$*, for* $j = \{fd, fs\}$*, is a normalization constant ruling out double counting of intermediate inputs when considering both factor input demand and supply network distances between sectors; absent any of these two cases, then* $\varkappa_d(s) = 1$*.*

A perfectly competitive representative firm in sector-s *maximizes total revenues from production net of costs of inputs of production:*

$$\max_{n(s),\; x(s,s')} \quad y(s) = z(s)\Big(n(s)\Big)^{\alpha(s)} \prod_{s'\in\Phi(s)} \Big(x(s,s')\Big)^{\alpha(s,s')} \varkappa_j(s)$$

$$\text{s.t.} \qquad \mathcal{C}(s) := w(s)\,n(s) + \sum_{s'\in\Phi(s)} p(s')\,x(s,s')$$

where $\alpha(s) + \sum_{s'} \alpha(s,s') = 1$*, and* $\varkappa_d(s) = 1$ *for* $j = \{fd, fs\}$*. Profit maximization then implies*

$$\max_{n(s),\ x(s,s')} D(s) := \ p(s)\, y(s) - \mathcal{C}(s)$$

Optimal competitive factor demands of sector-s relative to sector-s' are given by $n(s) = \alpha(s)\, \frac{p(s)\, y(s)}{w(s)}$ and $x(s,s') = \alpha(s,s')\, \frac{p(s)\, y(s)}{p(s')}$.

□

(Equilibrium characterization) *In equilibrium, the model should specify the clearing conditions of labour, circulating intermediate inputs, and goods markets. Starting from the labour market, equating labour demand and labour supply would deliver the labour market equilibrium condition, $n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ y(s)\right]^{\frac{1}{1+\phi}}$. Moreover, total labour demand is found by aggregating labour optimality condition of sectors, $N^{dem} = \sum_{s\in\Phi(s)} n^{dem}(s)$, and total labour supply is found by aggregating labour optimality condition of households, $N^{sup} = \sum_{s\in\Phi(s)} n^{sup}(s)$. Labour market clearing then requires that $N^{dem} = N^{sup}$.*

For what concerns equilibrium in the circulating intermediate inputs market, total demand of intermediates from sectors is $x^{dem}(s) = \sum_{s'\in\Phi(s)} x(s,s')$. Analogously, its total supply of intermediate inputs is $x^{sup}(s) = \sum_{s'\in\Phi(s)} x(s',s)$ so that, in equilibrium it must be true that $x^{dem}(s) = x^{sup}(s)$ which, in aggregate, simply states that $\sum_{s\in\Phi(s)} x^{dem}(s) = \sum_{s\in\Phi(s)} x^{sup}(s) \equiv X^{sup} = X^{dem}$.

The equilibrium in the good market is given by summing over all sectors the sectoral equilibrium condition in which total production must equal its total final consumption (by households) and its circulating intermediate input bought by other sectors: $\sum_{s\in\Phi(s)} y(s) = \sum_{s\in\Phi(s)} \left(\int_i c_i(s)\, \mathrm{d}\, i\right) + \sum_{s\in\Phi(s)} \left(\sum_{s'\in\Phi(s)} x(s,s')\right)$.

Finally, by aggregating the households' budget constraints over households and sectors, and imposing the clearing conditions so far, the aggregate resource constraint for this economy reads as $P^c C = WN + D$ which equals total output, defined by $Y \propto \boldsymbol{\beta}_s \boldsymbol{Y}$. According to the structure of the model, equilibrium conditions read as follows. A competitive equilibrium for this efficient economy is defined by a set of sectoral prices for labour and intermediate inputs, $\{w(s), p(s)\}_{s\in\Phi(s)}$, a set of sectoral production input quantities, $\{n(s), x(s,s')\}_{s,s'\in\Phi(s)}$, exogenous sectoral productivities, $\{z(s)\}_{s\in\Phi(s)}$, and a set of aggregate quantities, $\Omega = (Y, C, N, X, D)$, such that (i) each household satisfies its optimality conditions, (ii) the representative firm of each sector maximizes profits, and (iii) all markets clear, shaping Ω.

□

(Derive aggregate fluctuations result of eq. (1.4)) *Following exactly the same operating steps in vom Lehn and Winberry (2022), to characterize the dynamical changes in aggregate Gross Domestic Product (GDP), first it is necessary to determine the existing relation between sectoral intermediate output, $y(s)$, and the Divisia Index.*[38]

[38] This indicator provides a theoretically consistent measure of growth. It is a composite index, particularly suited to analyse variables made of multiple and differentiated elements, measuring changes in certain aggre-

Notice that constant returns to scale and homogeneity of degree one in Assumption 1 and in the production function in eq. (A.3) implies the following zero-profit condition

$$p(s)\,y(s) = w(s)\,n(s) + \sum_{s' \in \Phi(s)} p(s')\,x(s,s')$$

where $p(s)\,y(s)$ is the (gross) nominal output of sector-s, and from where the national accounting definition of nominal value added is given by $p(s)\,y(s) - p^s x^s = w(s)\,n(s)$, where I define $p^s x^s = \sum_{s' \in \Phi(s)} p(s')\,x(s,s')$ the total expenditure of sector-s for intermediate inputs from other connected sectors. Define the left-hand side of the previous equation as $p^Y(s)\,y^Y(s) \equiv p(s)\,y(s) - p^s x^s$, i.e., the nominal value-added.

In order to construct a single, aggregate measure of real output (or value added) across multiple sectors, I use a Divisia index. This index combines the growth rates of individual sectoral outputs into an overall growth rate, but does so in a way that accounts for each sector's economic importance – measured by its share in total nominal value added. Specifically, the Divisia index computes a weighted average of the growth rates of sectoral real outputs, where the weights are each sector's share in total nominal value added. This allows us to track how the economy's output is changing over time, adjusting dynamically as the composition of output across sectors evolves.

Henceforth, the Divisia Index requires to differentiate the nominal value added under constant price levels associated to variables:

$$
\begin{aligned}
& p^Y(s)\; d\,log\,y^Y(s) = p(s)\; d\,log\,y(s) - p^s\,d\,log\,x^s \\
\rightarrow\quad & p^Y(s)\,y^Y(s)\; d\log y^Y(s) = p(s)\,y(s)\; d\log y(s) - p^s x^s\,d\log x^s \\
\rightarrow\quad & \alpha(s)\; d\log y^Y(s) = d\log y(s) - \alpha^s\,d\log x^s \\
\rightarrow\quad & \alpha(s)\; d\log y^Y(s) = \Big(d\log z(s) + \alpha(s)\; d\log n(s) + \alpha^s\,d\log x^s\Big) - \alpha^s\,d\log x^s \\
\longrightarrow\quad & d\log y^Y(s) = \frac{1}{\alpha(s)}\,d\log z(s) + d\log n(s)
\end{aligned}
$$

where $\alpha^s = \sum_{s \in \Phi(s)} \alpha(s,s')$. Plug the above sector-level nominal value added growth, $d\log y^Y(s)$, into the Divisia Index, $d\log Y = \sum_{s \in \Phi(s)} \left(\frac{p^Y(s)y^Y(s)}{P^Y Y}\right) d\log y^Y(s)$, so that it is possible to obtain a non-complete equation for real GDP growth:

$$d\log Y = \sum_{s \in \Phi(s)} \left(\frac{p^Y(s)\,y^Y(s)}{P^Y Y}\right)\left[\frac{1}{\alpha(s)}\,d\log z(s) + d\log n(s)\right] \tag{A.4}$$

gate quantity of a given variable, and weighting all its defining components according to their relevance on that variable, that is it expresses the overall rate of output change as a weighted average of the growth rates of individual components, with time-varying weights corresponding to each component's share in total output. Conceptually, it captures the notion that aggregate GDP growth reflects both the differential growth of its components and their evolving relative importance within the economy. This formulation offers an improvement over fixed-weight indices by accommodating structural changes in the composition of economic activity over time. Refer to Oulton (2022) for further details.

Summing over all sectors the sectoral optimality conditions for intermediate inputs, $\sum_{s'\in\Phi(s)} x(s,s') = \sum_{s'\in\Phi(s)} \alpha(s,s') \left[p(s)\ \ y(s)\right] \left(p(s')\right)^{-1}$, noticing that $\alpha(s) = 1-\alpha^s$, and inserting the zero-profit condition would yield $\alpha(s) = \frac{p^Y(s) y^Y(s)}{p(s) y(s)}$, which is the ratio of sectoral value-added to sectoral intermediate (gross) output.

By plugging into the eq. (A.4) the condition for $\alpha(s)$, and making all the adjustments to simplify the elements multiplying the sectoral productivity it is then possible to get eq. (1.4), which determines the drivers of real aggregate GDP fluctuations.

□

Insert network distances into labour market equilibrium

(Labour market equilibrium with network "economic" distances) *In addition to the equilibrium definition, rearranging optimality conditions for labour market variables from both households and sectors, the following equilibrium condition for sectoral labour force is detected:*

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ \ y(s)\right]^{\frac{1}{1+\phi}} \tag{A.5}$$

It defines that employment is a positive function of both output and consumption (recall that total consumption C increases with sector-specific consumption).

To account for the role of changes to other sectors' employment on sectoral employment level, notice that when sector-s buys intermediate inputs from sector-s', it is introjecting also the quantities of production inputs in such sector. Henceforth, each element in the intermediate inputs bundle of sector-s can be taught as $x(s,s') = \vartheta(s,s')\ \ y(s')$, $\forall s' \in \Phi(s)$, which states that each intermediate input is just a given share $\vartheta(s,s')$ of other sector's output. Labour market equilibrium in eq. (A.5) is thus

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ \ z(s) \left(n(s)\right)^{\alpha(s)} \prod_{s'\in\Phi_s} \left(\vartheta(s,s')\ \ y(s')\right)^{\alpha(s,s')}\right]^{\frac{1}{1+\phi}} \tag{A.6}$$

so that, once plugging in the output of sector-s', sectoral employment levels are transparently related through the production network structure. As a result, the above labour market condition allows to study the general equilibrium comovement of employment levels across sectors.

Once inserting the production function of eq. (A.3) in the labour market equilibrium condition of eq. (A.6), then optimal quantities of a sector are related to that of the other ones. To incorporate factor input demand distance, in Appendix A I show that manipulation in general equilibrium would result in

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ z(s)\ n(s)^{\alpha(s)}\ \ \aleph^{fd}\ \boldsymbol{\Lambda}^{fd}\ \boldsymbol{\Xi}^{fd}\right]^{\frac{1}{1+\phi}} \tag{A.7}$$

where $\aleph^{fd} = \prod_{s'\in\Phi(s)} \left(\prod_{s\in\Phi(s)} x(s',s)^{\alpha(s',s)}\right)^{\frac{\alpha(s,s')}{1+\phi}}$ identifies the set of intermediate inputs bought by sector-s and all the other sectors connected to it, the element $\boldsymbol{\Lambda}^{fd} = \prod_{s'\in\Phi(s)} \left(\vartheta(s,s')\, z(s')\, n(s')^{\alpha(s)}\right)^{\alpha(s,s')}$ captures the relevance of other sectors' productivities and employment levels on sector-s working through the production network, *and the component* $\Xi^{fd} = \prod_{s' \in \Phi(s)} \left[\prod_{s \in \Phi(s)} \left(\frac{x(s',s)}{x(s,s)} \right)^{\frac{\alpha(s',s)}{\alpha(s,s)}} \right]$ *determines the ratio among the intermediate inputs between sector-s and each of the other sectors, when these are buying from the same upstream sector.*

Manipulation in general equilibrium to include factor input supply distance for pairs of sectors would result in

$$n(s) = \left[\alpha(s) \; \frac{C}{c(s)} \; \beta(s) \; z(s) \; n(s)^{\alpha(s)} \;\; \aleph^{fs} \; \Lambda^{fs} \; \Xi^{fs} \right]^{\frac{1}{1+\phi}} \tag{A.8}$$

where $\aleph^{fs} = \prod_{s' \in \Phi(s)} \left(\prod_{s' \in \Phi(s)} x(s',s')^{\alpha(s',s')} \right)^{\frac{\alpha(s,s')}{1+\phi}}$ *is the compounded set of intermediate inputs,* $\Lambda^{fs} = \prod_{s' \in \Phi(s)} \left(\vartheta(s,s') \, z(s') \, n(s')^{\alpha(s)} \right)^{\alpha(s,s')}$ *considers sectoral productivities and employment levels, and* $\Xi^{fs} = \prod_{s' \in \Phi(s)} \left[\prod_{s \in \Phi(s)} \left(\frac{x(s',s)}{x(s',s')} \right)^{\frac{\alpha(s',s)}{\alpha(s',s')}} \right]$ *determines the ratio among the intermediate inputs of paired sectors, when these are selling to the same downstream sector.*

□

Network-based propagation to sectoral employment

PROPOSITION 4 (Propagation of variations to intermediate inputs) *Consider a market economy characterized by Input-Output linkages as in eq. (A.3), whose labour market equilibrium is eq. (A.5). Then, the response of sectoral employment to changes in sectoral intermediate inputs is a first-order (log-linear) approximation*

$$d\,log\,\mathbf{N} = \Theta \left\{ d\,log\,C + \underbrace{d\,log\,\mathbf{z} - d\,log\,\mathbf{C}}_{d\,log\,\boldsymbol{\mathcal{S}}} + \mathbf{H}\,d\,log\,\mathbf{X} \right\} \tag{A.9}$$

where:

(a) $\mathbf{N}$ *identifies sectoral employment levels;*

(b) $\Theta = \frac{1}{1+\phi+\boldsymbol{\alpha}}$ *is a compound of structural parameters, and* $\boldsymbol{\alpha} = [\alpha(s), \ldots, \alpha(m)]$*;*

(c) C *is aggregate consumption;*

(d) $\boldsymbol{\mathcal{S}}$ *identifies sector-specific changes in productivity and consumption;*

(e) $\mathbf{H}$ *is the Input-Output matrix, comprising sectoral weight on other sectors, influencing the elements of* $\mathbf{X}$*, a matrix with intermediate inputs purchases.*

Proof in Appendix A.

PROPOSITION 5 (Direct propagation of variations to sectoral employment) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.6). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation*

$$d\,log\,\mathbf{N} = \Theta \left\{ d\,log\,C + d\,log\,\boldsymbol{\mathcal{S}} + \underbrace{\mathbf{H}\left(\Psi_{s,s'}\right) \left[d\,log\,\mathbf{z} + \boldsymbol{\alpha}\; d\,log\,\mathbf{N} \right] + \boldsymbol{\mathcal{E}}\; d\,log\,\mathbf{X}}_{d\,log\,\boldsymbol{\mathcal{V}}} \right\} \tag{A.10}$$

where:

(a) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(e) $\boldsymbol{\mathcal{V}}$ *identifies the production network effect of other sectors' changes in productivities, employment levels, and intermediate inputs usage, where:*

- $\boldsymbol{H}(\Upsilon_{s,s'})$ *is the Input-Output matrix, comprising the weight of each sector on other sectors, whose entries are set to 0 whenever* $s = s'$*;*
- $\boldsymbol{\mathcal{E}} = \boldsymbol{H}(\Upsilon_{s,s'})'\boldsymbol{H}(\Upsilon_{s',s})$ *is a compounded network effect, made of the inner product of the Input-Output matrix, influencing the elements of* $\boldsymbol{X}$*.*

Proof in Appendix A.

PROPOSITION 6 (Direct propagation under factor input demand distance) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.7). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation*

$$d\log\boldsymbol{N} = \Theta\left\{ d\log C + d\log\boldsymbol{\mathcal{S}} + d\log\boldsymbol{\mathcal{V}} + \underbrace{\boldsymbol{\mathcal{D}}^{fd}\left[d\log\boldsymbol{N} - \left(\boldsymbol{\mathcal{L}}'_{\Phi(s)}\, d\log\boldsymbol{N}\right)\boldsymbol{1}\right]}_{d\log\boldsymbol{\mathcal{D}}(fd)} \right\} \tag{A.11}$$

where:

(a) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(f) $\boldsymbol{\mathcal{D}}^{fd}$ *identifies all the demand-based distances across any pair of sectors;*

(g.i) $\boldsymbol{\mathcal{D}}(fd)$ *identifies the production network distance effect of other sectors' variations in employment levels.*

Proof in Appendix A.

PROPOSITION 7 (Direct propagation under factor input supply distance) *Consider an Input-Output economy defined by a labour market equilibrium as stated in eq. (A.8). Then, the response of sectoral employment to other sectors' employment is a first-order (log-linear) approximation*

$$d\log\boldsymbol{N} = \Theta\left\{ d\log C + d\log\boldsymbol{\mathcal{S}} + d\log\boldsymbol{\mathcal{V}} + \underbrace{\boldsymbol{\mathcal{D}}^{fs}\left[d\log\boldsymbol{P} - \left(\boldsymbol{\mathcal{L}}'_{\Phi(s)}\, d\log\boldsymbol{P}\right)\boldsymbol{1}\right]}_{d\log\boldsymbol{\mathcal{D}}(fs)} \right\} \tag{A.12}$$

where:

(a) $\boldsymbol{N}$ *identifies sectoral employment levels;*

...

(f) $\boldsymbol{\mathcal{D}}^{fs}$ *identifies all the supply-based distances across any pair of sectors;*

(g.ii) $\boldsymbol{\mathcal{D}}(fs)$ *identifies the production network distance effect of other sectors' variations in employment levels.*

Proof in Appendix A.

Proofs for model derivation

(Derive labour market equilibrium of eqs. (A.7) and (A.8)) *From the sector-level equilibrium definition of the labour market,* $n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ y(s)\right]^{\frac{1}{1+\phi}}$*, including the condition* $x(s,s') = \vartheta(s,s')\ y(s')$ *would imply eq. (A.6), that is*

$$n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ z(s)\left(n(s)\right)^{\alpha(s)} \prod_{s' \neq s}\left(\vartheta(s,s')\ y(s')\right)^{\alpha(s,s')}\ \varkappa_j(s)\right]^{\frac{1}{1+\phi}} \tag{A.13}$$

where it should be inserted the production function of eq. (A.3) for sector-s'*, and* $\varkappa_j(s) \neq 1$*, with* $j = \{fd, fs\}$*, for the purpose of this derivation. The resulting equation can be simplified by imposing an infinite inverse Frisch elasticity of labour,* $\phi \to \infty$*. In fact, in the model of Section 1.3, only the extensive margin of employment (i.e., total number of workers) is important, while it is not the intensive margin (i.e., total hours worked), which is ruled out under* $\phi \to \infty$*; refer to Rogerson (1988). However, for the sake of generality, I will proceed with* $\phi \in [0, \infty)$*.*

Factor input demand.*– Assume the non-common (constant) element in the production function to be given by*[39]

$$\varkappa_{fd}(s) = \tau_{fd}(s) \prod_{s'}\left[\frac{\prod_s x(s',s)^{\frac{1}{\alpha(s,s)}+\alpha(s',s)}}{\prod_s x(s,s)^{\frac{\alpha(s',s)}{\alpha(s,s)}}}\right]^{\frac{\alpha(s,s')}{1+\phi}} \tag{A.14}$$

Then, multiply and divide both sides of the above labour market equilibrium, in-

---

[39] In eq. (A.14), the component $\tau_{fd}(s) = \Pi_{s' \in \Phi(s)}\left[\Pi_{s \in \Phi(s)}\left(\frac{x(s',s)}{x(s,s)}\right)^{\frac{\alpha(s',s)}{\alpha(s,s)}}\right]^{\frac{1}{\alpha(s,s')}}$ allows to ensure that the distance effects are not influenced by the Leontief inverse weight. Specifically, incorporating the condition $x(s,s') = \vartheta(s,s')\ y(s')$ in the labour market equilibrium of eq. (A.13) would result in a double counting of the values from the Input-Output matrix: one of them is used to capture sectoral distances, while the other would have been multiplied by distance matrix $\mathcal{D}^{fd}$ when analysing the effect of sectoral propagation in Proposition 6. Within this context, it is essential to highlight that, starting from the baseline labour market equilibrium in eq. (A.5), and deriving the results presented in this section and in Proposition 6 without including the exogenous condition $x(s,s') = \vartheta(s,s')\ y(s')$ would produce an outcome reflecting the effect of network distances *without* the influence of the Input-Output Leontief inverse weights. In other words, as discussed in the main text (Theorem 2), the effect of the network "economic" distance across pairs of sectors is independent of inter-sectoral trade intensities characterizing the production network; imposing the $\tau_{fd}(s)$ term due to the condition $x(s,s') = \vartheta(s,s')\ y(s')$ allows to avoid that such concept is violated. Otherwise, without $\tau_{fd}(s)$, the component Ξ^{fd} in eq. (A.7) would be equal to

$$\Xi^{fd} = \prod_{s' \in \Phi(s)}\left[\prod_{s \in \Phi(s)}\left(\frac{x(s',s)}{x(s,s)}\right)^{\frac{\alpha(s',s)}{\alpha(s,s)}}\right]^{\alpha(s,s')}$$

Analogous is the rationale beyond component $\tau_{fs}(s) = \Pi_{s' \in \Phi(s)}\left[\Pi_{s \in \Phi(s)}\left(\frac{x(s',s)}{x(s',s')}\right)^{\frac{\alpha(s',s)}{\alpha(s',s')}}\right]^{\frac{1}{\alpha(s,s')}}$, proper of eq. (A.15) that delivers Proposition 7. Avoiding to consider it, the component Ξ^{fs} in eq. (A.8) is

$$\Xi^{fs} = \Pi_{s' \in \Phi(s)}\left[\prod_{s \in \Phi(s)}\left(\frac{x(s',s)}{x(s',s')}\right)^{\frac{\alpha(s',s)}{\alpha(s',s')}}\right]^{\alpha(s,s')}$$

cluding the production function, by $\prod_{s'}\left\{\left[\prod_s x\left(s',s\right)^{\frac{1}{\alpha(s,s)}}\right]\Big/\left[\prod_s x\left(s,s\right)^{\frac{\alpha(s',s)}{\alpha(s,s)}}\right]\right\}^{\frac{\alpha(s,s')}{1+\phi}}$.
Finally, making all the required adjustments once including $\varkappa_{fd}\left(s\right)$*, the resulting condition will deliver eq. (A.7). All the multiplications are over* $\Phi\left(s\right)$*.*

***Factor input supply**.– Assume now the non-common (constant) element in the production function to be given by*

$$\varkappa_{fs}\left(s\right)=\tau_{fs}\left(s\right)\prod_{s'}\left[\frac{\prod_s x\left(s',s\right)^{\frac{1}{\alpha(s,s)}}}{\prod_{s'} x\left(s',s'\right)^{\frac{\alpha(s',s)}{\alpha(s',s')}-\alpha(s',s')}}\right]^{\alpha(s,s')} \tag{A.15}$$

Multiply and divide both sides of the above labour market equilibrium, including the production function, by $\prod_{s'}\left\{\left[\prod_s x\left(s',s\right)^{\frac{1}{\alpha(s',s')}}\right]\Big/\left[\prod_{s'} x\left(s',s'\right)^{\frac{\alpha(s',s)}{\alpha(s',s')}}\right]\right\}^{\frac{\alpha(s,s')}{1+\phi}}$*. Making all the required adjustments once including* $\varkappa_{fs}\left(s\right)$*, the resulting condition will deliver eq. (A.8). All the multiplications are over* $\Phi\left(s\right)$*.*

□

(Proof of Proposition 3) *The proof directly follows Propositions 6 and 7.* □

(Proof of Proposition 4) *The interest is in characterizing the way in which sectoral employment changes in response to variations in sector-specific intermediate inputs usage. In order to inspect these changes, it is necessary to express the equilibrium conditions in terms of log-linear conditions. Starting from the sector-*s *labour market equilibrium of eq. (A.5), i.e.,* $n\left(s\right)=\left[\alpha\left(s\right)\ \frac{C}{c(s)}\ \beta\left(s\right)\ y\left(s\right)\right]^{\frac{1}{1+\phi}}$*, the associated log-linear form is* $\widetilde{n}\left(s\right)=\frac{1}{1+\phi}\left\{\widetilde{y}\left(s\right)+\widetilde{C}-\widetilde{c}\left(s\right)\right\}$*, with tilded variables,* $\widetilde{\cdot}$*, expressing the deviation from their respective steady state. Such log-linearized equation contains the log-linearized output, obtained by log-differentiating the production function in eq. (A.3):* $\widetilde{y}\left(s\right)=\widetilde{z}\left(s\right)+\alpha\left(s\right)\widetilde{n}\left(s\right)+\sum_{s'\in\Phi(s)}\alpha\left(s,s'\right)\widetilde{x}\left(s,s'\right)$*. Substituting out these two equations, one easily gets that log-linearized sectoral employment is a function of*

$$\widetilde{n}\left(s\right)=\frac{1}{1+\phi}\left\{\widetilde{z}\left(s\right)+\alpha\left(s\right)\widetilde{n}\left(s\right)+\sum_{s'\in\Phi(s)}\alpha\left(s,s'\right)\widetilde{x}\left(s,s'\right)+\widetilde{C}-\widetilde{c}\left(s\right)\right\}$$

expressing the elements whose variations induce changes in employment level of sector-s*. In vectorial notation, one can rewrite the summation as*

$$\widetilde{n}\left(s\right)=\frac{1}{1+\phi}\left\{\widetilde{z}\left(s\right)+\alpha\left(s\right)\widetilde{n}\left(s\right)+\boldsymbol{h}\left(s\right)\widetilde{\boldsymbol{x}}\left(s\right)+\widetilde{C}-\widetilde{c}\left(s\right)\right\}$$

which, when stacked over all sectors, can be written in matrix form:

$$\widetilde{\mathbf{N}}=\frac{1}{1+\phi}\left\{\widetilde{C}-\widetilde{\boldsymbol{C}}+\widetilde{\boldsymbol{z}}+\boldsymbol{\alpha}\widetilde{\mathbf{N}}+\boldsymbol{H}'\widetilde{\boldsymbol{X}}\right\}$$

where $\widetilde{\mathbf{N}}$ *is an* $S\times 1$ *vector of changes in sector-specific employment levels, identifies changes in aggregate consumption,* $\widetilde{\boldsymbol{C}}$ *is an* $S\times 1$ *vector of changes in sector-specific* *final consumption by households,* $\widetilde{\mathbf{z}}$ *is an* $S \times 1$ *vector of changes in sectoral productivities,* $\mathbf{H}$ *is an* $S \times S$ *squared Input-Output matrix identifying the structure of intersectoral trade, and* $\widetilde{\mathbf{X}}$ *is an* $S \times 1$ *vector of changes in sector-specific set of intermediate inputs bought within the production network. Finally,* ϕ *is a scalar for aggregate inverse Frisch elasticity of labour supply to wage level, and* $\boldsymbol{\alpha} = \left[\alpha(s), \alpha(s'), \ldots, \alpha(S)\right]$ *comprises sector-specific labour force as a share of its intermediate output.*

Bringing the vector of sectoral employments, $\widetilde{\mathbf{N}}$*, on the left-hand side, rewriting* $\widetilde{\cdot} = d\log(\cdot)$*, and rearranging terms, then one obtains eq. (A.9) in Proposition 4.*

□

(Proof of Proposition 5) *The interest is in characterizing the way in which sectoral employment changes in response to variations in other sectors' employment levels. In order to inspect these changes, it is necessary to express the equilibrium conditions in terms of log-linear conditions. Starting from the sector-*s *labour market equilibrium of eq. (A.5), i.e.,* $n(s) = \left[\alpha(s)\ \frac{C}{c(s)}\ \beta(s)\ y(s)\right]^{\frac{1}{1+\phi}}$*, the associated log-linear form is* $\widetilde{n}(s) = \frac{1}{1+\phi}\left\{\widetilde{y}(s) + \widetilde{C} - \widetilde{c}(s)\right\}$*, with tilded variables,* $\widetilde{\cdot}$*, expressing the deviation from their respective steady state. Then, use the condition* $x(s,s') = \vartheta(s,s')\ \ y(s')$*, which states that each intermediate input is just a given share* $\vartheta(s,s')$ *of other sector's output. Inserting its log-linearized form* $\widetilde{x}(s,s') = \widetilde{y}(s')$*, along the log-differentiation of the production function in eq. (A.3),* $\widetilde{y}(s) = \widetilde{z}(s) + \alpha(s)\,\widetilde{n}(s) + \sum_{s' \in \Phi(s)} \alpha(s,s')\,\widetilde{x}(s,s')$*, into the log-linear labour market equilibrium, one gets*

$$\widetilde{n}(s) = \frac{1}{1+\phi}\left\{\begin{array}{l} \widetilde{C} - \widetilde{c}(s) + \widetilde{z}(s) + \alpha(s)\,\widetilde{n}(s) \quad + \\ + \sum\limits_{s' \in \Phi_s} \alpha(s,s')\left[\widetilde{z}(s') + \alpha(s')\,\widetilde{n}(s') + \sum\limits_{s \in \Phi_{s'}} \alpha(s',s)\,\widetilde{x}(s',s)\right] \end{array}\right\}$$

expressing the elements whose variations induce changes in employment level of sector-s*. Note that using the above condition* $x(s,s') = \vartheta(s,s')\ \ y(s')$ *implies that summations are not over the whole set of sectors, but rather it should be excluded the sector whose one is summing for. To this aim, denote* $\Phi(s)$ *the set of all sectors; then, both* Φ_s *and* $\Phi_{s'}$ *denotes improper subsets of* $\Phi(s)$*, since they are excluding sector-*s *and sector-*s'*, respectively. In other words, all the elements in* $\{\Phi_s, \Phi_{s'}\}$ *are contained in* $\Phi(s)$ *but* $\{\Phi_s, \Phi_{s'}\}$ *and* $\Phi(s)$ *are not equal,* $\Phi_s \subseteq \Phi(s)$ *with* $\Phi_s \nsubseteq \Phi(s)$*, and* $\Phi_{s'} \subseteq \Phi(s)$ *with* $\Phi_{s'} \nsubseteq \Phi(s)$*. When stacking the above condition over all sectors (i.e., matrix notation), the Input-Output matrix from* $\alpha(s,s')$ *and* $\alpha(s',s)$ *are not full but rather are set to zero whenever* $s = s'$*, i.e., using the* $\Psi_{s,s'}$ *as a matrix indicator.*

Using such notation, and expressing the above log-linearized labour market equation in vectorial form and then in matrix form (as in the Proof of Proposition 4), and solving for $d\log n(s)$*, one obtains eq. (A.10) of Proposition 5.*

□

(Proof of Proposition 6) *The interest is in characterizing the way in which sectoral employment changes in response to variations in other sectors' employment levels ac-*

cording to their factor input demand network distance relationships. Taking the logarithm of eq. (A.7) would result in

$$\log n(s) = \frac{1}{1+\phi}\left\{\begin{array}{l}\log\alpha(s) + \log C + \log z(s) + \alpha(s)\,\log n(s) - \log c(s) \quad + \\ + \sum_{s'\in\Phi(s)} \alpha(s,s')\left[\log\vartheta(s,s') + \log z(s') + \alpha(s')\,\log n(s')\right] \quad + \\ + \sum_{s'\in\Phi(s)} \alpha(s,s')\left\{\sum_{s\in\Phi(s)} \alpha(s',s)\log x(s',s)\right\} + \log\beta(s) \quad + \\ + \sum_{s'\in\Phi(s)}\left\{\frac{\sum_{s\in\Phi(s)}\alpha(s',s)}{\sum_{s\in\Phi(s)}\alpha(s,s)}\log\left[\frac{\sum_{s\in\Phi(s)}x(s',s)}{\sum_{s\in\Phi(s)}x(s,s)}\right]\right\}\end{array}\right\}$$

Inserting the condition in eq. (A.1) used to derive Lemma 1 in the $log[\cdot]$ *component in the last row, compute the associate total differentiation, following closely the steps to derive Proposition 5, and using the notation associated to network distance matrix* $\mathcal{D}^{fd}$*, then one obtains eq. (A.11) characterizing Proposition 6.*

□

(Proof of Proposition 7) *The interest is in characterizing the way in which sectoral employment changes in response to variations in other sectors' employment levels according to their factor input supply network distance relationships. Taking the logarithm of eq. (A.8) would result in*

$$\log n(s) = \frac{1}{1+\phi}\left\{\begin{array}{l}\log\alpha(s) + \log C + \log z(s) + \alpha(s)\,\log n(s) - \log c(s) \quad + \\ + \sum_{s'\in\Phi(s)} \alpha(s,s')\left[\log\vartheta(s,s') + \log z(s') + \alpha(s')\,\log n(s')\right] \quad + \\ + \sum_{s'\in\Phi(s)} \alpha(s,s')\left\{\sum_{s'\in\Phi(s)} \alpha(s',s')\log x(s',s')\right\} + \log\beta(s) \quad + \\ + \sum_{s\in\Phi(s)}\left\{\frac{\sum_{s'\in\Phi(s)}\alpha(s',s)}{\sum_{s'\in\Phi(s)}\alpha(s',s')}\log\left[\frac{\sum_{s'\in\Phi(s)}x(s',s)}{\sum_{s'\in\Phi(s)}x(s',s')}\right]\right\}\end{array}\right\}$$

Inserting the condition in eq. (A.2) used to derive Lemma 2 in the $log[\cdot]$ *component in the last row, compute the associate total differentiation, following closely the steps to derive Proposition 5, and using the notation associated to network distance matrix* $\mathcal{D}^{fs}$*, then one obtains eq. (A.12) characterizing Proposition 7.*

□

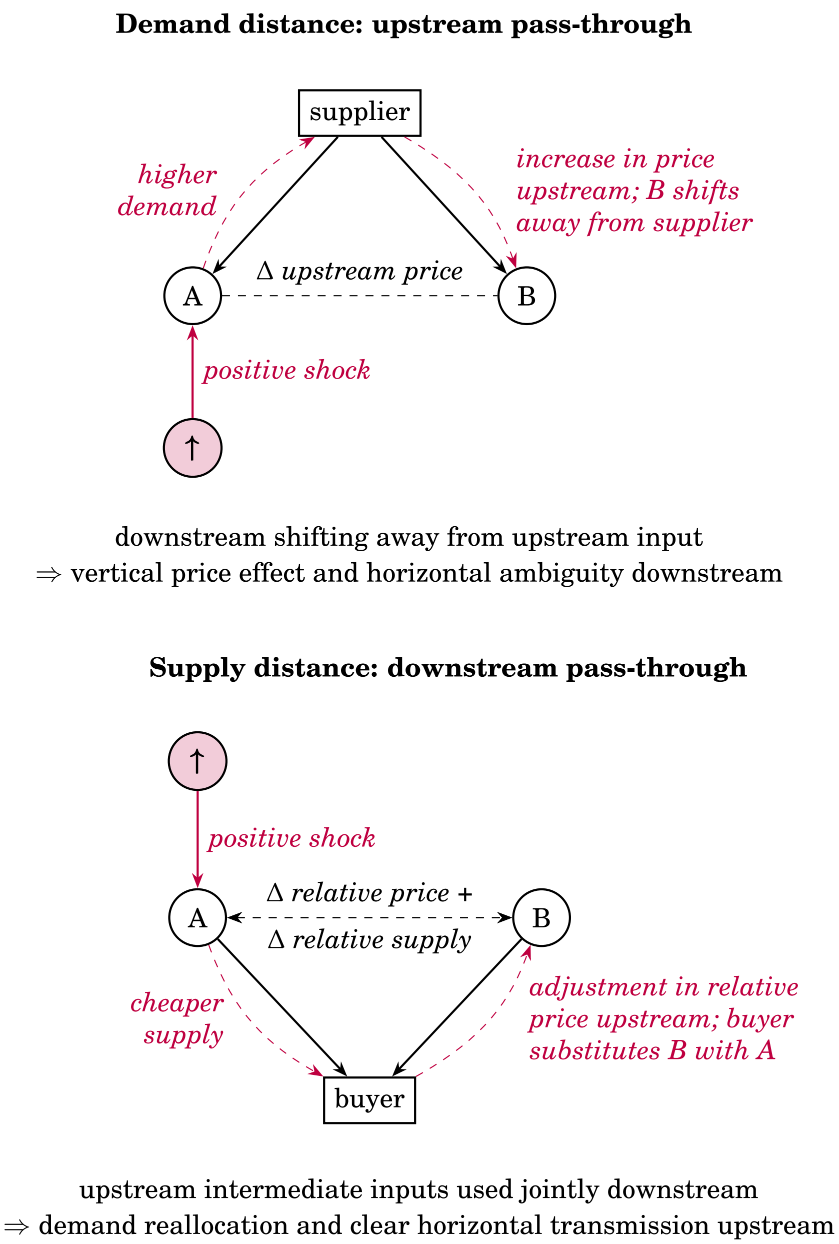


FIGURE A.1: DISTANCE-LEAD COMOVEMENT

Note: graphical visualization of Propositions 1-2 for comovement patterns within a production network based on network "economic" distances from resembling Input-Output structure across participants.

B. Data, figures and tables

(Data details) *In order to build the main dataset with a panel of 65 private 3-digit U.S. 2017 North American Industry Classification System (NAICS) sectors used in Sections 1.4-1.5, I rely on different data sources from the Bureau of Economic Analysis (BEA).*[40] *For years from 1998 to 2022, the features of these information are:*

(a) employment (Table 6.8D. - Persons Engaged in Production by Industry, in thousands. Last revised on: September 29, 2023);

(b) value-added (Value-Added by Industry, in billions of dollars. Last revised on: December 19, 2024);

(c) inter-sectoral trade and Input-Output linkages, in particular:

- *sectoral share of own production, and thus the sectoral share of intermediate inputs from other sectors (Industry-by-Commodity Market Share Matrix, After Redefinitions - Summary, in producers' prices);*
- *total inputs required (directly and indirectly) in order to deliver one dollar of output to final users (Commodity-by-Commodity Total Requirements, After Redefinitions - Summary, in producers' prices);*
- *sector-specific imports and exports of goods and services (The Use of Commodities by Industries, Before Redefinitions - Summary, in producers' prices and in millions of dollars).*

Note that "Summary" defines the level of sectoral disaggregation considered in the analysis (3-digit). The BEA also provides information on Input-Output matrices to a more detailed level (6-digit), but these are on a five-yearly basis (thus it would impossible to run the analysis in Subsection 1.5.1), and data on other variables are not collected. Moreover, the sample begins in 1998 so to have all sectors classified given the NAICS system, and ends in 2022, the last year available when I started this project.[41] *Last time I accessed these online data was in March 2025.*

Turn to the manipulation of Input-Output matrices. To recover the production share of intermediate inputs of a given sector coming from the production of other sectors, each cell of the yearly squared matrix contains the intermediate output share from a given sector to another and, on the main diagonal, the share of intermediate output of a given sector directly produced by that sector. Henceforth, labelling by $\mathbf{X} = \left[x(s,s')\right]$, *with* $x(s,s') \geq 0$, *this Input-Output matrix I compute, for each sector,* $x(s) = 1 - x(s,s)$. *In other words, to extract the total share of intermediate output that sector-*s *buys from other sectors, I compute* $\boldsymbol{x} = \mathbf{1} - diag(\mathbf{X})$, *where* $\mathbf{1}$ *an* $m \times 1$ *vector of ones; the resulting* $m \times 1$ *vector* $\boldsymbol{x}$ *reports, for every considered year, the total production*

[40] Note that I refer to 3-digit, but four sectors are at 2-digit ("construction", "management of companies and enterprises", "educational services", and "other services, except government"), while five sectors related to finance and insurance are at 4-digit level.

[41] The 3-digit classification is the most granular if one wants to keep track of the evolution in the set of intermediate inputs, and 1998 is the first year in which the NAICS system has been adopted. Before 1998, classification follows the U.S.Standard Industrial Classification (SIC) system.

of a sectoral good which is due to intermediate inputs bought through Input-Output relationships.

For what concerns the production network structure, I investigate its characteristics through the main Input-Output matrix (Commodity-by-Commodity Total Requirements, After Redefinitions) which, consistently with the main text, I label as $\boldsymbol{H} = \left[\alpha\left(s, s'\right) \geq 0\right]$. *Leontief inverse matrix (labelled with* $\boldsymbol{\mathcal{H}}$ *in the main text), upstream and downstream sectors, and the weight associated to each of them (as in Subsections 1.5.1-1.5.2) are identified through it.*[42]

Differently, in order to compute sectoral distances, I follow the literature (e.g., Acemoglu et al. 2012, Carvalho 2014) and disregard small transactions across sectors: this allows me to compute a finite set of production network distances at the extensive margin due to the removal of bias from meaningless inter-sectoral trade intensity. Such adjusted matrix is labelled as $\mathbf{H}^{j} = \left[\alpha^{j}\left(s, s'\right)\right]$ *whose generic element is then* $\alpha^{j}\left(s, s'\right) > 0.01$ *for* $j = \{fd, fs\}$ *identifying whether the matrix allows to compute factor input demand or factor input supply network distances. In particular, I set to 0 all the linkages which are below 1% of sector's total purchases (in case of factor input demand distance) and total sales (in case of factor input supply distance). Disregarding small transactions implies that*

$$\alpha^{j}\left(s, s'\right) = \frac{\alpha\left(s, s'\right)}{\mathcal{K}^{j}\left(s\right)} \quad \text{is set to} \quad 0 \quad \text{if} \quad \alpha^{j}\left(s, s'\right) \leq 0.01$$

where $\mathcal{K}^{fd}\left(s\right) = \sum_{s' \in row} \alpha\left(s, s'\right)$ *is sector-s's total input purchases (i.e., row sum of matrix* **H***) for factor input demand distance, or* $\mathcal{K}^{fs}\left(s\right) = \sum_{s' \in column} \alpha\left(s, s'\right)$ *is sector-s's total input sales (i.e., column sum of matrix* **H***) for factor input supply distance. It follows that, by defining* $\boldsymbol{\mathcal{K}}^{j} = \left[\mathcal{K}^{j}\left(s\right), \mathcal{K}^{j}\left(s'\right), \ldots, \mathcal{K}^{j}\left(m\right)\right]'$, *the considered Input-Output structures are*

$$\underset{m \times m}{\mathbf{H}^{j}} = \underset{m \times m}{\mathbf{H}}{}' \underset{m \times 1}{\boldsymbol{\mathcal{K}}^{j}} \qquad \text{for} \quad j = \{fd, fs\}$$

in which each cell, $\alpha^{j}\left(s, s'\right)$, *is greater than 0.01, i.e., larger than 1%. From these I compute intensive margin values for both network "economic" distance metrics.*

(Shortest path algorithm, technical but suitable for coding) *Given element* $\boldsymbol{\mathcal{G}}$ *being an* $m \times m$ *adjacency matrix with non-negative discrete values, the idea is to compute a distance matrix* $\boldsymbol{\mathcal{D}}$ *where each cell* $d\left[s, s'\right]$ *is the length of the shortest path between node-*s *and node-*s'. *The numerical algorithm is as follows:*

(a) *from the Input-Output matrix* **H***, build an associated identity matrix,* **I***;*

(b) *set the initial length of the path,* $n = 1$, *and build a length matrix* $\boldsymbol{\mathcal{G}}_{n} = \boldsymbol{\mathcal{G}}$*;*

(c) *build a Boolean matrix,* **B***, whose elements are* $b\left(s, s'\right) = 1$ *if* $\boldsymbol{\mathcal{G}}_{n} \neq 0$*;*

[42] Row sectors identify (upstream) suppliers, and column sectors identify (downstream) buyers. Basically, in matrix H each entry in the upper-triangular part corresponds to the dollar value of one unit of sector-s that is bought by sector-s', while each entry in the lower-triangular part identifies the dollar value of sector-s in order to buy one unit from sector-s'. Hence, row sector is the origin and column sector is the destination of the circulating intermediate input.

(d) as long as matrix **B** *contains a value different from zero (i.e., there are connections among nodes not considered):*

- *update identity matrix* **I** *by adding* n *whenever the condition on* **B** *is true, that is,* $i(s,s') = i(s,s') + n$ *if* $b(s,s') = 1$*;*
- *update lengths,* $n = n + 1$*;*
- *algebraic matrix calculation:* $\boldsymbol{\mathcal{G}}_n = \boldsymbol{\mathcal{G}}'_n \boldsymbol{\mathcal{G}}$*;*
- *update matrix* **B** *with new paths: set* $b(s,s') = 1$ *whenever* $\mathcal{G}_n(s,s') \neq 0$ *and* $d[s,s'] = 0$*;*

(e) once this loop concludes, set to infinite all the disconnected nodes, that is $d(s,s') = \infty$ *if* $d[s,s'] = 0$*, to check consistency;*

(g) set to the minimum distance the elements on the main diagonal, $\boldsymbol{\mathcal{D}} = \boldsymbol{\mathcal{D}} - \mathbf{I}$*.*

Outside this routine set to zero all the disconnected nodes (those at ∞*). Matrix* $\boldsymbol{\mathcal{D}}_{ext}$*, whose values are at the extensive margin and identify network distances among any pair* (s,s')*, is built. The generic element identifying the existence of a link between node-*s *and node-*s' *in the distance matrix* $\boldsymbol{\mathcal{D}}$ *is then* $d_{ext}[s,s'] = \{1, 2, \ldots, d_{max}\}$*.*

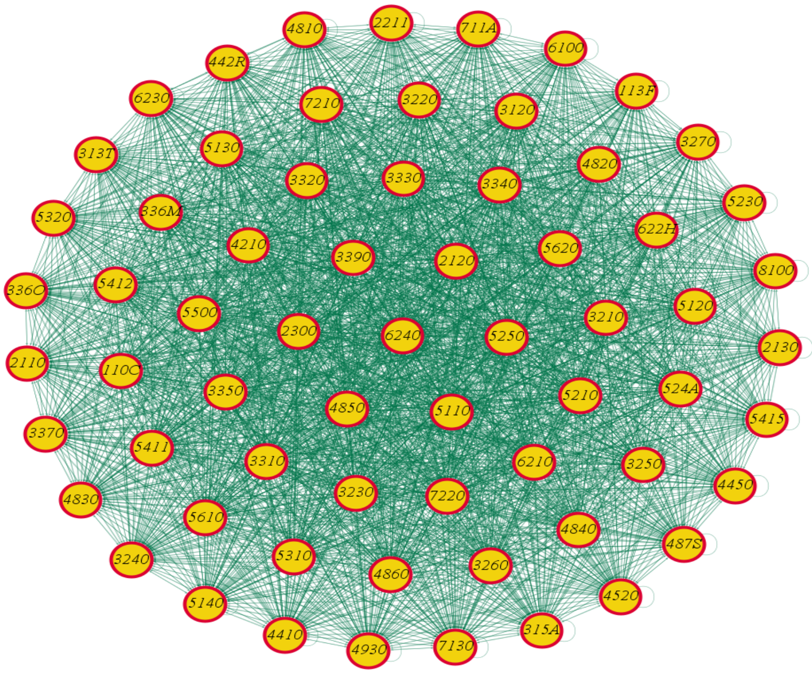


FIGURE B.1: PRODUCTION NETWORK OF THE U.S. ECONOMY IN 2007

Note: the figure shows the production network associated to the U.S. Input-Output system in year 2007 for 3-digit U.S. 2017 NAICS sectors. Each node corresponds to a sector, while green curves connecting vertexes trace any inter-sectoral trade connection in the detailed commodity-by-commodity direct requirements table. A linkage between a sector and all the other buyers of its intermediate input are removed for every transaction below 5% of its total input sales through its inter-sectoral trade. Figure drawn with the software package *Gephi*, version 0.10, exploiting the *Fruchterman Reingold* layout algorithm. *Source*: BEA and own calculations.

TABLE B.1: LIST OF CONSIDERED SECTORS AND THEIR CODES

denomination	*codes* NAICS 2017	*codes* BEA
Farms	111, 112	111CA
Forestry, Fishing and Related Activities	113, 114, 115	113FF
Oil and Gas Extraction	211	211
Mining, except Oil and Gas	212	212
Support Activities for Mining	213	213
Utilities	22	22
Construction	23	23
Wood Products	321	321
Nonmetallic Mineral Products	327	327
Primary Metals	331	331
Fabricated Metal Products	332	332
Machinery	333	333
Computer and Electronic Products	334	334
Electrical Equipment, Appliances, and Components	335	335
Motor Vehicle, Bodies and Trailers, and Parts	3361-3	3361MV
Other Transportation Equipment	3364-9	3364OT
Furniture and Related Products	337	337
Miscellaneous Manufacturing	339	339
Food and Beverage and Tobacco Products	311, 312	311FT
Textile Mills and Textile Product Mills	313, 314	313TT
Apparel and Leather and Allied Products	315, 316	315AL
Paper Products	322	322
Printing and Related Support Activities	323	323
Petroleum and Coal Products	324	324
Chemical Products	325	325
Plastics and Rubber Products	326	326
Wholesale Trade	42	42
Motor Vehicle and Parts Dealers	441	441
Food and Beverage Stores	445	445
General Merchandise Stores	452	452
Other Retail	442-4, 446-8, 45 ex. 452	4AO
Air Transportation	481	481
Rail Transportation	482	482
Water Transportation	483	483
Truck Transportation	484	484
Transit and Ground Passenger Transportation	485	485

...

Sectors are defined according to the nomenclature used by the U.S. Bureau of Economic Analysis (BEA).

TABLE B.1: LIST OF CONSIDERED SECTORS AND THEIR CODES (*continued*)

denomination	***codes*** *NAICS 2017*	*BEA*
	...	
Pipeline Transportation	486	486
Other Transportation and Support Activities	487, 488, 492	487OS
Warehousing and Storage	493	493
Publishing Industries, except Internet (includes Software)	511	511
Motion Picture and Sound Recording industries	512	512
Broadcasting and Telecommunications	515, 517	513
Data Processing, Internet Publishing, and Other Information Service	518, 519	514
Federal Reserve Banks, Credit Intermediation, and Related Activities	521, 5221, 5222, 5223	521CI
Securities, Commodity Contracts, and Investments	523	523
Insurance Carriers and Related Activities	5241, 5242	524
Funds, Trusts, and Other Financial Vehicles	525	525
Housing	·	HS
Other Real Estate	531	ORE
Rental and Leasing Services and Lessors of Intangible Assets	532, 533	532RL
Legal Services	5411	5411
Computer Systems Design and Related Services	5415	5415
Miscellaneous Professional, Scientific, and Technical Services	541 ex. 5411, 5415	5412OP
Management of Companies and Enterprises	55	55
Administrative and Support Services	561	561
Waste Management and Remediation Services	562	562
Educational Services	61	61
Ambulatory Health Care Services	621	621
Hospitals	622	622
Nursing and Residential Care Facilities	623	623
Social Assistance	624	624
Performing Arts, Spectator Sports, Museums, and Related Activities	711, 712	711AS
Amusement, Gambling, and Recreation industries	713	713
Accommodation	721	721
Food Services and Drinking Places	722	722
Other Services, except Government	81	81

Sectors are defined according to the nomenclature used by the U.S. Bureau of Economic Analysis (BEA).

(Network distances at the intensive margin) *In Section 1.4 of the main text I characterize distance matrices at their "extensive margin", $\mathcal{D}^{fd}_{ext}$ and $\mathcal{D}^{fs}_{ext}$, whose values are $d^{j}_{ext}[s,s'] = \{1,2,\ldots,d_{max}\}$ for $j = \{fd, fs\}$, exploiting the shortest path algorithm on the "intensive margin" distances from the directed production network,* $\mathbf{H}$*, that accounts for direct sectoral connections only.*

Indeed, to characterize the "intensive margin" matrices $\mathcal{D}^{fd}$ and $\mathcal{D}^{fs}$ I exploit the system of Cartesian coordinates typical of any production network (e.g., Conley and Dupor 2003). Given $\alpha^{fd}(s,s') = \frac{\alpha(s,s')}{\sum_{s' \in row} \alpha(s,s')}$, the generic element of factor input demand distance matrix is computed as follows:

$$d^{fd}\left[s,s'\right] = \left\{ \sum_{s^* \in \Phi(s)} \left[\alpha^{fd}\left(s,s^*\right) - \alpha^{fd}\left(s',s^*\right) \right]^2 \right\}^{\frac{1}{2}} \tag{B.1}$$

which defines the length of the line segment (i.e., the Euclidean distance) connecting sectors s and s' when both are buying from the same other sector-s^*, $\forall s^* \in \Phi(s)$, so that $\mathcal{D}^{fd} = \left[d^{fd}[s,s']\right]$ with generic element $d^{fd}[s,s'] > 0$. Demand distance is constructed by dividing each cell by its row sum. Why? A row sum represents the total sales of the sector in question, so dividing each entry by its associated row sum yields the share of that (row) sector's output purchased by each of the column sectors. This normalization allows for a meaningful comparison across dyads of sectors: by examining these shares, one can assess the relative distance between two (column) sectors in terms of their purchases from common upstream suppliers, thereby pointing to the strength of the upstream pass-through (share of total sales of the common sector).

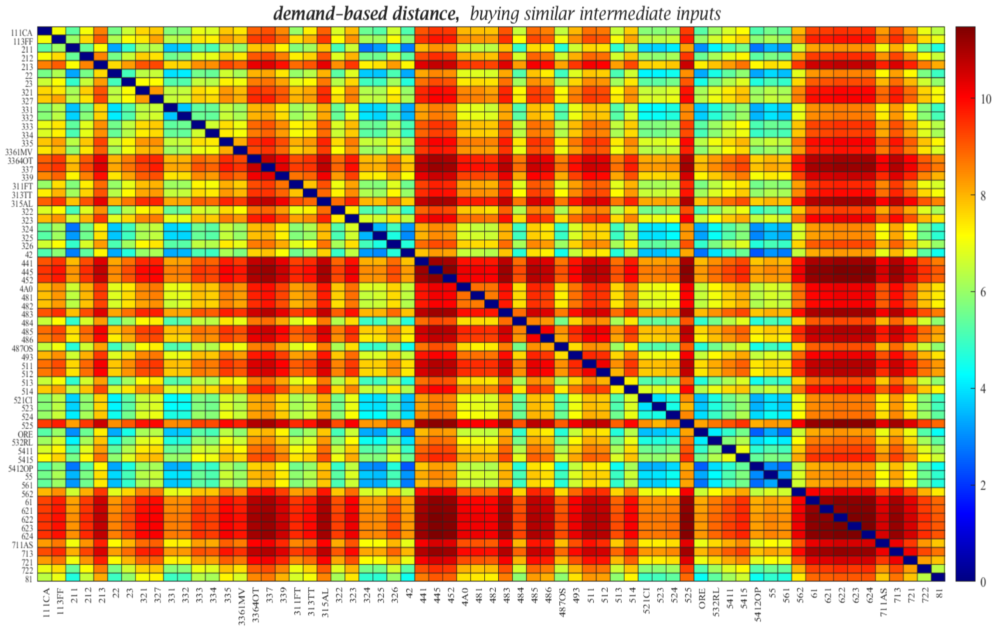
In an analogous way, given $\alpha^{fs}(s,s') = \frac{\alpha(s,s')}{\sum_{s' \in column} \alpha(s,s')}$, the generic element of factor input supply distance matrix is shaped by

$$d^{fs}\left[s,s'\right] = \left\{ \sum_{s^* \in \Phi(s)} \left[\alpha^{fs}\left(s^*,s\right) - \alpha^{fs}\left(s^*,s'\right) \right]^2 \right\}^{\frac{1}{2}} \tag{B.2}$$

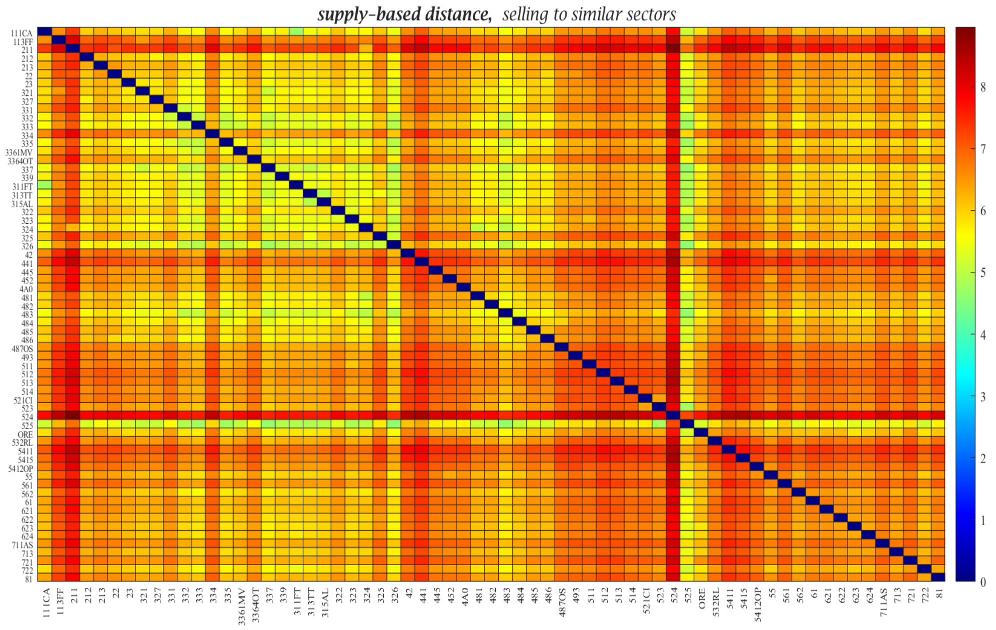
tracing a Euclidean distance between sectors $\{s,s'\}$ when both are selling to the same sector-s^*, $\forall s^* \in \Phi(s)$, so that $\mathcal{D}^{fs} = \left[d^{fs}[s,s']\right]$ with $d^{fs}[s,s'] > 0$. Supply distance is constructed by dividing each cell by its column sum. Why? A column sum represents the total purchases of the sector in question, so dividing each entry by its associated column sum yields the share of that (column) sector's input purchased from each of the row sectors. This normalization allows for a meaningful comparison across dyads of sectors: by examining these shares, one can assess the relative distance between two (row) sectors in terms of their traded quantities to common downstream buyers, thereby pointing to the strength of the downstream pass-through (share of total purchases of the common sector).

Both the distance matrices at the intensive margin are developed from the directed production network, $\mathbf{H} = \left[\alpha(s,s') \geq 0\right]$.[43]

[43] Recall that distance matrices are computed from an Input-Output structure in which each inter-sectoral trade linkage is greater than 1% of total purchases (or sales) of a sector.



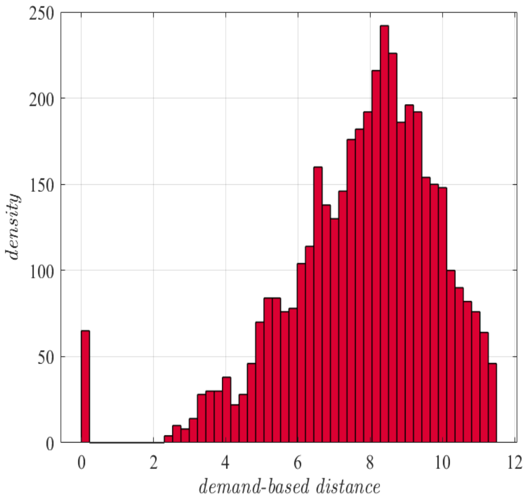
(A) Factor input demand



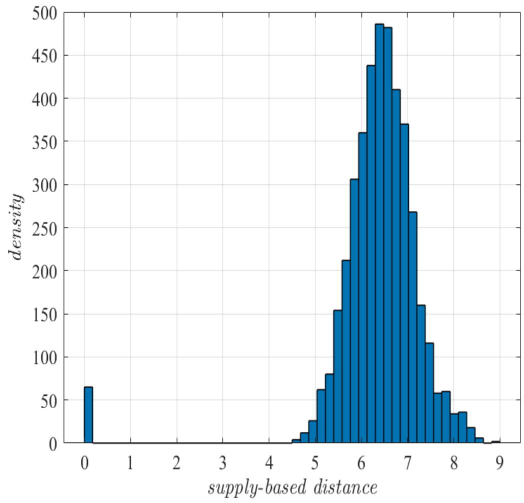
(B) Factor input supply

Figure B.2: Intensive margin production network distance matrices

Note: heatmaps of sectoral distances characterizing the U.S. production network in year 2007 for 3-digit U.S. 2017 NAICS sectors. Entry (s, s') represents the intensive marginal value, $d^j[s, s'] \geq 0$ for $j = \{fd, fs\}$, of the horizontal relationship between sector-s and sector-s' under *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers, Panel B.2a) and *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers, Panel B.2b). Diagonal sparsity (*i.e.*, lots of 0s), is a direct consequence of the fact that the difference of inter-sectoral trade intensities between a sector and itself is zero. Distance matrices are at the *intensive margin*, as in eqs. (B.1)-(B.2), computed from the *directed* sectoral production network. *Source*: BEA and own calculations.



(A) FACTOR INPUT DEMAND



(B) FACTOR INPUT SUPPLY

FIGURE B.3: DISTRIBUTION OF NETWORK ECONOMIC DISTANCES

Note: these figures depict the distributions for *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers, Panel B.3a) and *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers, Panel B.3b). Distance values on the x-axis are at the *intensive margin*, as in eqs. (B.1)-(B.2), computed from the *directed* sectoral production network *Source*: BEA and own calculations.

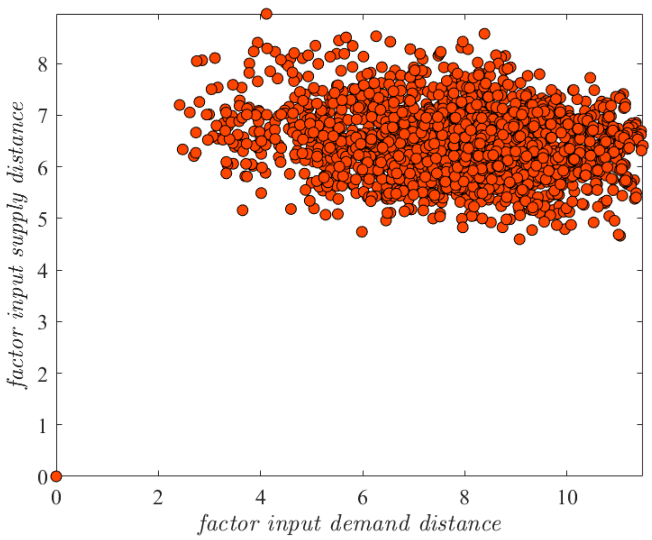
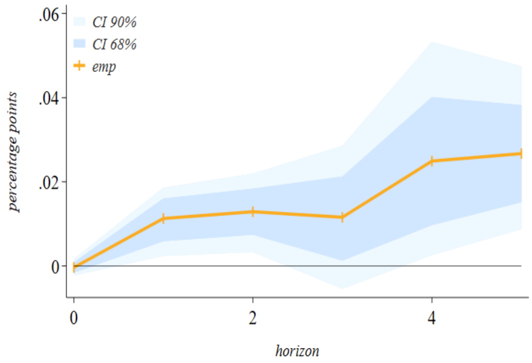


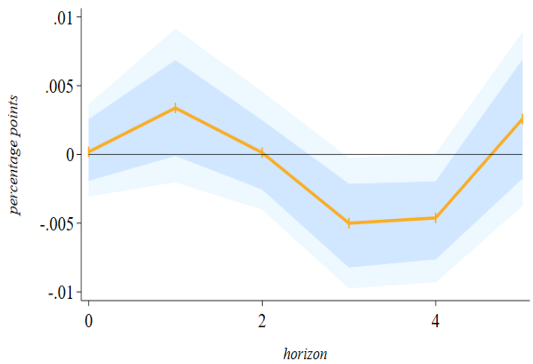
FIGURE B.4: CORRELATION OF NETWORK ECONOMIC DISTANCES

Note: the figure shows the correlation between the measures for *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers, x-axis) and *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers, y-axis). Distances are at the *intensive margin*, as in eqs. (B.1)-(B.2), computed from the *directed* sectoral production network *Source*: BEA and own calculations.

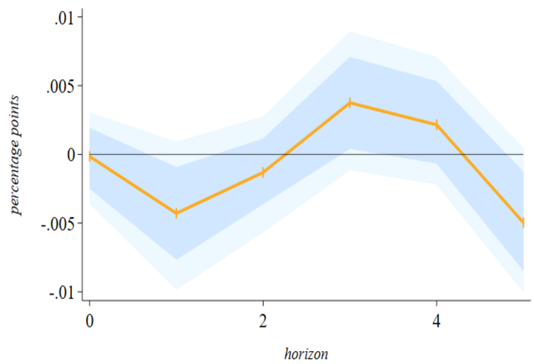
C. Complementary results



(A) Sector-specific



(B) $d = 1$

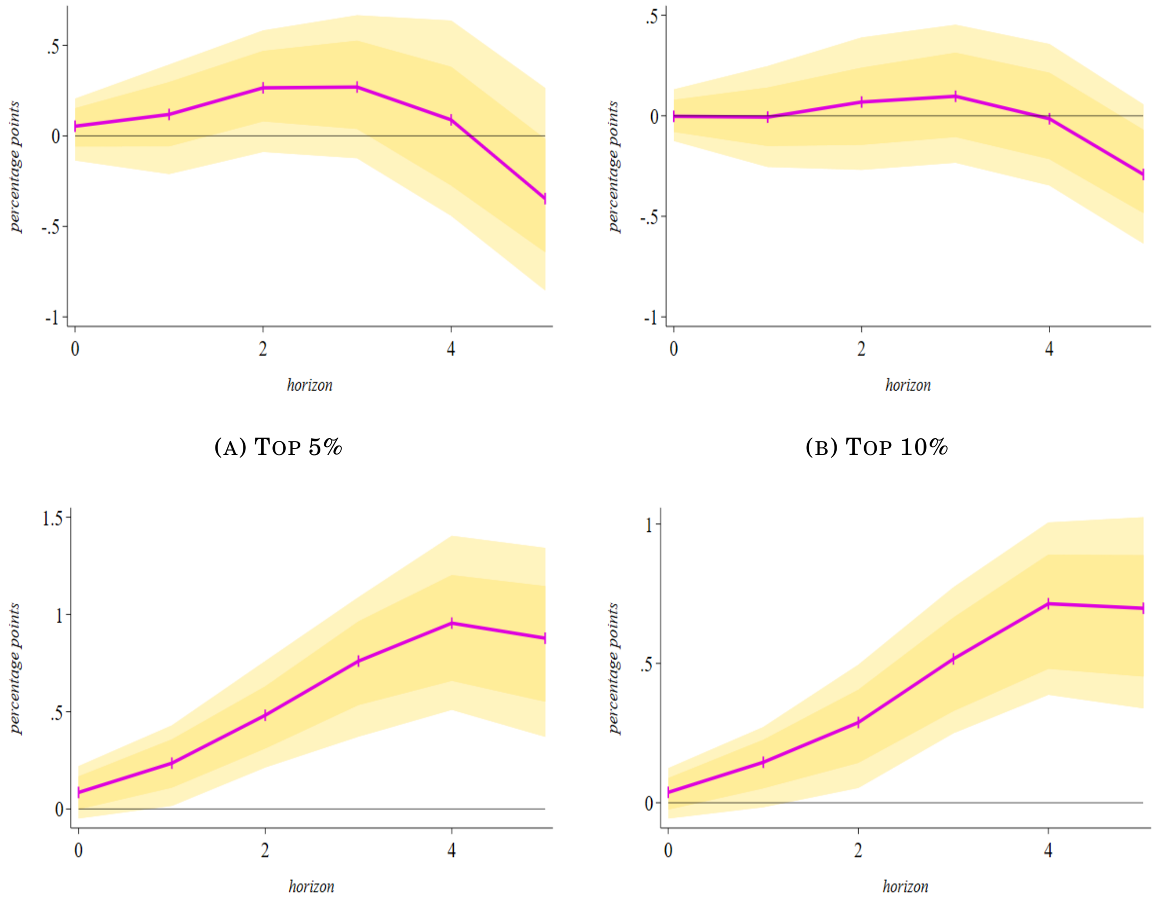


(C) $d = 2$

Figure C.1: Sectoral employment response to intermediate inputs

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment to a 1% increase in the sector-specific set of intermediate inputs as a share of its value-added for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. In particular, Panel C.1a represents the sector-level employment response to changes in its intermediate inputs, while Panel C.1b-C.1c to changes in the set of intermediates in closer (distance equal to 1) and further (distance equal to 2) sectors. The solid-orange line corresponds to the average response of employment across sectors, while shadow-blue and shadow-light blue areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eqs. (1.6)-(1.7). *Source*: BEA and own calculations.

(Plotting separated results for highly interlinked sectors) *Unpacking all the combined plots, below I report the estimated Local Projection (LP) dynamics for top 5%, top 10%, top 20%, and top 30% of more connected sectors in the production network (i.e., major number of Input-Output linkages) separately. The purpose is to show the significance of the responses appearing in Panels 1.10a-1.10b of Figure 1.10 in Subsection 1.5.2 of the main text, and in Panels 1.8a-1.8b of Figure 1.8 in Appendix C.*



(A) TOP 5%

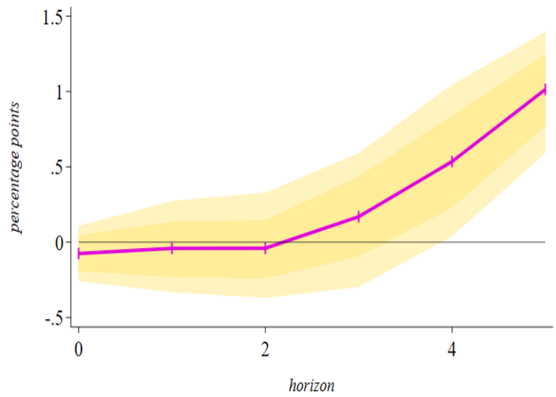
(B) TOP 10%

(C) TOP 20%

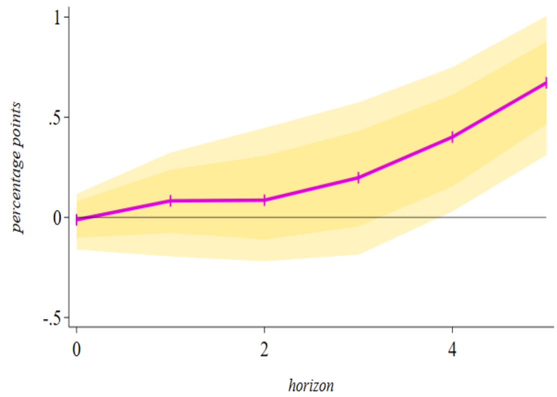
(D) TOP 30%

FIGURE C.2: DEMAND ($d = 1$) EFFECTS FOR INTERLINKED SECTORS

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel 1.8a of Figure 1.8, each panel of this figure shows the employment response of top 5% (Panel C.2a), 10% (Panel C.2b), 20% (Panel C.2c), and 30% (Panel C.2d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.



(A) TOP 5%



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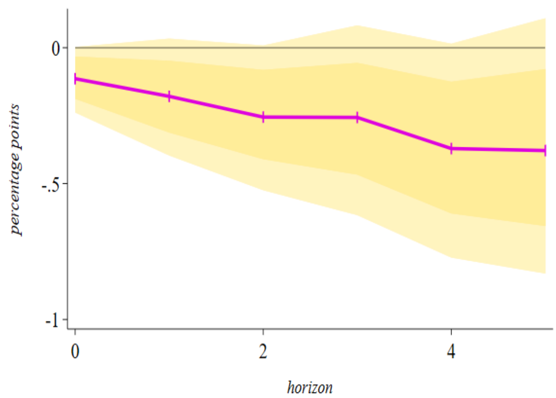
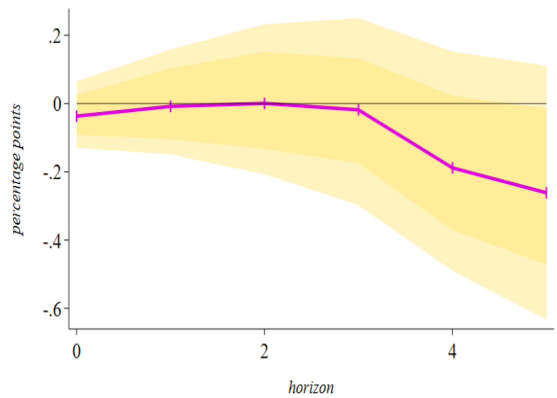


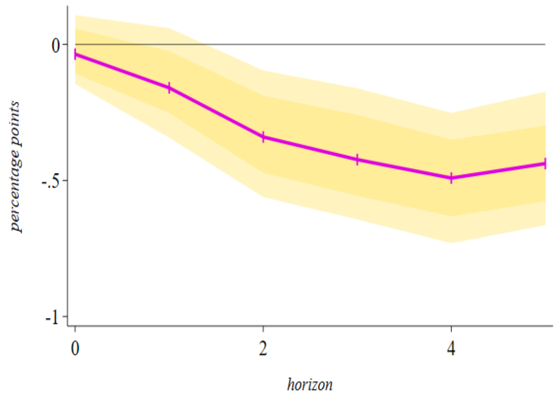
(C) TOP 20%



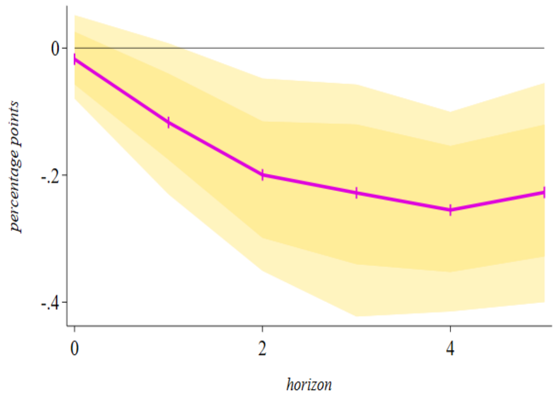
(D) TOP 30%

FIGURE C.3: DEMAND ($d = 2$) EFFECTS FOR INTERLINKED SECTORS

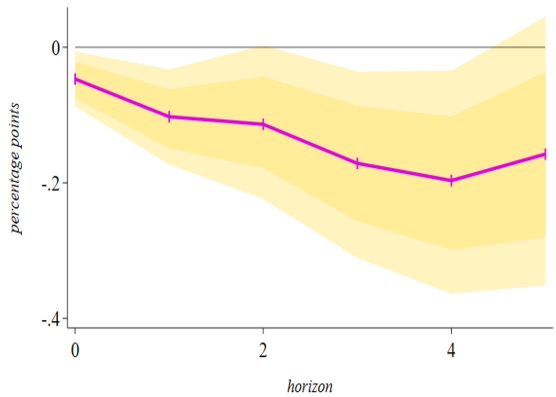
Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel 1.8b of Figure 1.8, each panel of this figure shows the employment response of top 20% (Panel C.3c) and 30% (Panel C.3d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) to employment changes in their further (distance equal to 2) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.



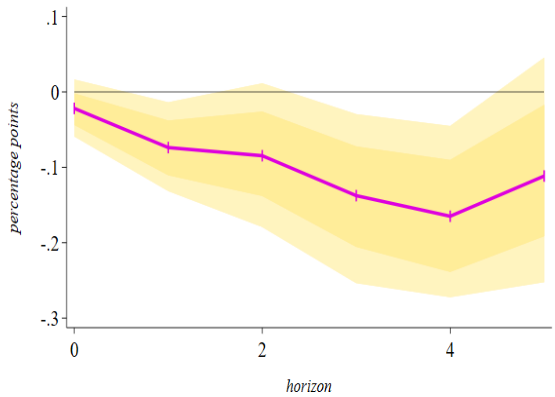
(A) TOP 5%



(B) TOP 10%



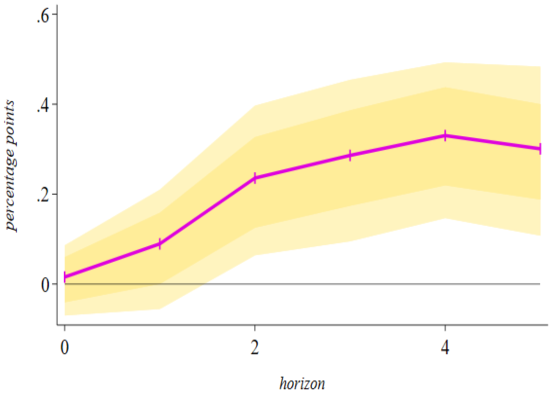
(C) TOP 20%



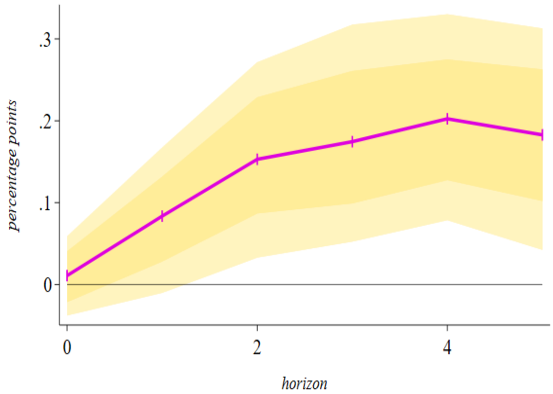
(D) TOP 30%

FIGURE C.4: SUPPLY ($d = 1$) EFFECTS FOR INTERLINKED SECTORS

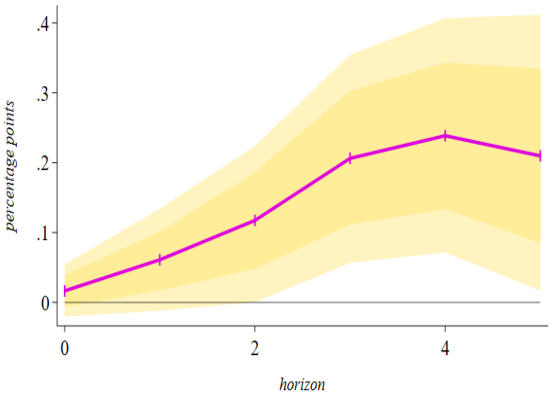
Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel 1.10a of Figure 1.10, each panel of this figure shows the employment response of top 5% (Panel C.4a), 10% (Panel C.4b), 20% (Panel C.4c), and 30% (Panel C.4d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.



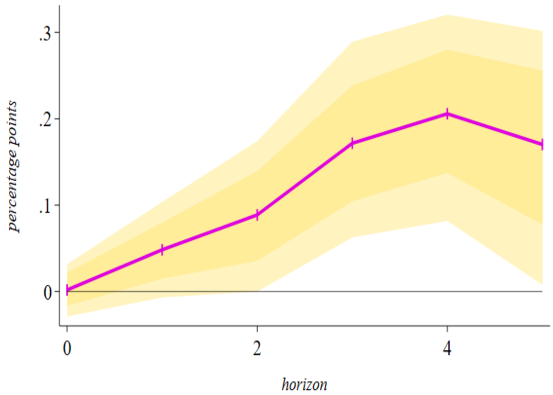
(A) TOP 5%



(B) TOP 10%



(C) TOP 20%



(D) TOP 30%

FIGURE C.5: SUPPLY ($d = 2$) EFFECTS FOR INTERLINKED SECTORS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Unpacking Panel 1.10b of Figure 1.10, each panel of this figure shows the employment response of top 5% (Panel C.5a), 10% (Panel C.5b), 20% (Panel C.5c), and 30% (Panel C.5d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their further (distance equal to 2) sectors. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.

(Robustness with centrality measures) *To identify the centrality degree of each sector, I follow the approach outlined in Carvalho (2014) to measure the sector-level Bonacich-Katz centrality.*[44] *Using the Leontief-inverse matrix $\boldsymbol{\mathcal{H}}$ for the U.S. production network as of 2007, the Bonacich-Katz centrality vector, $\boldsymbol{bk}$, is defined as:*

$$\boldsymbol{bk} = \sum_{k=0}^{\infty} \phi^k \boldsymbol{\mathcal{H}}^k \mathbf{1} = \left(\boldsymbol{I} - \omega \boldsymbol{\mathcal{H}}\right)^{-1} \mathbf{1}, \tag{C.1}$$

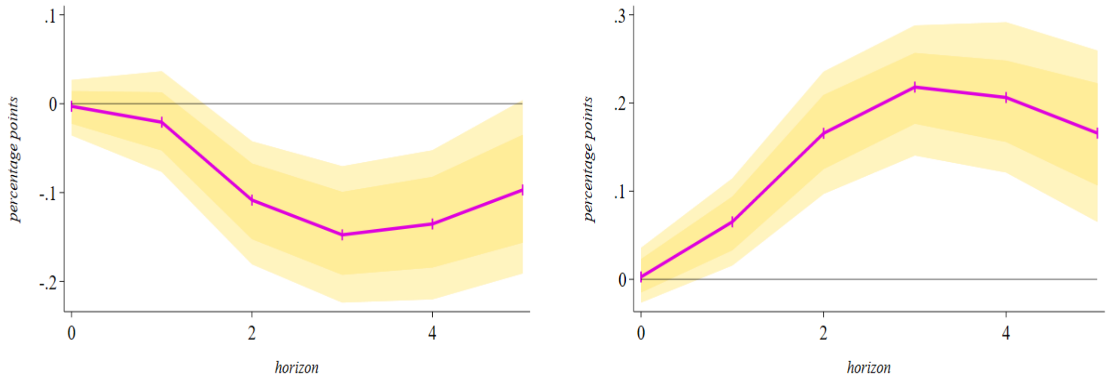
where index-k refers to the length of the paths in the production network – that is, the number of steps (or network "hops") connecting one sector to another – and, given a finite set of sectors $\{s, s', s'', \ldots, m\} \in \Phi(s)$:

- $\mathbf{1}$ *is an $m \times 1$ vector of ones, reflecting the base influence;*
- $\omega \in \left(0, \frac{1}{v_{max}}\right)$ *is a dampening parameter, with v_{max} being the largest eigenvalue of matrix $\boldsymbol{\mathcal{H}}$;*
- $\boldsymbol{I}$ *is the $m \times m$ identity matrix.*

The value of ω determines the weight assigned to indirect linkages: smaller values place more emphasis on direct connections, while larger values account for deeper propagation within the network. Following Carvalho (ibid.), a value of $\omega = 0.5$ is typically used, which lies safely below the inverse of the largest eigenvalue of most empirically observed Input-Output matrices, ensuring convergence of the series. This measure reflects the idea that a sector is central not only if it is directly connected to many others, but also if it is connected to sectors that are themselves highly central.

This formulation ensures that the centrality score of each sector captures both its direct connections and its indirect influence through other sectors due to the use of matrix $\boldsymbol{\mathcal{H}} = \left(\boldsymbol{I} - \boldsymbol{H}\right)^{-1} = \left[\ell(s, s') \geq 0\right]$, where $\boldsymbol{H} = \left[\alpha(s, s') \geq 0\right]$ is the directed production network matrix (i.e., that in the BEA Input-Output tables), defining the intensity of good produced by sector-s' in the total intermediate inputs used by sector-s, with $\alpha(s, s') = 0$ indicating that sector-s does not make use of the good produced by sector-s' in producing its own intermediate good.

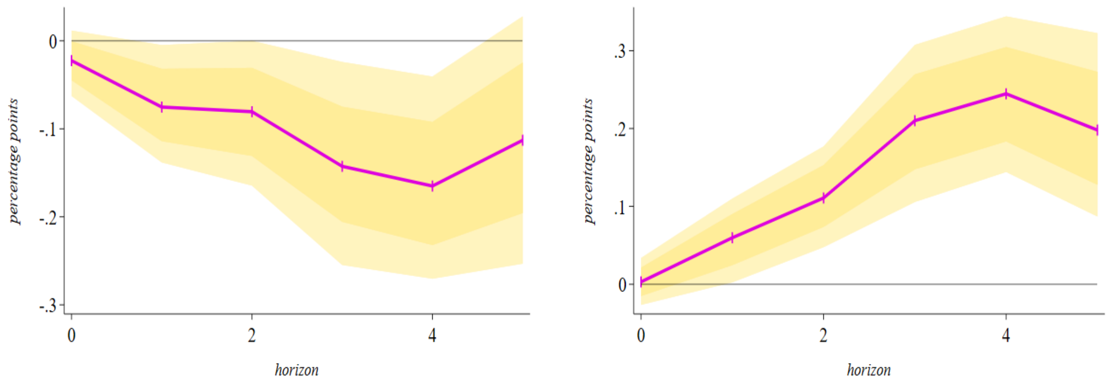
[44] Outlined by Bonacich (1987), it is a measure of a sector's overall importance within an Input-Output system, capturing not only its direct connections to other sectors but also the centrality of those it is connected to. In essence, a sector is considered central if it is linked to other central sectors, creating a recursive structure of influence. This measure goes beyond simple counts of linkages by incorporating indirect connections – weighted by a dampening factor – to reflect the diminishing influence of more distant sectors in the network. In the context of production network, a sector with high Bonacich-Katz centrality is one that plays a key role in the propagation of shocks, as its position allows it to influence (or be influenced by) large portions of the economy through both direct and indirect channels. Compared to other centrality measures, Bonacich-Katz is particularly well-suited for economic applications, as it accounts for the intensity of inter-sectoral relationships and the structure of the network in a realistic and nuanced way.



(A) $d = 1$ (B) $d = 2$

FIGURE C.6: DEMAND LINKAGES AND COMOVEMENT IN PERIPHERAL SECTORS

Note: given the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of most central sectors (with *centrality* defined according to sector's relevance in final consumption) to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel C.6a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel C.6b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.

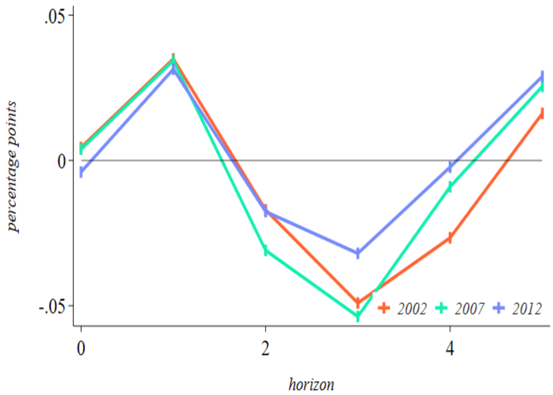


(A) $d = 1$ (B) $d = 2$

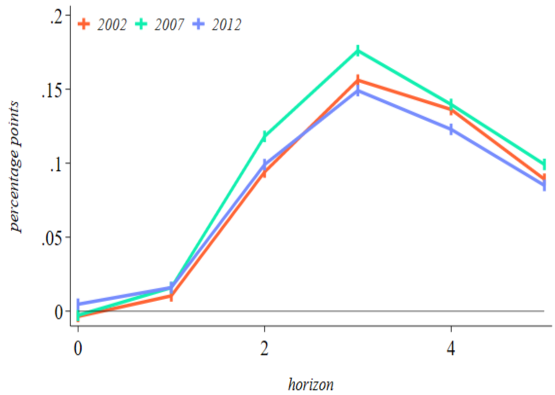
FIGURE C.7: SUPPLY LINKAGES AND COMOVEMENT IN PERIPHERAL SECTORS

Note: given the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network in year 2007, the figure shows the response of sectoral employment of most central sectors (with *centrality* defined according to sector's relevance in final consumption) to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Panel C.7a represents the sector-level employment response to employment changes in closer (distance equal to 1) sectors, while Panel C.7b plots the response to employment changes in further (distance equal to 2) ones. The solid-purple line corresponds to the average response of employment across sectors, while shadow-gold and shadow-light gold areas correspond to 68% and 90% significance levels of bootstrapped Confidence Interval (CI), respectively, computed from eq. (1.9). *Source*: BEA and own calculations.

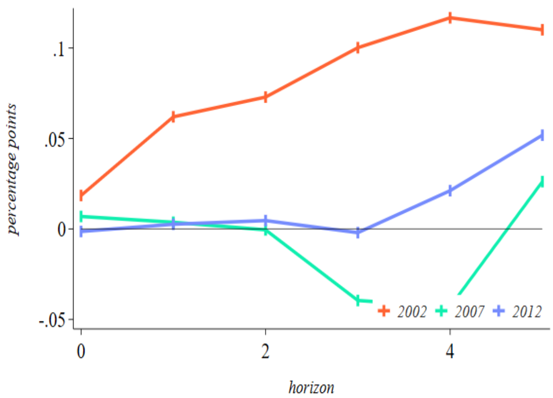
(Robustness with production network at different base years) *Responses for comovement in sectoral employment levels in the main analysis of Section 1.5 are centred on the implementation of a sectoral production network represented by an Input-Output matrix in the baseline year 2007. Differently, Figures C.8, C.9 and C.10 perform again all the main analysis by referring, instead, to a production network structure five years before (2002) and five years after (2012) the baseline year. Note that Input-Output matrices always correspond to their Leontief inverse configuration.*



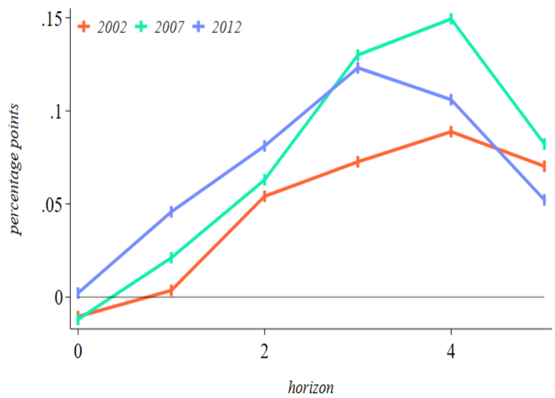
(A) DEMAND LINKAGES, $d = 1$



(B) DEMAND LINKAGES, $d = 2$



(C) SUPPLY LINKAGES, $d = 1$



(D) SUPPLY LINKAGES, $d = 2$

FIGURE C.8: COMOVEMENT OVER DIFFERENT BASELINE YEARS

Note: given both the *factor input demand* (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *factor input supply* (*i.e.*, supply linkages across sectors given their common downstream buyers) network distances, and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network over different years (2002, 2007, and 2012), the figure shows the response of sectoral employment of sectors to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Expanding to different production network structures the results in the main text: Panels C.8a-C.8b expand their relatives of Figure 1.7, while Panels C.8c-C.8d do so for their relatives in Figure 1.9. Solid lines corresponds to the average response of employment across sectors, while 68% and 90% significance levels of bootstrapped Confidence Interval (CI), computed from eq. (1.9), roughly correspond to the associated figures in the main text. Responses are referred to employment changes in closer (distance equal to 1) and further (distance equal to 2) sectors. *Source*: BEA and own calculations.

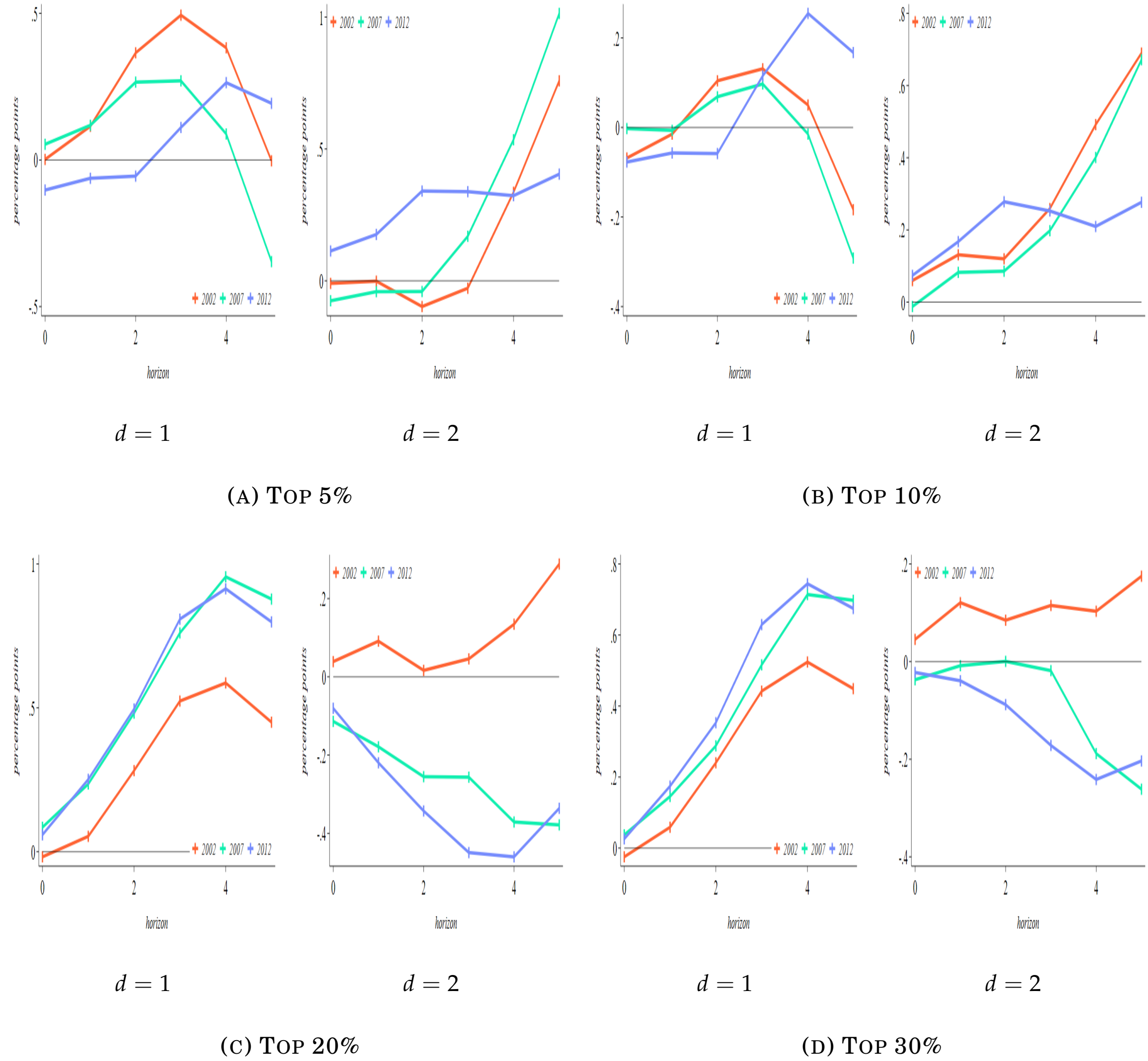
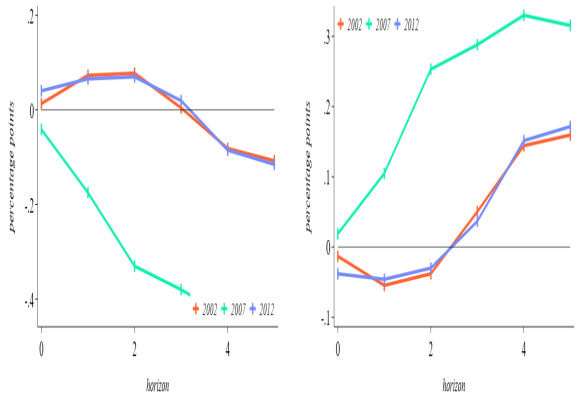


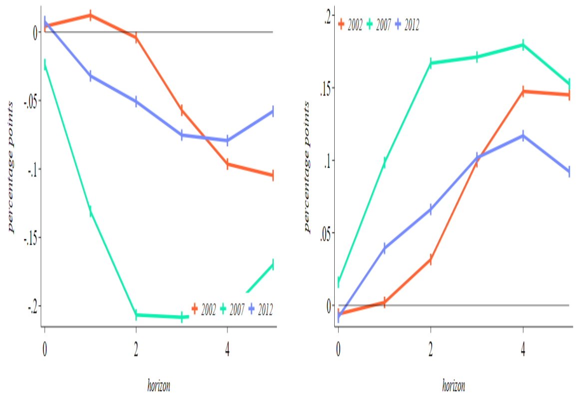
FIGURE C.9: DEMAND LINKAGES AND INTERLINKED SECTORS OVER THE YEARS

Note: given both the *factor input demand* network distance (*i.e.*, demand linkages across sectors given their common upstream sellers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network over different years (2002, 2007, and 2012), the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Expanding to different production network structures the responses of Panels 1.8a-1.8b of Figure 1.8, each panel of this figure shows the employment response of top 5% (Panel C.9a), 10% (Panel C.9b), 20% (Panel C.9c), and 30% (Panel C.9d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) or further (distance equal to 2) sectors. Solid lines corresponds to the average response of employment across sectors, while 68% and 90% significance levels of bootstrapped Confidence Interval (CI), computed from eq. (1.9), roughly correspond to the associated figure. If a plot is not appearing, it means there is no response for the specified distance value. *Source*: BEA and own calculations.



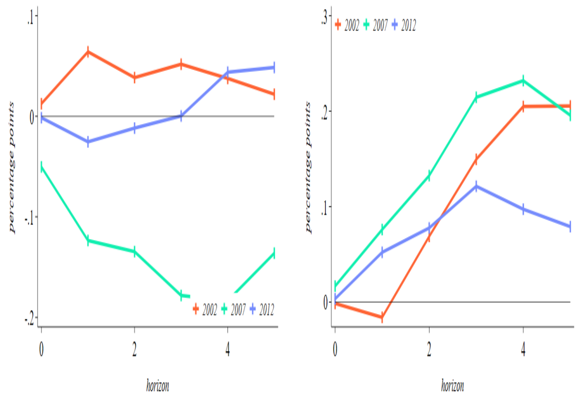
$d = 1$ $d = 2$

(A) TOP 5%



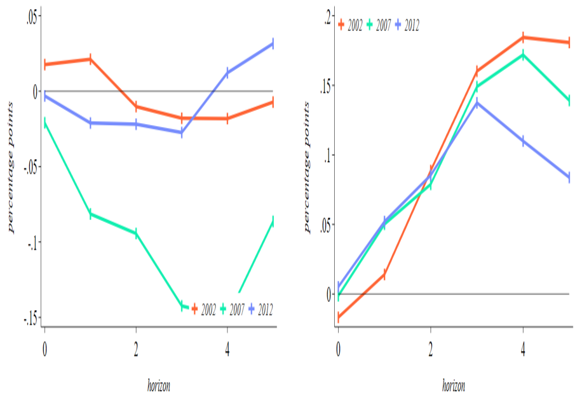
$d = 1$ $d = 2$

(B) TOP 10%



$d = 1$ $d = 2$

(C) TOP 20%



$d = 1$ $d = 2$

(D) TOP 30%

FIGURE C.10: SUPPLY LINKAGES AND INTERLINKED SECTORS OVER THE YEARS

Note: given both the *factor input supply* network distance (*i.e.*, supply linkages across sectors given their common downstream buyers) and the *Leontief inverse* transmission (*i.e.*, sectoral direct and indirect network exposure) characterizing the North American production network over different years (2002, 2007, and 2012), the figure shows the response of sectoral employment of sectors with most Input-Output linkages to a 1% increase in the employment level of other sectors for 3-digit U.S. 2017 NAICS sectors over the period 1998-2022. Expanding to different production network structures the responses of Panels 1.10a-1.10b of Figure 1.10, each panel of this figure shows the employment response of top 5% (Panel C.10a), 10% (Panel C.10b), 20% (Panel C.10c), and 30% (Panel C.10d) more connected sectors in the production network (*i.e.*, major number of I-O linkages) – from top-left to bottom-right, respectively – to employment changes in their closer (distance equal to 1) or further (distance equal to 2) sectors. Solid lines corresponds to the average response of employment across sectors, while 68% and 90% significance levels of bootstrapped Confidence Interval (CI), computed from eq. (1.9), roughly correspond to the associated figure. *Source*: BEA and own calculations.

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Chapter 2

Industry Contribution to U.S. Wage Inequality

by VALERIO DIONISI*

Abstract

Industry dimension is increasingly dominant to investigate the upward trend of inequality. This paper examines the key drivers of U.S. wage inequality, emphasising the role of heterogeneous capital-labour substitution elasticities across industries in shaping wage dispersion. Key is the distinction of a *quantity effect* (changes in the composition of capital and labour inputs) and a *structural effect* (reflecting technological transformations in inputs substitutability) from Skill-Biased Technological Change (SBTC). Findings suggest that industry-level transformations on the labour side – differentials in job tasks substitutability and workforce composition – constitute the principal drivers of real wage inequality, overshadowing the contribution of capital-side adjustments. A structural estimation of the model reveals that trend-asymmetries in sectoral elasticities of substitution between ICT capital, routine and non-routine workers account for 94% of observed wage variance, while stronger sorting and segregation effects further exacerbate such dispersion. Upon neutralising structural differences between industries, quantity adjustments reckon merely 6-15% of the observed wage inequality.

JEL: E24, J31, J82, L16

KEYWORDS: wage inequality, structural transformations, industry, tasks, labour force composition, elasticity of substitution

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2.1. INTRODUCTION

Wage inequality has been on the rise in United States over the last decades, and understanding the cause behind such turn is a secular challenge in economics. A large body of research has identified key drivers behind rising wage inequality. One prominent explanation focuses on employment polarization, largely attributed to new technologies and job tasks (*e.g.*, Autor, Katz, et al. 2006, Acemoglu and Autor 2011).[1] Complementing this view, recent evidence shows that more than 60% of the growth in U.S. wage dispersion over the past three decades reflects systematic differences across industries (*e.g.*, Haltiwanger et al. 2024). Thus, a thorough understanding of wage inequality requires considering both labour market characteristics and the broader organization of economic activity across sectors.

Addressing the issue, this paper argues that what matters for inequality is not the amount of capital or labour, but the structural transformation of substitution elasticities: some industries increasingly substitute routine for non-routine workers while complementing ICT capital, generating major wage dispersion. In fact, inequality in wages can be influenced along two dimensions. If elasticities of substitution among factors of production are uniform across industries, inequality arises primarily from differences in capital and labour accumulation, in consistency with the Skill Biased Technological Change (SBTC) framework.[2] If industries differ in their substitution elasticities, even similar production inputs growth lead to diverging wage outcomes: inequality is driven less by the *quantity* of inputs and more by the *structure of technology* and its interaction with factors accumulation.

Given these overlapping explanations, the analysis aims to quantify the relative importance of each factor contributing to wage inequality. While the "quantity" channel has been examined at length, largely unexplored remains the "structural" realm, whose effects are driven by *structural transformations* – hereafter interpreted as sectoral heterogeneous trends in the capital-labour substitution elasticities as in Alvarez-Cuadrado et al. (2018). Key result is that inequality emerges from industry-level technological heterogeneity rather than from factor accumulation alone.

The initial empirical analysis shows that technological change has uneven effects on the composition of the U.S. labour force across industries, and thus on real wages. As ICT technologies have become pervasive, changes in capital deepening are accompanied by systematic shifts in job tasks: the rising share of non-routine workers reflects the replacement of routine workers rather than an expansion in sectoral employment size. Wage differentials move with this reorganization of production, as industries contributing more to wage inequality adopt ICT capital more intensively

[1] Providing a sectoral perspective, Cerina et al. (2021) assert that the downward polarization of employment is characterized by the prevalence of routine tasks but it is fundamentally shaped by the type of sector (services and non-services sectors) rather than the performed job task.

[2] SBTC posits that changes in technological endowments disproportionately benefits high-skilled workers while displacing lower-skilled ones, contributing to wage inequality. Refer, for example, to Tinbergen (1974), Katz and Murphy (1992), Card and DiNardo (2002), and Acemoglu and Autor (2011).

and shift their task composition toward non-routine workers. Inequality thus arises from how industries deploy technology and reallocate tasks, linking capital structure directly to labour market outcomes.

To elucidate how differences in the industrial composition of capital and labour types affect wage dispersion, I build a general equilibrium model of industries differing in degrees of substitution among inputs of production. The market structure equips the economy with a block of industries populated by monopolistically competitive firms employing two types of both capital and labour: besides physical capital, non-routine workers are complementary to ICT capital while routine workers are substitutes, in the spirit of Krusell et al. (2000). As for heterogeneity, an important phenomena of the labour market is that workers are themselves of varying capacities depending on where they are employed in; accordingly, the structure of the labour market is designed around the endogenous Roy (1951)-style sorting of households into the firm-industry pair where it is most productive. This mechanism results in upward-sloping, firm-specific labour supply curves, linking industry wage premia to sorting and segregation effects in a competitive labour market.[3]

Quantitatively, I estimate the model on U.S. industry-level data. The key structural parameters include two substitution elasticities (between ICT capital and non-routine labour, and between routine and non-routine tasks) as well as the degree of labour market concentration (strength of workers' sorting and segregation effects). Despite its convoluted structure, the model's general equilibrium structure enables a clear identification of production technology parameters. Following Karabarbounis and Neiman (2014), industry-level elasticities are estimated using sectoral labour shares: estimates suggest "gross complementarity", with capital-labour elasticities of substitution ranging from 0.25 to 0.8.

Given the estimated set of parameters, the model successfully captures key features of the inter-industry wage structure and prevailing wage inequality levels. In the baseline cross-sectional calibration, substantial heterogeneity emerges in the estimated substitution elasticities: industries with the largest wage growth show the strongest impact of technological change, followed by those with minimal and moderate wage variation. A re-estimation over two distinct sub-periods confirms how these technological parameters evolved unevenly across industries both in direction and, more critically, in magnitude.[4]

Having provided reduced-form empirical support for the model's theoretical implications, I use the observed shifts in structural parameters to conduct counterfactual simulations. These exercises decompose the industry component of U.S. wage in-

[3] This framework aligns with insights from the class of "new monopsony models" (*e.g.*, Manning 2021). Appendix D shows that incorporating wage-setting power does not materially affect the model's predictions on industry wage inequality, consistent with the empirical findings of Card, Rothstein, et al. (2024a).

[4] Industries where wages increased the most (less) have experienced a decrease in the complementarity between technological capital and non-routine workers of about 25% (131%), while slightly increasing the complementarity between job tasks of 3% (11%). Differently, intermediate industries undergone a reduction (20%) in substitutability of ICT capital-non-routine tuple, along a pale increase (3%) in job tasks substitution.

equality since the early 2000s, accounting for the interaction between technological change, labour force composition, and labour market concentration. To isolate the key mechanisms, I examine how period-variations in individual or combined technological parameters affect key outcomes of the model related to observed moments. I also estimate sectoral productivity via industry-level production functions and assess its contribution – alongside capital and labour input changes – once structural heterogeneity in technological parameters is excluded.

My main findings highlight that structural transformations are the primary determinants of U.S. between-industry wage inequality, far outweighing the role of changes in factor inputs typically emphasized by SBTC dimension. The analysis allows to pin down four main takeaways. Firstly, *structural effects dominate quantity effects*: when allowing for cross-industry trend-differences in substitution elasticities, the model accounts for 94% of the observed between-industry wage dispersion. By contrast, when holding these elasticities uniform across industries, changes in production input quantities explain only 6% to 15% of the observed inequality. The "structural effect" (heterogeneous shifts in capital-labour elasticities of substitution across industries) thus accounts for the bulk of observed between-industry wage dispersion, while the "quantity effect" (industry-specific changes in ICT capital, labour force, and productivity) plays a minor role.

Secondly, *substitution across job tasks is the dominant channel*. The "indirect effect" of SBTC – namely, the reorganization of tasks in response to technological change – is way more impactful than its "direct effect" – changes in non-routine workers due to changes in technological capital. Rising inequality is therefore tightly linked to the *worker side* than the *capital side* of production across industries.

Thirdly, *labour market concentration magnifies structural effects*. Incorporating heterogeneous sorting and segregation effects into the model further amplifies inequality, explaining up to 98% of the total between-industry wage dispersion when interacted with the evolving technological structure, since they are reinforcing the role of structural transformations through workforce allocation dynamics.

These findings indicate that while Skill-Biased Technological Change is necessary in explaining U.S. wage inequality, it is not sufficient on its own. What matters most is not the scale of factor input quantities accumulation, but the sector-specific evolution of technology and production structures. As such, explaining wage inequality requires moving beyond quantity-based narratives toward a clearer focus on structural heterogeneity and the dynamic nature of technological change across industries.

Related literature.– This research contributes to several strands of the literature. A primary connection is with the study of wage inequality in the U.S., traditionally emphasizing *worker-level* (*e.g.*, Katz and Autor 1999, Autor, Katz, et al. 2008, Lemieux 2010) or *between-firm* (*e.g.*, Leonardi 2007, Card, Heining, et al. 2013, Song et al. 2019, Bingley and Cappellari 2022) differences. Anyhow, recent evidence unveils how differentials are primarily clustered at the industry level. While earlier studies had already shown that inter-industry wage differentials were substantial but stable throughout much of the 20th century (*e.g.*, Slichter 1950, Cullen 1956, Krueger and Summers 1988, Allen 1995),[5,6] the industry component of wage inequality has grown markedly over the past three decades. Central is the work of Haltiwanger et al. (2024): wage differentials across industries drive the rise in U.S. wage inequality from 1998 to 2018, with a small number of industries playing a disproportionate role.[7] Existing investigations overlook the key determinants of industry-driven wage dispersion; my contribution is a comprehensive framework that incorporates structural variations in capital-labour substitutability across U.S. industries to explain observed trends in between-industry wage inequality.

Second, my analysis directly dives into the task-based literature.[8] The role of tasks in shaping wage inequality is emphasized by Autor, Katz, et al. (2006), who highlight how technological change alters the distribution of job task demands, thus driving job polarization. Building on this, I examine how industry-driven technological change reshapes labour force composition through tasks division and reallocation, contributing to U.S. wage inequality.

Third, the paper builds on structural transformations studies (reallocation of economic activity across broadly defined sectors), in particular on the role of elasticity of substitution between capital and labour (*e.g.*, Buera and Kaboski 2012, Herrendorf, Rogerson, et al. 2014, Eden and Gaggl 2018). Sectoral differences in capital-labour elasticities of substitution, as in Herrendorf, Herrington, et al. (2015), shape factor reallocation across broadly defined sectors. This perspective intersects my results: ample differences in industry-level elasticities of substitution and their asymmetric shifts majorly explain wage inequality across industries.

Roadmap.– This introduction is succeeded by Section 2.2, which provides the motivating evidence at which the model presented in Section 2.3 refers to. Section 2.4 shows and assess the performance of the calibration strategy, while Section 2.5 quantifies the relevance of given parameters in accounting for the observed level of U.S. wage inequality. Section 2.6 determines the importance of capital and labour quantities and of an estimated measure of sectoral productivity. Last section concludes.

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[5] Refer to Krueger and Summers (1987) and Dickens and Katz (1987) for a discussion of the findings. A complementary literature explores the role of inter-industry dispersion in capital-labour ratios on wage inequality (*e.g.*, Montgomery 1991, Caselli 1999), while sectoral differences are important also to interpret both the racial (*e.g.*, Card, Rothstein, et al. 2024b) and the gender (*e.g.*, Fields and Wolff 1995) pay gaps, and wage differentials under a behavioural economics perspective (*e.g.*, Thaler 1989).

[6] Allen (2001) notices how the "stability" argument may be misleading since important within-industry factors changes over time, while the constancy depends on a strong autocorrelation over a long time period. Under a cross-country comparisons, substantial but stable inter-industry wage differentials exist among also E.U. (*e.g.*, Genre, Kohn, et al. 2011, Genre, Momferatou, et al. 2005) and OECD (*e.g.*, Gittleman and Wolff 1993) countries.

[7] Rising between-industry wage inequality is a well documented fact (*e.g.*, Davidson and Reich 1988, Bell and Freeman 1991, Howell and Wolff 1991, Allen 2001). Haltiwanger et al. (2024)'s contribution is to show its importance on the increase in total U.S. wage inequality relative to both individual and firm components. Similar results for Italy are in Briskar et al. (2022).

[8] Strong emphasis in measuring employment trends by job task content: rapid employment growth for non-routine tasks from early 1990s relative to routines (*e.g.*, Goos and Manning 2007, Autor and Dorn 2013, Cortes et al. 2017, Jaimovich and Siu 2020, Cerina et al. 2021, Siena and Zago 2024, Cossu et al. 2024).

2.2. Motivating Evidence

To account for trends in wage dispersion I employ annual U.S. data, and the 3-digit U.S. 2017 North American Industrial Classification System (NAICS) is adopted for the industry level definition. The (balanced) panel data built is a combination of two different data sources: first, information on the stocks of physical and technological capital types are recovered from the Bureau of Economic Analysis (BEA), in the *Detailed Data for Fixed Assets* tables; the second source is the *Occupational Employment and Wage Statistics* (OEWS) program of the Bureau of Labor Statistics (BLS), which contains information on occupational employment and wage rates.

Capital inputs are grouped into four categories using BEA data for 1998-2022: physical capital, digital equipment, intangible capital; the sum of the last two measurers ICT capital. Occupational data from the BLS cover routine and non-routine tasks for private U.S. industries from 2003 onward. BEA and BLS data are harmonized at the 3-digit NAICS level, yielding a balanced panel of 62 private industries observed annually from 2003 to 2022. For additional details refer to Appendix A.

2.2.1. Validating the sample

From Haltiwanger et al. (2024), total wage variance growth across industries can be decomposed as each industry-$s \in \mathcal{S}$ contribution to wage inequality

$$\underbrace{\Delta\, var\Big(w(s)-\overline{w}\Big)}_{\substack{\text{between-industry}\\ \text{wage variance growth}}} = \sum_s \overbrace{\Delta \underbrace{\left(\frac{\ell(s)}{\ell}\right)}_{\substack{\text{employment}\\ \text{share}}} \underbrace{\Big(w(s)-\overline{w}\Big)^2}_{\substack{\text{relative}\\ \text{wage}}}}^{\substack{\text{industry-}s\text{ contribution to}\\ \text{between-industry wage variance}}} \tag{2.1}$$

in which the employment share component allows to attach the proper weight to each industry. In each year, $w(s)$ is the average real log-wage of industry-s, $\overline{w}$ the associated economy-wide period mean, while $\ell(s)$ and ℓ represent sectoral and aggregate employment, respectively. A shift-share analysis provides further insights on industry factors accounting for such variance growth, inspecting the importance of relative wage changes versus employment share changes:

$$\underbrace{\Delta\left(\frac{\ell(s)}{\ell}\right)\Big(w(s)-\overline{w}\Big)^2}_{\substack{\text{industry }s\text{ contribution to}\\ \text{between-industry wage variance}}} = \underbrace{\overline{\Big(w(s)-\overline{w}\Big)^2}\,\Delta\left(\frac{\ell(s)}{\ell}\right)}_{\text{shift share: employment}} + \underbrace{\overline{\left(\frac{\ell(s)}{\ell}\right)}\,\Delta\Big(w(s)-\overline{w}\Big)^2}_{\text{shift share: wage}} \tag{2.2}$$

The two decompositions are reported in Table 2.1. Wage inequality growth is highly concentrated across industries: 15 out of 62 sectors account for 93% of the total increase in wage dispersion, despite employing less than 40% of the workforce. Concentration is even sharper at the top, with just three industries explaining more than half of the total growth in real log-wage variance while representing only 8% of total employment. Contributions to wage dispersion are further examined through a shift-share analysis. An expansion in industries' labour force plays a limited role,

TABLE 2.1: CONTRIBUTION TO BETWEEN-INDUSTRY WAGE VARIANCE

			share		shift-share	
contribution	*industries*	*variance*		*employment*	*wage*	*employment*
> 5%	3	.26	54%	8%	90%	10%
1% to 5%	7	.14	29%	18%	76%	24%
.05% to 1%	6	.05	10%	13%	124%	-24%
-.05% to .05%	46	.03	7%	61%		·
quantiles						
0-25	15	.24	50%	31%	91%	9%
25-50	16	.04	8%	28%	77%	23%
50-75	16	.02	4%	15%		·
75-100	15	.18	38%	26%	98%	2%

Estimates are referred to eq. (2.1) *for 3-digit U.S. 2017 NAICS industries. The last two columns report a quantification of the components in eq.* (2.2)*; not reported estimates '·' imply that the shift-share for employment is highly less than zero. Operator* Δ *in the equations is* $x_t - x_{t-1}$*, and not a percentage change. Industries are grouped according to their own contribution to between-industry wage inequality in the first part of the table while, in the second part, grouping follows the overall percentage change in real log-wage per capita of each industry.* Source*: BEA and own calculations.*

whereas relative wage dynamics dominate. As shown in the last two columns of Table 2.1, the bulk of wage variance growth stems from shifts in relative industry wages rather than from changes in employment composition.

FACT 1 (Contribution) *A small subset of industries drives the rise in wage inequality; these are in the tails of the industry-level wage growth distribution.*

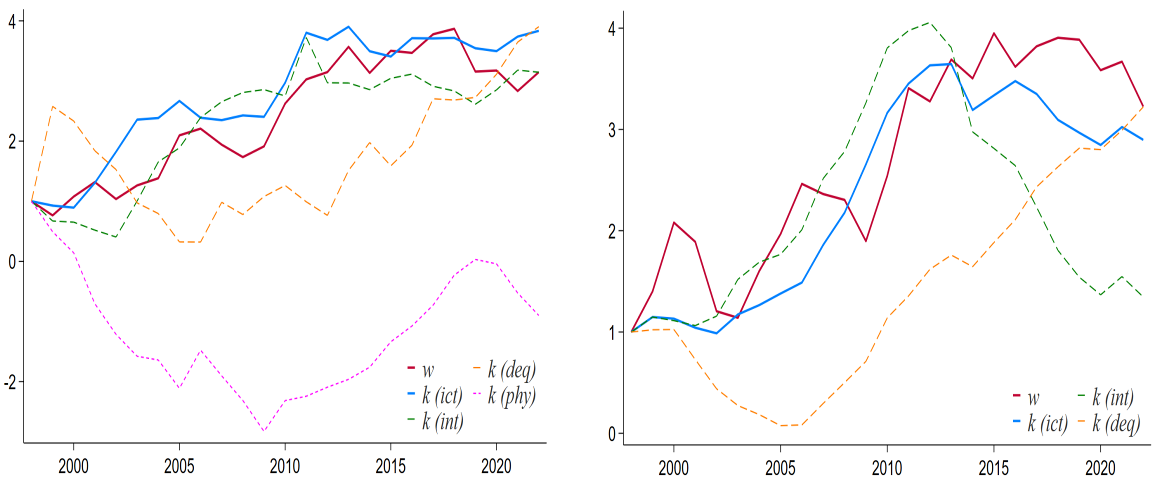
A similar exercise groups industries by changes in real log-wages (second panel of Table 2.1). Top and bottom 25% together account for 88% of the growth in wage inequality while employing nearly 60% of the workforce. As before, relative wage movements significantly dominate: changes in industry employment shares contribute little to wage dispersion, thus reconciling with Fact n. 3. These results align with Haltiwanger et al. (2024).[9]

What intrinsic characteristics underpin the dispersion of wages across industries? Why does the employment size of industries seem to play a minimal role? These questions are explored in the following subsections.

2.2.2. U.S. INDUSTRIES DIGITALIZATION

As a first step, the analysis shall examine whether cross-industry differences in capital stocks are reflected in wage differentials. To capture the rise in dispersion, Figure 2.1 traces the evolution of the differentials in capital intensities and real wages

[9] According to their estimates, 10% of 4-digit U.S. 2017 NAICS industries account for nearly 98% of the total increase in between-industry wage dispersion from 1996 to 2018, as well located at the tails of the industry-level earnings distribution.



(A) INTERQUARTILE RANGE, 90^{th}-10^{th} (B) LOG-VARIANCE

FIGURE 2.1: CROSS-INDUSTRY DISPERSIONS OF CAPITAL TYPES

Note: the figure depicts dispersion across industries of average real log-wage, physical, ICT, intangible capital types and digital equipment per capita. Solid red and blue lines are related to wages and ICT capital, while dashed green, orange and purple lines are intangible capital, digital structures, and physical capital, respectively, all taken in per capita log-terms. Panel 2.1a plots the yearly difference between top and bottom 10% of each component, while Panel 2.1b plots the associated log-variance. Series are standardized and indexed to 1 in 1998, so that both y-axis indexes the respective measure given the initial value at unity. Plots are referred to 3-digit U.S. 2017 NAICS industries. *Source*: BEA and own calculations.

across industries. Panel 2.1a measures industry-level dispersion using the interquartile range – the gap between the top and bottom 10% of industries in each log-variable. Dispersion in ICT-related capital ratios increases steadily over time, indicating a widening gap between industries with high and low ICT capital per worker. Intangible assets and digital equipment display similar, though less uniform, patterns. By contrast, dispersion in physical capital declines, suggesting a convergence in physical capital stocks across industries.

FACT 2 (Industry capital gaps) *Dispersion in physical capital-labour ratio is decreasing across industries, while it is not that of ICT capital-labour ratio.*

Comparing inter-industry differentials in capital intensities with those in labour income reveals that real wage gaps are primarily associated with disparities in ICT capital, rather than with other forms of capital. This pattern highlights the role of a macro-complementarity:[10] as shown in Panel 2.1b, dispersion in intangibles or digital equipment alone fails to match the evolution of wage dispersion unless the two are combined into a unified ICT capital measure. The interaction between intangible capital and digital equipment is therefore central: taken separately, intangibles loosely track wage dispersion until around 2012, while digital equipment becomes relevant only thereafter.

[10] A growing empirical and methodological debate emphasizes the importance of such complementarities. For instance, Corrado et al. (2017) show that returns to ICT depend critically on otherwise "unmeasured" intangible assets, documenting ICT-intangible complementarity in both U.S. and EU data. A similar argument appears in Crouzet et al. (2022), who note that intangible capital, lacking physical embodiment, requires digital infrastructure or storage medium to operate.

Further evidence on the role of ICT capital on real wages is provided in Table A.1. Fixed-effects panel regressions largely confirm the preceding patterns. While increases in physical capital per worker are associated with higher real wages, the effect of ICT capital alone is weaker (column 1), suggesting substantial heterogeneity across industries. Intangible capital plays a central role, accounting for a large share of wage variation (column 2), whereas digital equipment on its own appears less influential. Crucially, the interaction between intangibles and digital equipment (column 3) remains significant, and continues to do so when all capital types are included jointly (column 4), underscoring the importance of ICT complementarities in explaining industry-level real log-wage differences.

2.2.3. Accounting for workforce composition

Since their advent, advances in technical and technological processes have raised the issue of their underlying effects on employees' qualification requirements at both aggregate and industry layers (*e.g.*, Horowitz and Herrnstadt 1966, Rumberger 1981, Autor 2022), and that the mechanism through which these impact workers does not transmit uniquely on the absolute level of occupational employment, but especially in its relative term.[11] A natural intuition to analyse is whether changes in the relative workforce of each task can be attributed to changing sizes of industries or to the reallocation of worker-types within those industries. Denoting the set of tasks as $\{a, a'\} \in \mathcal{A}$ and with $s \in \mathcal{S}$ that of industries, the following labour force decomposition provides the answer to this inquiry:

$$\Delta\left(\frac{\ell\,(a)}{\ell\,(a')}\right) = \underbrace{\sum_s \overline{\left(\frac{\ell\,(a,s)}{\ell(s)}\right)} \Delta\left(\frac{\ell\,(a,s)}{\ell\,(a',s)}\right)}_{\textit{within-industry: substitution effect}} + \underbrace{\sum_s \overline{\left(\frac{\ell\,(a,s)}{\ell\,(a',s)}\right)} \Delta\left(\frac{\ell\,(a,s)}{\ell(s)}\right)}_{\textit{between-industry: size effect}} \tag{2.3}$$

Shifts in the economy-wide ratio of task-a to task-a' can be decomposed into within- and between-industry components. The within component captures changes in task intensity within industries – thereby capturing the substitution and reallocation of tasks –, while the between component reflects changes in the relative size of task-specific employment across industries. Figure A.3 illustrates the dynamics of fitted routine and non-routine labour shares obtained from Fixed Effects (FE) regressions with industry and year effects, used to implement equation (2.3). The share of non-routine workers rises steadily over time as routine employment declines, raising the question of whether this shift reflects adjustments in industry size or substitution in task composition within industries. This structural change is exactly the underlying logic behind eq. (2.3).

Table 2.2 reports the results of the decomposition. The increase in the non-routine-to-routine task ratio is driven almost entirely by within-industry adjustments, indicating that industries are replacing routine tasks with non-routine ones rather than

[11] Refer to Melman (1951) and Braverman (1974) for an analysis over the period 1820-1970.

TABLE 2.2: LABOUR FORCE COMPOSITION

	routine		***non-routine***	
interval	*within*	*between*	*within*	*between*
2003-2008	83%	17%	38%	62%
2009-2015	81%	19%	58%	42%
2016-2022	78%	22%	69%	31%

Quantification of eq. (2.3); *changing components (Δ) are linear trends predicted by a Fixed Effects (FE) regression of the form* $y_t = \beta_c + \beta_t \mathcal{X}_t + u_t$, *with* y *representing the task ratio, and* $\mathcal{X}$ *a vector of industry and year fixed effects, at 3-digit U.S. 2017 NAICS industries. Source: BLS and own calculations.*

expanding their workforce. Conversely, changes in the routine-to-non-routine ratio are associated with modest between-industry effects, reflecting limited employment expansion rather than increased reliance on routine tasks.[12]

FACT 3 (Labour force composition) *Increases in non-routine relative share are determined by a substitution effect rather than by the employment size of industries.*

The analysis next combines the evidence on capital accumulation with changes in labour force composition. The key question is whether industries experiencing faster real wage growth are also those that expand ICT capital and increase their reliance on non-routine labour. In this framework, non-routine workers are those whose tasks are more ICT-intensive, whereas routine workers primarily perform manual tasks, as formalized later in the model. To investigate this relationship, a set of industry-level fixed-effects panel regressions is estimated, grouping industries by their position in the real log-wage growth distribution

$$\Delta\, log\; w_{\omega t}\,(s) = \beta_{c,\omega} + \beta_{i,\omega} \mathcal{X}_{i,\omega t} + \delta_{j,\omega} \mathcal{Z}_{j,\omega t} + u_{\omega t} \tag{2.4}$$

with ω identifying each quantile in Table 2.1, $\mathcal{X}_i$ bundles percentage changes in both ICT-to-physical capital (k) and non-routine-over-routine workers (ℓ) ratios, and $\mathcal{Z}_j$ serves as controls. The interaction between ICT capital accumulation and task reallocation is particularly pronounced in industries at both ends of the wage growth distribution – those at the top and those at the bottom –, as shown in Table 2.3, thereby aligning with stylized Fact n. 1. High-growth industries exhibit large joint increases in ICT capital and non-routine employment, translating into substantial wage gains (1.56), whereas low-growth industries show milder adoption of these factors, resulting in weaker wage performance (0.876). No significant – and in some cases negative – effects are observed for industries in the middle, which contribute relatively little to overall wage inequality. Table A.2 reinforces this pattern. Column 3 shows that the interaction term $\beta_{\Delta k \times \Delta \ell}$, capturing joint changes in ICT capital and task composition, has a large and statistically significant effect on real log-wage

[12] A link between industry affiliation and occupational distribution is also documented in Haltiwanger et al. (2024), although that analysis does not address task substitution within industries.

TABLE 2.3: COMBINED REGRESSIONS BY GROUPS, PERCENTAGE CHANGES

	$\Delta\, log(w(s))$			
	0-25 quantile	*25-50 quantile*	*50-75 quantile*	*75-100 quantile*
$\beta_{\Delta k}$	-.693	.302***	.078	-.082*
	(.618)	(.072)	(.179)	(.042)
$\beta_{\Delta \ell}$	.041***	-.297	.015***	.052*
	(.004)	(.247)	(.003)	(.025)
$\beta_{\Delta k \times \Delta \ell}$	.876***	-1.304	-.643	1.560*
	(.100)	(2.945)	(.470)	(.789)

*Significance level at * (p<0.05), ** (p<0.01), *** (p<0.001). Standard error in parentheses. Analysis at 3-digit U.S. 2017 NAICS industries in 2003-2022. Each Fixed Effects (FE) regression – performed on groups of industries clustered in quantiles (ω) according to their overall growth in real wage – , is of the form of eq. (2.4), with $\mathcal{X}_i$ representing the percentage change in both ICT-to-physical capital and non-routine-over-routine workers ratios, and $\mathcal{Z}_j$ being a set of time-varying controls. Variables are all in log format. Constant not reported to save space. Source: BEA, BLS and own calculations.*

growth (.919), far exceeding the impact of isolated changes in either ICT capital (.009) or labour composition (.035). Therefore, wage inequality is driven primarily by the concentrated performance of industries at the extremes of the wage growth distribution, where the combined evolution of ICT capital and non-routine labour is most pronounced. The evidence underscores that the interaction of these two factors, rather than isolated changes, plays a central role in shaping wage dispersion.

An argument strongly substantiated by a bulk of quantile regressions of the form $\mathcal{Q}_\omega(log\ w_{\omega t}(s)) = \beta_{i,\omega}\mathcal{X}_{i,\omega t} + \delta_{j,\omega}\mathcal{Z}_{j,\omega t} + u_{\omega t}$, where ω represents each quantile (defined on the independent variable), and $\mathcal{X}_i$ and $\mathcal{Z}_j$ as before. The results point to a positive effect of both the task ratio and the ICT-to-physical capital ratio, with the impact strengthening from the lower to the higher end of the industry wage growth distribution. Table A.3 shows that the task ratio (β_ℓ) has a substantially larger effect on real log-wage growth than the ICT ratio (β_k, column 1). Effects are uneven across industries: the magnitude of both ratios increases along the distribution, with industries farther from the top experiencing smaller impacts (columns 2-5). Importantly, the interaction term reaches its largest values in regressions based on changes ($\beta_{\Delta k \times \Delta \ell}$) as opposed to those in levels ($\beta_{k \times \ell}$), suggesting that the joint evolution of ICT capital and task composition is most informative for explaining over-time variation in real log-wages, rather than cross-sectional differences.

FACT 4 (Structural transformations) *Industries marked by highest changes in real wages experience a substantial rise in their non-routine workers relative share along an increasing ICT capital ratio dynamics.*

When jointly considered, the presented stylized facts reveal some important characteristics of the U.S. inter-industry wage structure: the dispersed increase in industries' real average wage is influenced by a growing dispersion in technology (Fact n. 2); in turn, such technological differences reflect variations in the labour force composition within each industry (Fact n. 3). In other words, industries that have experienced a larger and more significant increase in the stock of ICT capital relative to physical capital tend to exhibit a higher rate of substitution of routine tasks with non-routine tasks (Fact n. 4). Reminiscent from the introduction, these findings are opened to two possible interpretations: wage inequality across industries could be the result of *(i)* a *quantity* effect, under different trajectories in factor of productions; or *(ii)* a *structural* effect, stemming from changes in the technological parameters that govern the relationship between capital and labour types. These general equilibrium considerations provide the motivating background to build a model with industry-specific capital-labour substitutability and heterogeneous labour force composition to account for trends in wage inequality in the United States.

2.3. Structural Model

To rationalize the presented empirical regularities, I build and simulate a structural general equilibrium model. The households block features a skill-related heterogeneity: households are *ex-ante* divided in routine and non-routine workers, sorting into firms and industries given heterogeneous productivities. Final and sectoral outputs are competitively aggregated, while firms in each industry compete monopolistically; they all use ICT and non-ICT capital types, non-routine labour is complementary to ICT capital, while routine workers are substitutable. Hence, the economy identifies the structure of production functions to be analogous in all industries except for their degrees of substitutability across factors of production. To deepen the role of substitution parameters, productivity differences across industries are excluded.[13]

2.3.1. Households

There is a unit mass of households, each labelled as $i \in \mathcal{I}$, split in tasks $a = \{rt, nrt\}$. Household-i of type-a gets utility from consumption, $\mathcal{C}_i$, and it is increasing in a known idiosyncratic factor, $\wp_h^i(a,s)$, drawn once from a specified distribution, measuring its efficiency level when working with firm-$h \in [1,H]$ in industry-$h \in [1,S]$:

$$\mathcal{U}_h^i(a,s) := log\ \mathcal{C}^i + log\ \mathcal{B}_{h,t}(a,s) + \wp_h^i(a,s) \tag{2.5}$$

Increasing utility in $\wp_h^i(a,s)$ suggests that household-i is optimally allocated (*i.e.*, endogenously sort) to a workplace that maximizes its productivity.[14] This outcome directly follows from assuming a self-selection based labour market, involving individuals choosing the firm-industry combination that maximizes their utility based on

[13] Sectoral productivity and structural changes are theoretically analysed by Ngai and Pissarides (2007), linking the elasticity of substitution among intermediate inputs to the sectoral reallocation of employment. In Section 2.6 I consider the effect of estimated industry-level productivity series on wage dispersion.

[14] An high value for $\wp_h^i(a,s)$ indicates that household-i is utilitarianism better off, as it reflects the household's ability to achieve its maximum productivity when working as type-a within the given firm and industry (h,s) tuple. This way of designing the utility function is common in the discrete choice literature (*e.g.*, Card, Cardoso, Heining, et al. 2018, Hsieh et al. 2019). Due to the absence of a non-employment option and unemployment, the terms household(s) and worker(s) are then used interchangeably.

their spectrum of productivities across different sectors. In other words, individuals self-select into workplaces by comparing the potential outcomes of various firm-industry pairs. Moreover, borrowing from the spatial economics literature, I include a firm-h, industry-s amenity for household type-a, entering multiplicatively in the utility function (*e.g.*, Kleinman et al. 2023). It is defined as $\mathcal{B}_{h,t}(a,s) = \left[g_{h,t}(a/a',s)\right]^{-\varsigma}$, where $g_{h,t}(a/a',s)$ reflects the relative share of task-a in task-a' (keeping the same notation of eq. 2.3), negatively scaled by the elasticity parameter ς (an indicator of the degree of substitutability among job tasks).[15]

An household can invest either in bonds (b^i, which are in zero net supply) at the rate r_t, and in capital types $k^i(j), \forall j = \{phy, ict\}$, while receiving dividends $\mathcal{D}^i$ from firms, so that its expected utility problem displays the budget constraint to be

$$C_t^i + I_t^i(phy) + I_t^i(ict) + b_{t+1}^i - (1+r_t)b_t^i =$$
$$w_{h,t}(a,s)\,\ell_h^i(a,s) + R_t(phy)\,k_t^i(phy) + R_t(ict)\,k_t^i(ict) + \mathcal{D}_t^i$$

where the wage rate associated to type-a in firm-industry pair (h,s) is $w_h(a,s)$, with labour $\ell_h^i(a,s)$ inelastically supplied and set to unity. Any capital stock depreciates at a rate δ and accumulates over time according to a quantity-adjusted law of motion: it is a negative function of ζ_t^i (the relative ICT capital share) that scales new capital investment, $I_t^i(j)$: as the capital stock owned by household-i becomes more technologically advanced (higher ζ_t^i), a higher rate of capital investment is required to maintain the future stock of that particular capital type.[16] ζ_t^i thus represents the technology level of the total capital stock owned by household-i.

Inter-temporal utility maximization implies a standard Euler condition for future path of marginal utility from consumption, and an equation displaying the evolution of the capital rental rate

$$R_t = (1+r_{t+1})\,\zeta_t - (1-\delta)\,\zeta_{t+1} \tag{2.6}$$

in which changes across states of R_t are determined by changes in the (aggregate) relative quantities across capital types, $\zeta_t = \int_i \zeta_t^i \,\mathrm{d}i$. In the spirit of Karabarbounis and Neiman (2014), eq. (2.6) determines that investing in capital types is profitable as long as the marginal benefit of investment is at least lower than its marginal cost.

In a market economy workers do not randomly sort across workplaces. Rather, in a Roy (1951)-style model of self-selection, households locate based on how their idiosyncratic productivity varies across firms and industries. Dropping time-t subscript, the idiosyncratic *productivity* of household-i, task-a working in firm-industry (h,s) tuple is drawn once from a multivariate Frechét-type cumulative distribution

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[15] Below, the component $\mathcal{B}_{h,t}(a,s)$ allows to characterize the equilibrium wages as a function of the task ratio too, as in Section 2.2. Its relevance in determining the endogenous labour supply of eq. (2.7) – *i.e.*, the outcome of the sorting choice by the part of households – will be empirically tested and validated in Subsection 2.4.1.

[16] Household-i's capital dynamics follows a law of motion as $k_{t+1}^i(j) = \frac{I_t^i(j)}{\zeta_t^i} + (1-\delta_j)k_t^i(j)$, $\forall j = \{phy, ict\}$. A higher ζ_t^i is an improvement in the technological sophistication of the capital bundle, symptom that any additional unit of invested capital faces diminishing marginal returns, as described by Inada (1963)'s conditions.

$$F_i\left(\wp^i_{h,\dots,H}(a,1),\dots,\wp^i_{h,\dots,H}(a,s),\dots,\wp^i_{h,\dots,H}(a,s)\right) = exp\left[-\sum_s\left(\int_h \wp^i_h(a,s)\,\mathrm{d}h\right)^{-\theta}\right]$$

Its shape parameter, $\theta > 1$, governs the degree of dispersion in idiosyncratic productivities' draws (*i.e.*, the labour supply elasticity), and lower values of θ imply major degrees of dispersion (and thus a more elastic labour supply response to wage differences);[17] without loss of generality, location and scale parameters are normalized to 1. Such Type-I Extreme Value Distribution identifies a discrete choice on how household of type-a sorts into firm (h,s) given heterogeneous productivity levels across different firm-industry pairs.[18]

Central to this modelling framework is that workers are potentially employable in any firm within any industry, but their productivity varies across firm-industry pairs. This structure captures both absolute and comparative advantage. Absolute advantage implies that any worker performing a task can, in principle, work in any firm-industry combination. Comparative advantage, however, determines actual employment: workers choose to be employed in the firm and industry where their relative productivity in that task is highest. Consequently, labour allocation reflects not only wage differences but also task-specific efficiency across different environments. This mechanism generates a "segregation" effect, whereby firms increasingly hire workers whose comparative productivity aligns closely with their operational and technological requirements.

The labour market, while competitive and frictionless in a traditional sense, is thus structurally imperfect due to this idiosyncratic productivity heterogeneity. The distribution of households' productivities – and thus the self-decision around employment choices – makes the labour supply to become endogenous, shaped by both firm-industry level wages and resulting task-specific comparative advantages. Described sorting preferences allows to derive analytical form for the measure of each worker-type in firms and industries given the respective wage level: in Appendix B I show that the labour supply curve of each (a,h,s) takes the form of

$$\ell_h(a,s) = \left(\frac{w_h(a,s)}{\mathcal{W}_{\mathcal{H}}(a,\mathcal{S})}\,\frac{\mathcal{B}_h(a,s)}{\mathcal{B}_{\mathcal{H}}(a,\mathcal{S})}\right)^{\theta} \tag{2.7}$$

[17] It measures the centrality of a change in labour supply of task-a induced by a change in its firm-h, industry-s wage level. Rogerson (2024) stresses the importance of the (aggregate) labour supply elasticity in macroeconomics, pointing to both intensive (average hours worked) and extensive (employed people in the total population) margins. In my model only the latter margin (relative task-a share of total employment) is considered, since households are assumed to supply inelastically one unit of work (so that $\ell^i_h(a,s) = 1$), linking this elasticity to a certain degree of workers' concentration in the labour market; refer to Figure 2.2.

[18] Self-selection concept dates back to Roy (1951)'s verbal-model, whose scope is to describe how the subjective choice for a given occupation would affect the distribution of wages: it is not only a function of gaps in potential wages, but rather it includes also occupational sorting choices. Roy's idea has been substantially tested in the literature (right after Heckman and Sedlacek 1985 and Borjas 1987), and the macroeconomic implications of such microeconomics sorting choice have been firstly introduced by Hsieh et al. (2019).

with $\mathcal{W}_{\mathcal{H}}(a,\mathcal{S}) = \sum_{h,s} w_h(a,s)$ and $\mathcal{B}_{\mathcal{H}}(a,\mathcal{S}) = \sum_{h,s} \mathcal{B}_h(a,s)$. Labour supply is modelled as depending not only on the absolute wage offered to task-specific workers but also on its relative attractiveness, captured by a "pay premia" effect – that is, how the wage compares with those offered in other firms and industries. Additionally, to attract a larger share of workers with similar productivity levels (the "sorting" effect), a firm must offer higher wages.[19]

Such labour supply allows to summarize the role of wage premium, sorting and segregation, all effects of pivotal importance in shaping wage inequality. Haltiwanger et al. (2024) highlight how wage dispersion across industries is increasingly driven by sorting – referred to as the covariance of industry wage premium and the segregation effect across industries – associated to wage premium.[20] Card, Rothstein, et al. (2024a) find how the correlation between workers sorting across industries and the associated wage premium is high.[21] In other words, this constitutive self-selection directly enhances a mechanism on the critical interaction of the workplaces' labour force composition with sorting and segregation effects, both associated to the firm-industry wage premium from not perfectly elastic labour supplies.

REMARK 1 (Labour market elasticity, sorting and segregation) *Selected specification of households' productivities allows to characterize firms and industries workforce composition of tasks as determined by a not fully elastic labour supply elasticity, shaped by comparative advantage, interacting with equilibrium wage premium designed through perfect competition in the labour market.*

Labour supply curves in eq. (2.7) are upward-sloping not due to frictions, but due to the structure of comparative advantage and the imperfect substitutability of workers across jobs.[22] In this sense, workers in the same task are "highly rival factors" (Hicks 1932) since they can be freely substituted for one another, yet ensuring that high-wage workers do sort in high-wage firms and industries, and also that more

[19] As shown in Appendix B, the endogenous sorting decision is determined by a comparison of the combination of wages and idiosyncratic productivities (thus the total earnings) of working in a given workplace compared to other ones, *i.e.*, $Pr\left[w(a,s)\,\mathcal{B}(a,s)\,\wp^i(a,s) > w(a,s')\,\mathcal{B}(a,s')\,\wp^i(a,s')\right]$.

[20] In continuity with Card, Heining, et al. (2013) for Germany, several empirical works (*e.g.*, Alvarez et al. (2018) for Brazil, Morin (2023) for Denmark, Briskar et al. (2022) for Italy, Card, Cardoso, and Kline (2016) for Portugal, Håkanson et al. (2020) for Sweden, Barth et al. (2016), Song et al. (2019) and Haltiwanger et al. (2023) for US) detect how these major effects are increasingly impacting the distribution of wages.

[21] Using a cross-sectional approach at sectoral level, Gibbons et al. (2005) show how the high-skill occupational sorting is determined by differentials in return to skills, and the comparative advantage motive, rather than learning, seems to affect the resulting wage premiums.

[22] An important observation is that the presented model traces these effects back to a deeper micro-structural foundation: endogenous heterogeneity in worker productivity and endogenous allocation of labour in a competitive but non-random labour market in the sense of "new classical monopsony" (Manning 2021). The not-fully-flexible labour supply (due to parameter $\theta \neq 1$) is central to the understanding of how wage premia emerge and interact with sorting and segregation forces. In this perspective monopsony is not a dynamic search-and-matching friction; rather, its static nature ensures the labour supply elasticity a notable role, affecting labour in a fully competitive labour market. Even if I will not allow firms to take their respective labour supply of each task-a as given (not encompassing a proper monopsony power), the equilibrium wage level of firm (h,s) directly determines the number and the type of workers working there through sorting and segregation effects.

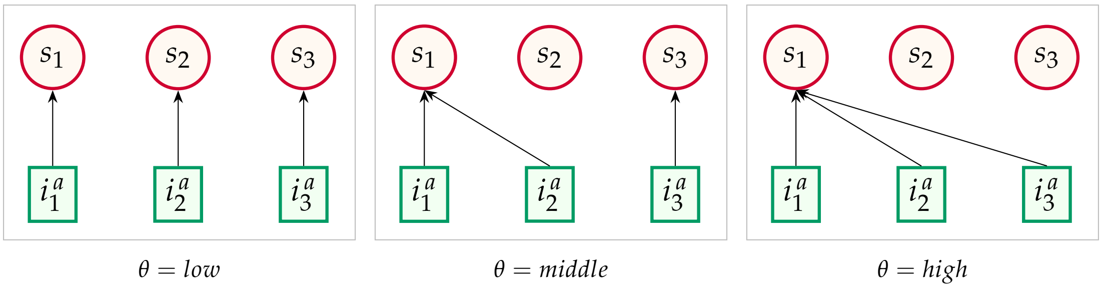


FIGURE 2.2: PRODUCTIVITY DISPERSION, SORTING AND SEGREGATION

Note: these graphs display some stylized examples on how productivity dispersion of households relates to labour market concentration at industry level. Each i^a represents an household of task-a, while s is a specific industry. The higher is the value of θ, the lower is the dispersion of households' productivities, so that the larger are the effects of sorting and segregation, thus the higher is the degree of labour market concentration of workers across industries.

efficient people would be preferred to get a job in high-wage workplaces than ones who are less productive.

ASSUMPTION 1 *Since households' productivities are Frechét-distributed, workers are perfectly mobile across firms within an industry, and immobile across industries.*

Parameter θ governs the elasticity of labour supply and, since wage differentials primarily drive employment choices, it is interpreted as a structural measure of labour market concentration. As idiosyncratic productivities become more dispersed, the strength of these mechanisms diminishes, reducing labour market polarization. This concentration arises from the distribution of household productivities through sorting and segregation mechanisms. Figure 2.2 provides a conceptual illustration of how variations in productivity dispersion shape employment allocations.

EXAMPLE 1 (Labour market concentration) *Consider a stylized economy with* $\{s_1, s_2, s_3\} \in \mathcal{S}$ *industries, each represented by a single firm, and three households of unique type-a. Households' productivities may acquire only three values,* $\wp^i(a,s) = \{low, middle, high\}$ *depending where employed, with parameter* θ *determining even dispersion. When* $\theta = low$*, productivity levels are highly dispersed across households and industries, leading each household to sort into a different industry, and resulting in low labour market concentration. Conversely when* $\theta = high$*: productivities result to be very clustered, with all households concentrating in the same industry, thereby generating strong polarization. An intermediate case occurs when* $\theta = middle$*, yielding to partial sorting pattern and reflecting moderate labour market concentration.*

Under a broader perspective, I am interpreting the degree of type-a workers' concentration at industry level to be directly proportional to the degree of variability in households' productivities (and therefore to the labour supply elasticity).

2.3.2. INDUSTRIES

The supply side is made of a countable set of industries $s \in \mathcal{S}$, each producing a specific variety $y(s)$. A perfectly competitive final good producer combines intermediate outputs from industries using a Constant Elasticity of Substitution (CES) technology

$$Y = \left(\sum_s y(s)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where $\eta > 1$ is the elasticity of substitution among varieties of differentiated goods/services from industries. Final producer's optimizing behaviour gives birth to a relative demand of each industry-s output, given by $\frac{y(s)}{Y} = \left(\frac{p(s)}{P}\right)^{-\eta}$, with aggregate price index $P = \left(\sum_s p(s)^{1-\eta} \right)^{\frac{1}{1-\eta}}$ since final good is competitively supplied.

A unitary mass of monopolistically competitive homogeneous firms, indexed with $h \in \mathcal{H}$, is comprised in all industries. Given market power of firms, aggregation for industry-s bundles together different varieties by a new CES aggregating function

$$y(s) = \left(\int_h y_h(s)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}h \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the sectoral elasticity of substitution among firms' varieties. In a way analogous to final producer, the conditional demand of firm (h, s) arises from competitive profit maximization at industry level:

$$y_h(s) = \left(\frac{p_h(s)}{p(s)} \right)^{-\epsilon} y(s) \tag{2.8}$$

with the sectoral price index as the *numeraire*, $p(s) = \left(\int_0^1 p_h(s)^{1-\epsilon} \, \mathrm{d}h \right)^{\frac{1}{1-\epsilon}} = 1$. Firm-$h$ in industry-s comprises two types of workers, $a = \{rt, \, nrt\}$: non-routine (nrt) workers are complementary to ICT capital, while routine (rt) labour force is substitutable with the ICT composite good produced under the non-routine-ICT capital complementarity. The production function $y = f\left(k^{phy}, k^{ict}, \ell^{rt}, \ell^{nrt}\right)$ of firm (h, s) exploits that in Krusell et al. (2000), that is

$$y_h(s) = \left(k_h(phy, s)\right)^{\alpha} \left[\mu \left(\ell_h(rt, s)\right)^{\varsigma} + (1-\mu) \left(q_h(s)\right)^{\varsigma} \right]^{\frac{1-\alpha}{\varsigma}} \tag{2.9}$$

with

$$q_h(s) = \left[\lambda \left(k_h(ict, s)\right)^{\varrho} + (1-\lambda) \left(\ell_h(nrt, s)\right)^{\varrho} \right]^{\frac{1}{\varrho}}$$

where $k_h(phy, s)$ and $k_h(ict, s)$ are non-ICT and ICT capital, $\ell_h(rt, s)$ and $\ell_h(nrt, s)$ are measures of routine and non-routine workers, while μ and λ are just weighting parameters that govern output share of each firm (h, s); parameter α is the physical capital share of output. Elasticity indicators (ϱ, ς) are $\varrho = \frac{\rho-1}{\rho}$ and $\varsigma = \frac{\sigma-1}{\sigma}$.

ASSUMPTION 2 *Substitution elasticities, $\rho_s, \sigma_s \in [0, \infty)$, physical capital and distributive shares, $\alpha_s, \mu_s, \lambda_s \in (0, 1)$, are industry-specific. Since elasticities are finite, each industry adopts strictly positive levels for capital and labour quantities.*

Parameter ρ is the elasticity of substitution between ICT capital and non-routine labour input (*i.e.*, the CES composite), while σ identifies the elasticity of substitution between routine labour input and the CES composite. The CES structure imposed above yields a symmetry constraint in the interpretation of σ: the elasticity of substitution between routine and non-routine workers is the same as that between routine labour force and the ICT composite. Capital-task complementarity requires $\sigma > \rho$: this condition resorts the idea that technological process (namely an increase in ICT capital relative to non-ICT one) widens the relative demand of non-routine labour force, thus lessening that of routine workers.

Given the market structure, the formation of wages is decided at firm level considering monopolistic competition among firms in the same industry. Profit maximizing behaviour thus implies the Cobb Douglas-nested CES production function in eq. (2.9) to be subject on the conditional firm demand in eq. (2.8):

$$\max_{p_h(s), k_h(j,s), \ell_h(a,s)} \left[\mathcal{D}_h\left(s\right) \mid y_h\left(s\right) \right]$$

with $\mathcal{D}_h\left(s\right)$ being profits. Within an industry, firms are all homogeneous; thus, any wage decision by firm (h, s) is not altering the sectoral wage level.[23]

ASSUMPTION 3 *Firms are atomistic, so that any wage level offered by a firm* (h, s) *only does not affect the sectoral one:* $\frac{\partial w(a,s)}{\partial w_h(a,s)} = 0 \ \forall a, h, s$.

Under this assumption, equilibrium wages paid by firm (h, s) are set, for routine and non-routine workers, respectively, to

$$\begin{aligned} w\left(rt,s\right) &= \left[\Lambda\left(s\right) \chi\left(rt,s\right) \left(k\left(phy,s\right)\right)^{\alpha} \mathcal{V}\left(s\right)^{\frac{1-\alpha-\varsigma}{\varsigma}} \left(\frac{\mathcal{B}\left(rt,s\right)}{\mathcal{WB}\left(rt,\mathcal{S}\right)} \right)^{\theta(\varsigma-1)} \right]^{\frac{1}{1+\theta-\theta\varsigma}} \\ w\left(nrt,s\right) &= \left[\Lambda\left(s\right) \chi\left(nrt,s\right) \left(k\left(phy,s\right)\right)^{\alpha} \mathcal{V}\left(s\right)^{\frac{1-\alpha-\varsigma}{\varsigma}} \mathcal{Q}\left(s\right)^{\frac{\varsigma-\varrho}{\varrho}} \left(\frac{\mathcal{B}\left(nrt,s\right)}{\mathcal{WB}\left(nrt,\mathcal{S}\right)} \right)^{\theta(\varrho-1)} \right]^{\frac{1}{1+\theta-\theta\varrho}} \end{aligned} \tag{2.10}$$

where industry-specific composite parameters are given by $\chi\left(rt,s\right) = \left(1-\alpha\right)\mu$ and $\chi\left(nrt,s\right) = \left(1-\alpha\right)\left(1-\mu\right)\left(1-\lambda\right)$, while $\Lambda\left(s\right) = p\left(s\right)\ \mathcal{M}^{\epsilon,-1}$ is an indicator combining the industry-specific price level multiplied by the price mark-up, $\mathcal{M}^{\epsilon} = \frac{\epsilon}{\epsilon-1}$, and the aggregate element $\mathcal{WB}\left(a,\mathcal{S}\right) = \sum_s \mathcal{W}_{\mathcal{H}}\left(a,s\right) \mathcal{B}_{\mathcal{H}}\left(a,s\right)$; refer to Appendix B for the definition of the other components. Aggregate industry wage is thus found by averaged aggregation among job tasks: $w\left(s\right) = \left(\mathcal{A}\right)^{-1} \sum_a w\left(a,s\right)$.

Before delving into the intuition behind the equations, it is important to establish a clear understanding of why wages are at industry level: gaining this foundational insight will help to better contextualize eq. (2.10).

PROPOSITION 1 (Firm and industry layers) *In an economy characterized by sorting and segregation effects and a not perfectly elastic labour supply where the measure of workers in a given firm is determined by its wage relative to the others, as long as*

(a) firms within an industry have the same size; or

(b) workers are perfectly mobile across firms within an industry,

[23] This assumption is a natural outcome in the proof of Proposition 1 in Appendix B.

$$\frac{\partial \ell_h(a,s)}{\partial w_h(a,s)} = -\frac{\partial \ell_{h'}(a,s)}{\partial w_h(a,s)} \quad and \quad \frac{\partial w_h(a,s)}{\partial \ell_h(a,s)} = \frac{\partial w_{h'}(a,s)}{\partial \ell_{h'}(a,s)} \quad ,$$

profit-maximizing wages set by firms in a specific industry are equal, and thus the unique optimal wage level can be directly written under industry notation. Moreover,

(c) workers are immobile across firms between industries.

Proof in Appendix B.

Besides formal proofs, the movement of workers across firms and industries is directly influenced by the assumed distribution of households' idiosyncratic productivity parameter $\wp_h^i(a,s)$. In fact, the *max-stability property* of the Frechét distribution ensures that a worker, once choosing a workplace, will not move nor across firms neither among industries: since households' productivities are so distributed, such property guarantees that its maximum is as well distributed (Mc Fadden 1974), and each household always picks its utility-maximizer productivity level, $\wp_h^i(a,s)_{max}$. Note that the imposed within-industry firm-homogeneity implies that each worker performs its job task content with the same productivity among firms in the same industry, *i.e.*, $\wp_h^i(a,s) \equiv \wp_{h'}^i(a,s)$ with $h' = \{1,\ldots,h,\ldots,H\} \in s$.

REMARK 2 (Max stability and workers' movement) *As from eq. (2.5), a worker has no incentive to choose a workplace where performing worse since*

$$\mathcal{U}_h^i(a,s) \mid \wp_h^i(a,s)_{max} \gg \mathcal{U}_h^i(a,s) \mid \forall\, \wp_h^i(a,s) \in \left[\wp_h^i(a,s)_{min}\, , \wp_h^i(a,s)_{max}\right)$$

is ensured by each worker's selection of its maximal productivity for the (h,s) tuple. In addition, since firms are symmetric within an industry, the sorting choice is uniquely driven by the dispersion of households' productivities between industries so that workers are free to move across firms within an industry while getting the same utility.

Interpreting eq. (2.10), both wage rise with non-ICT capital and the price level, and each wage is a function of the economy-wide wage for the corresponding task. Due to the production structure, the wage of non-routine workers is directly influenced by the ICT composite – the joint evolution of ICT capital and non-routine employment –, while routine wages are not. Task differences also manifest through the distinct roles of the two elasticities of substitution, ρ and σ: routine wages are primarily affected by substitution among themselves and non-routine tasks, whereas non-routine wages respond mainly to the elasticity governing the ICT composite, *i.e.*, the substitution between ICT capital and non-routine labour. Both elasticities further affect wages indirectly through total industry output, $\mathcal{V}(s)$. Consistently, an increase in the industry-specific amenity of a task, $\mathcal{B}(a,s)$, driven by substitution effects (ς) following a higher relative share of the other task, raises the wage of that task: for instance, a rise in $\mathcal{B}(rt,s)$ increases the ratio of non-routine to routine workers, thereby increasing routine wages. Finally, wages are non-decreasing in their aggregate task wage $\mathcal{W}(a,\mathcal{S})$, but rather increasing if $(\varrho,\varsigma) < 1$ (as estimated in Subsection 2.4.2), since $\left[\sum_s \mathcal{W}_{\mathcal{H}}(a,s)\right]^{-\theta([\varrho,\varsigma]-1)}$ with $-\theta([\varrho,\varsigma]-1) > 0$.

Cross-sectional differences in ρ and σ reflect not only divergences across comparable tasks but, importantly, differences across industries. In fact, if the distribution of capital and worker types were identical across industries, wage premia would be the same. This highlights that industry-specific characteristics, particularly their production plans and the degree of complementarity or substitutability among factors of production, play a central role in explaining real wage differentials.

Finally, parameter θ is common across all industries, capturing the strength of sorting and segregation forces and serving as a proxy for economy-wide labour market concentration (Figure 2.2). It represents a single structural distortion: as sorting and segregation intensify, the workforce composition within each industry becomes less diverse, generating a systematic allocation of tasks toward selected industries.

Closing the model, profit maximization by firm-h in industry-s not only displays the optimal amounts of each type-a's wage rates, but also a choice on the optimal quantities of both ICT and non-ICT capital stocks:

$$k_h(j,s) \ : \ p_h(s) \, f_{k_h(j,s)} = \mathcal{M}^{\epsilon} \, R(j) \tag{2.11}$$

where $\mathcal{M}^{\epsilon} = \frac{\epsilon}{\epsilon - 1}$ is the (constant) price mark-up, $f_{k_h(j,s)}$ being marginal products of both capital types from the production function specified in eq. (2.9), and $R(j)$ the capital-specific rental rate, for $j = \{phy, ict\}$.

According to the structure of the model, equilibrium conditions read as follows.

(Equilibrium) *An equilibrium for this economy is defined as an households' choice of job place, a combination of factors' prices* $\{w(a,s), R(phy), R(ict)\}$*, and a set of aggregate quantities* $\Omega = \{Y, K(phy), K(ict), L(rt), L(nrt)\}$*, such that: (a) each household picks the firm-industry tuple that maximizes eq. (2.5); (b) according to the occupational choice, each household maximizes its expected-utility version of the utility in eq. (2.5); (c) final and sectoral good producers maximize their revenues; (d) given the availability of workers in each job task as in eq. (2.7), optimal wages are determined by the equilibrium of labour demand and supply; (e) firms choose also capital bundles to maximize their profits; and (f) all markets clear, shaping* Ω*.*

Proof in Appendix B.

2.4. From Theory to Data

In this section I quantitatively evaluate the model and bring it to the data. To gauge the performance and the consistency of the model in addressing salient features of the inter-industry wage structure of the United States economy, I split the universe of industries into three groups (following the "contribution" result – Fact n. 1 – of the data motivating section), namely the top 25%, the middle 50% and the bottom 25% of industries according to the overall growth in their real log-wage. The time window considered is the usual, spanning from 2003 to 2022.

TABLE 2.4: INDUSTRY WAGE AND RELATIVE TASK SIZE

	$log(w(s))$		
	(1)	(2)	(3)
$g(rt/nrt,s)$	.016***(.006)	.001 (.007)	-.020***(.002)
$g(nrt/rt,s)$	.129***(.036)	.250***(.084)	.318***(.043)
Industry FE	✓	✓	✗
Time FE	✗	✓	✗

*Significance level at * (p<0.05), ** (p<0.01), *** (p<0.001). Standard error in parentheses. Analysis at 3-digit U.S. 2017 NAICS industries in 2003-2022 on* $N = 1240$ *observations. The Fixed Effects (FE) regressions are of the form* $log\ w_t(s) = \beta_c + \beta_i \mathcal{X}_{i,t} + \delta_j \mathcal{Z}_{j,t} + u_t$, *with* $\mathcal{X}_i$ *being the regressors, and* $\mathcal{Z}_j$ *a set of controls. All series are in logs. Constant not reported to save space. Source: BEA, BLS and own calculations.*

2.4.1. VALIDATING THE MODEL IMPLICATIONS

As a first step to validation, I quantify the relevance of the labour force composition on the industry-specific wage level. Beside the usual role of capital and labour types in both $\mathcal{V}(s)$ and $\mathcal{Q}(s)$, the industry-specific wage level in eq. (2.10) considers a sizeable role of the industry benefit of each task, namely $\mathcal{B}(a,s)\ \forall s$, which points directly to the substitution among job tasks at the industry layer. To offer a better understanding, I now quantify its role by providing reduced-form evidence on how it impacts industry wages.[24] A set of panel Fixed Effects (FE) regressions

$$log\ w_t(s) = \beta_c + \beta_i \mathcal{X}_{i,t} + \delta_{j,t} \mathcal{Z}_{j,t} + u_t$$

where $\mathcal{X}_i$ comprises the regressors – task ratio and its inverse – and $\mathcal{Z}_j$ is a set of employment-related controls, is performed. Results are presented in Table 2.4. Column 1 presents the baseline estimation with industry FE only. A positive effect of increases in both ratios is observed, with the non-routine-to-routine worker ratio exerting a stronger role on industry real log-wages, consistent with stylized Fact 4 and the theoretical wage-setting equations. Column 3 examines cross-sectional variation by omitting industry FE. In this specification, an increase in the routine-to-non-routine ratio reduces wages, contrasting with the industry-aggregated prediction of equation (2.10). Nonetheless, the positive impact of the relative non-routine task ratio remains robust and quantitatively substantial compared with the baseline.

Employment and relative wages.– Another central hypothesis of the model concerns the labour market. Since the employment measure of a task in a given industry is defined as in equation (2.7), linking task-level employment to relative nominal task wages, the same regression procedure used for the wage equation can be applied. Results, reported in Table C.1, show strong empirical support: employment in both routine and non-routine tasks rises with the associated relative wage. The effect

[24] As a reminder, the model distinguishes between two types of workers, $a = \{rt, nrt\}$, so that $\mathcal{B}(rt,s) = \left[\ell(rt,s)/\ell(nrt,s)\right]^{-\varsigma}$ and $\mathcal{B}(nrt,s) = \left[\ell(nrt,s)/\ell(rt,s)\right]^{-\varsigma}$, where fractions are identified by $g(\cdot,s)$.

is substantial for routine workers and even larger for non-routine workers, indicating that industries offering higher wages for a given task relative to the rest of the economy attract more workers to that task.

2.4.2. Calibration

The vector of structural parameters to be calibrated is $\Theta = \left(\alpha_s, \epsilon, \lambda_s, \mu_s, \theta, \rho_s, \sigma_s\right)_{\forall s}$, and comprises features of both households and industries. A total of 17 parameters is computed, and the strategy can be summarized in three steps: *(i)* some parameters are calibrated directly from the data, and some are externally taken; *(ii)* the elasticities of substitution are directly and individually estimated via panel robust regressions at industry level; finally, *(iii)* other parameters are internally estimated by moment-matching of key features of the U.S. economy.

Data and external calibration.– The elasticity of substitution among different varieties (ϵ) is externally set to 6 (so to have an annual mark-up of 20%). Differently, the production function's share of non-ICT capital (α) is directly taken by manipulating data from BEA: as in Arvai and Mann (2022), for each group of industries, I compute one minus the share of labour in total output, adjusted by considering the weight of non-ICT capital in the total capital stock. This procedure reports $\alpha = \left\{0.263, 0.195, 0.514\right\}$, order from bottom to top groups.[25]

Elasticities of substitution.– The key model parameters, namely $(\rho_s, \sigma_s)_{\forall s}$, are estimated through the general equilibrium propoerties of the model as in Karabarbounis and Neiman (2014). For each industry group, labour, capital, and profits are expressed as income shares of output and blended with the industry-side F.O.C.s in eq. (2.11) to exploit the evolution of the capital rental rate (eq. 2.6). This allows the resulting condition to be expressed in differences between two periods. Using a linear approximation around a zero over-time trend, this approach yields two separate estimating equations that both relate the industry-level labour shares, $s_\ell\left(a, s\right)$ and $s_\ell\left(s\right)$, to technological capital:

$$\frac{s_{\ell,t}\left(nrt, s\right)}{1 - s_{\ell,t}\left(nrt, s\right)}\widehat{s}_\ell\left(nrt, s\right) = \beta_c + \left(\rho - 1\right)\widehat{\zeta}\left(s\right) + u_t \tag{2.12}$$

and

$$\frac{s_{\ell,t}\left(s\right)}{1 - s_{\ell,t}\left(s\right)}\widehat{s}_\ell\left(s\right) = \beta_c + \left(\sigma - 1\right)\widehat{\zeta}\left(s\right) + \beta_k\left(\widehat{\frac{\ell\left(nrt, s\right)}{k\left(ict, s\right)}}\right) + u_t \tag{2.13}$$

where *hatted* variables identify their own percent change between arbitrary t and $t-1$ periods, and ζ is the industry-related quantity of ICT capital relative to physical capital. The first equation identifies the elasticity of substitution between ICT capital and non-routine labor (ρ) by treating the ICT composite as a nested intermediate input. By contrast, equation (2.13) estimates the elasticity of substitution

[25] Estimates are consistent with the argument against the usual "alpha equal one-third" rule – that the elasticity of output with respect to capital (*i.e.*, the capital share of output in a neoclassical production function) is 0.33 – by Vollrath (2024) for the U.S. economy in 1948-2018.

between routine and non-routine workers (σ), derived from the full production structure in eq. (2.9). Importantly, the estimated coefficients from these regressions do not correspond to $\beta_{\zeta}^{(\rho,\sigma)} = \left([\rho,\sigma] - 1\right)$; instead, the specification directly identifies the elasticity parameters (ρ,σ), since the left-hand-side variable (the labour share trend) is augmented by the trend in the relative ICT capital stock, $\widehat{\zeta}\,(s)$.

The intuition behind the two equations is straightforward. A negative relationship between trends in labour shares and trends in relative capital quantities arises only when the estimated elasticities imply sufficient complementarity, that is, when $(\rho,\sigma) < 1$. Abstracting from other possible economic factors (such as output productivity, capital-or labour-augmenting technology, profit share's or mark-up's growth), an increase in the relative stock of ICT capital then leads to a consequential decline in the labour share of each occupational group.[26] Estimated overall correlations between trends in both labour shares and relative ICT stock among 3-digit U.S. 2017 NAICS industries are $corr_{\rho} = -0.76$ and $corr_{\sigma} = -0.75$ between years 2003 and 2022.

Given the relatively short time dimension, the estimation may be sensitive to outliers. A robust regression procedure is therefore employed, iteratively down-weighting observations that lie far from the fitted regression line.[27] The resulting estimates of equations (2.12) and (2.13) are reported in Table 2.5. A key finding is that all industry groups except the middle one exhibit complementarity between ICT capital and non-routine tasks, as indicated by $\sigma_s > \rho_s$ for any $s = \{bot, top\}$. This pattern reflects a broadly similar adoption of technological change across industries, coupled with heterogeneous effects on labour force composition, as the magnitude of the (ρ,σ) gap varies across groups. Consistent with the evidence in Section 2.2, industries at the top of the distribution benefit the most from task reallocation driven by technological change. All estimated elasticities are statistically significant.[28]

Interpreting ρ, top industries exhibit the strongest complementarity between ICT capital and non-routine workers, followed by bottom and then middle industries. Values of $\rho \approx 1$ indicate greater gross substitutability, implying weaker complementarity. Accordingly, top industries sustain larger stocks of ICT capital while employing relatively more non-routine workers than other industry groups. Turning to σ, the elasticity of substitution between routine workers and the ICT composite (or, by sym-

[26] This framework differs from Karabarbounis and Neiman (2014), who require elasticities greater than one so that a decline in the user cost of capital – driven by falling investment prices – reduces the labour share through substitution away from labour. Here, quantities rather than prices are used, as capital stocks are directly observable at the industry level in BEA data.

[27] Following Karabarbounis and Neiman (ibid.), this approach mitigates potential violations of OLS assumptions arising from small samples by reducing the influence of high-residual observations. At each iteration, observations with Cook's distance greater than one – indicative of high leverage – are removed, so that the weight of each observation is no longer uniform at $\frac{1}{n}$ in a data sample with n observations.

[28] Estimating the elasticity of substitution between capital and labour is inherently challenging, as it reflects both demand- and supply-side forces, and there is no consensus on its precise value. Existing aggregate and sectoral estimates range from 0.3 to 0.9 (Arrow et al. 1961, Klump et al. 2007, Herrendorf, Herrington, et al. 2015, Alvarez-Cuadrado et al. 2018, Oberfield and Raval 2021), from unity (Berndt 1976), to between 1.25 and 1.6 (Piketty 2013, Karabarbounis and Neiman 2014). Moreover, as emphasized by Oberfield and Raval (2021), it remains unclear whether time-series or micro-level data are more appropriate for identification.

TABLE 2.5: BASELINE ESTIMATION

	ρ	*Std.Err.*	*95% CI*	σ	*Std.Err.*	*95% CI*
bottom	.329	.04	[.250, .407]	.634	.08	[.482, .785]
middle	.420	.02	[.375, .466]	.400	.05	[.310, .491]
top	.249	.03	[.188, .310]	.766	.06	[.656, .877]

Estimation of the elasticities of substitution as given in eqs. (2.12) and (2.13), for 3-digit U.S. 2017 NAICS industries over the period 2003-2022. ρ refers to the estimate of the pair $\left(\ell\left(nrt,s\right);k\left(ict,s\right)\right)$, and it exploits the degree of substitutability between non-routine workers and ICT capital; σ relates to the pair $\left(\ell\left(rt,s\right);\left[\ell\left(nrt,s\right),k\left(ict,s\right)\right]\right)$, and it is the degree of substitutability between routine workers and the joint combination of non-routine workers and ICT capital.

metry, between routine and non-routine tasks), values closer to one imply a lower propensity to retain routine workers as ICT capital and non-routine employment expand. This pattern is precisely what emerges from Table 2.5: top industries display the lowest tendency to employ routine workers at rates comparable to non-routine workers, followed by bottom industries, with middle industries showing the weakest response. Together, the estimated elasticities point to substantial heterogeneity in the degree of structural transformation across industries, implying uneven reallocation of economic activity toward selected sectors. This reallocation is driven by differences in industry-level prices and quantities of capital-labour combinations, as reflected in the observed trends in labour shares. Finally, the estimates align with the wage-setting mechanism in eq. (2.10): industries characterized by lower ρ and higher σ are those that command higher wage premia.

Matching moments.– To conclude the calibration, the remaining parameters are estimated by matching salient moments in the data using the Method of Simulated Moments (MSM), discussed in Mc Fadden (1989). These parameters include the industry-group-specific weights $(\mu_s, \lambda_s)_{\forall s}$ in the production function, as well as θ, which governs the dispersion of households' idiosyncratic productivities and, in turn, the strength of sorting and segregation effects. Given the parameters estimated thus far, the calibration leaves seven parameters to be identified by matching seven moments. Each λ_s is pinned down by the ICT capital share of the corresponding industry group, while each μ_s is identified using the group-specific share of routine workers, exploiting its theoretical definition in eq. (2.7). Finally, since θ captures productivity dispersion and the resulting degree of labour market polarization, it is calibrated to match the real log-wage premium of routine workers in the top industry group relative to the bottom group.[29]

To match key moments of the production structure and households' productivities specification, the method estimates the vector of parameters $\widetilde{\Theta} = \left\{\lambda_s\,,\,\mu_s\,,\,\theta\right\}$ for $s = \left\{bot, mid, top\right\}$, by minimizing the *loss function*

[29] In the model, heterogeneity in household productivity is intrinsically linked to sorting and segregation choices. Moreover, the dynamics of labour market concentration in aggregate employment are closely mirrored by those of routine workers at the 3-digit U.S. 2017 NAICS industry level; see Figure 2.4.

TABLE 2.6: SUMMARY OF CALIBRATION

	parameter	***value***				***source***
		bottom	*middle*	*top*	*global*	
α	*physical capital, share of $y(s)$*	0.263	0.195	0.514		*data*
ϵ	*demand elasticity across firms*				6	*external*
μ	*weight of routine workers in $y(s)$*	0.676	0.495	0.337		*MSM*
λ	*ICT capital share in $Q(s)$*	0.457	0.465	0.451		*MSM*
θ	*households' productivities dispersion*				11.3	*MSM*
ρ	*EoS, ICT capital and non-routine*	0.329	0.420	0.249		*estimation*
σ	*EoS, routine and ICT composite*	0.634	0.400	0.766		*estimation*

Set of estimated parameters of the model. "data" implies that the values are directly computed from data sources, while in "external" I choose standard calibrated values from the literature. "MSM" refers to the Methods of Simulated Moments. "estimation" refers to previously estimated values under a specific procedure; these values are taken from Table 2.5.

$$\mathcal{L}\left(\widetilde{\Theta}\right) = \left(\widehat{m}(\widetilde{\Theta}) - \widetilde{m}\right)' \mathbf{W} \left(\widehat{m}(\widetilde{\Theta}) - \widetilde{m}\right)$$

to reckon $\widetilde{\Theta}$ that is, to minimize the distance between the estimated moments $\widehat{m}(\widetilde{\Theta})$ and the data moments counterpart, $\widetilde{m}$. Element $\mathbf{W}$ is an *efficient weighting* matrix that allows to implement an efficient estimator.[30] Table C.2 summarizes the resulting values, and compares the targeted data moments (expressed as mean values throughout the period 2003-2022) with that in the model.

Of particular interest is the estimate of θ. As discussed, it can be interpreted as a measure of labour market concentration arising from sorting and segregation mechanisms operating through the dispersion of households' productivities, and thus as the (average) elasticity of labour supply faced by firms at the industry level. Since $\theta > 1$, a higher value indicates less heterogeneity in individual productivity and, consequently, stronger incentives for workers to sort into specific industries rather than move across them. Put differently, the farther θ lies from unity, the more polarized the labour market becomes in terms of workforce characteristics, reflecting stronger sorting and segregation forces. The estimated value, $\theta = 11.3$, points to a highly concentrated labour market (as in Figure 2.2), consistent with recent evidence for the U.S. labour market in Yeh et al. (2022).[31]

Comparative discussion of the calibration.– A summary of the parametrization of the model is in Table 2.6. The calibration highlights that top industries – those experiencing the largest increases in real wages over the sample period – are characterized by the strongest complementarity between ICT capital and non-routine labour

[30] Practically, I first set it to be an identity matrix whose diagonal elements are the mean values of each moment in the data, to then update these values using the estimated vector valued function with the distance between data and simulated moments.

[31] Authors document how concentration – due to employer market power, referred to as aggregate markdown (ratio between wages and marginal revenue product of labour) –, decreased over 1977-2002, but thereafter it has been sharply increased. Such turn is well captured even at industry level (refer to Figure 2.4).

TABLE 2.7: IMPLIED FIT OF WAGE VARIANCES

moment	***data***	***model***
routine wage variance, across industries	2.285	2.289
non-routine wage variance, across industries	2.314	2.311
between-industry wage variance	2.299	2.300

Comparison of the model fitting of the data for moments related to 3-digit U.S. 2017 NAICS between-industry real log-wage variance structure over the period 2003-2022; variances are computed according to eq. (2.1). The first two rows compute this measure for routine and non-routine tasks, while the last row directly reports the wage dispersion across industries.

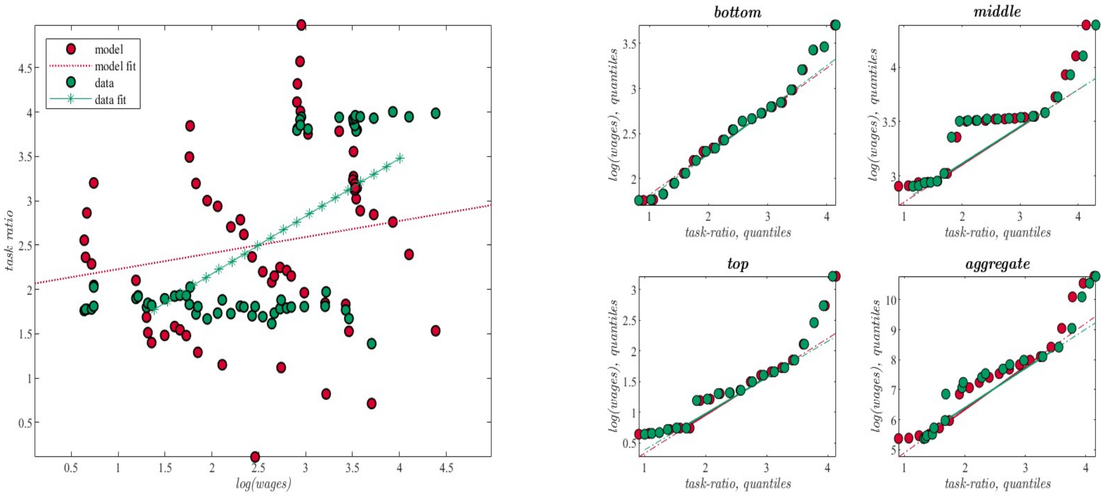
(lowest ρ) and, simultaneously, by a greater propensity to substitute routine workers with non-routine tasks (highest σ). By contrast, bottom industries also display meaningful ICT-non-routine complementarity, but this is accompanied by weaker substitutability between routine and non-routine tasks, limiting the reallocation away from routine labour. These results closely align with the stylized facts documented in Section 2.2. While the non-routine-to-routine task ratio evolves primarily within industries (Fact 3), real wage growth is muted unless this shift is accompanied by an increase in the ICT-to-non-ICT capital ratio (Fact 4).

Moreover, top industries feature a substantial share of physical capital (α) in their production structure, alongside the lowest weight assigned to routine labor (μ). The prominent role of physical capital may partly reflect automation forces, whose effects differ across industry groups. For instance, bottom industries display an intermediate reliance on non-ICT capital but assign the largest weight to routine labour. Such heterogeneity suggests that automation operates unevenly across sectors.[32] A further latent indicator of automation emerges from the estimated ICT-capital weights in the ICT composite (λ), which indicate that industries with the strongest real log-wage growth place relatively less weight on ICT capital within the composite. Overall, the MSM calibration is internally consistent and mirrors the estimated elasticities of substitution across industry groups.

2.4.3. MODEL FIT

The primary scope of the model is to detect the industry's determinants that account for between-industry wage inequality, moment not directly targeted in the MSM parameters' estimation. Table 2.7 reports the fitting of real log-wage variance across industries for routine and non-routine workers, as well as that for industry-specific wage rates. Tasks' specific variances are computed according to eq. (2.1) given the analytical wage series sparking from eq. (2.10), while the between-industry wage variance is found by averaging these two dispersion indicators. Given the calibrated values of parameters, the model built has the virtue of well targeting all the consid-

[32] Automation (industrial robots and artificial intelligence) tends to displace routine tasks while increasing the employment share of non-routine workers. At the industry level, however, the evidence on employment growth versus displacement remains mixed and context-dependent; see Filippi et al. (2023) for a detailed review. In this model, automation is therefore captured only indirectly through changes in $(\alpha_s, \mu_s, \lambda_s)_{\forall s}$.



(A) TASK RATIO VS. WAGES

(B) QUANTILES

FIGURE 2.3: MODEL AND DATA COMPARISON

Note: the figure shows the comparison between the model series and the related series in the data. Panel 2.3a represents the correlation in the baseline equilibrium between the task ratio (non-routine over routine tasks) with the empirical and model-implied real log-wage level, and the corresponding lines show the linear fit of the correlation; Panel 2.3b plots the parametric curves associated to quantiles of the considered HP-filtered series (model vs. data) one against each other. Series are scaled to be in the same range for graphical comparison. All plots consider industries to be grouped in terms of bottom, middle, and top industries' groups. Red circles refer to the model, while red ones to the data.

ered measures of inequality on real log-wages.

Another central hypothesis to validate in the cross-section is the model's ability to capture an increase in industry real log-wage following a rise in the task ratio, as discussed in Subsection 2.4.1. To this end, Panel 2.3a plots the correlation between the share of non-routine workers over the mass of routine workers and the industry wage as both in the data and in the model. Imposed model structure is able to successfully replicate the upward-sloping correlation implied by the data.

Finally, the validity of the calibration strategy can be assessed by checking how the model performs in addressing some other data moments, in particular those not targeted throughout the estimation procedure. I pick all the moments related to the wage structure in the economy, and evaluate the fitting also at the industry-group levels. Table C.3, Appendix C, reports the outcome. The direction of the untargeted moments in the data is captured by the model. An important observation is that almost all the appraised wage-moments have negative slope, thus linking my model to the secular decline in real wages (*e.g.*, Massenkoff and Wilmers 2023). The only increasing moments are the "top/bottom wage ratio" and the "aggregate task premium" (reflecting the wage premium of non-routine over routine tasks), making the model suitable for studying the evolution of this wage gap.

To conclude, the model performs well in replicating both targeted and untargeted moments related to the wage structure of the U.S. economy over the period 2003-2022: fitting is detected either at the aggregate and industry levels, and also for differences among routine and non-routine workers.

2.5. Counterfactual Analysis

The purpose of the following analysis is to understand whether heterogeneity in the industrial composition of the U.S. economy contributes to observed trends in wage inequality. To do so, a counterfactual analysis evaluates the role of variations in structural parameters on between-industry wage inequality. Specifically, the model's parameters are fully re-estimated over two equal periods, and the associated moments are computed by sequentially "switching on" changes in one or more parameters while holding the others fixed.[33] The primary goal is to identify the key drivers of wage variance across industries and quantify the share of observed inequality explained by the "structural" or by the "quantity" effects.

In the set of counterfactual exercises the core parameters are three: the elasticity of substitution between ICT capital and non-routine labour, namely ρ; the elasticity of substitution between routine and non-routine labour, namely σ; and the household productivity dispersion, namely θ, which captures sorting, segregation, and overall labour market concentration. Their contributions are evaluated both individually and jointly. Variations in industry-specific elasticities , $(\rho_s, \sigma_s)_{\forall s}$, isolate the role of structural transformations across industries, independent of changes in input quantities. Changes in θ reflect the degree of labour market concentration: with imperfectly elastic labour supplies, higher value implies stronger sorting and segregation tuple, making labour supply more industry-specific.

2.5.1. Selecting the parameters

Table 2.8 summarizes the evolution of the two substitution elasticities (ρ, σ) across two periods, 2003-2012 and 2013-2022. Top industries display a pronounced change in only one elasticity: the complementarity between ICT capital and non-routine workers declines, while the "gross substitution" between routine and non-routine workers shows only a modest increase. In the middle group, non-routine workers strengthen their complementarity with ICT capital, accompanied by a slight reduction in complementarity with routine workers. The bottom group experiences a simultaneous drop in both the substitutability across job types and the complementarity between ICT capital and non-routine labour. These patterns highlight heterogeneous structural adjustments across industries over time.

Figure 2.4 provides intuition on the evolution of labour market concentration by plotting the Herfindahl-Hirschman Index (HHI) for both worker types and aggregate employment.[34] Since θ is estimated via MSM, it reflects not only a cross-sectional measure but also the variation of sorting and segregation: higher θ implies that similar workers cluster more tightly within industries. A clear pattern emerges: labour market concentration rises steadily in the first half of the sample, then flattens in

[33] Tables 2.9 and C.8 report the results for overall between-industry wage inequality. In Appendix C, the same procedure is applied separately to routine (Table C.9) and non-routine (Table C.10) workers.

[34] Further details are provided in Appendix C.

TABLE 2.8: TIME-VARYING ESTIMATION

	2003-2012		***2013-2022***	
	ρ	σ	ρ	σ
bottom	.355 [.12]	.366 [.12]	.819 [.05]	.326 [.06]
middle	.431 [.04]	.429 [.08]	.345 [.04]	.438 [.09]
top	.408 [.04]	.367 [.12]	.508 [.06]	.358 [.05]

Estimation of the elasticities of substitution as given in eqs. (2.12) *and* (2.13) *over different time spans (2003-2012 and 2013-2022), in absolute values, for 3-digit U.S. 2017 NAICS industries. Standard errors in parenthesis,* $[\cdot]$, *and* 95% *confidence interval significant but not reported.* ρ *refers to the estimate of the pair* $\left(\ell(nrt,s)\,;\,k(ict,s)\right)$, *and it exploits the degree of substitutability between non-routine workers and ICT capital;* σ *relates to the pair* $\left(\ell(rt,s)\,;\,[\ell(nrt,s),\,k(ict,s)]\right)$, *and it is the degree of substitutability between routine workers and the joint combination of non-routine workers and ICT capital.*

the second half (green line), driven by declining concentration among non-routine workers (blue line). Re-estimation over the two sub-periods yields $\theta_{2003\text{-}2012} = 7.26$ and $\theta_{2013\text{-}2022} = 7.79$, indicating that variation in industry-level worker concentration was stronger in the first period and more moderate in the second, yet overall concentration remains high.[35] Hence, tracking concentration of routine tasks suffices to capture the economy-wide pattern, consistent with using the top/bottom industry wage ratio for routine jobs to calibrate θ. Together, heterogeneous changes in (ρ, σ), coupled with variations in θ, effectively capture the reallocation of capital and labour types across industries, reflecting both structural transformations and rising labour market concentration for routine and non-routine tasks.

Key is to assess the role of differential effect of structural changes. As a preview of the results, heterogeneous shifts in both elasticities matter: changes in ρ and σ individually explain a substantial portion of real log-wage variance, but it is their joint evolution that accounts for the largest share of U.S. wage inequality. In addition, higher labour market concentration – reflected in stronger sorting and segregation effects – amplifies the inequality captured by these structural differences.

2.5.2. INSPECTING THE MECHANISM: MODEL RE-CALIBRATION

The core exercise consists in re-estimating the full set of model parameters over two equally sized subperiods (2003-2012 and 2013-2022) and quantifying the effects of allowing one – or a subset – of parameters to change at a time. The new estimates are reported in Table C.6. This approach provides a direct bridge between data-driven shifts in structural transformations across U.S. industries and the observed evolution of wage inequality, as disciplined by the model. The objective is to isolate the contribution of each structural parameter by computing counterfactual wage variance levels

[35] Recall that $\theta > 1$: as it approaches to 1, labour market concentration decreases. Hence, the two estimates are consistent both with the theory and the figure: the concentration of workers at industry level is higher in the second half $(\theta_{2013\text{-}2022})$ than in the first one $(\theta_{2003\text{-}2012})$.

FIGURE 2.4: LABOUR MARKET CONCENTRATION AS A STRUCTURAL CHANGE

Note: the figure represents the evolution in labour market concentration as measured by the Herfindahl-Hirschman Index (HHI) – on the y-axis – by routine and non-routine (red and green lines, respectively) job tasks, and by total employment (yellow line). Series are normalized relative to their initial value in 2003, set to 1. *Source*: BLS and own calculations.

in which only a given parameter moves from its first-period to its second-period value, while all remaining parameters are held fixed at their initial levels. Formally, given the re-estimated parameters, consider the following model:

$$\Delta var\Big(w(s) - \overline{w}\Big) = f\Big(\Phi_s\left(x, \tau_1\right), \Theta_s(p, \tau_1), \Phi_s\left(x, \tau_2\right) \mid \Delta\Theta_s\left(p_{\tau_2}^{\{\rho,\sigma,\theta\}}, -p_{\tau_1}\right)\Big) \quad \text{(Model A)}$$

where the change in between-industry wage variance is a function of the input factor series in both periods, $\Phi_s\left(x, \tau_1\right)$ and $\Phi_s\left(x, \tau_2\right)$, the whole set of parameters in period one, $\Theta_s(p, \tau_1)$, and a subset of parameters in the second period keeping fixed the others, $\Theta_s\left(p_{\tau_2}, -p_{\tau_1}\right)$, for any industry-$s$. Practically, given the change in capital and labour types, wage levels and variances are estimated under two set of parameters: *(i)* full set related to period one, and *(ii)* full set related to period one with one (or a combination) related to its period two level; these parameters are those selected in Subsection 2.5.1.

Results are reported in Table 2.9. Column 2 displays the between-industry variance in the second period (τ_2) for both the data and the model specification described in column 1. To assess the contribution of individual structural parameters, column 3 reports the model-implied wage variance obtained by holding all parameters at their first-period values, except for the parameter(s) of interest, which are set at their second-period levels. The resulting counterfactuals highlight the central role of industry-heterogeneous changes in the substitutability of production factors in shaping U.S. between-industry wage inequality. In particular, the joint evolution of the two substitution elasticities – ρ, governing complementarity between ICT capital and non-routine labour, and σ, governing substitution between routine and non-routine workers – emerges as pivotal. Isolated changes in ρ alone would have generated a non-negligible increase in wage dispersion (.88), while changes in σ alone would have

TABLE 2.9: MODEL COUNTERFACTUAL, CHANGE

industry wages	$var(w)_{\tau_2}$	Δ ***model*** $\mid \Delta\, m\Big(\Phi(x,\tau_2),\Theta = \{p_{\tau_2}, -p_{\tau_1}\}\Big)$ *level*	*share, model*	*share, data*
DATA	1.18			
MODEL	1.09			
$\Delta\sigma$		2.37	2.18	1.99
$\Delta\rho$		.88	.81	.74
$\Delta(\sigma,\rho)$		1.12	1.03	.94
$\Delta\theta$		1.34	1.23	1.13
$\Delta(\sigma,\rho,\theta)$		1.16	1.07	.98

Quantification of Model A. Model implied between-industry real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2),\Theta(-p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2)\right]}$*, where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2)*, are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

implied an even larger surge in inequality (2.37). Strikingly, when both elasticities are allowed to change simultaneously, their opposing magnitudes partially offset each other, yielding an implied between-industry real log-wage variance (1.12) that closely aligns with both the full model calibration (1.09) and the observed data (1.18). This evidence underscores that it is the coordinated – and uneven – restructuring of task substitutability across industries, rather than movements in any single elasticity in isolation, that accounts for the observed evolution of wage inequality.

Column 5 quantifies the contribution of each parameter to the observed rise in U.S. wage inequality by reporting the share of the empirical real log-wage variance explained by the variance implied by parameter shifts in the model. Isolated changes in ρ account for about 74% of the observed between-industry wage dispersion, while heterogeneous and joint shifts in both elasticities explain as much as 94% of the data-implied variance (corresponding to 103% of the variance generated by the fully calibrated model for the 2013-2022 period). These results indicate that large wage differentials across industries are driven primarily by industry-specific differences in the substitutability between routine and non-routine workers, rather than by variation in the substitutability between ICT capital and non-routine labour alone. Crucially, it is the interaction of these two structural dimensions – how industries reorganize tasks internally and how they combine labour with technology – that accounts for the bulk of the observed increase in wage inequality.

These conclusions naturally extend to the interpretation of changes in θ, which rises from $\theta_{2003\text{-}2012} = 7.26$ to $\theta_{2013\text{-}2022} = 7.79$. Taken in isolation, this increase – signaling stronger worker concentration through intensified sorting and segregation – would substantially amplify real log-wage dispersion (1.34). However, once these sorting-segregation forces interact with the industry-heterogeneous shifts in substitution elasticities documented in Table 2.8, their effect becomes more nuanced: labour market concentration slightly sharpens the economy-wide level of between-industry wage inequality, raising the model-implied variance (1.16). Put differently, when industries differ in how they reorganize tasks and combine labour with technology, a more concentrated labour market magnifies existing wage gaps rather than creating them on its own. When variations in industry-specific structural characteristics – captured by joint changes in both substitution elasticities – are considered together with changes in labour market concentration, the combined component $\Delta(\sigma, \rho, \theta)$ accounts for nearly the entire observed increase in U.S. between-industry real *log*-wage variance, explaining about 98% of its data-implied level.

Sensitivity.– So far, the analysis has focused on changes in the three core parameters of the model, $(\rho_s, \sigma_s, \theta)_{\forall s}$, while holding fixed the production function parameters $(\alpha_s, \mu_s, \lambda_s)_{\forall s}$. The results are further strengthened by the complementary, reverse exercise. In this case, the counterfactual asks what the level of between-industry wage variance would have been had one parameter remained fixed at its initial (period-1) value, while all other parameters were allowed to evolve from period one to period two. Formally, this amounts to the following experiment:

$$\Delta var\Big(w(s) - \overline{w}\Big) = f\Big(\Phi_s\left(x, \tau_1\right), \Theta_s(p, \tau_1), \Phi_s\left(x, \tau_2\right) \mid \Delta\Theta_s\left(p_{\tau_2}, -p_{\tau_1}^{\{\rho,\sigma,\theta\}}\right)\Big) \quad \text{(Model B)}$$

whose results are presented in Table C.8.[36] Absent any change in σ, thereby assigning a first-order role to the joint evolution of ρ, θ, and the industry-specific production weights and income shares, the model would explain about 79% of the observed rise in wage inequality. By contrast, fixing both elasticities simultaneously, $\left(\Delta\Theta_{|(\sigma,\rho)}\right)$, leads the model to overpredict dispersion: the implied variance (1.31) exceeds that observed in both the data and the baseline model. A similar overstatement emerges when all structural parameters are held fixed, leaving only changes in income shares to operate. Conversely, holding constant the productivity dispersion parameter θ while allowing both substitution elasticities and production weights to vary captures most of the observed inequality. In line with the stylized facts in Section 2.2, industry-heterogeneous shifts in substitution elasticities combined with changes in factor weights alone explain roughly 88% of the data-implied between-industry wage variance (96% in the model). This result underscores that wage inequality is driven primarily by uneven structural transformation across industries, rather than by changes in labour market concentration per se.

Investigations in this section highlight a central feature of the U.S. wage structure: structural heterogeneity across industries is the dominant source of wage inequality.

[36] This counterfactual exercise differs from that in Table 2.9 by allowing production function weights to vary as well. Specifically, it admits industry-specific changes not only in the structural parameters $(\rho_s, \sigma_s)_{\forall s}$, but also in the production parameters $(\alpha_s, \mu_s, \lambda_s)_{\forall s}$, alongside shifts in households' productivity dispersion captured by θ. In doing so, the analysis jointly accounts for structural transformation, changes in factor intensities, and evolving labour market concentration, providing a more comprehensive assessment of the forces shaping between-industry wage inequality.

When the model is disciplined by the data, most of the observed dispersion is traced back to industry-specific differentials in the elasticities of substitution between capital and labour types (94%, Table 2.9), and this share remains large when combined with heterogeneity in factor weights within production (88%, Table C.8). Conditional on these structural differences, the increase in labour market concentration through stronger sorting and segregation across industries acts as an amplifier of inequality. Once both mechanisms are allowed to interact, the model explains up to 98% of the observed between-industry variance in real log wages over the past two decades (Table 2.9).[37] Overall, the evidence points to uneven structural transformation across industries – rather than quantity-based technological change – as the key force shaping the evolution of U.S. wage inequality.

The role of labour market power.– Model specification allows also to inspect whether monopsony power by the part of firms plays a significant role in between-industry wage inequality. Labour market failures due to employer market power in wage-setting is a widely discussed channel (*e.g.*, Prager and Schmitt 2021, Dodini et al. 2021), though Card, Rothstein, et al. (2024a) show that, in many manufacturing industries, industry-level wage premiums are not positively correlated with wage markdowns. In Appendix D, the previous exercises are replicated under a version of the model where monopolistically competitive firms (h, s) optimally exploit monopsonistic power. In this setting, eq. (2.10) would display an average wage markdown over the marginal revenue product of labour, $\mathcal{M}^{\theta} = \frac{\theta}{1+\theta}$, which is an increasing function of the labour supply elasticity θ (*e.g.*, Card, Cardoso, Heining, et al. 2018). As illustrated in Figure 2.2, when $\theta \to 1$ household productivities are more dispersed, labour supply is more elastic, and monopsony power – and the associated wage markdown – declines. Results for $\Delta\theta$ and $\Delta(\sigma, \rho, \theta)$ under monopsony remain largely unchanged in both direction and magnitude relative to the baseline, indicating that changes in between-industry wage inequality are not primarily driven by variations in aggregate wage markdowns.

2.5.3. Skill-Biased Technological Change

Finally, the analysis turns to the systematic impact of Skill-Biased Technological Change (SBTC). The model can evaluate how simultaneous changes in capital-labour series affect between-industry wage inequality when the evolution of production technology parameters $(\alpha, \mu, \lambda, \rho, \sigma)$ and labour market concentration (θ) is held fixed. Given the re-estimation procedure, consider the following counterfactual setup:

$$\Delta var\Big(w(s) - \overline{w}\Big) = f\Big(\Phi_s\left(x, \tau_1\right), \Delta\Phi_s\left(x_{\tau_2}, -x_{\tau_1}\right) \bigm| \Theta_s\left(p, \tau_1\right)\Big) \qquad \text{(Model C)}$$

where the change in between-industry wage variance is a function of industry-specific factor series, $\Phi_s\left(x, \tau_1\right)$, and parameters, $\Theta_s(p, \tau_1)$, taken in period one, while

[37] These conclusions are robust when wage dispersion is examined separately for routine and non-routine tasks (Tables C.9 and C.10, Appendix C). Substitution elasticities exert a more balanced effect for routine workers, while sorting and segregation forces are relatively more important for non-routine workers.

considering one (or a combination of) series at the second period level while keeping fixed the others, $\Phi_s\left(x_{\tau_2}, -x_{\tau_1}\right)$ to first period.

Table C.11 presents the results. Column (2) reports the between-industry variance in the second period (τ_2) for both the data and the model, while columns (3)-(5) show, for each series in column (1), the implied wage variance, its share relative to the "all-series" model, and the fraction of the observed variance explained. Differences in routine workers alone substantially inflate wage inequality (2.62), with ICT capital contributing to a lesser extent (1.69). Combinations of both labour types, and in combination with ICT capital, account for more than the actual real log-wage variance (112-113%). Compared to Table 2.9, these "changing-series" shares of data-driven wage variance are considerably higher than when also including substitution elasticities and sorting/segregation effects. This implies that the "quantity effect" of the SBTC, by itself, tends to overstate between-industry wage inequality. Changes in capital and labour series are necessary but not sufficient to fully explain observed real log-wage variance: the key drivers are the heterogeneous structural parameters, which vary unevenly across industries. The next section illustrates this more clearly by recomputing between-industry wage inequality while fully neutralizing structural differences, isolating the contribution of factor input variations alone.

2.6. Factor Quantities and Productivity Changes

As a concluding step in the analysis to disentangle the main channels underlying U.S. between-industry wage inequality, I examine the extent to which divergences in production inputs matter when industries are constrained to share the same structural parameters. Taking τ_0 as the initial year of the sample (2003),

$$var\Big(w\left(s\right) - \overline{w}\Big) = f\Big(\Delta\Phi_s\left(x\right), \Phi_s\left(-x, \tau_0\right) \mid \Theta_{=\forall s}\left(p\right)\Big) \qquad \text{(Model D)}$$

Put differently, the exercise conjectures what the cross-industry variance in real log-wages would have been if only a selected set of inputs – capital types, labour types, or their combinations – had evolved over time, $\Delta\Phi_s\left(x\right)$, while all remaining inputs were held fixed at their initial levels, $\Phi_s\left(-x, \tau_0\right)$, and all industries were constrained to share common structural parameters, $\Theta_{=\forall s}\left(p\right)$. In the counterfactuals presented below, these parameters are set to their economy-wide means: $\mu = .503$, $\lambda =, 458$, and $\theta = 11.3$, as estimated via the MSM procedure, together with a physical-capital income share of $\alpha = .324$ and a price-markup parameter fixed at $\epsilon = 6$. Treating all industries as a single group, the elasticities of substitution are re-estimated and set to $\rho = .333$ and $\sigma = .595$.[38]

Table 2.10 presents a model-based decomposition of Table 2.7, following the framework in Model D. The counterfactual analysis reveals that shifts in industry-specific

[38] These values drawn Figure C.1. Associated standard errors are $Std.Err._{(\rho)} = .02$ and $Std.Err._{(\sigma)} = .03$, with 95% confidence bands of $[.301, .366]_{(\rho)}$ and $[.545, .645]_{(\sigma)}$. Results in Table 2.10 are robust to re-estimating all parameters for the full economy (Table C.13), rather than imposing the mean calibration from Table 2.6.

TABLE 2.10: MODEL VS. DATA COUNTERFACTUAL, SERIES AND MEAN PARAMETERS

		model $\mid \Delta\,\Phi\,(x)$						
							$\Delta\,(tfp)$	
	data	$\Delta\ell\,(rt)$	$\Delta\ell\,(nrt)$	$\Delta\,(\ell)$	$\Delta k\,(ict)$	$\Delta\,(tech)$	*mean*	*baseline*
WAGES, VARIANCE								
routine	2.285	.35	.16	.22	.24	.15	.25	.22
non-routine	2.314	.32	.12	.17	.23	.13	.21	.23
industry	2.299	.33	.14	.19	.23	.14	.23	.23

Quantification of Model D. Changes in variances in real log-wages in the model induced by variations in one or more series, keeping fixed the others, and imposing the mean parameters to be homogeneous across industries. $\Delta\,(\ell)$ refers to joint variations in routine and non-routine series, $\Delta\,(tech)$ is associated to simultaneous changes in both ICT capital and non-routine workers, while changes in estimated industry-specific Hicks-neutral exogenous total factor productivity (TFP), given eq. (2.14), are captured by $\Delta\,(tfp)$; TFP series are taken under the same (mean) parameters' values, or given the baseline calibration (Table 2.4.2). In all the columns, the variation is computed throughout the period-by-period percentage differential thus identifying overall changes in empirical trends implied both by the data and the model; same values differ in terms of decimals.

routine workers explain the largest portion of the variation, while dynamics in non-routine employment, $\Delta\ell\,(nrt)$, contribute little – even when combined with changes in ICT capital, $\Delta\,(tech)$. Divergent ICT capital across industries has a non-negligible effect, but it remains quantitatively minor in terms of observed real log-wage variance. Overall, industry-level differences in ICT adoption and in the composition of routine and non-routine employment account for roughly 6-15% of observed wage inequality across industries and tasks.[39]

Productivity differentials.– So far all the analysis has been centred on excluding total factor productivity (TFP). Such patterns cannot be directly observed in the data, but sectoral series can be recovered from the theoretical model. In fact, Proposition 1 allows to write the industry-level counterpart of eq. (2.9) as

$$y\,(s) = z\,(s)\; f\Big(k\,(phy,s)\,, k\,(ict,s)\,, \ell\,(rt,s)\,, \ell\,(nrt,s)\;\Big|\;\Theta_s\Big)$$

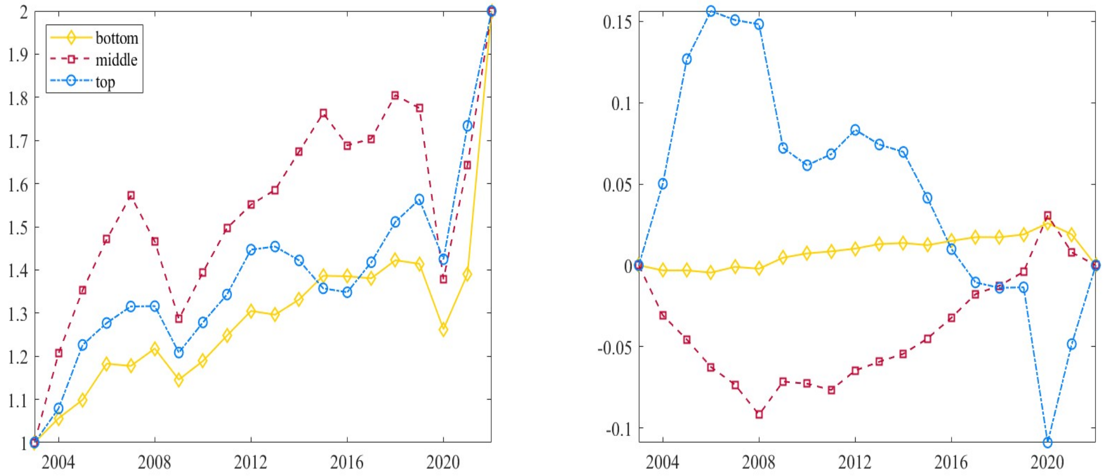
where $z\,(s)$ is an exogenous measure of product-augmenting Hicks-neutral TFP for industry-s, with $k\,(j,s)$ and $\ell\,(a,s)$, for $j = \{phy, ict\}$ and $a = \{rt, nrt\}$, being its quantities of capital and labour types, and Θ_s identifying the vector of calibrated and estimated parameters.[40] Using industry-by-industry data on the annual, seasonally adjusted value added extracted from BEA to measure output, $y\,(s)$, it is possible to recover the year-by-year series for sectoral productivity from

$$log\;\; z_t\,(s) = log\;\; y_t\,(s) - \alpha_s\; log\;\; k_t(phy,s) - (1-\alpha_s)\;\; log\;\; v_t\,(s) \tag{2.14}$$

where $v_t\,(s)$ comprises the relation among ICT capital, routine and non-routine

[39] A similar exercise, weighting each capital and labour series according to heterogeneous structural parameters (Table C.7), produces comparable results: routine labour and ICT capital slightly dominate, explaining approximately 5-20% of the variance. Yet, even when all series are held constant over time, the model only marginally overestimates variance, indicating that structural parameters play a more important role in determining the overall level of wage inequality across industries and tasks.

[40] Neutrality in the sense of Hicks (1932) implies that the marginal rate of substitution between capital and labour inputs is not altered, and hence including productivity does not pose threads to the parameters' identification strategy of Subsection 2.4.2, which may be confounded by other forms of neutrality (labour-augmenting Harrod-neutrality, or capital-augmenting Solow-neutrality).



(A) UNIQUE CALIBRATION, MEAN (B) DIFFERENCE WITH BASELINE

FIGURE 2.5: ESTIMATED PRODUCTIVITIES UNDER MEAN PARAMETERS

Note: the figure shows the estimated Hicks-neutral exogenous total factor productivity (TFP) measures estimated from the model. Panel 2.5a plots the series given a calibration where all the parameters are evenly set at the same *mean* values for all the industry-groups, while Panel 2.5b shows the difference between such series and the estimated TFP measures using the baseline calibration reported in Table 2.6. Series are scaled to be in the same range for graphical comparison.

workers along the parameters governing their associated weights, the different elasticities of substitution, and the degree of labour market concentration.

Estimated series for $s = \{bot, mid, top\}$ are shown in Figure 2.5. Panel 2.5a reports the estimated series under uniform industry-level calibrated parameters, while Panel 2.5b shows the differences relative to the baseline calibration from Table 2.6. Across both estimations and all industry groups, TFP rises over time but exhibits clear drops during the 2008 Great Financial Crisis and the 2020 COVID-19 shock. Differences between the two TFP measures are minimal, though slightly more pronounced for top and middle industries. The right-hand side of Table 2.10 reports the magnitude of shifts in industry-specific productivity for both calibrations. As with factor quantities, isolated changes in TFP have little explanatory power for the observed cross-industry variance in real log wages.

Overall, this section highlights the contribution of industry-specific patterns in capital, labour, and productivity to wage inequality, holding all structural parameters constant. Notably, both individual and combined shifts in these series account for only 6-15% of the observed variance, reinforcing the conclusion from Section 2.5 that cross-industry wage inequality is predominantly driven by heterogeneous structural parameters, both across industries and over time.

2.7. CONCLUSIONS

Over the past decades, U.S. wage inequality has risen sharply, drawing renewed attention on its origins. Evidence increasingly points to differences across industries as a dominant force, with structural transformations serving as a key mechanism to interpret observed changes in U.S. labour market and in its wage structure. This paper formalizes this intuition within a general equilibrium framework, featuring heterogeneous substitution between capital and labour types and a labour market characterized by concentration, which amplifies sorting and segregation effects due to imperfect labour mobility.

A structural estimation of the model successfully replicates the U.S. wage structure across industries. Critically, inequality is largely driven by structural differences between industries: trend-heterogeneous substitution elasticities between routine and non-routine workers emerge as the main determinant of between-industry wage variance, whereas variation in ICT-non-routine substitutability plays a secondary role. Combined, these channels explain 94% of the observed real log-wage variance. By contrast, when differences in structural parameters are neutralized, only 6-15% of observed wage variance can be accounted for through changes in capital, labour inputs, or sectoral TFP alone. These findings strongly suggest that differences in structural parameters across industries serve as the predominant force governing the observed trend of wage inequality.

The model also illuminates broader labour market dynamics. Trends in sorting and segregation, captured through labour market concentration, have a marginal effect on wage dispersion once structural transformations are considered. Variations in substitution elasticities and labour concentration together explain nearly 98% of industry-specific wage inequality. When combined with heterogeneous weights of capital and labour inputs, the model accounts for 88% of the between-industry wage variance. These conclusions remain robust even when the analysis is disaggregated to examine wage inequality separately for routine and non-routine workers.

Overall, the analysis underscores that wage inequality is primarily shaped by the reallocation of economic activity across industries, driven by differences in the substitutability between routine and non-routine tasks. Shifts in capital-labour substitutability are secondary, suggesting that structural changes on the labour side – how tasks are organized and how workers are employed – are the main drivers of the secular rise in the U.S. wage inequality.

Two elements emerge as fundamental: *(i)* structural technology parameters, and *(ii)* the composition of capital and labour within industries. While both are essential for understanding wage dynamics, the persistent increase in inequality appears largely driven by an "indirect" effect of Skill Biased Technological Change – specifically, the evolving substitutability across tasks and the reallocation of employment within industries. Further explanations on what determines structural differences in technological parameters are called for.

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Appendix

(Outline) *In this appendix I report all the material complementary to the main text; it is made of additional tables and figures, and of discussions on further analytical results. Section A complements and enriches the exposition of the empirical findings in Section 2.2; Section B derives all the salient elements of the structural general equilibrium model outlined in Section 2.3, while Section C consolidates and further discusses the calibration strategy and the counterfactual analysis of the model of Sections 2.4-2.5. A replication is in Section D under both monopsony and monopolistically competitive power of firms.*

A. MOTIVATING EVIDENCE: DATA, FIGURES AND TABLES

(Data details) *Data are for United States (US). For what concerns data from Bureau of Economic Analysis (BEA), private non-residential capital types are in net stocks evaluated at current costs by detailed industry dating back from 1925 to 2022.*[41] *Gross categories are "total equipment", "total structures", and "total intellectual property products", and each contains different types of assets according to their own National Income and Product Accounts (NIPA) asset type code; in total there are 96 different asset types. Data are provided for 74 industries at different layers.*[42] *Among all these types of capital assets I am going to classify them accordingly: digital equipment comprises all the electronic structures that are useful to process technological issues, that is "Mainframes", "PCs", "DASDs", "Printers", "Terminals", "Tape Drives", "Storage Devices", and "System Integrators"; the stock of intangible capital coincides with that of "Total Intellectual Property Products" (IPP);*[43] *while physical (or non-*

[41] Starting year is 1998 so that industries are classified with the latest available system, the U.S. 2017 NAICS; before, classification follows the U.S. Standard Industrial Classification (SIC) system. Assets' value is expressed in millions of U.S. dollars, and last update was in November 3, 2023.

[42] Classified according the BEA industry code system. Most are classified at 3-digit U.S. 2017 NAICS level (some 3-digit industries may fall in the same BEA code), four at 2-digit ("construction", "management of companies and enterprises", "educational services", and "other services, except government"), while five industries related to finance and insurance are at 4-digit level.

[43] A recent discussion analyses how the BEA collects data on IPP. Koh et al. (2020) argue that the effects of IPP capital on the U.S. labour share only emerged since 2013, when the BEA revised its methodology by re-classifying the notion of capital; now, the capital income comprises the rents arising from IPP investment. The authors compare the IPP effect on the labour share using both pre- and post-2013 data, finding out a negative effect of the rise in IPP on the U.S. labour share.

ICT) capital is the sum of all the remaining asset types. In addition to the aggregated capital types, I further define industry-specific ICT capital to be the combination of intangibles and digital equipment.[44]

BEA also collects data on wage levels of industries, together with their total employment. Herby, I extract also annual data on (seasonally adjusted) value added, wages and salaries, and of persons engaged in production (effective workforce) for each industry. Complementing the analysis with real wages, data on annual (seasonally adjusted) Consumer Pirce Index (CPI) for all items (indexed at 2015 = 100) is taken from Federal Reserve Economic Data (FRED) database. Not exploiting BLS data throughout, the time window considered is 1998-2022. Both capital stocks and real wages are taken in industry per capita log-terms.

Production does not occur with capital only. A thorough examination of wage dispersion necessitates a careful consideration of the composition of the labour force within each industry. To this end, reference is made to the U.S. Bureau of Labor Statistics (BLS)' Occupational Employment and Wage Statistics (OEWS) programme, which provides comprehensive data covering over one hundred occupational categories. Regarding the data extracted from BLS, information are available for a total of more than 80 industries at 3-digit U.S. 2017 NAICS level starting from 2003 onwards.[45] *At workers' level, each industry displays information on a set of several and granular occupations classified according to the U.S. Office of Management and Budget's Standard Occupational Classification (SOC) system, i.e., occupations are categorized based on the type of job and on required skills, and employees are assigned to an occupation based on the work they perform and not on their education or training. However, since detailed occupations may vary across industries, for my purposes I classify occupations by considering their "major" group membership:*[46] *I divide such major occupations in both routine and non-routine tasks. The latter group considers occupations such as "Management", "Business and Financial Operations", "Computer and Mathematical", "Architecture and Engineering", "Life, Physical, and Social Science", "Community and Social Service", "Legal", "Educational Training and Library", and "Arts, Design, Entertainment, Sports, and Media", while the left-outside occupations are comprised in the group of routine worker tasks.*

In the merging process (BEA and BLS), the 4-digit industries in the BEA data have been clustered into 3-digit category. The final dataset employed in the conducted analysis comprises information on a complete sample of 62 private-sector industries at the 3-digit U.S. 2017 NAICS system over an annual time window spanning from 2003 to 2022. This dataset forms the empirical foundation for the conduction of the main empirical analysis and subsequent investigation.

---

[44] Similar classification in Arvai and Mann (2022), as well coherent with Eden and Gaggl (2018).

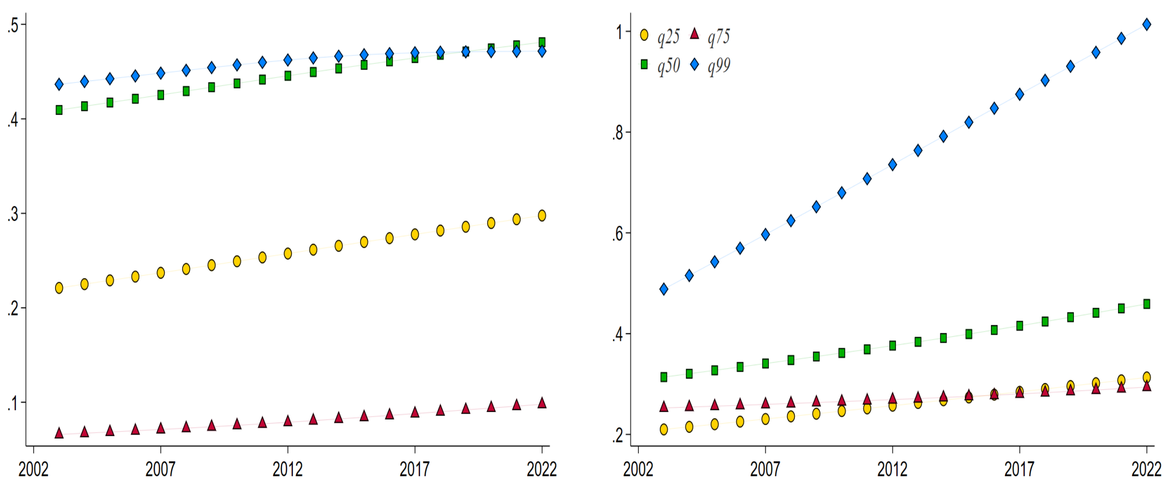
[45] The number of industries is variable year by year due to issues of data sampling. Moreover, prior 2003, these are classified according to the Standard Industrial Classification (SIC) system.

[46] This choice arises from the fact that there are missing group definitions for some more detailed occupations. In fact, the number of occupations in each industry is not fixed: there are from 80 to 180 types of occupations for each industry, and these are clustered in common 22 major groups.

TABLE A.1: REGRESSIONS, CAPITAL STOCKS

	$log(w(s))$			
	(1)	(2)	(3)	(4)
$\beta_{k^{phy}}$	.134***	.122**	.164***	.114***
	(3.48)	(2.94)	(9.05)	(3.52)
$\beta_{k^{ict}}$	.052*			.049*
	(2.00)			(2.27)
$\beta_{k^{int}}$		.057*		
		(2.03)		
$\beta_{k^{deq}}$		-.003		
		(-.26)		
$\beta_{k^{int}} \times \beta_{k^{deq}}$			.009**	.009**
			(2.99)	(2.80)
R^2	.461	.491	.287	.497

*t-statistics in parentheses. * (p<0.05), ** (p<0.01), *** (p<0.001). Analysis at 3/4-digit U.S. 2017 NAICS industries over 1998-2022 on N = 1650 observations. Fixed Effects (FE) regressions are of the form* $log\ w_t(s) = \beta_c + \beta_i \mathcal{X}_{i,t} + u_t$*, with* $\mathcal{X}_i$ *representing the different capital-labour ratios considered. Results are robust when controlling for the log size of industries, or when taking capital series directly in levels. Variables are in log format. Constant not reported to save space. Source: BEA and own calculations.*

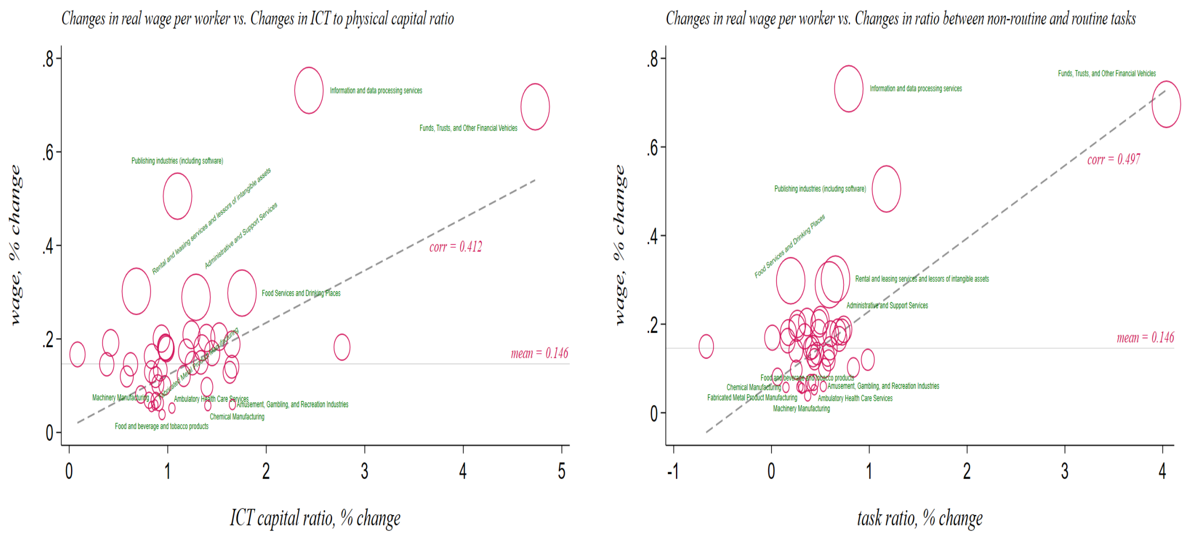


(A) ICT CAPITAL RATIO

(B) TASK RATIO

FIGURE A.1: CHANGES BY PERCENTILES

Note: each subplot draws the HP-filtered trend in ICT capital ratio (ICT capital stock in physical capital quantity, Panel A.1a), and in task ratio (fraction of non-routine workers of routine ones, Panel A.1b). Series are divided according the growth in group-specific industry wage (*i.e.*, total $\Delta_{\%}$ in industry wage per worker). Industries are classified at 3-digit U.S. 2017 NAICS level. *Source*: BEA, BLS and own calculations.



(A) Real wages vs. ICT capital ratio

(B) Real wages vs. task ratio

Figure A.2: Industry correlations

Note: each subplot of the figure represents the correlation of overall percentage change in industry-specific real log-wage per capita with both ICT capital ratio (ICT over non-ICT capital) in Panel A.2a, and task ratio (non-routine over routine workers) in Panel A.2b. The oblique (dashed-grey) line represents the fitting curve with its associated degree of correlation, while the horizontal (solid-black) line identifies the mean value of all industries' overall percentage change in real log-wage per capita. For a better graphical visualization, the ICT ratio takes constant the initial level of physical capital. Each circle is referred to a specific industry, and I report the label only for more and less virtuous (top and bottom 10%) industries; circles' size captures different groups of industries, each expressing the group's position in the distribution of overall percentage change in their real log-wage. Plots are referred to 3-digit U.S. 2017 NAICS industries over the period 2003-2022. *Source*: BEA, BLS and own calculations.

Table A.2: Combined regressions, percentage changes

	$\Delta\, log(w(s))$		
	(1)	(2)	(3)
$\beta_{\Delta k}$	.054***		.009
	(.002)		(.008)
$\beta_{\Delta \ell}$	.035*		.035***
	(.018)		(.007)
$\beta_{\Delta k \times \Delta \ell}$		.982	.919***
		(.649)	(.152)
R^2	.030	.021	.037
N	1178	1178	1178

*t-statistics in parentheses. * (p<0.05), ** (p<0.01), *** (p<0.001). Analysis at 3-digit U.S. 2017 NAICS industries over 2003-2022. Fixed Effects (FE) regressions are of the form $\Delta\, log\ w_t(s) = \beta_c + \beta_i \mathcal{X}_{i,t} + \delta_j \mathcal{Z}_{j,t} + u_t$, with $\mathcal{X}_i$ representing the percentage change in both ICT-to-physical capital and non-routine-over-routine workers ratios, and $\mathcal{Z}_j$ being a set of time-varying controls. Variables are in log format. Constant not reported to save space. Source: BEA, BLS and own calculations.*

TABLE A.3: COMBINED REGRESSIONS, LEVELS

	$log(w(s))$				
	Fe	*25th quantile*	*50th quantile*	*75th quantile*	*100th quantile*
β_k	.102**	.084***	.102***	.119***	.171*
	(.040)	(.029)	(.022)	(.027)	(.074)
β_ℓ	.299***	.269***	.300***	.328***	.416***
	(.078)	(.040)	(.030)	(.037)	(.104)
$\beta_{k\times\ell}$	.031	.029***	.031***	.034***	.041
	(1.89)	(.010)	(.008)	(.009)	(.025)

*Significance level at * (p<0.05), ** (p<0.01), *** (p<0.001). Standard error in parentheses. Analysis at 3-digit U.S. 2017 NAICS industries in 2003-2022 on* $N = 1240$ *observations. The Fixed Effects (FE) regression is of the form* $log\ w_t(s) = \beta_c + \beta_i\mathcal{X}_{i,t} + \delta_j\mathcal{Z}_{j,t} + u_t$, *with* $i = k, \ell$, *and* $\mathcal{Z}_j$ *being a set of controls, and associated with* $R^2 = .348$ *. Analogously, the conditional quantile regressions are then* $\mathcal{Q}_\omega\Big(log\ w_{\omega t}(s)\Big) = \beta_{i,\omega}\mathcal{X}_{i,\omega t} + \delta_{j,\omega}\mathcal{Z}_{j,\omega t} + u_{\omega t}$, *where* ω *represents each quantile (defined on the independent variable). Variables are all in log format. Constant not reported to save space, while quantile regressions do not have the constant term. Source: BEA, BLS and own calculations.*

TABLE A.4: COMBINED REGRESSIONS, STANDARD DEVIATIONS

	$sd\left[log(w(s))\right]$				R^2	
	(1)		(2)			
	$\beta_{sd(k)}$	β_ℓ	β_k	$\beta_{sd(\ell)}$	(1)	(2)
25th quantile	.256***	.070***	.008***	.403***		
	(.054)	(.015)	(.002)	(.010)		
50th quantile	.121***	.051***	.008***	.404***		
	(.034)	(.009)	(.002)	(.008)		
75th quantile	.031	.038***	.008***	.405***		
	(.039)	(.011)	(.002)	(.010)		
100th quantile	-.127	.015	.008	.408***		
	(.112)	(.024)	(.005)	(.021)		
Fe	.149***	.055***	.008***	.404***	.324	.793
	(.031)	(.010)	(.002)	(.007)		

*Significance level at * (p<0.05), ** (p<0.01), *** (p<0.001). Standard error in parentheses. Analysis at 3-digit U.S. 2017 NAICS industries in 2003-2022 on* $N = 1240$ *observations. The Fixed Effects (FE) regression is of the form* $sd\left[log\ w_t(s)\right] = \beta_c + \beta_i\mathcal{X}_{i,t} + \delta_j\mathcal{Z}_{j,t} + u_t$, *with* $i = k, \ell$, *and* $\mathcal{Z}_j$ *being a set of controls. Analogously, the conditional quantile regressions are then* $\mathcal{Q}_\omega\Big(sd\left[log\ w_{\omega t}(s)\right]\Big) = \beta_{i,\omega}\mathcal{X}_{i,\omega t} + \delta_{j,\omega}\mathcal{Z}_{j,\omega t} + u_{\omega t}$, *where* ω *represents each quantile (defined on the independent variable). Variables are all in log or sd format. Constant not reported to save space, while quantile regressions do not have the constant term. Source: BEA, BLS and own calculations.*

(Group decomposition) *Total change in wage inequality can be written also as*

$$
\begin{aligned}
\underbrace{\Delta\,\widetilde{var}\Big(w_t(s)-\overline{w}_t\Big)}_{\text{total, wages}} = \;& \underbrace{\left(\frac{\ell_0(g)}{\ell_0}\right)\left[\Delta\,\widetilde{var}\Big(w_t(s\in g)-\overline{w}_t(g)\Big)\right]}_{\text{within-group, wages}} \\
& + \underbrace{\sum_g \left[\Delta\,\widetilde{var}\Big(\ell_t(s)-\overline{\ell}_t(g)\Big)\right]\widetilde{var}\Big(w_0(s)-\overline{w}_0(g)\Big)}_{\text{between-groups, employment}} \\
& + \underbrace{\sum_g \left[\Delta\,\widetilde{var}\Big(w_t(s)-\overline{w}_t(g)\Big)\right]\left[\Delta\,\widetilde{var}\Big(\ell_t(s)-\overline{\ell}_t(g)\Big)\right]}_{\text{between-groups, interaction}} \qquad \text{(A.1)} \\
& + \underbrace{\underbrace{\sum_{\mathcal{G}_{/g}} \left(\frac{\ell_0\left(\mathcal{G}\right)}{\ell_0}\right)\left[\Delta\,\widetilde{var}\Big(w_t(s)-\overline{w}_t\left(\mathcal{G}\right)\Big)\right]}_{\text{within-other groups, wages}}
+ \underbrace{\sum_g \left[\Delta\left(\frac{\ell_t(g)}{\ell_t}\right)\widetilde{var}\Big(\overline{w}_t(g)-\overline{w}_t\Big)\right]}_{\text{between groups, wages}}}_{\text{residual}}
\end{aligned}
$$

where industries are partitioned in $g \in \mathcal{G}$ groups, and variances are employment-weighted. Subscript $t = 0$ indicates the initial level (starting year of the sample). Total change in variance can be decomposed in several components: (i) rise in variance for a particular group g; (ii) reallocation of employment across groups, keeping constant the variance of each group at its base level; (iii) cross-changes of wages and employment; (iv) rising variance within all other groups but g; and (v) rising variance between groups. Quantification of each component is in Table A.5; this decomposition is borrowed from Kleinmann (2023).

TABLE A.5: DECOMPOSITION OF THE RISE IN WAGE INEQUALITY

	industry group g				
	(1)	(2)	(3)	(4)	(5)
share of the increase, wage variance	*tails*	*middle*	*services*	*manuf.*	*other*
rising variance within the group	79%	32%	58%	11%	27%
employment reallocation across groups	34%	34%	17%	51%	10%
comovement (variance, employment)	7%	7%	3%	4%	5%
residual	-20%	27%	-22%	-34%	58%
total change across all industries	*100%*	*100%*	*100%*	*100%*	*100%*

Estimates of each component in eq. (A.1) for 3-digit U.S. 2017 NAICS industries between 2003 and 2022 related to $\log(w(s))$. *Operator* Δ *in the equation is* $x_t - x_{t-1}$, *and not a percentage change. The first row shows the share of total increase in variance due to rising variance in the group of industries; the second row shows the share due to changes in employment between that group and the other industries in the sample (employment reallocation), holding constant the change in variance in each group; the third row shows the share that is due to the cross-product of rising variance and rising employment share; the fourth row is so that the sum for each column is 100%. "tails" and "middle" are referred to overall percentage changes distribution in industry real wage per capita; "manuf." stands for manufacturing industries, while "other" does not consider services and manufacturing industries. Source: BEA and own calculations.*

(A) ROUTINE WORKERS

(B) NON-ROUTINE WORKERS

FIGURE A.3: LABOUR SHARES PATTERN

Note: the figure shows the fitted values and the associated linear trend for the labour share of both routine (Panel A.3a) and non-routine (Panel A.3b) workers. The fitted values are extracted from a Fixed Effects (FE) regression for both the labour share measures with year and industry fixed effects. *Source*: BLS and own calculations.

TABLE A.6: INDUSTRY CONTRIBUTION TO WAGE VARIANCE GROWTH

industry	***contribution share*** (%)	***group***
Administrative and Support Services	0.01	*top*
Agriculture	0.12	*top*
Ambulatory Health Care Services	0.23	*bottom*
Amusement, Gambling, and Recreation industries	0.62	*bottom*
Chemical Manufacturing	-0.26	*bottom*
Computer and Electronic Product Manufacturing	0.94	*top*
Fabricated Metal Product Manufacturing	0.14	*bottom*
Federal Reserve banks, Credit Intermediation, and related activities	0.47	*top*
Food Services and Drinking Places	2.36	*top*
Food and Beverage Stores	1.34	*bottom*
Food and Beverage and Tobacco Products	0.39	*bottom*
Funds, Trusts, and Other Financial Vehicles	0.10	*top*
General Merchandise Stores	1.29	*bottom*
Information and Data Processing Services	5.90	*top*
Machinery Manufacturing	-0.02	*bottom*
Management of Companies and Enterprises	2.71	*top*
Oil and Gas Extraction	0.26	*top*
Other Services, except government	0.43	*top*
Other Transportation and Support Activities	0.67	*bottom*
Paper Manufacturing	-0.02	*bottom*
Performing Arts, Spectator Sports, Museums, and related activities	-0.02	*top*
Printing and Related Support Activities	0.08	*bottom*
Professional, Scientific, and Technical Services	11.7	*bottom*
Publishing industries (including software)	2.23	*top*
Rail Transportation	-0.10	*bottom*
Real Estate	-0.04	*top*
Rental and Leasing Services and Lessors of Intangible Assets	-0.08	*top*
Securities, Commodity Contracts, and Other Financial Investments and Related Activities	8.59	*top*
Transportation Equipment Manufacturing	1.17	*bottom*
Warehousing and Storage	0.86	*bottom*

Contribution to between-industry wage variance growth of industries comprised in top and bottom (in terms of overall growth in real log-wage per capita, as reported by Table 2.1) groups over the period 2003-2022. Variance contribution follows the definition proposed in eq. (2.1). Industries are at 3-digit U.S. 2017 NAICS level. Source: BEA and own calculations.

B. Model derivation

(Household inter-temporal problem) *The household utility problem in eq. (2.5) can be rewritten inter-temporally, future-discounted by factor β, as*

$$\max_{\mathcal{C}_t^i,\, b_{t+1}^i,\, \{k_{t+1}^i(j)\}_{\forall j}} \mathcal{U}_{h,t}^i(a,s) = \sum_{t=0}^{\infty}\left[\beta^t \log\left(\mathcal{C}_t^i\right)\right] + \sum_{t=0}^{\infty}\left[\log \mathcal{B}_{h,t}(a,s)\right] + \wp_h^i(a,s)$$

$$\begin{aligned} s.t.\quad & \mathcal{C}_t^i + I_t^i(phy) + I_t^i(ict) + b_{t+1}^i - (1+r_t)\, b_t^i = \\ & = w_{h,t}(a,s)\ell_h^i(a,s) + R_t(phy)\, k_t^i(phy) + R_t(ict)\, k_t^i(ict) + \mathcal{D}_t^i \\ with\quad & k_{t+1}^i(phy) = \frac{I_t^i(phy)}{\zeta_t^i} + \left(1-\delta_{phy}\right) k_t^i(phy) \\ and\quad & k_{t+1}^i(ict) = \frac{I_t^i(ict)}{\zeta_t^i} + (1-\delta_{ict})\, k_t^i(ict) \end{aligned}$$

where the investment schedule comprises both assets, b_t^i (which are in zero net supply, thus exchanged only across households), and physical and ICT capital holdings, $k_t^i(j)$, with $j = \{phy, ict\}$. Household-specific productivity when working as type-a for firm-h in industry-s is denoted with $\wp_h^i(a,s)$. Each household inelastically supplies one unit of work, so that $\ell_h^i(a,s) = 1$. The firm benefit of household-a is defined to be $\mathcal{B}_{h,t}(a,s) = \left[g_{h,t}(a,s)\right]^{-\varsigma}$, where $g_{h,t}(a,s)$ reflects the relative share (i.e., the task ratio) of a given task, which is negatively scaled by the elasticity parameter ς, while $\mathcal{D}_t^i$ is the share of firms' profits that goes to household-i. Capital stock of household-i depreciates at a rate $\delta_{(j)}$ and accumulates over time by a law of motion which is function of $\zeta_{i,t}$, namely the quantity of ICT capital relative to that of non-ICT capital, that enters negatively in new capital investment, $I_t^i(j)$, under the idea that as the stock of capital owned by household-i becomes more sophisticated (larger ICT relative to non-ICT capital when ζ_t^i increases), it is required an higher rate of capital investment to keep constant the future stock of given capital type. Modelling the dynamics of capital in this way serves only to derive analytical solutions for the equations (2.12) and (2.13) estimating the pair (ρ,σ) of Section 2.4.

Utility maximization implies the Lagrangian function to be

$$\begin{aligned} \mathcal{L}_{\mathcal{C}_t^i,\, b_{t+1}^i,\, \{k_{t+1}^i(j)\}_{\forall j}} = & \sum_t \beta^t\left[\log \mathcal{C}_t^i\right] + \sum_t\left[\log \mathcal{B}_{h,t}(a,s)\right] + \wp^i(a,s) + \\ & - \sum_t \beta^t \psi^t\Big[\mathcal{C}_t^i + I_t^i(phy) + I_t^i(ict) + \\ & + b_{t+1}^i - (1+r_t)\, b_t^i - w_{h,t}(a,s) + \\ & - R_t(phy)\, k_t^i(phy) - R_t(ict)\, k_t^i(ict) - \mathcal{D}_t^i\Big] \end{aligned}$$

with ψ^t being the penalty multiplier. Optimality conditions are in order

$$\frac{1}{C_t^i} = \beta^t\,(1 + r_{t+1})\,\frac{1}{C_{t+1}^i}$$
$$R_t = (1 + r_{t+1})\,\zeta_t - (1-\delta)\zeta_{t+1}$$

The first is the usual Euler condition, which displays future path of consumption. The second implies that, aggregating across households, the path on interest rates on capital types, $R_t(phy) = R_t(ict) \equiv R_t$, is linked to the path of aggregate relative quantity of ICT capital, $\zeta_t = \int_i \zeta_t^i \,\mathrm{d}i$, when $\delta_{phy} = \delta_{ict} \equiv \delta$;[47] *in other words, given the constancy of both the discount factor and the capital depreciation rate(s), changes across states of the capital rental rate are determined by changes in the relative quantities across capital types. In the spirit of Karabarbounis and Neiman (2014), eq. (2.6) determines that investing in capital types is profitable as long as the marginal benefit of investment (the capital rental rate) is at least lower than its marginal cost (interest rate, r_t and depreciation rate, δ).*

□

(Labour supply derivation) *The probability of worker-a choosing firm-$h = 1$ in industry-$h = 1$ can be written as*

$$\Big(\mathcal{P}_1(a,1) \equiv\Big) \Longleftrightarrow \mathcal{P}(a,1) \qquad \text{for notation purposes, abstract from firm subscript}$$

$$\begin{aligned}\mathcal{P}(a,1) &= Pr\Big[w\,(a,1)\,\mathcal{B}\,(a,1)\,\wp^i\,(a,1) > w(a,s)\mathcal{B}(a,s)\wp^i(a,s)\Big],\ \forall s \neq 1\\ &= Pr\left[\frac{w\,(a,1)\,\mathcal{B}\,(a,1)}{w(a,s)\mathcal{B}(a,s)}\wp^i\,(a,1) > \wp^i(a,s)\right],\ \forall s \neq 1\end{aligned}$$

For $s \in [2, S]$, the partial derivative of $\mathcal{P}(a,1)$ with respect to $\wp^i\,(a,1)$ is

$$\left\{\wp^i\,(a,1)\,,\ \frac{w\,(a,1)\,\mathcal{B}\,(a,1)}{w\,(a,2)\,\mathcal{B}\,(a,2)}\wp^i\,(a,1)\,,\ \ldots,\ \frac{w\,(a,1)\,\mathcal{B}\,(a,1)}{w(a,S)\mathcal{B}\,(a,S)}\wp^i\,(a,1)\right\}$$

so that $\mathcal{P}(a,1)$ can be re-written as

$$\begin{aligned}\mathcal{P}(a,1) &= F_{\wp^i(a,1)}\Big(\wp^i\,(a,1)\,,\ \alpha_2\wp^i\,(a,1)\,,\ \ldots,\ \alpha_s\wp^i\,(a,1)\Big),\ \forall\wp^i(a,s)\\ &= \int F_{\wp^i(a,1)}\Big(\wp^i\,(a,1)\,,\ \alpha_2\wp^i\,(a,1)\,,\ \ldots,\ \alpha_s\wp^i\,(a,1)\Big)\mathrm{d}\wp^i\,(a,1)\end{aligned}$$

for $\alpha_s = \frac{w(a,1)\mathcal{B}(a,1)}{w(a,s)\mathcal{B}(a,s)}$. Now, recall that the parameter $\wp^i(a,s)$ is drawn from a multivariate Frechét-type cumulative distribution,

[47] Alternatively, the common depreciation rate might be a weighted average of all the capital-types' depreciation rates.

$$F_i\Big(\wp^i_{h,\ldots,H}\,(a,1)\,,\ldots,\wp^i_{h,\ldots,H}(a,s),\ldots,\wp^i_{h,\ldots,H}\,(a,S)\,\Big) = exp\left[-\sum_s\left(\int_h \wp^i_h(a,s)\,\mathrm{d}h\right)^{-\theta}\right]$$

which becomes, in this framework (i.e., not considering firm's notation),

$$\begin{aligned}F\Big(\alpha_1\wp^i\,(a,1)\,,\ldots,\alpha_s\wp^i(a,s),\ldots,\alpha_S\wp^i\,(a,S)\,\Big) &= exp\left[-\sum_s\alpha_s^{-\theta}\,\sum_s\wp^i(a,s)^{-\theta}\right]\\ &= exp\left[-\sum_s\Big(\alpha_s\,\wp^i(a,s)\Big)^{-\theta}\right]\end{aligned}$$

Taking its derivative with respect to $\wp^i(a,s)$ *turns to write that*

$$F_{\wp^i(a,1)}\Big(\wp^i\,(a,1)\,,\ \alpha_2\wp^i\,(a,1)\,,\ \ldots,\ \alpha_s\wp^i\,(a,1)\,\Big) = -\,(-\theta)\,\wp^i(a,s)^{-\theta-1}exp\left[\overline{\alpha}\wp^i(a,s)^{-\theta}\right]$$

with $\overline{\alpha} = \sum_s\alpha_s^{-\theta}$*. Evaluating the integral in* $\mathcal{P}\,(a,1)$ *yields to*

$$\mathcal{P}\,(a,1) = \int\theta\,\underbrace{\wp^i(a,s)^{-\theta-1}}_{A}\,\underbrace{exp\left[\overline{\alpha}\wp^i(a,s)^{-\theta}\right]}_{B}\,\mathrm{d}\wp^i(a,s)$$

By multiplying and dividing by $\overline{\alpha}$ *(so that* A *can be integrated with* B*), one gets*

$$\begin{aligned}\mathcal{P}\,(a,1) &= \frac{\overline{\alpha}}{\overline{\alpha}}\int\theta\wp^i(a,s)^{-\theta-1}exp\left[\overline{\alpha}\wp^i(a,s)^{-\theta}\right]\mathrm{d}\wp^i(a,s)\\ &= \frac{1}{\overline{\alpha}}\int\overline{\alpha}\theta\wp^i(a,s)^{-\theta-1}exp\left[\overline{\alpha}\wp^i(a,s)^{-\theta}\right]\mathrm{d}\wp^i(a,s)\\ &= \frac{1}{\overline{\alpha}}\int\mathrm{d}F\Big(\wp^i\,(a,1)\,,\ldots,\wp^i(a,s),\ldots,\wp^i(a,S)\Big)\\ &= \frac{1}{\overline{\alpha}}\\ &= \frac{1}{\sum_s\alpha_s^{-\theta}}\end{aligned}$$

By recalling that $\alpha_s = \frac{w(a,1)\mathcal{B}(a,1)}{w(a,s)\mathcal{B}(a,s)}$*, it is possible to obtain that,* $\forall s \neq 1$*,*

$$\mathcal{P}\,(a,1) = \frac{\Big(w\,(a,1)\,\mathcal{B}\,(a,1)\,\Big)^{\theta}}{\Big(\sum_s w(a,s)\mathcal{B}(a,s)\Big)^{\theta}}$$

where, including firm's subscript-h*, it becomes*

$$\mathcal{P}_1\,(a,1) = \frac{\Big(w_1\,(a,1)\,\mathcal{B}_1\,(a,1)\,\Big)^{\theta}}{\Big(\sum_{h,s} w_h(a,s)\mathcal{B}_h(a,s)\Big)^{\theta}}$$

Taking in general notation, $\forall h \in [1,H]$ *and* $\forall s \in [1,S]$*, it can be written as*

$$\mathcal{P}_h(a,s) = \frac{\Big(w_h(a,s)\mathcal{B}_h(a,s)\Big)^{\theta}}{\Big(\sum_{h,s} w_h(a,s)\mathcal{B}_h(a,s)\Big)^{\theta}} \quad , \quad with \quad \mathcal{B}_h(a,s) = \big[g_h(a,s)\big]^{-\varsigma}$$

which denotes the fraction of type-a households choosing to work in firm-h, industry-s as in eq. (2.7).

To interpret the role of θ – the shape parameter of the Frechét distribution –, Figure B.1 plots the distribution of productivities under different values of θ, interpreted as being the degree of dispersion of households-specific efficiencies in working in firm-h in industry-s. Basically, the larger is the value of θ the lower is the variability of the productivity distribution (i.e., the lower is the dispersion of households' productivities), thus the higher is the degree of labour market concentration of workers across firms and industries.

Moreover, as explained in the main text to interpret eq. (2.7), "the number of workers of a given task-a willing to be employed in a certain firm-industry pair is determined by different optimal wage levels by firms in industries (wage premium), jointly with workforce composition in terms of both the firm-specific relative amount of workers of a certain type (sorting) and workers having the same efficiency in conducting that job task content (segregation). In this sense, workers in the same task are "highly rival factors" (Hicks 1932) since they can be freely substituted for one another, and this possibility ensures that high-wage workers do sort in high-wage firms and industries, and also that more efficient people would be preferred to get a job in high-wage workplaces than ones who are less productive".[48]

□

(Firm optimization) *Given $y_h(s)$ being the production function in eq. (2.9), and the firm's conditional demand given by eq. (2.8), the problem of a monopolistically competitive firm (h,s) choosing capital and labour endowments is*

---

[48] To build intuition, such idea can be formalized using an Horvath (2000)'s aggregator: the aggregate type-a workforce of industry-s can be taught as a CES aggregator of firm-specific labour measures of eq. (2.7),

$$\ell(a,s) = \left[\int_h \Big(\ell_h(a,s)\Big)^{\frac{1+\psi}{\psi}} \mathrm{d}h\right]^{\frac{\psi}{1+\psi}}$$

since each worker-i is endowed with $\ell^i_h(a,s) = 1$ unit inelastically supplied. Here, as $\psi \to \infty$, type-a workers become perfect substitutes; conversely, for any given value of parameter ψ comprised in $[0,\infty)$ workers won't be freely substituted across firms in the same industry. As a natural consequence, each worker would work in the firm (h,s) paying the highest wage according to its $\wp^i_h(a,s)$. More, at the intensive margin, firms in the same industry would pay the same wage level.

This is a crucial aspect to consider when moving from firm to industry dimension: free labour mobility within an industry – materialized when $\psi = \infty$ – allows to impose firms' optimal wages to be even, so to have a unique industry-s wage in equilibrium; refer to Proposition 1 and Remark 2.

Analogously, worker-a wage can be seen as $w(a,s) = \left[\int_h \Big(w_h(a,s)\Big)^{\frac{1+\psi}{\psi}} \mathrm{d}h\right]^{\frac{\psi}{1+\psi}}$ with $\psi = \infty$.

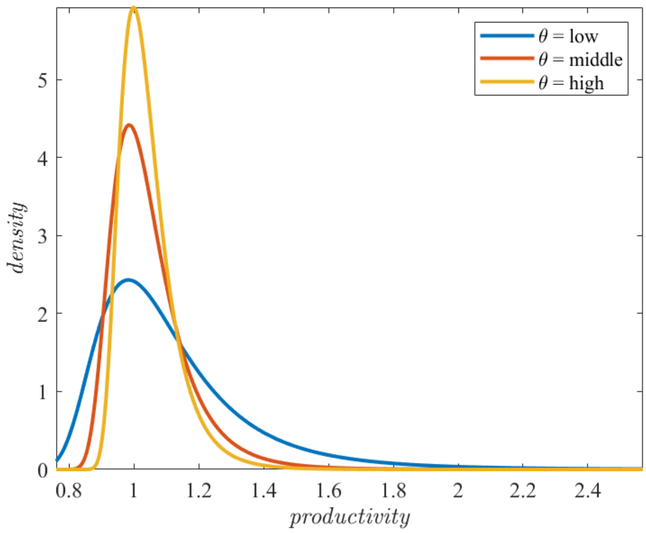


FIGURE B.1: VARIABILITY AND THE SHAPE PARAMETER

Note: the plot represents how the distribution of an arbitrary variable (*e.g.*, productivity) changes along different values of the shape parameter, keeping unchanged the scale parameter; lower shape parameter results in major variability.

$$\max_{p_h(s),\{k_h(j,s)\}_{\forall j},\{\ell_h(a,s)\}_{\forall a}} p_h(s)y_h(s) - \left(\sum_j R(j)\,k_h(j,s) + \sum_a w_h(a,s)\,\ell_h(a,s)\right)$$

$$s.t. \quad y_h(s) = \left(\frac{p_h(s)}{p(s)}\right)^{\epsilon} y(s)$$

with $a = \{rt, nrt\}$ *and* $j = \{phy, ict\}$. *The inter-temporal Lagrangian function for firm-*h *industry-*s *takes the form of*

$$\begin{aligned}\mathcal{L}_{p_h(s),\{k_h(j,s),\ell_h(a,s)\}_{\forall j,a}} = p_h(s)y_h(s) - \Big(&R(phy)\,k_h(phy,s) + R(ict)\,k_h(ict,s) + \\ &+ w_h(rt,s)\,\ell_h(rt,s) + w_h(nrt,s)\,\ell_h(nrt,s)\Big) + \\ &- \psi^t\left[y_h(s)p_h(s)^{\epsilon} - y(s)\right]\end{aligned}$$

with ψ^t *being the penalty multiplier. Optimality conditions are in order*

$$\frac{\partial\mathcal{L}}{\partial p_h(s)} : \psi = \frac{1}{\epsilon}p_h(s)^{1-\epsilon}$$

$$\frac{\partial\mathcal{L}}{\partial k_h(phy,s)} : p_h(s)f_{k_h(phy,s)} = \mathcal{M}^{\epsilon}R(phy)$$

$$\frac{\partial\mathcal{L}}{\partial k_h(ict,s)} : p_h(s)f_{k_h(ict,s)} = \mathcal{M}^{\epsilon}R(ict)$$

with the price mark-up being $\mathcal{M}^{\epsilon} = \frac{\epsilon}{\epsilon-1}$, *and first order conditions of firm-industry* *specific output relative to capital-types are*

$$f_{k_h(phy,s)} = \alpha \Big(k_h(phy,s)\Big)^{\alpha-1} \left[y_h(s)\Big(k_h(phy,s)\Big)^{-\alpha}\right]$$

$$f_{k_h(ict,s)} = (1-\alpha)(1-\mu)\lambda \left[\Big(k_h(phy,s)\Big)^{\alpha} \mathcal{V}_h\,(s)^{\frac{1-\alpha-\varsigma}{\varsigma}}\, \mathcal{Q}_h\,(s)^{\frac{\varsigma-\varrho}{\varrho}}\right] \Big(k_h(ict,s)\Big)^{\varrho-1}$$

with

$$\mathcal{V}_h\,(s) = \mu\Big(\ell_h(rt,s)\Big)^{\varsigma} + (1-\mu)\,\mathcal{Q}_h\,(s)^{\frac{\varsigma}{\varrho}}$$

and

$$\mathcal{Q}_h\,(s) = \lambda\Big(k_h(ict,s)\Big)^{\varrho} + (1-\lambda)\Big(\ell_h(nrt,s)\Big)^{\varrho}$$

Note how aggregate capital rental rates, $R_t\,(j)$, are obtained by aggregating marginal product of capital types, $f_{k_h(j,s)}$, across firms and industries.

Optimality conditions for wages are found by deriving directly for $\ell_h(a,s)$; this results in computing what follows.

$$\frac{\partial \mathcal{L}}{\partial \ell_h(rt,s)} \;:\; p_h(s) f_{\ell_h(rt,s)} = \mathcal{M}^{\epsilon}\, w_h(rt,s)$$
$$\frac{\partial \mathcal{L}}{\partial \ell_h(nrt,s)} \;:\; p_h(s) f_{\ell_h(nrt,s)} = \mathcal{M}^{\epsilon}\, w_h(nrt,s)$$

with the price mark-up still being $\mathcal{M}^{\epsilon} = \frac{\epsilon}{\epsilon-1}$, and first order conditions of firm (h,s)'s output relative to labour-types are

$$f_{\ell_h(rt,s)} = (1-\alpha)\mu\Big(k_h(phy,s)\Big)^{\alpha} \mathcal{V}_h\,(s)^{\frac{1-\alpha-\varsigma}{\varsigma}} \Big(\ell_h(rt,s)\Big)^{\varsigma-1}$$

$$f_{\ell_h(nrt,s)} = (1-\alpha)(1-\mu)(1-\lambda)\Big(k_h(phy,s)\Big)^{\alpha} \mathcal{V}_h\,(s)^{\frac{1-\alpha-\varsigma}{\varsigma}}\, \mathcal{Q}_h\,(s)^{\frac{\varsigma-\varrho}{\varrho}} \Big(\ell_h(nrt,s)\Big)^{\varrho-1}$$

with

$$\mathcal{V}_h\,(s) = \mu\Big(\ell_h(rt,s)\Big)^{\varsigma} + (1-\mu)\,\mathcal{Q}_h\,(s)^{\frac{\varsigma}{\varrho}}$$

and

$$\mathcal{Q}_h\,(s) = \lambda\Big(k_h(ict,s)\Big)^{\varrho} + (1-\lambda)\Big(\ell_h(nrt,s)\Big)^{\varrho}$$

Now, note that labour supplies for both types of tasks $a = \{rt, nrt\}$ are all determined by eq. (2.7), namely $\ell_h(a,s) = f\Big(w_h(a,s),\, \mathcal{B}_h(a,s),\, \mathcal{W}_{\mathcal{H}}(a,\mathcal{S}),\, \mathcal{B}_{\mathcal{H}}(a,\mathcal{S})\Big)$. Therefore, equating demand and supply of labour for each task-a results in deriving the associated optimal wage level in general equilibrium, that is

$$\begin{cases} p_h(s) f_{\ell_h(rt,s)} = \mathcal{M}^{\epsilon}\, w_h(rt,s) \\ \\ \ell_h(rt,s) = \left(\dfrac{w_h(rt,s)\mathcal{B}_h(rt,s)}{\sum_{h,s} w_h(rt,s)\mathcal{B}_h(rt,s)} \right)^{\theta} \end{cases}$$

and

$$\begin{cases} p_h(s) f_{\ell_h(nrt,s)} = \mathcal{M}^{\epsilon}\, w_h(nrt,s) \\ \\ \ell_h(nrt,s) = \left(\dfrac{w_h(nrt,s)\mathcal{B}_h(nrt,s)}{\sum_{h,s} w_h(nrt,s)\mathcal{B}_h(nrt,s)} \right)^{\theta} \end{cases}$$

By writing down the extensive forms for each derivative of $\ell_h(a,s)$ and exploiting the calculations to solve for $w_h(a,s)$, together with proposition 1, optimal wages are those reported in eq. (2.10). Note that $\mathcal{V}_h(s)$ expresses the substitutability between routine workers $(\ell_h(rt,s))$ and the ICT composite good, while $\mathcal{Q}_h(s)$ identifies the substitutability between ICT capital $(k_h(ict,s))$ and non-routine workers $(\ell_h(nrt,s))$, considering each firm-$h \in \mathcal{H}$ in industry-s.

Finally, firm (h,s) profits can be found by including equilibrium optimality conditions for both capital and wages in

$$\mathcal{D}_h(s) = p_h(s) y_h(s) - \left(\sum_j R(j) k_h(j,s) + \sum_a w_h(a,s)\ell_h(a,s) \right)$$

where $j = \{phy, ict\}$ identifies the types of capital in the economy.

□

(Proof of Proposition 1) *This heuristic proof is centred around the definition of the workers' measures as given by eq. (2.7) with no worker-a benefit, $\mathcal{B}_h(a,s) = 1$, $\forall a,h,s$. Imagine an economy in which there is only one industry-$h \in \mathcal{S} = 1$ populated by two firms, $\{h,h'\} \in \mathcal{H}$, and define $\mathcal{W}_{\mathcal{H}}(a,s) = w_h(a,s) + w_{h'}(a,s)$. Then:*

(a) when firms are assumed to be homogeneous in their size, i.e., when $\ell_h(a,s) \equiv \ell_{h'}(a,s)$, then they should set the same optimal wage level since

$$\left(\frac{w_h(a,s)}{\mathcal{W}_{\mathcal{H}}(a,s)} = \right) \ \ell_h(a,s) \equiv \ell_{h'}(a,s) \ \left(= \frac{w_{h'}(a,s)}{\mathcal{W}_{\mathcal{H}}(a,s)} \right)$$

holds only if $w_h(a,s) = w_{h'}(a,s)$.

(b) if firms are heterogeneous in size eq. (2.7) predicts how, in order to have $\ell_h(a,s) \neq \ell_{h'}(a,s)$, it is necessary that wage levels are different, $w_h(a,s) \neq w_{h'}(a,s)$. Assume that firm (h',s) sets a wage rate higher than firm (h,s), so that the former is larger than the latter. Assume further an increase in the wage chosen by firm (h',s), labelling the new level as $w_{h'}(a,s)'$, while the other wage remains unchanged. Henceforth, it must be the case that

$$w_{h'}(a,s) \to w_{h'}(a,s)', \quad \text{so that} \quad \mathcal{W}_{\mathcal{H}}(a,s) < \mathcal{W}_{\mathcal{H}}(a,s)'.$$

For such firm, clearing the condition

$$\mathcal{W}_{\mathcal{H}}(a,s)' = \frac{w_{h'}(a,s)'}{\ell_{h'}(a,s)'} \tag{B.1}$$

means that there should be a related increase in its workforce, $\frac{\partial \ell_{h'}(a,s)}{\partial w_{h'}(a,s)} > 0$*, so that* $\ell_{h'}(a,s)' > \ell_{h'}(a,s)$*. Different scenario happens to the employees level of firm* (h,s)*: it turns out that* $\ell_h(a,s)' < \ell_h(a,s)$ *when* $w_{h'}(a,s) \to w_{h'}(a,s)'$ *since* $\frac{\partial \ell_h(a,s)}{\partial w_{h'}(a,s)} < 0$*. The implied mechanism is just*

$$\frac{\partial \ell_{h'}(a,s)}{\partial w_{h'}(a,s)} = -\frac{\partial \ell_h(a,s)}{\partial w_{h'}(a,s)}$$

However, the given change in the wage of firm (h',s) *not only has an impact on* $\ell_{h'}(a,s)$*, but it indirectly translates to the wage level of firm* (h,s) *due its negative effect on* $\ell_h(a,s)$*. The firm* (h,s) *version of the condition in eq. (B.1)*

$$\mathcal{W}_{\mathcal{H}}(a,s)' = \frac{w_h(a,s)'}{\ell_h(a,s)'} \tag{B.2}$$

implies that, after an increase in $\mathcal{W}_{\mathcal{H}}(a,s)$ *and a decrease in* $\ell_h(a,s)$ *due to a positive change in* $w_{h'}(a,s)$*, then the wage level of firm* (h,s) *must increase as well. It turns out that an increase in the wage level of firm* (h',s) *must determine an equal increase in the right-hand-side of both eqs. (B.1)-(B.2), which means*

$$\frac{w_h(a,s)'}{\ell_h(a,s)'} = \frac{w_{h'}(a,s)'}{\ell_{h'}(a,s)'}$$

so that $\frac{\partial w_h(a,s)}{\partial \ell_h(a,s)} = \frac{\partial w_{h'}(a,s)}{\partial \ell_{h'}(a,s)}$.

To sum up, when firms in a specific industry are of different size in terms of employed workers of type-a*, an increase in the wage level of a given firm causes: (i) an increase in the number of employed people in that firm,* $\frac{\partial \ell_{h'}(a,s)}{\partial w_{h'}(a,s)} > 0$*; (ii) a related decrease in the number of workers of the other firms,* $\frac{\partial \ell_h(a,s)}{\partial w_{h'}(a,s)} < 0$*, and thus an increase in other firms' wage levels,* $\frac{\partial w_h(a,s)}{\partial w_{h'}(a,s)} > 0$*. This is true as long as workers of each type-*a *are free to move across firms in the same industry at no cost and firms can hire new workers from the other firms;*

(c) by applying the same reasoning of point (b), if workers were perfectly mobile across industries, then it would have been the case that $\frac{w(a,s')}{\ell(a,s')} = \frac{w(a,s)}{\ell(a,s)}$*. This chance is ruled out by assuming a very high cost of transition from one industry to another such that no one among workers is keen to move.*

□

(Equilibrium characterization) *In equilibrium, the model should specify the clearing conditions of labour, capital, and goods markets. Starting from the labour market, since each household inelastically supplies one unit of labour, then it must hold that the total number of workers of type-a in firm (h, s) is $\ell_h(a, s) = \int_0^1 \ell_h^i(a, s)\mathrm{d}i$, so that the total labour supply is $L^S = \sum_a \sum_h \sum_s \ell_h(a, s)$. The measure of type-$a$ worker in firm (h, s) is given by eq. (2.7). Aggregating it across tasks and firms results in obtaining the industry-specific labour supply, $L(s) = \sum_{a,h} \ell_h(a, s)$. Analogously, aggregate labour supply of task-a is found by aggregating across firms and industries, $L(a) = \sum_{h,s} \ell_h(a, s)$. It follows that, considering $a = \{rt, nrt\}$, aggregate labour demand for this economy is just $L^D = \sum_a L(a) = L(rt) + L(nrt)$. Labour market clearing requires that $L^D = L^S$.*

For what concerns equilibrium in the capital market(s), total physical and ICT capital demands from industries are, respectively, $K^D(phy, s) = \sum_h k_h(phy, s)$ and $K^D(ict, s) = \sum_h k_h(ict, s)$, so that aggregate demands are simply determined: $K^D(phy) = \sum_s K^D(phy, s)$ and $K^D(ict) = \sum_s K^D(ict, s)$. By the part of supply, aggregating capital quantities over households would results in aggregate physical and ICT capital supplies: $K^S(phy) = \int_i k^i(phy)\mathrm{d}i$ and $K^S(ict) = \int_i k^i(ict)\mathrm{d}i$. Equilibrium in both markets requires $K(phy) \equiv K^D(phy) = K^S(phy)$ and $K(ict) \equiv K^D(ict) = K^S(ict)$, while market clearing in the capital market implies $K^D(phy) + K^D(ict) = K^S(phy) + K^S(ict)$.

Finally, aggregate profits to be given to households are $\mathcal{D}^D = \int_i \mathcal{D}_i \,\mathrm{d}i$, while $\mathcal{D}^S = \sum_s \mathcal{D}(s)$ are the total profits computed by aggregating industry-specific profits, $\mathcal{D}(s) = \sum_h \mathcal{D}_h(s)$. Equilibrium requires $\mathcal{D} \equiv \mathcal{D}^D = \mathcal{D}^S$. This implies that, by aggregating the households' inter-temporal budget constraints and imposing the clearing conditions so far, including also total quantities for

$$
\begin{aligned}
\mathcal{C}^i &= \int_i \mathcal{C}^i \,\mathrm{d}i \\
I(j) &= \int_i I^i(j)\,\mathrm{d}i\,, \quad \forall j \\
b &= \int_i b^i \,\mathrm{d}i \\
w &= \sum_a \sum_h \sum_s w_h(a, s) \\
\mathcal{B} &= \sum_a \sum_h \sum_s \mathcal{B}_h(a, s)
\end{aligned}
$$

where $\mathcal{B} = 1$ since it is the sum of relative quantities, the aggregate resource constraint for this economy at time t reads as

$$
\mathcal{C}_t + I_t(phy) + I_t(ict) + b_{t+1} - (1 + r_t)b_t = w_t \mathcal{B}_t L_t + R_t\Big(K_t(phy) + K_t(ict)\Big) + \mathcal{D}_t
$$

which equals the total output as defined by the final output CES aggregator, Y. Equilibrium conditions are described in Section 2.3.

□

C. MODEL ESTIMATION AND SIMULATION

(Estimating equations) *To derive the equations to estimate the elasticity of substitution between ICT capital and non-routine workers (ρ), and the elasticity of substitution between routine workers and ICT composite good (σ), i.e., that shown in eqs. (2.12) and (2.13), I apply the procedure implemented by Karabarbounis and Neiman (2014). The main steps to be implemented are:*[49]

1. *Define a CES production function, $y(\cdot)$ and compute the related F.O.C.s.; then, equate them to the aggregated (across firms and industries) F.O.C.s of the monopolistically competitive firms;*

2. *Define the following income shares. For a given labour force (ℓ), a given capital stock (k), and given profits ($\mathcal{D}$),*

$$s_\ell = \left(\frac{1}{\mathcal{M}^\epsilon}\right)\left(\frac{w(\ell)\ell}{w(\ell)\ell + Rk}\right) \quad , \quad s_k = \left(\frac{1}{\mathcal{M}^\epsilon}\right)\left(\frac{Rk}{w(\ell)\ell + Rk}\right) \quad , \quad s_{\mathcal{D}} = 1 - \frac{1}{\mathcal{M}^\epsilon}$$

3. *By combining the F.O.C. for capital (either for labour) with all the above shares, one gets an equation whose left-hand side is $1 - s_\ell \mathcal{M}^\epsilon$. Then, this should be written in changes between two arbitrary periods, whose resulting elements are labelled as $\widehat{x}$;*

4. *Use eq. (2.6) to substitute $\widehat{R}$;*

   → *use the Euler condition (from the households side) expressed in deviation between two arbitrary periods get $\widehat{(1+r)} = \frac{1}{\beta}$ so that, under constant β and δ, it holds that $\widehat{R} = \widehat{\zeta}$;*

5. *Once substituting out $\widehat{R}$, take a linear approximation of the resulting equation around $\widehat{\zeta} = 0$, thus obtaining the estimating equation.*

Apply this procedure for whatever CES functional form of the production function $y(\cdot)$. Note that, to carry out this procedure in the framework I propose it is necessary to assume equal marginal product of each type of capital. In fact, in eq. (2.6), trends in both capital rental rates are tied with trends in the quantity of ICT capital relative to non-ICT one, after aggregating optimal conditions in the firm problem. Here, the two capital rates are given by capitals' marginal productivity: thus, to have a unique capital rental rate, such that $R(phy) = R(ict) \equiv R$, equal marginal product of capital types are in order.

(Targeting moments for MSM) *Start from the weighting parameters, λ and μ. For each industry-$h \in \mathcal{S}$, the weight of ICT capital (λ) in the ICT composite is matched with the industry-specific ICT capital in the aggregate stock in the data.*

[49] Please refer to Karabarbounis and Neiman (2014) for further details and discussion.

Differently, the weight of routine workers (μ) in the production function is used to bridge the share of routine workers in the data with that predicted by the model, i.e., I implement the following identity using eq. (2.7): $\ell_{model}(a,s) = \left(\frac{w(a,s)}{\mathcal{W}(a,\mathcal{S})}\frac{\mathcal{B}(a,s)}{\mathcal{B}(a,\mathcal{S})}\right)^{\theta} \approx \ell_{data}(a,s)$. *Of course, since the model's measures of employment are determined by relative wages, some slight differences in the estimated matched moments are in order.*

Finally, I consider productivity dispersion parameter, θ, which directly relates to the between-industry wage difference for worker-a. Given $a = rt$, I employ the wage premium of type-a working in top industry (s) relative to its counterpart in the bottom industry (s'):

$$\frac{w(rt,s)}{w(rt,s')} = \frac{\left[\Lambda(s)\,\chi(rt,s)\,\Big(k(phy,s)\Big)^{\alpha(s)}\,\mathcal{V}(s)^{\frac{1-\alpha(s)-\varsigma(s)}{\varsigma(s)}}\,\mathcal{B}(rt,s)^{\theta(\varsigma(s)-1)}\,\mathcal{WB}(rt,\mathcal{S})^{\theta(1-\varsigma(s))}\right]^{\frac{1}{1+\theta-\theta\varsigma(s)}}}{\left[\Lambda(s')\,\chi(rt,s')\,\Big(k(phy,s')\Big)^{\alpha(s')}\,\mathcal{V}(s')^{\frac{1-\alpha(s')-\varsigma(s')}{\varsigma(s')}}\,\mathcal{B}(rt,s')^{\theta(\varsigma(s')-1)}\,\mathcal{WB}(rt,\mathcal{S})^{\theta(1-\varsigma(s'))}\right]^{\frac{1}{1+\theta-\theta\varsigma(s')}}}$$

If considering changes over time, only $k(phy,\cdot)$, $\mathcal{V}(\cdot)$, $\mathcal{B}(\cdot)$, and $\mathcal{WB}(\cdot)$ are time-varying, with all the other parameters previously calibrated, targeted and fixed: this leaves θ as the only free parameter to match this moment. Note how I choose to pin down labour market concentration for routine workers in accordance with Figure 2.4.

TABLE C.1: EMPLOYMENT MEASURES AND TASKS' RELATIVE WAGES

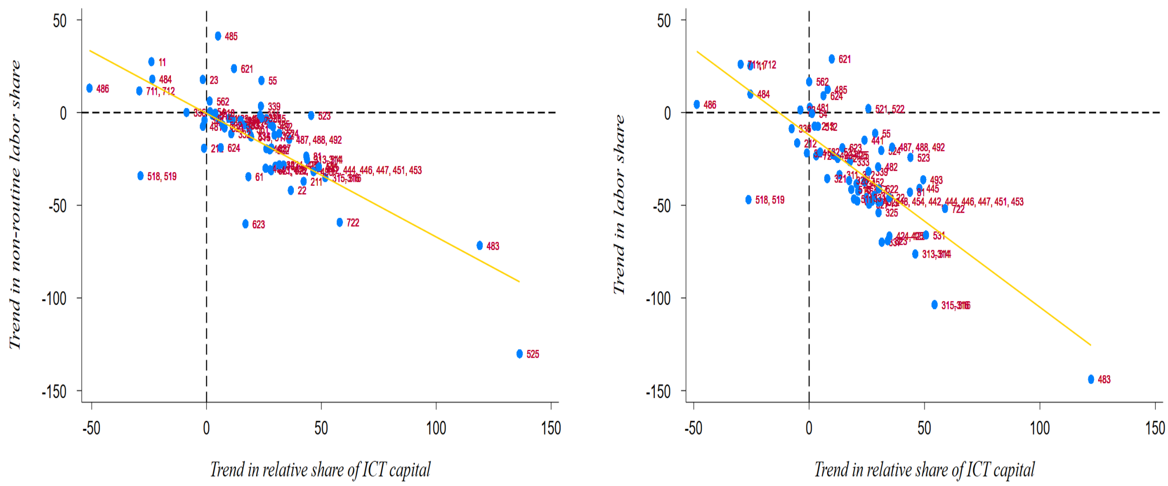
	$log(\ell(rt,s))$			$log(\ell(nrt,s))$		
	(1)	(2)	(3)	(1)	(2)	(3)
$\ell(rt,s\,\vert\,w\mathcal{B})$	.765*	.657*	3.55***			
	(.32)	(.28)	(.17)			
$\ell(nrt,s\,\vert\,w\mathcal{B})$				5.76***	3.71***	29.4***
				(1.9)	(1.1)	(2.2)
Industry FE	✓	✓	✗	✓	✓	✗
Time FE	✗	✓	✗	✗	✓	✗

*Significance level at * (p<0.05), ** (p<0.01), *** (p<0.001). Standard error in parentheses. Analysis at 3-digit U.S. 2017 NAICS industries in 2003-2022 on $N = 1240$ observations. All the regressions are of the form $y_t = \beta_c + \beta_i \mathcal{X}_{i,t} + \delta_j \mathcal{Z}_{j,t} + u_t$, with $\mathcal{X}_i$ being the regressors, and $\mathcal{Z}_j$ a set of controls. All series are in logs. Constant not reported to save space. Source: BLS and own calculations.*

TABLE C.2: METHOD OF SIMULATED MOMENTS

	parameter	*value*	***moment to match***	*fit* — *data*	*fit* — *model*
μ_{bot}	weight of routines in $y(bot)$	0.6763	routine share, bottom	.0858	.0434
μ_{mid}	weight of routines in $y(mid)$	0.4953	routine share, middle	.1357	.0458
μ_{top}	weight of routines in $y(top)$	0.3366	routine share, top	.0985	.0199
λ_{bot}	weight of ICT in $\mathcal{Q}(bot)$	0.4565	ICT share, bottom	.3968	.3968
λ_{mid}	weight of ICT in $\mathcal{Q}(mid)$	0.4645	ICT share, middle	.3042	.3042
λ_{top}	weight of ICT in $\mathcal{Q}(top)$	0.4514	ICT share, top	.2990	.2990
θ	productivity dispersion	11.302	wage premium, $w\left(a,[s,s']\right)$	.9945	.9945

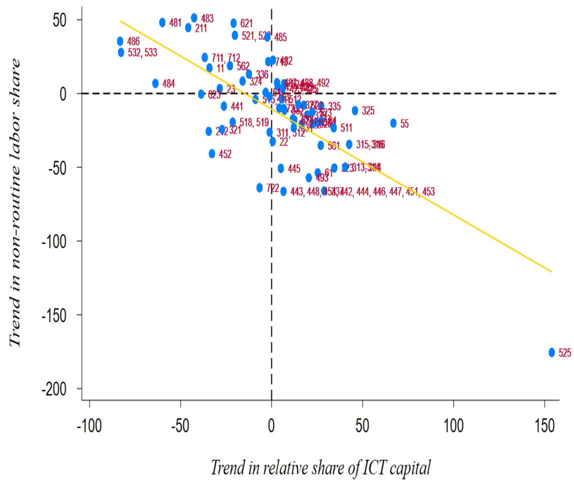
Estimated values and related matched moment using the Methods of Simulated Moments by Mc Fadden (1989).



(A) ESTIMATES FOR ρ (B) ESTIMATES FOR σ

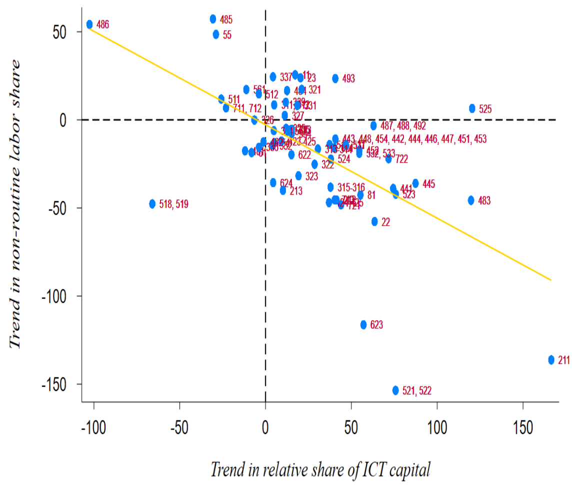
FIGURE C.1: CORRELATION BETWEEN ELASTICITIES AND RELATIVE ICT

Note: these scatterplots compute the correlation of trends in labour share with trends in the stock of ICT capital relative to physical capital, namely the left- and right-hand sides estimated through the elasticities of substitution as in eqs. (2.12) and (2.13), respectively. Each y-axis report the labour share, while common x-axis is referred to ICT capital. Panel C.1a refers the correlation of the elasticity of substitution between ICT capital and non routine workers, while Panel C.1b plots that of the entire labour share (considering both routine and non routine workers). Solid-gold negatively-sloped line is the estimated linear trend from robust regression. Labels to each point refer to the 3-digit U.S. 2017 NAICS code of the related industry. *Source*: BEA, BLS and own calculations.

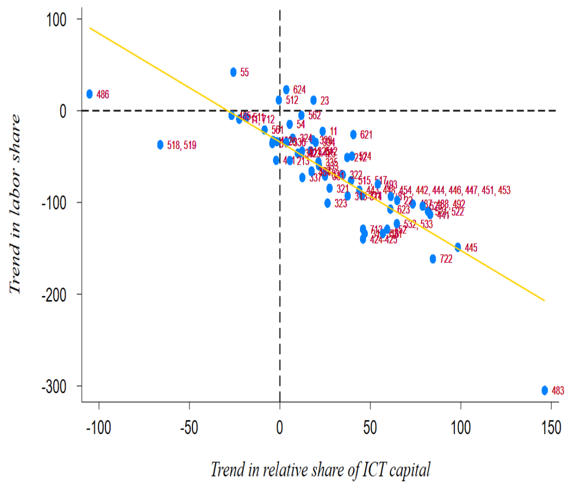


(A) Estimates for ρ, 2003-2012

(B) Estimates for σ, 2003-2012



(C) Estimates for ρ, 2013-2022



(D) Estimates for σ, 2013-2022

Figure C.2: Correlation between in-time elasticities and relative ICT

Note: these scatterplots compute the correlation of trends in labour share with trends in the stock of ICT capital relative to physical capital, namely the left- and right-hand sides estimated through the elasticities of substitution as in eqs. (2.12) and (2.13), respectively; first row displays correlation in the period 2003-2012, while the second row that in the period 2013-2022. Each y-axis report the labour share, while common x-axis is referred to ICT capital. Panels C.2a and C.2c refer the correlation of the elasticity of substitution between ICT capital and non routine workers, while Panels C.2b and C.2d plot that of the entire labour share (considering both routine and non routine workers). Solid-gold negatively-sloped line is the estimated linear trend from robust regression. Labels to each point refer to the 3-digit U.S. 2017 NAICS code of the related industry. *Source*: BEA, BLS and own calculations.

TABLE C.3: MODEL FIT, UNTARGETED MOMENTS

moment	***fit***	
	data	*model*
aggregate task-premium	.001	.005
aggregate wage	-.008	-.073
routine wage, bottom	-.016	-.070
routine wage, middle	-.001	-.051
routine wage, top	-.007	-.079
non-routine wage, bottom	-.019	-.060
non-routine wage, middle	.003	-.074
non-routine wage, top	-.010	-.046

Untargeted moments to match to validate the calibration strategy. All moments, referred to real log-wages, are taken as percentage changes throughout the series.

(Market concentration) *To evaluate the pattern in concentration at industry level, the standard measure Herfindahl-Hirschman Index (HHI) is computed to account for market concentration of labour force for industries. To compute the dynamics in each year, labour market-level concentration for task-*a *is defined as*

$$HHI_{\ell(a)} = \sum_{s_{|g}} \left(\frac{\ell(a, s_{|g})}{\ell(a)} \right)^2 \tag{C.1}$$

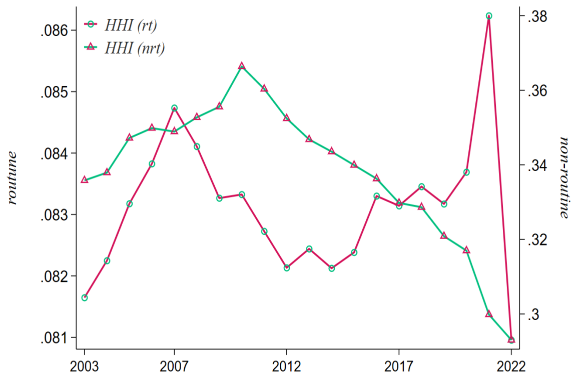
where the sum is over individual industries (s), or over a group of industries ($s_{|g}$*). Analogously, total employment concentration is defined by* $HHI_{\ell} = \sum_a HHI_{\ell(a)}$. *This measure is included in the range* $[0, 1]$*: a value of 1 identifies maximum market concentration, namely a single monopsonist in the labour market; conversely, a value of 0 results in a perfectly competitive environment. By definition, if industries have equal labour force size, the index would converge to the number of industries. An index below 0.15 points for an un-concentrated labour market, an index between 0.15 and 0.25 indicates a moderate concentration, while a value higher than 0.25 results in an highly concentrated labour market.*

Each subplot of Figure C.3 depicts the evolution in HHI score for both routine and non-routine workers. As a general observation, labour market concentration for both tasks has increased for top and bottom groups, but it has decreased for middle. With respect to bottom group, Panel C.3a indicates a joint increase in market concentration for both job tasks, with a very high concentration for non-routine workers $\left(HHI_{\ell(nrt)} > 0.25 \right)$*; routine workers are approaching to a moderate concentration.*

In relation to middle group, after a steady increase, labour market concentration for non-routine workers constantly drops, even if concentration is still high since $HHI_{\ell(nrt)} > 0.25$. *The score for routine workers is low, but if follows cyclical fluctuations, with a steady increase before a recession, and thereafter a substantial drop.*

Finally, top group has seen a marked and steady increased in market concentration for non-routine workers, which is approaching to a moderate concentration; for the part of routine workers, concentration is high and, after a steady rise in the early years, then a flat pattern has occurred.

(A) BOTTOM



(B) MIDDLE

(C) TOP

FIGURE C.3: LABOUR MARKET CONCENTRATION BY GROUPS OF INDUSTRIES

Note: the figure represents the evolution in labour market concentration as measured by the Herfindahl-Hirschman Index (HHI). Each subplot measures the dynamics in each broadly defined group of industries by routine (red line, left axis) and non-routine (green line, right axis) tasks. *Source*: BLS and own calculations.

TABLE C.4: SUMMARY OF CALIBRATION, 2003-2012

		value				
	parameter	*bottom*	*middle*	*top*	*global*	***source***
α	*physical capital, share of* $y(s)$	0.131	0.091	0.254		*data*
ϵ	*demand elasticity across firms*				6	*external*
μ	*weight of routine workers in* $y(s)$	0.196	0.486	0.902		*MSM*
λ	*ICT capital share in* $Q(s)$	0.650	0.530	0.185		*MSM*
θ	*households' productivities dispersion*				7.26	*MSM*
ρ	*EoS, ICT capital and non-routine*	0.355	0.431	0.408		*estimation*
σ	*EoS, routine and ICT composite*	0.366	0.429	0.367		*estimation*

Set of estimated parameters of the model, first-half of the sample. "data" implies that the values are directly computed from data sources, while in "external" I choose standard calibrated values from the literature. "MSM" refers to the Methods of Simulated Moments as in Mc Fadden (1989). "estimation" refers to previously estimated values under a specific procedure; these values are taken from Table 2.8.

TABLE C.5: SUMMARY OF CALIBRATION, 2013-2022

		value				
	parameter	*bottom*	*middle*	*top*	*global*	***source***
α	*physical capital, share of* $y(s)$	0.131	0.104	0.260		*data*
ϵ	*demand elasticity across firms*				6	*external*
μ	*weight of routine workers in* $y(s)$	0.415	0.506	0.608		*MSM*
λ	*ICT capital share in* $Q(s)$	0.786	0.490	0.327		*MSM*
θ	*households' productivities dispersion*				7.79	*MSM*
ρ	*EoS, ICT capital and non-routine*	0.819	0.345	0.508		*estimation*
σ	*EoS, routine and ICT composite*	0.326	0.438	0.357		*estimation*

Set of estimated parameters of the model, second-half of the sample. "data" implies that the values are directly computed from data sources, while in "external" I choose standard calibrated values from the literature. "MSM" refers to the Methods of Simulated Moments as in Mc Fadden (1989). "estimation" refers to previously estimated values under a specific procedure; these values are taken from Table 2.8.

TABLE C.6: METHOD OF SIMULATED MOMENTS, SUB-PERIODS

	moment to match	*2003-2012* value	data	model	*2013-2022* value	data	model
μ_{bot}	routine share, bottom	0.196	0.087	0.097	0.415	0.085	0.051
μ_{mid}	routine share, middle	0.486	0.132	0.057	0.506	0.140	0.030
μ_{top}	routine share, top	0.902	0.100	0.013	0.608	0.097	0.083
λ_{bot}	ICT share, bottom	0.650	0.399	0.399	0.786	0.395	0.395
λ_{mid}	ICT share, middle	0.530	0.314	0.314	0.490	0.295	0.295
λ_{top}	ICT share, top	0.185	0.287	0.287	0.327	0.311	0.311
θ	wage premium, $w(a, [s, s'])$	7.259	0.994	0.994	7.787	0.995	0.995

Estimated values and related matched moment using the Methods of Simulated Moments by Mc Fadden (1989) for first and second half of the sample.

TABLE C.7: VARIATIONS IN THE SERIES AND STRUCTURAL HETEROGENEITY

	data	**model**	***model*** $\mid \Delta\Phi(x)$: $\Delta\ell(rt)$	$\Delta\ell(nrt)$	$\Delta(\ell)$	$\Delta k(ict)$	$\Delta(\ell, ict)$	$(all)_{\tau_0}$
WAGES, VARIANCE								
routine	2.285	2.289	2.55	2.42	2.24	2.78	2.45	2.75
non-routine	2.314	2.311	2.78	2.44	2.37	2.78	2.39	2.83
industry	2.299	2.300	2.66	2.43	2.30	2.78	2.42	2.79

Changes in variances in real log-wages in the model induced by variations in one or more series, keeping fixed the others, and imposing the parameters to be that in the baseline calibration (i.e., quantification of Model D under industry-specific parameters). $\Delta(\ell)$ *refers to joint variations in routine and non-routine series, while* $\Delta(\ell, ict)$ *is associated to simultaneous changes in both ICT capital and both types of workers.* $(all)_{\tau_0}$ *keeps fixed all the series (physical capital included), at their initial level,* $\tau_0 = 2003$. *In all the columns, the variation is computed throughout the period-by-period percentage differential thus identifying overall changes in empirical trends implied both by the data and the model; same values differ in terms of decimals.*

(Further results of Subsection 2.5.2) *As in the main text, below I have performed an opposite exercise of Table 2.9 where the main structural parameters in the model, namely* $(\rho_s, \sigma_s, \theta)_{\forall s \in \{bot,mid,top\}}$ *are keeping fixed one or more at time, and let to change the non-ICT capital share and the weighting parameters in the production function, respectively* $(\alpha_s, \mu_s, \lambda_s)_{\forall s \in \{bot,mid,top\}}$*; the output of such exercise is shown in Table C.8.*

The two analysis performed are conducted by considering industries as a whole, that is, keeping the total employment in terms of both routine and non-routine workers, in order to address the determinants of between-industry wage inequality. However, as noticed, these two exercises can be performed also by distinguish the effects of changing parameters on separate job tasks: Table C.9 reports the two for routine workers, while Table C.10 the same but for non-routine workers.

Below it is possible to find a detailed comment for each employment group and, further below, a comparative comment that shows how the patterns holding for industry with aggregate employment hold also for routine and non-routine workers separately. Visually, these results are in Figures C.4a and C.5a for aggregate industries, and in Figures C.4b and C.5b by job task categories.

TABLE C.8: MODEL COUNTERFACTUAL, FIXING

		Δ ***model*** $\mid \Delta\ m\Big(\Phi(x,\tau_2), \Theta = \{p_{\tau_2}, -p_{\tau_1}\}\Big)$		
industry wages	$var(w)_{\tau_2}$	*level*	*share, model*	*share, data*
DATA	1.18			
MODEL	1.09			
$\Delta\Theta_{\mid\sigma}$		.94	.86	.79
$\Delta\Theta_{\mid\rho}$		2.21	2.04	1.87
$\Delta\Theta_{\mid(\sigma,\rho)}$		1.31	1.21	1.11
$\Delta\Theta_{\mid\theta}$		1.04	.96	.88
$\Delta\Theta_{\mid(\sigma,\rho,\theta)}$		1.30	1.20	1.10

Quantification of Model B. Model implied between-industry real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by fixing a specified parameter, $\frac{m\left[\Phi(x,\tau_2) \mid \Theta(p,\tau_2), \Theta(-p^{\{\rho,\sigma,\theta\}},\tau_1)\right]}{m\left[\Phi(x,\tau_2) \mid \Theta(p,\tau_2)\right]}$*, where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2)*, are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

TABLE C.9: MODEL COUNTERFACTUALS FOR ROUTINE WORKERS

		Δ ***model*** $\mid \Delta\ m\Big(\Phi(x,\tau_2),\Theta=\{p_{\tau_2},-p_{\tau_1}\}\Big)$		
routine wages	$var(w)_{\tau_2}$	*level*	*share, model*	*share, data*
		CHANGE		
DATA	1.14			
MODEL	.95			
$\Delta\sigma$		.99	1.04	.87
$\Delta\rho$		.35	.37	.30
$\Delta(\sigma,\rho)$		.66	.70	.58
$\Delta\theta$		.57	.60	.50
$\Delta(\sigma,\rho,\theta)$		.71	.76	.63
		FIXING		
DATA	1.14			
MODEL	.95			
$\Delta\Theta_{\mid\sigma}$		.71	.75	.62
$\Delta\Theta_{\mid\rho}$		1.15	1.22	1.01
$\Delta\Theta_{\mid(\sigma,\rho)}$		.84	.89	.74
$\Delta\Theta_{\mid\theta}$		.90	.95	.79
$\Delta\Theta_{\mid(\sigma,\rho,\theta)}$		.81	.86	.71

Quantification of Model A and Model B. Model implied industry-level between-routine workers real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2),\Theta(-p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2)\right]}$*, where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2)*, are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

TABLE C.10: MODEL COUNTERFACTUALS FOR NON-ROUTINE WORKERS

		Δ ***model*** $\mid \Delta\ m\Big(\Phi(x,\tau_2),\Theta=\{p_{\tau_2},-p_{\tau_1}\}\Big)$		
non-routine wages	$var(w)_{\tau_2}$	*level*	*share, model*	*share, data*
CHANGE				
DATA	1.21			
MODEL	1.22			
$\Delta\sigma$		3.74	3.06	3.09
$\Delta\rho$		1.41	1.15	1.16
$\Delta(\sigma,\rho)$		1.57	1.28	1.30
$\Delta\theta$		2.11	1.73	1.75
$\Delta(\sigma,\rho,\theta)$		1.61	1.31	1.33
FIXING				
DATA	1.21			
MODEL	1.22			
$\Delta\Theta_{\mid\sigma}$		1.16	.95	.96
$\Delta\Theta_{\mid\rho}$		3.27	2.67	2.70
$\Delta\Theta_{\mid(\sigma,\rho)}$		1.78	1.45	1.47
$\Delta\Theta_{\mid\theta}$		1.18	.97	.98
$\Delta\Theta_{\mid(\sigma,\rho,\theta)}$		1.79	1.46	1.48

Quantification of Model A and Model B. Model implied industry-level between-non-routine workers real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2),\Theta(-p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2)\right]}$*, where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2)*, are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

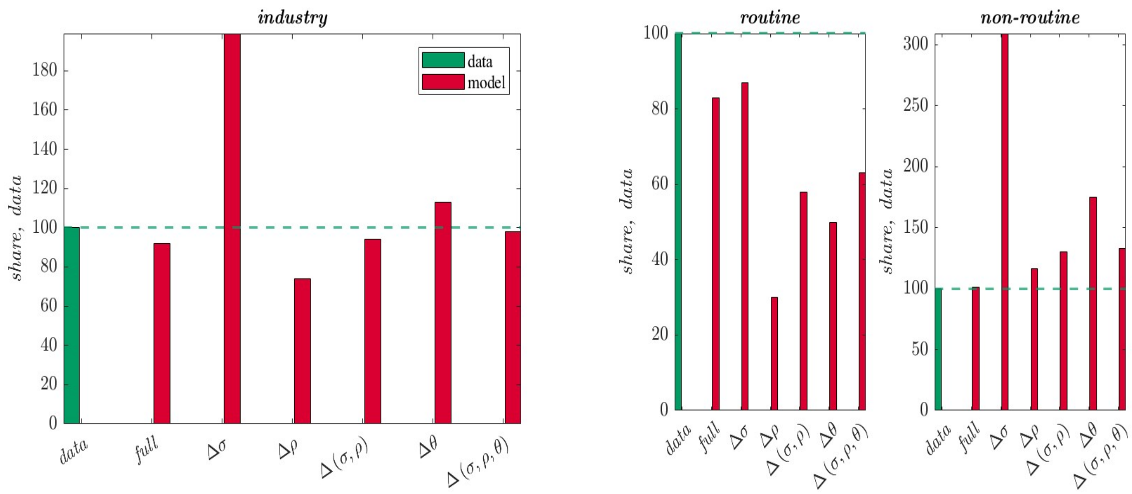
(Comparative comment) *Investigations on the contribution of structural parameters outline an important feature of the U.S. wage structure: observed structural differences among industries account for a sizeable fraction of wage inequality. All the effects can be summarized as follows:*

(a) ***routines****. Most of the share is accounted by trends in industry-heterogeneous differentials in elasticities of substitution among capital and worker types (58%, upper part of Table C.9), also taken in combination to different weights of factor inputs in production (79%, lower part of Table C.9). Considering these differences across industries, the rise in labour market concentration in terms of workers' complementarities (sorting and segregation effects) is an amplifier of U.S. wage inequality across routine job tasks in the last two decades, explaining 63% of the total between-industry routine real log-wage variance (upper part of Table C.9);*

(b) ***non-routines****. Most of the share is accounted by trends in industry-heterogeneous differentials in elasticities of substitution among capital and worker types (130%, upper part of Table C.10), also taken in combination to different weights of factor inputs in production (98%, lower part of Table C.10). Considering these differences across industries, the rise in labour market concentration in terms of workers' complementarities (sorting and segregation effects) is an amplifier of U.S. wage inequality across non-routine job tasks in the last two decades, explaining 133% of the total between-industry non-routine real log-wage variance (upper part of Table C.10);*

(c) ***industries****. Most of the share is accounted by trends in industry-heterogeneous differentials in elasticities of substitution among capital and worker types (94%, Table 2.9), also taken in combination to different weights of factor inputs in production (88%, Table C.8). Considering these differences across industries, the rise in labour market concentration in terms of workers' complementarities (sorting and segregation effects) is a amplifier of U.S. wage inequality across industries in the last two decades, explaining 98% of the total between-industry real log-wage variance (Table 2.9).*

Under a comparative perspective, routine workers wage inequality behaves similarly from that of non-routine and industries. All these variances are mostly explained by the substitution elasticity between routine and non-routine workers (σ) only and, jointly, the huge negative effect of σ on wage inequality is reduced due to changes in ρ. On the role of θ, again a clear division does not emerge: real wage dispersion increases after an increase in labour market concentration either for routine and non-routine workers and for industries. Henceforth, stronger workers' complementarities through stronger sorting and segregation effects result in negatively impacting all the considered categories, thus increasing real log-wage dispersion.

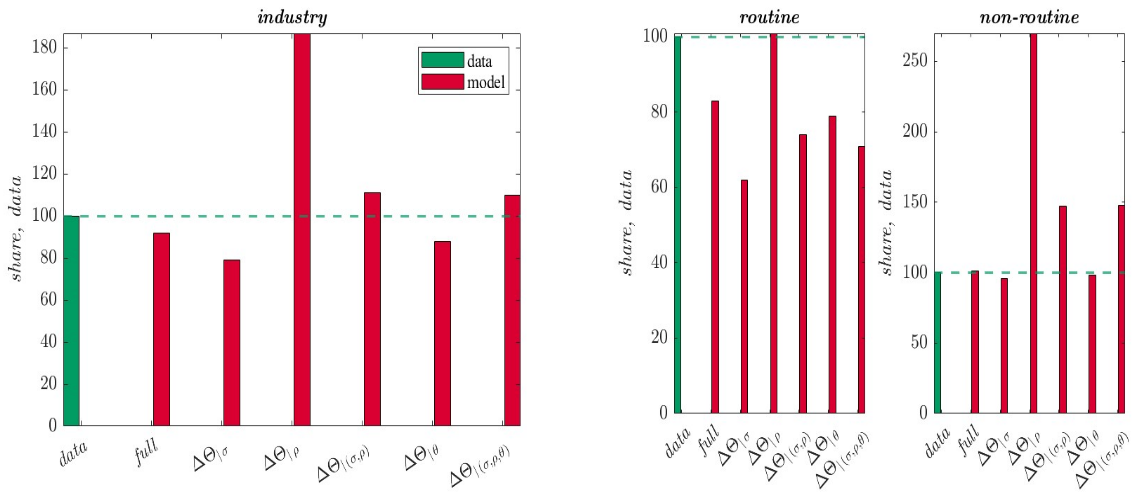


(A) INDUSTRY

(B) TASKS

FIGURE C.4: MODEL COUNTERFACTUAL, CHANGE

Note: these figures plot the second-period between-industry real log-wage variance, as a share of that in the data, implied by the model when changing one or more parameters at time as shown in Table 2.9 and in the upper part of Tables C.9 and C.10, for industry (total employment), routine and non-routine workers.



(A) INDUSTRY

(B) TASKS

FIGURE C.5: MODEL COUNTERFACTUAL, FIXING

Note: these figures plot the second-period between-industry real log-wage variance, as a share of that in the data, implied by the model when fixing one or more parameters at time as shown in Table C.8 and in the lower part of Tables C.9 and C.10, for industry (total employment), routine and non-routine workers.

(Comment for Skill-Biased Technological Change (SBTC) estimation) *As argued in the main text, Subsection 2.5.3, Skill-Biased Technological Change (SBTC) theory implies that wage differentials are shaped by different adoption rate of technology, so that dispersion in wages is mostly affected by how much an industry increases its technological capital and this, in turn, will increase its demand for high-skill or non-routine workers. Thus, below I have performed an exercise where the main structural parameters in the model, namely* $(\rho_s, \sigma_s, \theta)_{\forall s \in \{bot, mid, top\}}$*, and the production function weighting parameters, namely* $(\alpha_s, \mu_s, \lambda_s)_{\forall s \in \{bot, mid, top\}}$*, are keeping fixed at the first period, and let to change the series related to routine and non-routine workers, and that of ICT capital, as described in the main text by Model C; the output of such exercise is shown in Table C.11.*

The analysis performed is conducted by considering industries as a whole, that is, keeping the total employment in terms of both routine and non-routine workers, in order to address the determinants of between-industry wage inequality. As noticed, these two exercises can be performed also by distinguish the effects of changing parameters on separate job tasks; to this end, Table C.12 reports the SBTC analysis for both routine and non-routine workers. The impact of SBTC seems to have major effect on non-routine workers: in fact, while the variance explained by shifts in the series explain substantially less data-share compared to the case in which shifts in structural parameters are also considered, the wage variance of non-routine workers explodes. Therefore, as in the model, major impact of only the series is on non-routine workers.

TABLE C.11: MODEL COUNTERFACTUAL, SBTC

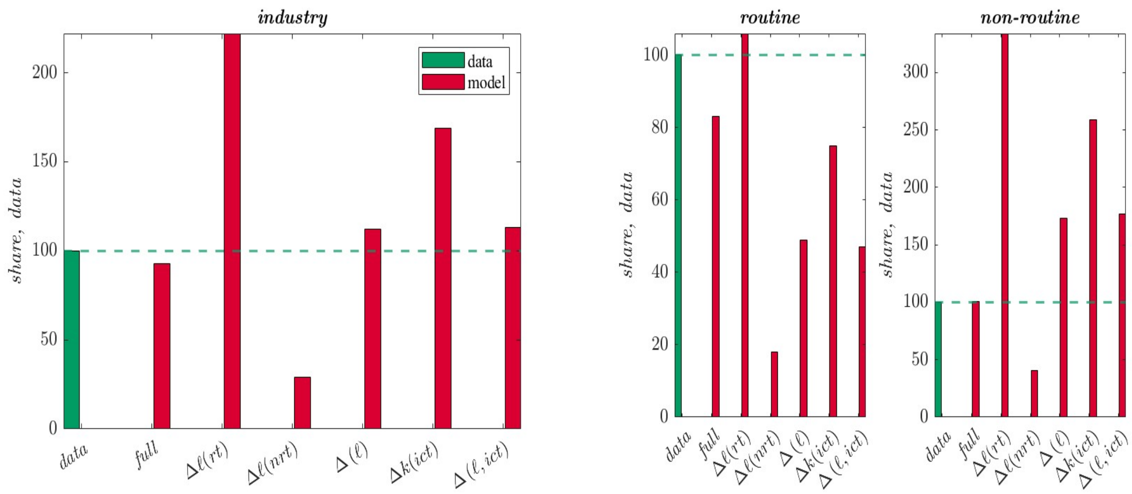
		Δ ***model*** $\mid \Delta\, m\Big(\Phi = \{x_{\tau_2}, -x_{\tau_1}\} \;\Big|\; \Theta(p, \tau_1)\Big)$		
industry wages	$var(w)_{\tau_2}$	*level*	*share, model*	*share, data*
DATA	1.18			
MODEL	1.09			
$\Delta\ell(rt)$		2.62	2.42	2.22
$\Delta\ell(nrt)$		.35	.32	.29
$\Delta\ell$		1.32	1.22	1.12
$\Delta k(ict)$		1.99	1.84	1.69
$\Delta(\ell, ict)$		1.34	1.23	1.13

Quantification of Model C. Model implied between-industry real log-wage variance changes between two time spans uniformly calibrated, with changes according to variations in some series; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2), \Phi(-x,\tau_1) \,\middle|\, \Theta(p,\tau_1)\right]}{m\left[\Phi(x,\tau_2) \,\middle|\, \Theta(p,\tau_2)\right]}$*, where* $\Theta(x, \tau_1)$ *identifies the set of parameters in the first period, and* Φ *the set of capital and labour series where some of them,* (x, τ_2)*, are taken in the second period, while* $(-x, \tau_1)$ *reflects the set of all the series in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

TABLE C.12: MODEL COUNTERFACTUAL, SBTC BY TASKS

wages	$var(w)_{\tau_2}$	Δ ***model*** $\mid \Delta\ m\Big(\Phi = \{x_{\tau_2}, -x_{\tau_1}\}\ \Big\vert\ \Theta(p,\tau_1)\Big)$ *level*	*share, model*	*share, data*
		ROUTINE		
DATA	1.14			
MODEL	.95			
$\Delta\ell(rt)$		1.21	1.28	1.06
$\Delta\ell(nrt)$		.20	.21	.18
$\Delta\ell$		.56	.59	.49
$\Delta k(ict)$		.86	.91	.75
$\Delta(\ell, ict)$		.53	.56	.47
		NON-ROUTINE		
DATA	1.21			
MODEL	1.22			
$\Delta\ell(rt)$		4.04	3.30	3.34
$\Delta\ell(nrt)$		.50	.40	.41
$\Delta\ell$		2.09	1.70	1.73
$\Delta k(ict)$		3.13	2.56	2.59
$\Delta(\ell, ict)$		2.14	1.75	1.77

Quantification of Model C for routine and non-routine workers separately. Model implied between-industry real log-wage variance changes between two time spans uniformly calibrated, with changes according to variations in some series; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2),\Phi(-x,\tau_1)\ \middle\vert\ \Theta(p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\ \middle\vert\ \Theta(p,\tau_2)\right]}$*, where* $\Theta(x,\tau_1)$ *identifies the set of parameters in the first period, and* Φ *the set of capital and labour series where some of them,* (x,τ_2)*, are taken in the second period, while* $(-x,\tau_1)$ *reflects the set of all the series in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*



(A) INDUSTRY

(B) TASKS

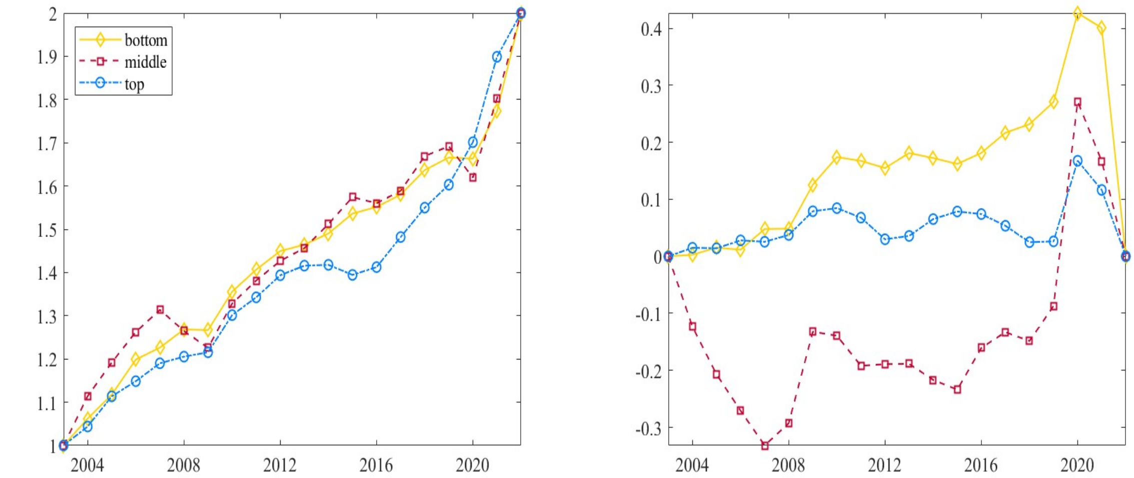
FIGURE C.6: MODEL COUNTERFACTUAL, SBTC

Note: these figures plot the second-period between-industry real log-wage variance, as a share of that in the data, implied by the model when changing one or more parameters at time while keeping fixed the all the parameters, as shown in Tables C.11 and C.12, for industry (total employment), routine and non-routine workers.

TABLE C.13: MODEL VS. DATA COUNTERFACTUAL, SERIES AND NEW PARAMETERS

		model $\mid \Delta\,\Phi(x)$						
							$\Delta\,(tfp)$	
	data	$\Delta\ell(rt)$	$\Delta\ell(nrt)$	$\Delta\ell$	$\Delta k(ict)$	$\Delta\,(tech)$	*newly*	*baseline*
WAGES, VARIANCE								
routine	2.285	.62	.41	.46	.54	.40	.57	.34
non-routine	2.314	.07	.02	.02	.04	.03	.02	.09
industry	2.299	.35	.22	.24	.29	.21	.30	.22

Quantification of Model D. Changes in variances in real log-wages in the model induced by variations in one or more series, keeping fixed the others, and imposing the newly estimated parameters to be homogeneous across industries. $\Delta\,(\ell)$ *refers to joint variations in routine and non-routine series,* $\Delta\,(tech)$ *is associated to simultaneous changes in both ICT capital and non-routine workers, while changes in estimated industry-specific Hicks-neutral exogenous total factor productivity (TFP), given eq. (2.14), are captured by* $\Delta\,(tfp)$*; TFP series are taken under the same (newly estimated) parameters' values, or given the baseline calibration (Table 2.4.2). In all the columns, the variation is computed throughout the period-by-period percentage differential thus identifying overall changes in empirical trends implied both by the data and the model; same values differ in terms of decimals.*

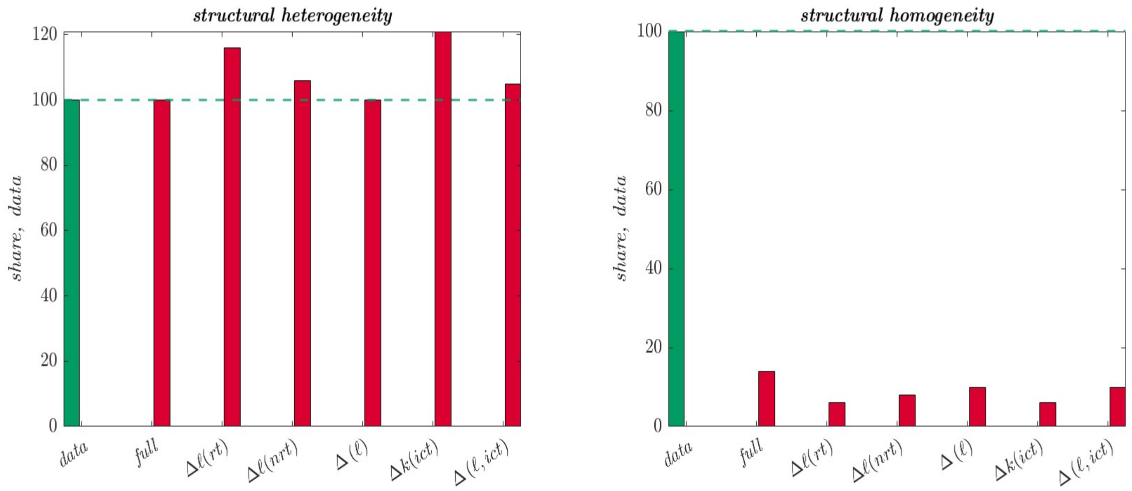


(A) UNIQUE CALIBRATION, NEWLY

(B) DIFFERENCE WITH BASELINE

FIGURE C.7: ESTIMATED PRODUCTIVITIES UNDER NEW PARAMETERS

Note: the figure shows the estimated Hicks-neutral exogenous total factor productivity (TFP) measures estimated from the model. Panel C.7a plots the series given a calibration where all the parameters are evenly set at the same *newly estimated* values for all the industry-groups, while Panel C.7b shows the difference between such series and the estimated TFP measures using the baseline calibration reported in Table 2.6. Series are scaled to be in the same range for graphical comparison.



(A) STRUCTURAL HETEROGENEITY

(B) HOMOGENEOUS PARAMETER

FIGURE C.8: MODEL COUNTERFACTUAL, SERIES

Note: these figures plot the between-industry real log-wage variance, as a share of the data, implied by the model when one or more series at time are changing, keeping fixed the parameters. Panel C.8a reports the result of Table C.7, where structural parameters are heterogeneous across industries. By contrast, Panel C.8b reports the outcome from Table 2.10, under industry-homogeneous structural parameters; note that a similar graph would appear if considering Table C.13.

D. Case under monopsony power

(Discussion) *In this appendix I am going to replicate all the analysis in the main text (calibration, estimation and counterfactuals from Model A and Model B) under the case in which firms are assumed to be monopolistically competitive and take the labour supply of each task-a as given. In this case, frictions due to a concentration in the labour market suggest that wages' formation is decided at firm level so that, taking labour supplies and industry variables as given, the profit-maximization problem for firm-h in industry-s becomes*

$$\max_{p_h(s),k_h(j,s),w_h(a,s)} \Big[\mathcal{D}_h(s) \mid y_h(s), \ell_h(a,s)\Big]$$

where firms have wage-setting power over employees. In other words, the profit maximization implies that the Cobb Douglas-nested CES production function in eq. (2.9) is subject to the labour supply curves specified by eq. (2.7), and to the conditional firm demand in eq. (2.8), thus exploiting both monopolistic and monopsonistic power, respectively, in combination. Whether one decides to assume or not a monopsony power by the part of firms (i.e., maximizing taking $\ell_h(a,s)$ as a constraint) does not change substantially the results and the conclusion of the main text. In fact, the only difference between optimal wages for routine and non-routine workers for a monopolistically competitive firm only, as in eq. (2.10) and in Appendix B, and that of a monopsonistic-monopolistically competitive firm is just the wage-markdown element $\mathcal{M}^\theta = \frac{\theta}{1+\theta}$ in both the composite parameters $\chi(a,s)$, a feature of firms when considering also a monopsonistic environment.

Since θ, measuring the degree of sorting and segregation effects in the economy, is the same for all industries, changes in variances due to changes in θ have an even effect in each industries, thus not changing the conclusion of the main text. This is because, as already noticed, the specification of the households' sorting choices comprises in itself the definition of monopsony power since the measure of workers of type-a in firm-h, industry-s is mostly determined by relative wages as in eq. (2.7).

TABLE D.1: SUMMARY OF CALIBRATION UNDER MONOPSONY

	parameter	***value***				***source***
		bottom	*middle*	*top*	*global*	
α	*physical capital, share of* $y(s)$	0.263	0.195	0.514		*data*
ϵ	*demand elasticity across firms*				6	*external*
μ	*weight of routine workers in* $y(s)$	0.676	0.490	0.333		*MSM*
λ	*ICT capital share in* $\mathcal{Q}(s)$	0.455	0.456	0.439		*MSM*
θ	*households' productivities dispersion*				11.4	*MSM*
ρ	*EoS, ICT capital and non-routine*	0.329	0.420	0.249		*estimation*
σ	*EoS, routine and ICT composite*	0.634	0.400	0.766		*estimation*

Set of estimated parameters of the model. "data" implies that the values are directly computed from data sources, while in "external" I choose standard calibrated values from the literature. "MSM" refers to the Methods of Simulated Moments as in Mc Fadden (1989). "estimation" refers to previously estimated values under a specific procedure; these values are taken from Table 2.5.

TABLE D.2: METHOD OF SIMULATED MOMENTS UNDER MONOPSONY

	parameter	*value*	***moment to match***	***fit***	
				data	*model*
μ_{bot}	weight of routines in $y(bot)$	0.6760	routine share, bottom	.0858	.1420
μ_{mid}	weight of routines in $y(mid)$	0.4902	routine share, middle	.1357	.1596
μ_{top}	weight of routines in $y(top)$	0.3331	routine share, top	.0985	.0637
λ_{bot}	weight of ICT in $\mathcal{Q}(bot)$	0.4552	ICT share, bottom	.3968	.3968
λ_{mid}	weight of ICT in $\mathcal{Q}(mid)$	0.4585	ICT share, middle	.3042	.3042
λ_{top}	weight of ICT in $\mathcal{Q}(top)$	0.4398	ICT share, top	.2990	.2990
θ	productivity dispersion	11.376	wage premium, $w(a, [s, s'])$	.9945	.9945

Estimated values and related matched moment using the Methods of Simulated Moments by Mc Fadden (1989).

TABLE D.3: MODEL FIT UNDER MONOPSONY, UNTARGETED MOMENTS

moment	***fit***	
	data	*model*
aggregate task-premium	.001	.005
aggregate wage	-.008	-.073
routine wage, bottom	-.016	-.070
routine wage, middle	-.001	-.051
routine wage, top	-.007	-.080
non-routine wage, bottom	-.019	-.060
non-routine wage, middle	.003	-.075
non-routine wage, top	-.010	-.046

Untargeted moments to match to validate the calibration strategy. All moments, referred to real log-wages, are taken as percentage changes throughout the series.

TABLE D.4: METHOD OF SIMULATED MOMENTS UNDER MONOPSONY, SUB-PERIODS

		2003-2012			2013-2022		
	moment to match	*value*	*data*	*model*	*value*	*data*	*model*
μ_{bot}	routine share, bottom	0.225	0.087	0.103	0.415	0.085	0.051
μ_{mid}	routine share, middle	0.492	0.132	0.061	0.506	0.140	0.030
μ_{top}	routine share, top	0.906	0.100	0.013	0.608	0.097	0.083
λ_{bot}	ICT share, bottom	0.647	0.399	0.399	0.786	0.395	0.395
λ_{mid}	ICT share, middle	0.518	0.314	0.314	0.490	0.295	0.295
λ_{top}	ICT share, top	0.170	0.287	0.287	0.326	0.311	0.311
θ	wage premium, $w(a, [s, s'])$	7.255	0.994	0.994	7.787	0.995	0.995

Estimated values and related matched moment using the Methods of Simulated Moments by Mc Fadden (1989).

TABLE D.5: SUMMARY OF CALIBRATION UNDER MONOPSONY, 2003-2012

		value				
	parameter	*bottom*	*middle*	*top*	*global*	***source***
α	*physical capital, share of* $y(s)$	0.131	0.091	0.254		*data*
ϵ	*demand elasticity across firms*				6	*external*
μ	*weight of routine workers in* $y(s)$	0.225	0.492	0.906		*MSM*
λ	*ICT capital share in* $Q(s)$	0.647	0.518	0.170		*MSM*
θ	*households' productivities dispersion*				7.26	*MSM*
ρ	*EoS, ICT capital and non-routine*	0.355	0.431	0.408		*estimation*
σ	*EoS, routine and ICT composite*	0.366	0.429	0.367		*estimation*

Set of estimated parameters of the model, first-half of the sample. "data" implies that the values are directly computed from data sources, while in "external" I choose standard calibrated values from the literature. "MSM" refers to the Methods of Simulated Moments as in Mc Fadden (1989). "estimation" refers to previously estimated values under a specific procedure; these values are taken from Table 2.8.

TABLE D.6: SUMMARY OF CALIBRATION UNDER MONOPSONY, 2013-2022

		value				
	parameter	*bottom*	*middle*	*top*	*global*	***source***
α	*physical capital, share of* $y(s)$	0.131	0.104	0.260		*data*
ϵ	*demand elasticity across firms*				6	*external*
μ	*weight of routine workers in* $y(s)$	0.415	0.506	0.608		*MSM*
λ	*ICT capital share in* $Q(s)$	0.786	0.490	0.326		*MSM*
θ	*households' productivities dispersion*				7.79	*MSM*
ρ	*EoS, ICT capital and non-routine*	0.819	0.345	0.508		*estimation*
σ	*EoS, routine and ICT composite*	0.326	0.438	0.357		*estimation*

Set of estimated parameters of the model, second-half of the sample. "data" implies that the values are directly computed from data sources, while in "external" I choose standard calibrated values from the literature. "MSM" refers to the Methods of Simulated Moments as in Mc Fadden (1989). "estimation" refers to previously estimated values under a specific procedure; these values are taken from Table 2.8.

TABLE D.7: MODEL COUNTERFACTUALS, MONOPSONY

industry wages	$var(w)_{\tau_2}$	Δ ***model*** $\mid \Delta\ m\Big(\Phi(x,\tau_2),\Theta=\{p_{\tau_2},-p_{\tau_1}\}\Big)$ *level*	*share, model*	*share, data*
		CHANGE		
DATA	1.18			
MODEL	1.08			
$\Delta\sigma$		2.36	2.19	1.99
$\Delta\rho$		.88	.82	.75
$\Delta(\sigma,\rho)$		1.12	1.03	.94
$\Delta\theta$		1.34	1.24	1.13
$\Delta(\sigma,\rho,\theta)$		1.16	1.07	.98
		FIXING		
DATA	1.18			
MODEL	1.08			
$\Delta\Theta_{\mid\sigma}$		.94	.87	.79
$\Delta\Theta_{\mid\rho}$		2.21	2.04	1.87
$\Delta\Theta_{\mid(\sigma,\rho)}$		1.31	1.21	1.11
$\Delta\Theta_{\mid\theta}$		1.04	.96	.88
$\Delta\Theta_{\mid(\sigma,\rho,\theta)}$		1.30	1.20	1.10

Quantification of Model A and Model B given both monopsonistic and monopolistic power by the part of firms. Model implied between-industry real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2),\Theta(-p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\mid\Theta(p,\tau_2)\right]}$, *where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2), *are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

TABLE D.8: MODEL COUNTERFACTUALS FOR ROUTINES, MONOPSONY

		Δ ***model*** $\mid \Delta\ m\Big(\Phi(x,\tau_2),\Theta=\{p_{\tau_2},-p_{\tau_1}\}\Big)$		
routine wages	$var(w)_{\tau_2}$	*level*	*share, model*	*share, data*
		CHANGE		
DATA	1.14			
MODEL	.94			
$\Delta\sigma$		.97	1.03	.85
$\Delta\rho$		.33	.35	.29
$\Delta(\sigma,\rho)$		.64	.68	.57
$\Delta\theta$		.55	.59	.49
$\Delta(\sigma,\rho,\theta)$		.70	.74	.61
		FIXING		
DATA	1.14			
MODEL	.94			
$\Delta\Theta_{\mid\sigma}$		.71	.75	.62
$\Delta\Theta_{\mid\rho}$		1.15	1.22	1.01
$\Delta\Theta_{\mid(\sigma,\rho)}$		.84	.89	.73
$\Delta\Theta_{\mid\theta}$		.89	.95	.78
$\Delta\Theta_{\mid(\sigma,\rho,\theta)}$		.81	.86	.71

Quantification of Model A and Model B given both monopsonistic and monopolistic power by the part of firms. Model implied industry-level between-routine workers real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2)\ \middle|\ \Theta(p,\tau_2),\Theta(-p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\ \middle|\ \Theta(p,\tau_2)\right]}$*, where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2)*, are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

TABLE D.9: MODEL COUNTERFACTUALS FOR NON-ROUTINES, MONOPSONY

		Δ ***model*** $\mid \Delta\, m\Big(\Phi(x,\tau_2), \Theta = \{p_{\tau_2}, -p_{\tau_1}\}\Big)$		
non-routine wages	$var(w)_{\tau_2}$	*level*	*share, model*	*share, data*
		CHANGE		
DATA	1.21			
MODEL	1.22			
$\Delta\sigma$		3.76	3.08	3.10
$\Delta\rho$		1.44	1.18	1.19
$\Delta(\sigma,\rho)$		1.59	1.30	1.31
$\Delta\theta$		2.12	1.74	1.75
$\Delta(\sigma,\rho,\theta)$		1.63	1.33	1.34
		FIXING		
DATA	1.21			
MODEL	1.22			
$\Delta\Theta_{\mid\sigma}$		1.17	.96	.97
$\Delta\Theta_{\mid\rho}$		3.27	2.68	2.70
$\Delta\Theta_{\mid(\sigma,\rho)}$		1.78	1.46	1.47
$\Delta\Theta_{\mid\theta}$		1.18	.97	.97
$\Delta\Theta_{\mid(\sigma,\rho,\theta)}$		1.79	1.47	1.48

Quantification of Model A and Model B given both monopsonistic and monopolistic power by the part of firms. Model implied industry-level between-non-routine workers real log-wage variance changes between two time spans differently calibrated, and changes also according to variations in some parameters; values are referred to variance levels considering bottom, middle, and top industries. Column 2 shows the level in the second period of the between-industry variance both in the data and in the period-two fully calibrated model. For the model specified in column 1: column 3 represents the variance level in the second period as implied by the change in the parameter(s), while column 4 computes the second period variance share of the period-two full model which is accounted by the change in a specified parameter, $\frac{m\left[\Phi(x,\tau_2)\,\middle|\,\Theta(p,\tau_2),\Theta(-p,\tau_1)\right]}{m\left[\Phi(x,\tau_2)\,\middle|\,\Theta(p,\tau_2)\right]}$*, where* $\Phi(x,\tau_2)$ *identifies the series in the second period, and* Θ *the set of parameters where some of them,* (p,τ_2)*, are taken in the second period, while* $(-p,\tau_1)$ *reflects the set of all the parameters in the first period but those considered in the second period. Column 5 reports the fraction of variance explained by changes in parameter(s) in the observed data-driven variance.*

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Conclusions

The research agenda beyond the presented doctoral thesis sits at the confluence of macroeconomics, network theory, and labour markets, seeking to understand how sectoral complementarities and interdependencies shape aggregate dynamics.

In my Job Market Paper, *The Horizontal Geometry of Production Networks*, I develop a framework that captures complementarities in intermediate inputs through shared inter-sectoral trade relationships. I propose new measures of "network economic distances" between sectors via shared upstream and downstream linkages that generates sectoral horizontal interdependencies and complements the classical propagation channel along vertical supply chains, yielding richer predictions on how idiosyncratic shocks diffuse across sectors. Empirically, using U.S. sector-level data, I show that shocks in closely connected sectors generate negative comovement, while shocks in further sectors generate stronger comovement. This highlights the critical role of horizontal geometry in shaping sectoral comovement due to asymmetric shocks. These patterns have important implications for aggregate dynamics: while vertical propagation amplifies shocks through cascading effects and relies on highly central sectors, horizontal transmission generates synchronized adjustments across sectors linked through common suppliers or customers, producing persistent aggregate fluctuations even in the absence of vertical cascades or dominant sectors.

In the second paper, *Industry Contribution to U.S. Wage Inequality*, I explore how industry heterogeneity in capital-labour substitution elasticities shapes wage inequality. By disentangling a "quantity" effect (changes in input composition) from a "structural" effect (changes in substitution technology), I find that structural heterogeneity – particularly on the labour side – dominates in driving inequality, re-framing the conventional view on Skill-Biased Technological Change: wage inequality arises less from technology being skill-biased per se – that is, favouring workers with more technological skills –, since labour market concentration barely amplifies inequality, and more from industry-specific differences in how technologies interact with labour and reshape task structures.