(Dis)Solving the Zero Lower Bound Equilibrium through Income Policy *

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October 26, 2021

Abstract

We investigate the possibility to reflate an economy experiencing a long-lasting zero lower bound episode with subdued or negative inflation by imposing a minimum level of wage inflation. The income policy under investigation is formalized as a downward nominal wage *growth* rigidity (DNWGR), such that wage inflation cannot be lower than a fraction of the inflation target. This policy allows dissolving the zero lower bound steady state equilibrium in an OLG model featuring "secular stagnation" and in an infinite-life model, where this equilibrium emerges due to deflationary expectations.

Keywords: Zero lower bound, Wage indexation, Inflation expectations.

JEL classification: E31, E52, E64.

*We thank the editor, Kenneth D. West, and two anonymous referees for their comments on previous versions of the paper. We are also thankful to Klaus Adam, Carlos Carvalho, Giovanni Crea, Takushi Kurozumi, Eric Leeper, Neil Mehrotra, and Martín Uribe.

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"But what one can make today - what central banks, what economists can make - is an indisputable case for the benefits of having higher wage growth."

Mario Draghi, President of the ECB, Q&A Press Conference, 8 September 2016

1 Introduction

Given the persistence of binding zero lower bound (ZLB) and subdued inflation in the pre-COVID 19 era, some economists made a case for income policies to reflate the economy. We study the theoretical underpinnings of this proposal by defining an income policy imposing minimum wage inflation, i.e., minimum wage growth.

The problem at the ZLB is to stop the downward inertia in inflation. Low inflation resulting from binding ZLB feeds into wages, possibly through formal or informal backward-looking indexation practices, and then into prices again, creating inertia in the inflation dynamics.[1] To break the *downward* wage-price spiral, a possible solution is to "re-index" the economy by imposing a floor on wage inflation. We study a "reflationary" income policy based on a downward nominal wage growth rigidity (DNWGR) such that wage inflation cannot be lower than a fraction of the inflation target.

We show how this income policy works in two different frameworks: the OLG model of Eggertsson et al. (2019) (EMR, henceforth), in which a ZLB equilibrium arises when the natural interest rate is negative, and the infinite-life model of Schmitt-Grohé and Uribe (2017) (SGU, henceforth), in which a ZLB equilibrium originates from deflationary expectations, as in Benhabib et al. (2001).

Both papers also feature a downward nominal wage rigidity (DNWR), that we replace by a DNWGR, which can also be seen as a form of income policy based on wage indexation to the inflation target. This reflationary income policy eliminates the ZLB equilibrium in both theoretical frameworks, provided that the inflation target is sufficiently high. If wage inflation is high enough, agents cannot coordinate on a deflationary or a secular stagnation equilibrium because expectations of deflation/low inflation and ZLB are not consistent with rational expectations. In equilibrium, the DNWGR constraint does not bind; hence, it is not the case that it is mechanically imposed. Moreover, both price and wage inflation is equal to the intended target, and there is full employment in the unique equilibrium that survives. The income policy acts as a coordination device that destroys the ZLB equilibrium.

Our paper relates to the debate regarding the implementation of income policy to reflate an economy in ZLB. Arbatli et al. (2016) advocate such a policy as a "fourth arrow" to the "Abenomics". However, they simulate the IMF Flexible System of Global Models (FSGM) without formalizing a proper income policy but adding shocks to price and wage inflation expectations. Instead, da Silva and Mojon (2019) propose an increase in the nominal unit labor costs consistent with the 2% inflation target. Still, they do not investigate the effectiveness of this proposal explicitly.

This paper is also linked to the ZLB literature. Specifically, Mertens and Ravn (2014) and Bilbiie (2018) study the effects of several policies depending on whether the liquidity trap is fundamental or expectation-driven and find that the policies beneficial in the former are detrimental in the latter and vice versa. Instead, the reflationary income policy is "robust" because it can address the ZLB problem independently of its source. Cuba-Borda and Singh (2019) find a similar result for minimum wage policy by considering a unified framework accommodating the secular stagnation hypothesis and the expectation-driven liquidity trap. However, the minimum wage policies can only *mitigate* the secular stagnation equilibrium, imposing a floor on the nominal wage *level*. By contrast, the income policy *eliminates* both the expectations trap and the secular stagnation equilibrium, imposing a floor on the *growth rate* of nominal wage. Moreover, in Cuba-Borda and Singh (2019), as in EMR and SGU, increasing the inflation target cannot eliminate any of the two bad equilibria. Instead, it does so in our framework, and thus there is no issue of credibility of the target due to multiple steady states.

The paper proceeds as follows. Sections 2 and 3 present how the income policy works, respectively, in the EMR and SGU model. Section 4 concludes.

2 Reflation in the EMR OLG model

We study a three-period OLG economy. Young households borrow up to the debt limit D_t through a one-period riskless bond sold to the middle-aged ones. The middle generation earns a positive income, $Y_t = \frac{W_t}{P_t} L_t + \frac{Z_t}{P_t}$, by supplying inelastically its labor endowment $\bar{L}$ for a wage W_t and running a firm, whose nominal profits are Z_t. Old agents consume their wealth. The household's problem is

$$\max_{C^m_{t+1}, C^o_{t+2}} E_t \left\{ \ln C^y_t + \beta \ln C^m_{t+1} + \beta^2 \ln C^o_{t+2} \right\}$$

s.t.

$$C^y_t = B^y_t = \frac{D_t}{(1+r_t)} \tag{1}$$

$$C^m_{t+1} = Y_{t+1} - (1+r_t) B^y_t - B^m_{t+1} \tag{2}$$

$$C_{t+2}^{o} = (1 + r_{t+1}) B_{t+1}^{m}. \tag{3}$$

β is the subjective discount factor. C_t and B_t denote, respectively, the real consumption of the generations and the real value of bonds where the superscripts y, m, o stands for young, middle-aged and old, respectively. In Equation (1), the young-age consumption is constrained by the debt limit, which binds because EMR assume $D_{t-1} < Y_t / [1 + (1 + \beta) \beta]$. The optimality condition for the problem is the Euler equation

$$\frac{1}{C_t^m} = \beta (1 + r_t) E_t \frac{1}{C_{t+1}^{o}}, \tag{4}$$

where the real return on bonds, r_t, is linked to the nominal interest rate, i_t, and (expected) future inflation, $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$, via the Fisher equation: $1 + r_t = (1 + i_t) E_t \Pi_{t+1}^{-1}$.

The financial market clears when the total demand of loans from young households equals the supply from middle-aged ones: $(1 + g_t) B_t^y = B_t^m$, where g_t is the population growth rate. We can denote the loan demand with L_t^d and express it as

$$L_t^d = \left(\frac{1 + g_t}{1 + r_t} \right) D_t \tag{5}$$

by using (1). Combining (1), (2), (3), and (4) yields the loan supply L_t^s:

$$L_t^s = B_t^m = \frac{\beta}{1 + \beta} (Y_t - D_{t-1}). \tag{6}$$

The market clearing real interest rate that equates (5) and (6) is

$$(1 + r_t) = \frac{(1 + g_t)(1 + \beta) D_t}{\beta (Y_t - D_{t-1})}. \tag{7}$$

At the potential level of output, $Y_t = Y^f$, (7) defines the natural interest rate r_t^f.

Each middle-aged household runs a firm that is active for only one period in a perfectly competitive market. The production technology of firms exhibits decreasing returns to labor, $Y_t = L_t^{\alpha}$ with $0 < \alpha < 1$. Profits, $Z_t = P_t Y_t - W_t L_t$, are maximized when labor is remunerated at its marginal productivity, $\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}$.

Finally, there are two key frictions in the model. The DNWR in the labor market and the ZLB in the monetary policy rule. We focus on the steady state of the EMR model, conveniently depicted in Figure 1, using an aggregate supply (AS) and aggregate demand (AD) diagram. Note that the DNWR frictions determines the features of the AS curve, and the ZLB the features of the AD curve.

DNWR and *AS* curve. The labor market is perfectly competitive, but workers are unwilling to supply labor for a nominal wage lower than a minimum level:

$$W_t = \max\left\{W_t^*, W_t^{flex}\right\}. \tag{8}$$

$W_t^{flex} \equiv P_t \alpha \bar{L}^{\alpha-1}$ is the "flexible" wage compatible with full employment $\bar{L}$, and W_t^* is the minimum wage that is proportional to that in the previous period (Schmitt-Grohé and Uribe, 2016):

$$W_t^* = \gamma W_{t-1}, \tag{9}$$

where $0 < \gamma \leq 1$. The labor market does not necessarily clear due to the DNWR (8). If market clearing requires a wage larger than γW_{t-1}, $W_t = W_t^{flex}$ and the labor market clears, $L_t = \bar{L}$. The AS curve is accordingly vertical at the potential output, $Y^f = \bar{L}^{\alpha}$, for $\Pi^W = \Pi \geq \gamma$. Instead, if labor supply exceeds labor demand at $W_t = \gamma W_{t-1} > W_t^{flex}$, the wage cannot decrease further, so that involuntary unemployment arises, $L_t < \bar{L}$. In this case, the AS is thus flat at the wage/price inflation $\Pi^W = \Pi = \gamma$ for $\forall L \leq \bar{L}$, with the level of employment/output that is demand-determined along the AS^{WR}.

ZLB and ***AD*** **curve.** The central bank follows a standard Taylor rule:

$$1+i_t = \max\left[1, \left(1+r_t^f\right)\Pi^*\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right], \tag{10}$$

that responds only to inflation and where $\phi_\pi > 1$ and Π^* is the gross inflation target. Whether or not the ZLB constraint (10) is binding defines two regimes for the AD curve. When the ZLB does not bind, $1+i > 1$, we can combine (7), (10) and the Fisher equation to derive the AD curve in steady state as:

$$Y_{AD}^{TR} = D + \left(\frac{1+\beta}{\beta}\right)\left(\frac{1+g}{1+r^f}\right)\left(\frac{\Pi^*}{\Pi}\right)^{\phi_\pi - 1} D. \tag{11}$$

Equation (11) defines a negative relationship between inflation and output, represented by the downward-sloping curve AD^{TR} in Figure 1. By contrast, the ZLB, $1+i=1$, implies a positive relationship between inflation and output:

$$Y_{AD}^{ZLB} = D + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi D, \tag{12}$$

shown by the upward-sloping AD^{ZLB} curve in Figure 1.[1] We denote Π^{kink} the inflation rate at which AD^{TR} and AD^{ZLB} cross. It determines when the ZLB becomes binding and can be computed from the two arguments in (10):

$$\Pi^{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi_\pi}} \Pi^{*\frac{\phi_\pi - 1}{\phi_\pi}}. \tag{13}$$

To prepare the ground for our main result, Figure 1 shows that the *AD* curve shifts from $AD^{TR,0}$ to $AD^{TR,1}$ when the inflation target, Π^*, increases. The higher the inflation target, the lower the risk of hitting the ZLB for a given natural interest rate r^f in (10). Therefore, a higher Π^* shifts out the kink in the *AD* curve, and thus the downward-sloping *AD* curve, but it does not affect the upward-sloping one.

Steady State. The crossing between the *AS* and the *AD* curves identifies the steady state in Figure 1. A "secular stagnation" equilibrium arises for $r^f < 0$, but there can be two different cases depending on the inflation target. First, there is a unique steady state at point A, given by the intersection between AD^{ZLB} and AS^{WR} (see the dashed line $AD^{TR,0}$). It is a demand-determined and stagnant steady state because ZLB and DNWR are both binding. Second, there are three different steady states (see the solid line $AD^{TR,1}$): (A) the *ZLB-U* equilibrium just described featuring ZLB, inflation below the target and unemployment: $i = 0, \Pi = \gamma < \Pi^*, Y < Y^f$; (B) a *ZLB-FE* equilibrium that occurs at the intersection of the AD^{ZLB} and the AS^{FE}, and it features ZLB, inflation below the target and full employment: $i = 0, 1 < \Pi = \frac{1}{1+r^f} < \Pi^*, Y = Y^f$; (C) a *TR-FE* equilibrium that occurs at the intersection of the AD^{TR} and the AS^{FE}, and it features a positive nominal interest rate, inflation at the target and full employment: $i > 0, \Pi = \Pi^*, Y = Y^f$. The equilibria *ZLB-U* and *TR-FE* are determinate, while the equilibrium *ZLB-FE* is an indeterminate, but not deflationary, steady state á la Benhabib et al. (2001).[2]

As explained above, an increase in the inflation target moves AD^{TR}, but moves neither the AD^{ZLB} nor the *AS*. Hence, if the natural real interest rate is negative, a *ZLB-U* equilibrium always exists no matter what the inflation target is.[3]

2.1 Dissolving the ZLB Equilibrium

We now present an income policy capable of avoiding a secular stagnation even if $r^f < 0$. The secular stagnation equilibrium *ZLB-U* vanishes with this policy, which is a simple modification of equation (9) that defines W_t^* in equation (8) to

$$W_t^* = \delta \Pi^* W_{t-1}, \tag{14}$$

where $0 < \delta \leq 1$. From an economic point of view, (14) implies that wage inflation cannot be lower than a certain fraction δ of the inflation target. Hence, it puts a *floor* on wage deflation to break the *downward* wage-price spiral, and it links wage growth to targeted inflation to re-anchor inflation expectations.

Although this modification might seem minimal, it underlies a substantial change. Equation (9) establishes a lower bound on the nominal wage *level*, with γ measuring how much, if anything, the wage can be cut downward. By contrast, (14) imposes a lower bound to the nominal wage *growth rate*, where δ can be interpreted as the degree of wage indexation to the inflation target. Therefore, the reflationary income policy implementation requires a switch from DNWR to DNWGR.

From an analytical point of view, comparing Figure 2 with Figure 1 reveals how this modification changes the results in the previous section. The main point is that (14) *makes the AS curve to shift with the inflation target*, because the AS^{WR} curve is now equal to $\delta\Pi^*$, rather than γ, as in the EMR case. Hence, an increase in the inflation target shifts the AS^{WR} curve upward. As the AD curve is unchanged with respect to the previous section, raising the inflation target shifts out AD^{TR}, as in Figure 1. We are now in the position to state our main result. [4]

Proposition 1. *Assume* $r^f < 0$ *and* $\delta \leq 1$. *Then, if* $\Pi^* > \frac{1}{\delta(1+r^f)}$, *there exists a unique, locally determinate,* $TR-FE$ *equilibrium, where the ZLB is not binding, inflation is at the target and output is at full employment,* $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.

There always exists a sufficiently high level of the inflation target, such that the unique and locally determinate equilibrium features full employment and inflation at the target without binding ZLB. Figure 3 displays the intuition very clearly. As the inflation target increases, the economy moves from Panel A to Panel E. The key thing to note is that the AD curve moves as described in the previous section, but now the AS^{WR} shifts upward too. For an enough high Π^*, the economy reaches the situation in Panel E, where the $ZLB-U$ equilibrium disappears for $\Pi^* > \frac{1}{\delta(1+r^f)} > \frac{1}{1+r^f}$ because the level of output on the AD^{ZLB} corresponding to $\Pi = \delta\Pi^*$ is greater than Y^f, so that the AD^{ZLB} never crosses the AS^{WR}. This condition guarantees the uniqueness of the $TR-FE$ equilibrium, which, instead, emerges for $\Pi^* > \Pi^{kink}$, and thus $\Pi^* > \frac{1}{1+r^f}$.[5]

Contrary to EMR where it is powerless, now monetary policy can wipe out the ZLB equilibrium by choosing an adequate inflation target. However, the degree of wage indexation, δ, also plays a crucial role: the higher this parameter, the lower the inflation target necessary to prevent binding ZLB. Finally, the next proposition presents another important implication of the reflationary income policy with respect to EMR.

Proposition 2. *Assume* $r^f < 0$, $\delta \leq 1$, *and that the economy is trapped in a secular stagnation equilibrium,* $ZLB-U$ *(Panel A). Then, an increase in the inflation target is always beneficial because steady state output and inflation increase, irrespective of this increase being sufficient or not to escape from the secular stagnation.*

Any, however small, increase in the target shifts the AS^{WR} upward, and thus it moves the secular stagnation equilibrium along the AD^{ZLB} increasing the level of output and inflation. This is depicted in Figure 3, where the $ZLB-U$ equilibrium A in Panel A moves up in Panels B, C and D. This does not happen in the EMR model. In Figure 1 both AD^{ZLB} and AS^{WR} curves do not change with the inflation target. As a result, a mild increase in the target does not affect the secular stagnation equilibrium $ZLB-U$ at point A, capturing Krugman's (2014) idea of "timidity trap". Only sufficiently large changes in the target make the $TR-FE$ equilibrium appear. Our model has a similar flavor but has a quite different implication: while it is still true that the policy is subject to a "timidity trap" to escape from the secular stagnation, in the sense that the inflation target should be sufficiently high to avoid it, an increase in the target is always beneficial. EMR originally assume $W_t^* = \gamma W_{t-1} + (1-\gamma) W_t^{flex}$, whereby only a fraction γ of workers is subject to DNWR, while the remanining fraction has flexible wages. Our modification does not alter the model but allows for better comparability with the reflationary income policy and the DNWR in SGU. In the **Online Appendix**, we show that Proposition 1 and 2 hold in the EMR model even if we assume an encompassing constraint defined as $W_t = \max\left\{\gamma^{WL} W_{t-1} + \gamma^{WG} \Pi^* W_{t-1} + \left(1-\gamma^{WL}-\gamma^{WG}\right) W_t^{flex}, W_t^{flex}\right\}$.

3 Reflation in the SGU infinite-life model

We turn to a different model, where the source of binding ZLB is a negative shock to inflation expectations. SGU employ a flexible-price, infinite-life representative agent model to study the dynamics leading to a liquidity trap and a jobless recovery.[1] Unless otherwise mentioned, the notation is identical to that of the EMR model. The representative household maximizes the utility function $U_t = E_0 \sum_{t=0}^{\infty} \beta^t \ln C_t$, subject to the budget constraint $P_t C_t + B_t = W_t L_t + Z_t + (1+i_{t-1}) B_{t-1}$ and the no-Ponzi-game condition $lim_{j\to\infty} E_t \left[\prod_{s=0}^{j} (1+i_{t+s})^{-1}\right] B_{t+j+1} \geq 0$, where C_t denotes the real consumption expenditure, and B_t is the value of risk-free bonds in nominal terms. The optimality conditions for the household's problem are the Euler equation

$$C_t^{-1} = \beta (1+i_t) E_t \left(\frac{C_{t+1}^{-1}}{\Pi_{t+1}}\right), \tag{15}$$

where $1+r_t = (1+i_t) E_t \Pi_{t+1}^{-1}$, and the no-Ponzi-game condition holding with equality. The problem of the firm is the same as illustrated in the EMR model.

As before, we explain the steady state equilibria using an *AS*-*AD* diagram, where the DNWR shapes the *AS* curve, and the monetary policy shapes the *AD* curve.

DNWR and *AS* curve. With respect to the EMR model in the previous section, SGU employ a different specification of the DNWR constraint:

$$W_t \geq \gamma_0 (1-u_t)^{\gamma_1} W_{t-1} = \gamma_0 \left(\frac{L_t}{\bar{L}}\right)^{\gamma_1} W_{t-1}. \tag{16}$$

The DNWR implies that the lower bound on the nominal wage depends on the level of unemployment, $u_t = (\bar{L} - L_t)/\bar{L}$, or on the employment ratio $L_t/\bar{L}$. When $L_t = 0$, i.e., $u_t = 1$, the lower bound is zero, then it increases with employment with elasticity γ_1. At full employment nominal wages cannot be lower than $\gamma_0 W_{t-1}$ as in (9). However, SGU impose the following important assumption on γ_0: $\beta < \gamma_0 \leq \Pi^*$. For simplicity, we assume $\gamma_0 = \Pi^*$, as SGU do in their quantitative calibration. The

DNWR (16) implies the following complementary slackness condition

$$(\bar{L}-L_t)\left[W_t-\gamma_0\,(1-u_t)^{\gamma_1}\,W_{t-1}\right]=0 \tag{17}$$

that ties down quite strictly the type of equilibrium under unemployment. If $L_t<\bar{L}$, then in steady state it follows $W_t/W_{t-1}\equiv\Pi^W=\Pi=\gamma_0\,(1-u_t)^{\gamma_1}<\gamma_0=\Pi^*$. Hence, steady state inflation is below the target whenever there is positive unemployment.

Similar to the previous model, there are accordingly two regimes characterizing the AS in steady state. First, for $\Pi\geq\gamma_0=\Pi^*$, the AS is still vertical at the potential output $Y^f=\bar{L}^{\alpha}$. Second, for $\Pi<\gamma_0=\Pi^*$, the AS^{WR} is upward-sloping in the presence of unemployment due to the binding DNWR constraint. Using (16), (17), and the production function, $Y_t=L_t^{\alpha}$, yields:

$$Y_{AS}^{WR}=\left(\frac{\Pi}{\gamma_0}\right)^{\frac{\alpha}{\gamma_1}}Y^f. \tag{18}$$

The two branches of the AS schedule meet at the kink $\Pi=\gamma_0=\Pi^*$.

ZLB and ***AD*** **curve.** The demand side is shaped by the monetary policy rule:

$$1+i_t=\max\left\{1,1+i^*+\alpha_\pi\,(\Pi_t-\Pi^*)+\alpha_y\ln\left(\frac{Y_t}{Y^f}\right)\right\}, \tag{19}$$

where $1+i^*=\Pi^*/\beta>1$. For $1+i>1$, we can compute AD from (19) by substituting $1+i$ for its steady state value $\frac{\Pi}{\beta}$:

$$\ln Y_{AD}^{TR}=\ln Y^f-\frac{\beta\alpha_\pi-1}{\beta\alpha_y}\,(\Pi-\Pi^*)\,. \tag{20}$$

This equation expresses a negative steady state relationship between output and inflation if monetary policy is active ($\beta\alpha_\pi > 1$), as in the EMR model. The main difference between the EMR and SGU models lies in the steady state determination of the equilibrium/natural real interest rate. Given the Euler equation, the inverse of β pins down the natural real interest rate in an infinite-life model, so the latter does not depend on the supply and demand of assets as in an OLG model. This implies that, while the AD^{TR} is downward-sloping as in the EMR model, the AD^{ZLB} is now horizontal in this model, rather than upward-sloping. If the ZLB is binding ($i = 0$) and given $1 + r = 1/\beta$, the steady state inflation rate must equal β due to the Fisher equation, whatever the level of steady state output. AD is therefore flat at $\Pi = \beta$, and the AS determines the steady state output for that inflation level.

Steady State. Figure 4 shows the AS-AD diagram for the SGU model. The assumption in SGU $\beta < \gamma_0 \leq \Pi^*$ guarantees that there does not exist either an intersection between AS^{FE} and AD^{ZLB} or an intersection between AD^{TR} and AS^{WR}.[2] Given these assumptions, there are always two equilibria.[3] Point A is a $ZLB-U$ type of equilibrium, where both the ZLB and the DNWR constraints are binding, and point C is a $TR-FE$ one, where none of the two constraints is binding, the economy is at full employment and inflation at target. However, the $ZLB-U$ equilibrium in the SGU model does not reflect the idea of secular stagnation as described in Summers (2015) that entails $r^f < 0$. By contrast, it is an (indeterminate) expectation-driven deflationary equilibrium á la Benhabib et al. (2001). Therefore, we define it as a *deflationary* equilibrium, because $\Pi = \beta < 1$, rather than a *secular stagnation* one.

3.1 Dissolving the ZLB equilibrium

We now apply the reflationary income policy to the SGU model by replacing the DNWR (16) with the DNWGR given by (8) and (14). Recall that the idea is to reflate the economy by using the DNWGR constraint to impose *a floor to the growth rate of nominal wages that depends on the inflation target*. (16) does not do that because wage inflation is bounded by zero when employment is zero. Figure 5 plots the *AS-AD* diagram for the SGU model with the DNWGR. In this case, instead of being upward-sloping, the AS^{WR} is flat at the wage/price inflation $\Pi = \delta\Pi^*$.

Figure 5 shows how the DNWGR yields similar implications as in Section 2. Two are the stark differences compared to the original SGU model. First, if the inflation target is not high enough, $\Pi^* < \beta/\delta$, the DNWGR makes a new equilibrium arise at point B (Panel A). This new equilibrium replaces the original deflationary one at point A, and thus the economy runs at the potential level even when the ZLB is binding due to deflation. Indeed, though Π^* is not high enough to destroy the ZLB equilibrium, the DNWGR sets a minimum level of wage/price inflation, $\delta\Pi^*$, that is lower than actual inflation, β, allowing the real wage to fall and thus stimulating employment. The equilibrium B in Panel A of Figure 5, though deflationary because of $\Pi = \beta$, resembles the $ZLB-FE$ one in Figure 1. Second, while in the original SGU model two equilibria always exist, here instead, for a sufficiently high inflation target, $\Pi^* > \beta/\delta$, deflationary expectations cannot be supported in equilibrium so that the $ZLB-U$ equilibrium A dissolves (Panel B). Intuitively, by forcing the increase in wage inflation above a certain threshold, no level of inflation/deflation supports the ZLB equilibrium. Our DNWGR constraint acts as a coordination device for agents on the now unique $TR-FE$ equilibrium.[4]

The two following propositions formalize these results.

Proposition 3. *Assume $\delta \leq 1$. Then, if $\Pi^* > \beta/\delta$, there exists a unique, locally determinate, $TR-FE$ equilibrium, where the ZLB is not binding, inflation is at the target and output is at full employment, $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.*

Proposition 4. *Assume $\delta \leq 1$, and that the economy is trapped in a deflationary $ZLB-U$ equilibrium (point A). Then, the introduction of the DNWGR is always beneficial because steady state output increases, irrespective of the economy escapes from deflation.*

As a further extension, we assume that minimum wage inflation depends negatively on unemployment, combining the original DNWR constraint in SGU, (16), with the DNWGR in (14):

$$\frac{W_t}{W_{t-1}} \geq \delta\Pi^* + \gamma(u_t) = \delta\Pi^* + \gamma_0 (1-u_t)^{\gamma_1}. \tag{21}$$

We also assume that $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$ (and thus $\delta < 1$), which is the equivalent assumption to $\beta < \gamma_0 \leq \Pi^*$ in the SGU case. Accordingly, the AS^{WR} becomes

$$Y_{AS}^{WR} = \left(\frac{\Pi - \delta\Pi^*}{\gamma_0}\right)^{\frac{\alpha}{\gamma_1}} Y^f. \tag{22}$$

Figure 6 shows how the modification of the DNWGR affects the results in the SGU model. Panel A displays the two equilibria, $ZLB-U$ and $TR-FE$, with our modified DNWGR, (21), along with the original deflationary equilibrium A in the SGU model. The other two panels show what happens when the inflation target increases. In Panel A, for $\Pi^* < \beta/\delta$, the introduction of the DNWGR no longer improves the deflationary equilibrium as in Figure 5 (Panel A) but rather worsens it reducing output for the same level of deflation, $\Pi = \beta$. On the other hand, Panel B shows that if the inflation target increases but is not sufficient to escape from the ZLB, the deflationary equilibrium implies a further reduction in output/employment. Only when the inflation target increases sufficiently, so that $\Pi^* > \beta/\delta$, the $ZLB-U$ equilibrium A disappears, leaving as unique equilibrium the $TR-FE$ one at point C. Putting differently, Krugman's (2014) timidity trap is *enhanced* in the SGU model, when the minimum wage inflation imposed by the DNWGR depends negatively on unemployment.

Proposition 5. ***Enhanced Timidity Trap.*** *Assume that the economy is trapped in a deflationary $ZLB-U$ equilibrium (Point A). Then, the introduction of the DNWGR (21) is detrimental because steady state output decreases in a ZLB equilibrium unless the DNWGR allows escaping from deflation. Furthermore, for $\Pi^* < \beta/\delta$, the output losses caused by the DNWGR are greater, the higher is the inflation target Π^*.*

The last result not only contradicts Proposition 4, but it is also the opposite of Proposition 2, which is robust to the specification of the DNWGR. This stark difference between the EMR and SGU models relies on the demand side, not on the different DNWGR. AD^{ZLB} is upward-sloping and steeper than the AS^{WR} in an OLG model because any increase in inflation decreases the real interest rate, spurring demand at the ZLB. Instead, in an infinite-life economy, the real interest rate is given by $1/\beta$, and thus inflation has to be $\Pi = \beta$ in a ZLB/deflationary equilibrium. The AD^{ZLB} is then flat and thus flatter than the upward-sloping AS^{WR} in Figure 6. As price/wage inflation is β, any attempt to increase the inflation target enlarges the inflation gap, Π/Π^*, and the binding DNWGR dictates higher unemployment in equilibrium. The increase in the inflation target is too timid; hence unless agents change their expectations by moving to the other $TR-FE$ equilibrium, the ZLB equilibrium survives and worsens.

4 Conclusions

Our theoretical exercise sheds light on the effect of income policies at the ZLB. Policy makers (see footnote 5) worries that a low inflation environment could unfold into a downward wage-price spiral by feeding into wage dynamics through expectations and formal or informal backward-looking indexation practices. The solution is to break this spiral by "indexing" wages to the target inflation rate, providing a floor to wage inflation.

One way of intuitively grasps the essence of this solution is simply to see it as the reversion of income policies used in the '80s to "de-index" the economy by breaking the *upward* wage-price spiral underlying high inflation persistence. This type of income policy was popular at the time, and many cases show that it could be very efficient in disinflating the economy (da Silva and Mojon, 2019). The disinflationary experience of Italy in the '90s provides a clear example of this very same idea, but in opposite direction. While the source of the Italian inflation was the oil crisis(es) of the '70s, its upward inertia in the following decade resulted from the automatic indexation to the past inflation mechanism, the so-called *scala mobile*. To bring down inflation, Italian institutions put in place a coordinated effort involving exchange rate policies (to enter again the European ERM), monetary restraint, and income policies. In particular, a key cornerstone of this policy mix was the Protocol signed by the employers and trade-union organizations on 23 July 1993. It marked the definite dismantling of the automatic indexation to the past inflation mechanism (*scala mobile*) replaced, exactly as in our theoretical model, by the targeted inflation by the government (*tasso dinflazione programmato*) as a reference for the indexation of collective contracts.[1] Fabiani et al. (1998) showed that the wage moderation induced by the Protocol was crucial to attain the disinflationary path (in the sense of gaining from 3 to 5 % less of inflation in 1997), in a period exhibiting two large exchange rate devaluations and where a further tightening of monetary policy would have been very costly and put in danger the process of fiscal consolidation (see also Destefanis et al., 2005 and Casadio et al., 2005).

Having said that, it should be clear that our theoretical solution could suffer of quite serious applicability and implementability issues as a policy proposal in actual economies. First, it requires centralized wage bargaining to impose wage indexation to target inflation, switching from DNWR to DNWGR. This limits the policy's applicability to a small set of countries with such an institutional framework, excluding those where firm-level wage negotiations prevail (low δ). Second, even if wage bargaining is centralized, our analysis is confined to steady state while we do not say anything about the possible cost of the transition, which to be modelled realistically would require a much richer framework. Third, and related, credibility issues regarding the income policy could arise during the transition. Indeed, the DNWGR can cause lower output in an expectation-driven liquidity trap if it forces the wages to increase, but inflation expectations do not adjust upwards, or they do slowly. Hence, if the economy can escape from the ZLB only gradually, as it could happen in a richer model (e.g., including price and wage stickiness), the short-run costs of the transition maybe be so high to undermine the credibility of the reflationary income policy or even to discourage its implementation altogether. Fourth, in an open economy context, wage increases may hurt the international competitiveness of firms, with consequent further employment/output losses and even weaker credibility of the income policy, or, alternatively, international competition might impede the transmission from wage inflation to price inflation, hurting firms' profit margins. Finally, the DNWGR could distort the allocation of labor input, generating misallocation of resources across low and high productivity sectors or firms.

References

Acocella, Nicola and Riccardo Leoni, eds, *Social Pacts, Employment and Growth. A Reappraisal of Ezio Tarantellis Thought* AIEL Series in Labour Economics, Physica-Verlag Heidelberg, 2007.

Arbatli, Elif, Dennis Botman, Kevin Clinton, Pietro Cova, Vitor Gaspar, Zoltan Jakab, Douglas Laxton, Constant Lonkeng Ngouana, Joannes Mongardini, and Hou Wang, "Reflating Japan; Time to Get Unconventional?," IMF Working Papers 16/157, International Monetary Fund August 2016.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe, "The Perils of Taylor Rules," *Journal of Economic Theory*, 2001, *96* (1-2), 40–69.

Bilbiie, Florin O., "Neo-Fisherian Policies and Liquidity Traps," CEPR Discussion Papers 13334, C.E.P.R. Discussion Papers November 2018.

Casadio, Piero, Maria Lamelas, and Giorgio Rodano, "Cambiamento istituzionale, salari e flessibiità: L'esperienza della concertazione in Italia," *Rivista Internazionale di Scienze Sociali*, 2005, *113* (2), 185–214.

Cuba-Borda, Pablo and Sanjay R. Singh, "Understanding Persistent Stagnation," Working Papers 1243, Board of Governors of the Federal Reserve System 2019.

da Silva, Luiz A. Pereira and Benoît Mojon, "Exiting Low Inflation Traps by "Consensus": Nominal Wages and Price Stability," 2019. Speech based on the keynote speech at the Eighth High-level Policy Dialogue between the Eurosystem and Latin American Central Banks, Cartagena de Indias, Colombia, 28-29 November 2019.

Destefanis, Sergio, Giuseppe Mastromatteo, and Giovanni Verga, "Wages and Monetary Policy in Italy Before and After the Wage Agreements," *Rivista Internazionale di Scienze Sociali*, 2005, *113* (2), 289–318.

Draghi, Mario, "Accompanying the economic recovery," Introductory speech by Mario Draghi, President of the ECB, at the ECB Forum on Central Banking, Sintra, 27 June 2017.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins, "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, January 2019, *11* (1), 1–48.

Fabiani, S., A. Locarno, G. Oneto, and P. Sestito, "Risultati e problemi di un quinquennio di politica dei redditi: una prima valutazione quantitativa," *Rivista*

Internazionale di Scienze Sociali, 1998. Bank of Italy, Temi di Discussione, No. 329.

Krugman, Paul, "The Timidity Trap," *The New York Times*, 2014. March 20, available at https://www.nytimes. com/2014/03/21/opinion/krugman-the-timidity-trap.html.

Mertens, Karel and Morten O. Ravn, "Fiscal Policy in an Expectations-Driven Liquidity Trap," *Review of Economic Studies*, 2014, *81* (4), 1637–1667.

Schmitt-Grohé, Stephanie and Martín Uribe, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

_ and _ , "Liquidity Traps and Jobless Recoveries," *American Economic Journal: Macroeconomics*, 2017, *9* (1), 165204.

Summers, Lawrence H., "Demand Side Secular Stagnation," *American Economic Review*, May 2015, *105* (5), 60–65.

5 Notes

Section 1

1. *"A prolonged period of low inflation is always likely to be exacerbated by backward-lookingness in wage and price formation that occurs due to institutional factors, such as wage indexation. This has plainly happened in the euro area."* (Draghi, 2017)

Section 2

1. As EMR, we depict AD^{TR} as linear in Figure 1 for clarity, despite it being non-linear. We will do the same for AD^{TR} in the SGU model. None of the results obviously depends on this.

2. Although EMR show it for a different DNWR specification, these results still hold, and a proof is available upon request. For $\gamma = 1$, the DNWR coincides with the minimum wage policy of Cuba-Borda and Singh (2019), and it cannot eliminate, as well as the secular stagnation equilibrium, the *ZLB-FE* one. Hence, that policy seems ineffective if the equilibrium á la Benhabib et al. (2001) features positive, below the target inflation as in the EMR model.

3. While there is always a minimum level of public debt that makes $r^f > 0$, this value might be not necessarily sustainable/achievable, as shown by EMR in their quantitative exercise. We refer to the **Online Appendix** for an analysis of the complementarity between income and fiscal policies.

4. A formal proof of Proposition 1 is given in the **Online Appendix**.

5. Figure 3 refers to the case $\delta < 1$, that guarantees that AD^{TR} does not intersect

AS^{WR} (which instead happens for values of $\delta > 1$). When $\delta = 1$, the unique $TR-FE$ equilibrium still lies on the AD^{TR}, but now it corresponds to the kink point of the AS in which AS^{FE} and AS^{WR} cross.

Section 3

1. Compared to the original model in SGU, we abstract from growth and fiscal policy, and we assume a logarithmic utility function instead of a more general CRRA specification. Our results are unaffected by these modifications.

2. $\beta < \gamma_0$ implies that the floor on wage inflation corresponding to $u = 0$ is higher than the minimum inflation so that AD^{ZLB} and AS^{FE} cannot cross. Instead, $\gamma_0 \leq \Pi^*$ implies that such a maximum floor is at most equal to the inflation target, preventing the crossing between AD^{TR} and AS^{WR}.

3. There are no restrictions on γ_1. Hence, we can distinguish three cases: if $\gamma_1 > \alpha$, the AS^{WR} is convex (Figure 4); it is concave for $\gamma_1 < \alpha$, and it is a straight line when $\gamma_1 = \alpha$. Whether the AS^{WR} is convex, concave, or a straight line does not affect our results qualitatively.

4. Unlike SGU, we do not prevent the intersection between the AD^{ZLB} and AS^{FE} because there is no economic reason to exclude $\delta\Pi^* < \beta$. On the other hand, the inflation target is the natural upper limit to the floor on wage inflation, implying $\delta \leq 1$. For this range of values, AD^{TR} intersects AS^{WR} only if $\delta = 1$. In this limiting case, the $TR-FE$ equilibrium C occurs at the kink point of the AS curve.

Section 4

1. This Protocol implemented the original Ezio Tarantelli's idea for which the young Italian economist was killed by Red Brigades in 1985. As Acocella and Leoni, eds (2007) put it: "He thought that trade unions could play a positive role by agreeing to set wages on the basis of a target rate of inflation. Therefore, they would contribute to economic and social stability through influencing future price expectations, protecting real wages."

Figure 1: Steady State equilibria in the EMR model

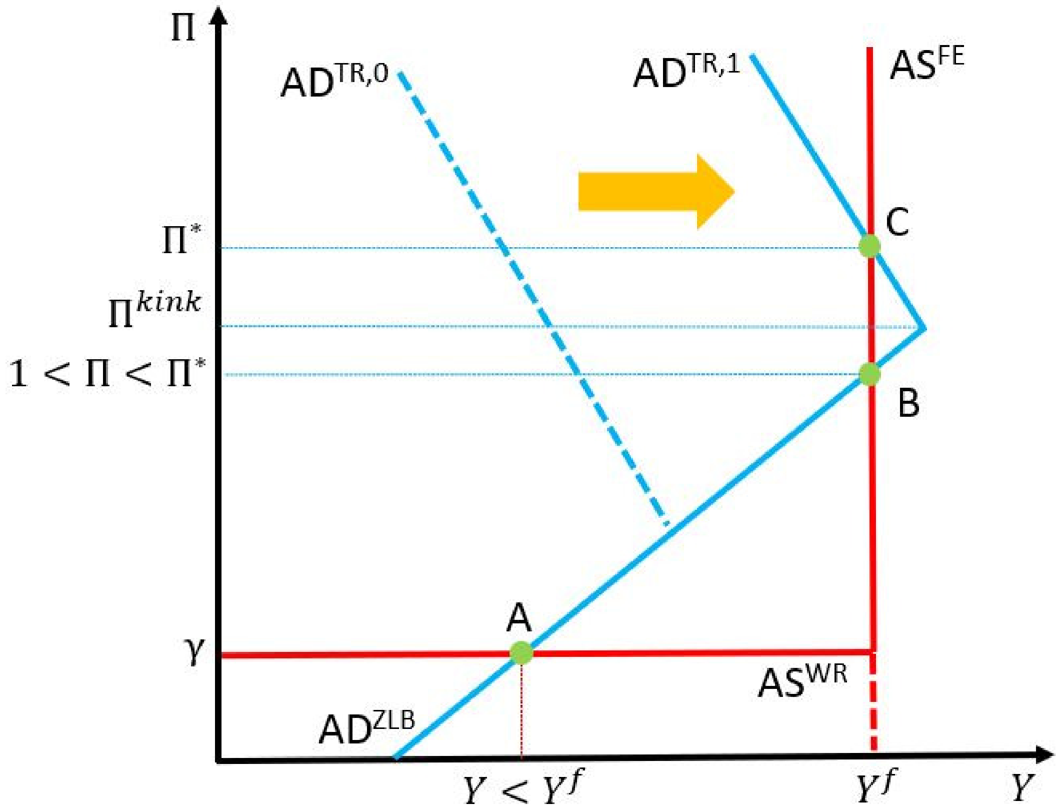

Figure 2: Unique steady state equilibrium in the EMR model with DNWGR

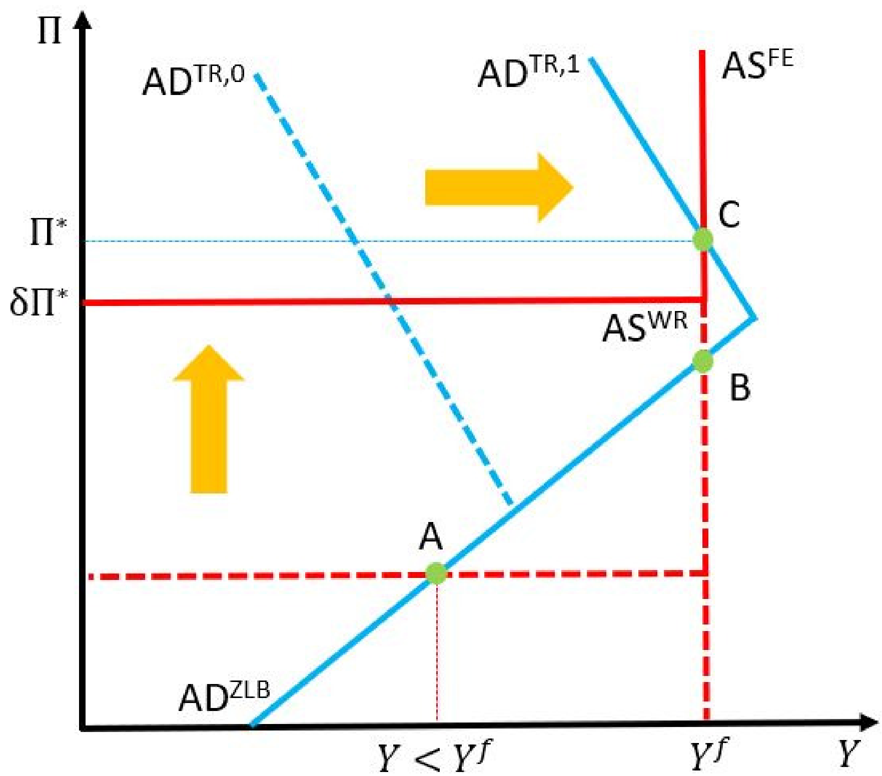

Figure 3: Steady state equilibria in the EMR model with DNWGR

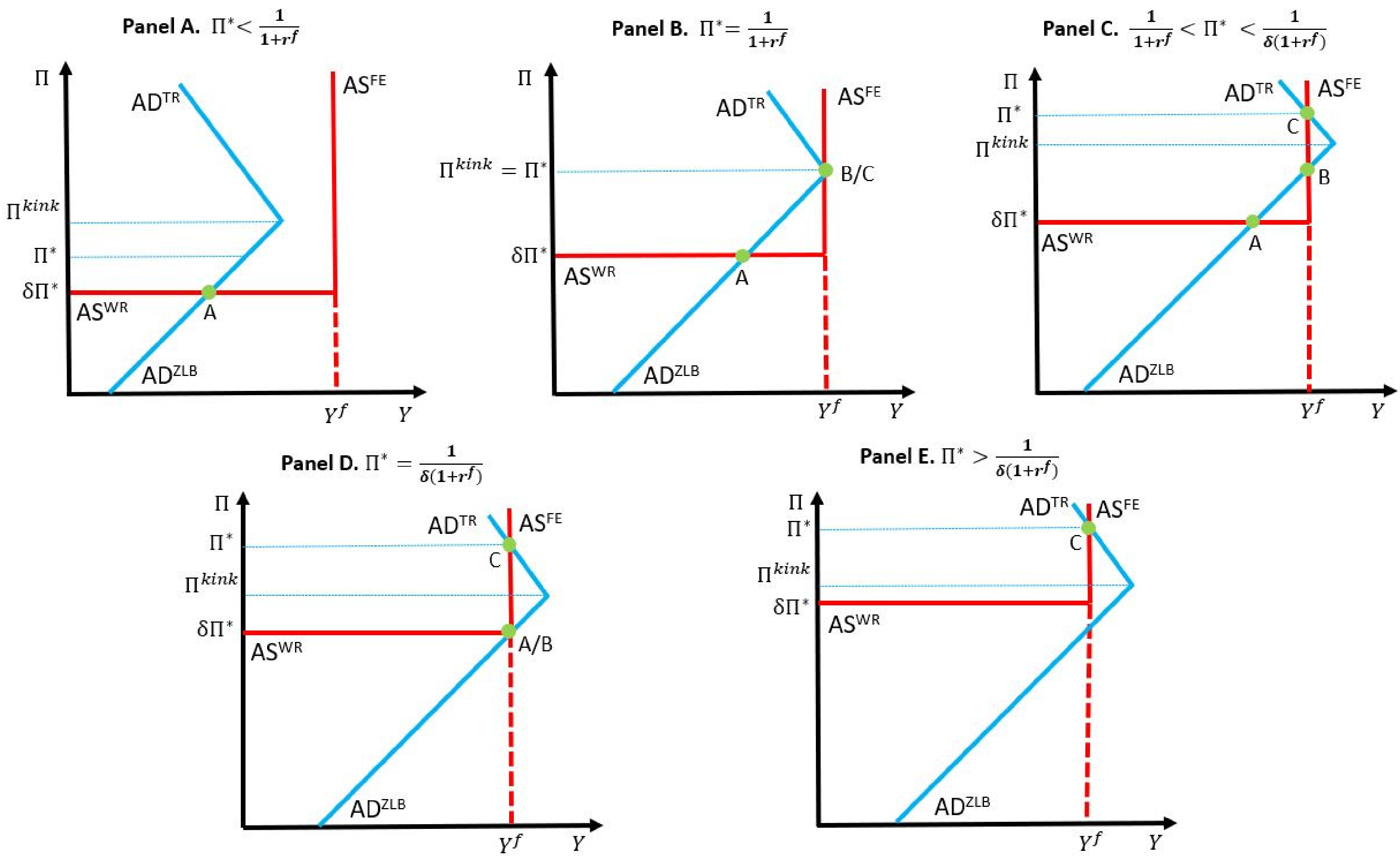

Figure 4: Steady state equilibria in the SGU model

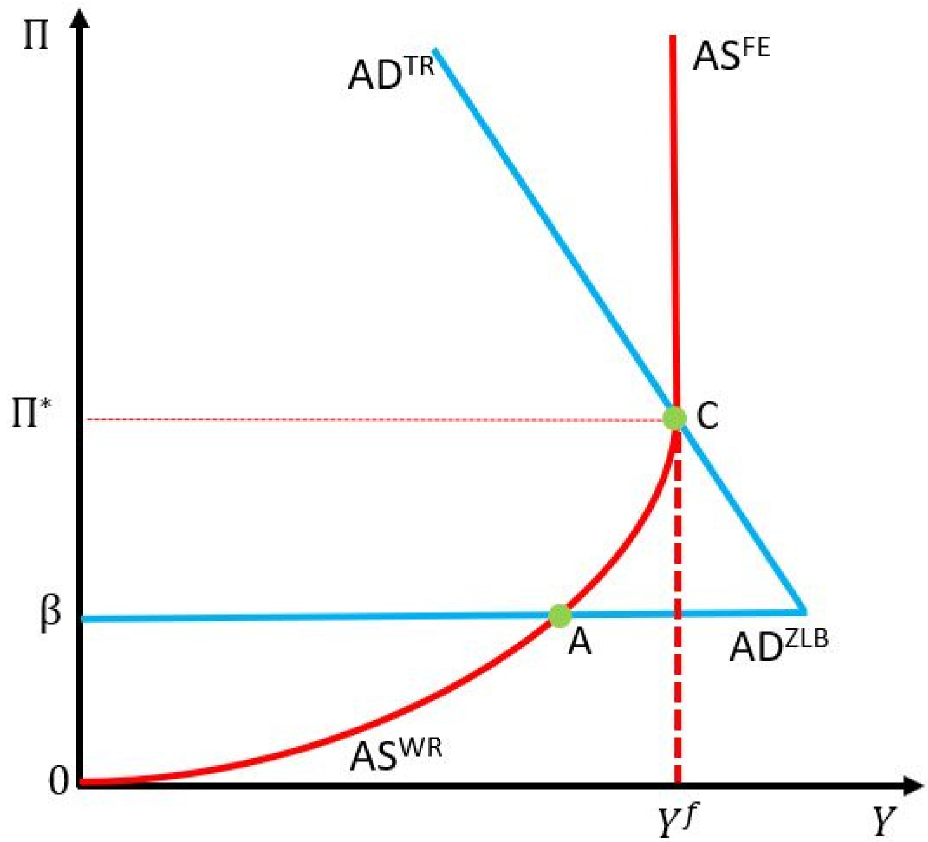

Figure 5: Steady state equilibria in the SGU model with DNWGR

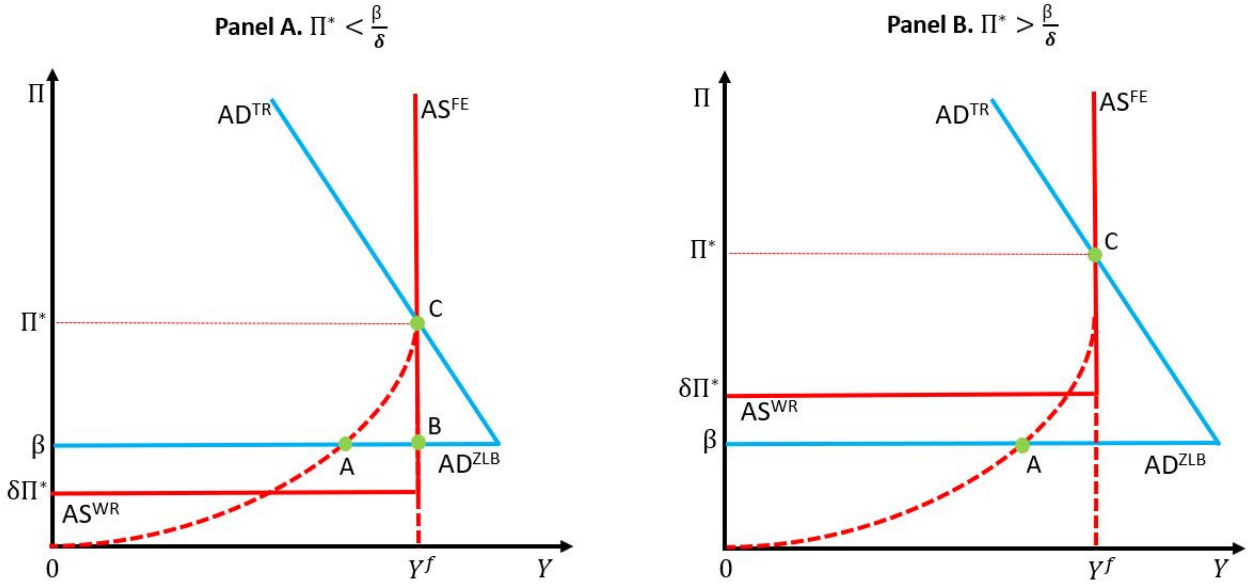

Figure 6: Steady state equilibria in the SGU model with different DNWGR