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G-structures for black hole near-horizon geometries

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Abstract: We derive necessary and sufficient conditions for warped AdS_2 solutions of Type II supergravity to preserve $\mathcal{N}=1$ supersymmetry, in terms of bispinors. Such solutions generically support an SU(3)-structure on their internal manifold M_8, which can experience an enhancement to a G_2-structure. We perform an SU(3)-structure torsion classes analysis and express the fluxes and other physical fields in terms of these, in general. We use our results to derive two new classes of AdS_2 solutions. In (massive) Type IIA supergravity we derive an $\mathcal{N}=1$ supersymmetric class for which M_8 decomposes as a weak G_2-manifold foliated over an interval and which is locally defined in terms of a degree three polynomial. In Type IIB supergravity we find a class of $\mathrm{AdS}_2\times \mathrm{S}^2\times \mathrm{CY}_2\times\Sigma_2$ solutions preserving small $\mathcal{N}=4$ supersymmetry, governed by a harmonic function on Σ_2 and partial differential equations reminiscent of D3-D7-brane configurations.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory, Flux Compactifications

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Contents

1 Introduction

Black hole mechanics has been a driving force and guide in the quest of a quantum theory of gravity. Quantum mechanically black holes behave as thermodynamic objects with temperature and entropy. The Bekenstein-Hawking formula expresses the entropy of a black hole in terms of the area of its event horizon, and the microscopic origin of the entropy is a problem that every quantum theory of gravity aspires to address. Extremal black holes play a prominent role as their quantum description is under better control.

All known supersymmetric extremal black holes possess an AdS_2 factor in their near-horizon geometry and one expects that the holographic correspondence between two-dimensional anti-de Sitter space (AdS_2) and superconformal quantum mechanics (SCQM) is of value in their study. The AdS_2/SCQM correspondence is less understood than its higher-dimensional counterparts owning to the special features of gravity in spacetimes that

JHEP06(2024)056

asymptote to AdS_2. Identifying AdS_2/SCQM pairs in string theory is thus desirable and motivates the analysis of the space of AdS_2 solutions.

Some interesting existing examples of AdS_2 solutions and their physical relevance are the following. There is the GK solution [1, 2] which appears prominently in the holographic dual to $\mathcal{I}$-extremisation [3, 4]. Various compactifications of higher-dimensional AdS solutions, dual to likewise compactified conformal field theories (CFTs), have been constructed; see for instance [5–8] for compactifications on Riemann surfaces and [9–13] for compactifications on higher-dimensional spaces. There are also various holographic duals to defects in higher-dimensional CFTs [14–21] exhibiting AdS_2 factors.

AdS_2 solutions in Type II supergravity can often be generated from existing AdS_3 solutions via Hopf fiber T-duality or Hopf fiber reduction from $d = 11$ supergravity [22]. This has been exploited in recent works to yield the first examples of new classes of supersymmetric AdS_2 solutions whose dual quantum mechanics where explored in [18–21, 23, 24, 54], see also [25]. Another way to generate AdS_2 solutions is via SL(2) non-Abelian T-duality [24, 26–28].

Several classifications of maximally supersymmetric AdS_2 solutions already exist [14, 29–32], though the $\mathcal{N} = (8, 0)$ AdS_3 classification of [33] suggests that likely, this only scratches the surface of such possibilities. Additionally, in [34], minimally supersymmetric AdS_2 solutions of $d = 11$ supergravity were classified under the assumption that they preserve an SU(4)-structure. We prove here that AdS_2 solutions not conforming to this assumption merely embed AdS_2 inside higher-dimensional AdS spaces, so [34] is actually general. See also [35, 53] for earlier partial classification.

In this paper we aim for a classification of supersymmetric AdS_2 solutions of Type II supergravity, leveraging the techniques involving bispinors and G-structures, that have been proven effective in the classification of other supersymmetric solutions. Supersymmetric AdS_2 solutions can support a wide variety of superconformal algebras. Those that can be embedded into $d = 10$ and $d = 11$ supergravities were classified in [36]. The possible algebras are actually the same as the (simple) chiral superconformal algebras that CFTs in $d = 2$ can support; see for instance [37]. Like their AdS_3 counterparts, sixteen real supercharges, so $\mathcal{N} = 8$, is maximal for solutions containing an AdS_2 factor. Our focus here will be on solutions that preserve at least minimal supersymmetry, i.e. $\mathcal{N} = 1$, which is two real supercharges.

The layout of the paper is as follows:

In section 2 we present necessary and sufficient conditions for AdS_2 solutions of Type II supergravity to preserve $\mathcal{N} = 1$ supersymmetry, under the assumption that they do not experience an enhancement to a higher-dimensional AdS space, as they do generically. We then move on to classify these conditions in terms of G-structures. We find that the internal 8-manifold M_8 generically supports an SU(3)-structure, though a limit exists where this gets enhanced to a G_2-structure. This section is supplemented by the technical appendix D, where many of these results are derived.

In section 3 we perform a non-trivial check of the conditions for supersymmetry derived earlier. All supersymmetric AdS_3 solutions of $d = 11$ supergravity and of Type II supergravity with purely magnetic NSNS flux, can be mapped to supersymmetric AdS_2 solutions by either dimensionally reducing or T-dualising on the Hopf fiber of AdS_3. Such AdS_3 solutions have been classified in the literature in terms of G-structures: see [33, 38, 39] for $d = 11$ and [40–43] for $d = 10$. Our AdS_2 conditions should then reproduce those of these AdS_3 classifications in certain limits and we show this is indeed the case.

Section 4 contains a general classification of the possible types of solutions in terms of SU(3)-structure torsion classes. It turns out that the possible internal manifolds and fluxes these solutions can support is quite broad. This is perhaps unsurprising as the internal space is large and the supersymmetry low. Nonetheless, this section will serve as a good road map of which backgrounds can be obtained for supersymmetric AdS_2.

The main text ends with section 5, where, as a display of the utility of the tools we provide in this work, we derive two new and non-trivial classes of AdS_2 solutions, one in each of Type IIA and IIB supergravities. The first, in section 5.1, is a generalisation of a class of $\mathcal{N} = 8$ solutions in massive Type IIA supergravity, with $\mathrm{AdS}_2\times\mathrm{S}^7$ foliated over an interval [31]. We generalise by replacing S^7 with an arbitrary weak G_2-manifold, allowing the fluxes to depend on its associated G_2-structure. The result is a class of $\mathcal{N} = 1$ solutions governed by two ordinary differential equations, which in two distinct cases can be solved in terms of a degree three polynomial. In section 5.2, we derive a class of solutions on a warped product of $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \Sigma_2$, governed by a harmonic function on Σ_2 and a system of partial differential equations that generalises those of localised D3-branes with D7-branes, that have CY_2 as their relative codimensions. It admits a limit in which the near-horizon of a $d = 4$ black hole is recovered, so it might be useful for the study of microstate counting in the future. This class generalises a class of solutions generated via T-duality in [24], and also has some partial intersection with the classification of [15, 16].

Finally our work is supplemented by several in depth appendices. Our conventions can be found in appendix A. In appendices B and C we lay the groundwork for the following appendix, by analysing Killing spinor bilinears on AdS_2 and refining (for generic spacetimes) the pairing constraints of [44], for scenarios where the $d = 10$ Killing spinors define a time-like Killing vector. As previously mentioned, appendix D derives most of the results of section 2. Finally, in appendix E, we prove that the classification of AdS_2 solutions in [34] is actually fully general if one insists that they are not merely the embedding of AdS_2 into some higher-dimensional AdS space.

2 Geometric conditions for $\mathcal{N} = 1$ supersymmetric AdS_2 in Type II supergravity

In this section we present a set of necessary and sufficient geometric conditions for an AdS_2 solution of Type II supergravity to preserve minimal supersymmetry. This section is largely a summary of appendix D, where these conditions are derived.

An AdS_2 solution of Type II supergravity has bosonic field content that can, by definition, be decomposed as

$$\begin{aligned} ds^2 &= e^{2A} ds^2(\mathrm{AdS}_2) + ds^2(\mathrm{M}_8), \\ F &= f_\pm + e^{2A}\mathrm{vol}(\mathrm{AdS}_2) \wedge \star_8 \lambda(f_\pm), \quad H = e^{2A}\mathrm{vol}(\mathrm{AdS}_2)\wedge H_1 + H_3, \end{aligned} \tag{2.1}$$

where H is the NSNS 3-form flux and F is the $d = 10$ RR polyform flux, with the upper/lower signs taken in Type IIA/IIB throughout, with $\pm$ labelling even/odd form degree when appearing on forms or chirality when appearing on spinors. The AdS_2 warp factor e^{2A}, dilaton Φ and the forms $(f_\pm, H_1, H_3)$ have support on the internal manifold M_8 alone, and the operator λ acts on a k-form as $\lambda(C_k) = (-1)^{\lfloor \frac{k}{2} \rfloor} C_k$. The fluxes, away from the loci of sources, should obey the following magnetic

$$dH_3 = 0, \quad d_{H_3} f_\pm = 0 \tag{2.2}$$

and the following electric

$$d(e^{2A} H_1) = 0, \quad d_{H_3}(e^{2A} \star_8 \lambda(f_\pm)) = e^{2A} H_1 \wedge f_\pm. \tag{2.3}$$

Bianchi identities. Further details of our conventions can be found in appendix A.

When an AdS_2 solution preserves supersymmetry it does so in terms of the Killing spinors of AdS_2. These come in two chiral variants (ζ_+, ζ_-), which can be taken to be Majorana without loss of generality, and obey the Killing spinor equations

$$\nabla_\mu^{\mathrm{AdS}_2} \zeta_+ = \frac{m}{2} \gamma_\mu^{(2)} \zeta_-, \quad \nabla_\mu^{\mathrm{AdS}_2} \zeta_- = \frac{m}{2} \gamma_\mu^{(2)} \zeta_+, \tag{2.4}$$

where m is the inverse AdS_2 radius; more details are given in appendix B. In $d = 10$ each of these couple to Majorana-Weyl spinors on M_8 (χ^i_+, χ^i_-) for $i = 1, 2$ such that the $d = 10$ Majorana-Weyl spinors decompose as[1]

$$\epsilon_1 = \zeta_+ \otimes \chi_{1+} + \zeta_- \otimes \chi_{1-}, \quad \epsilon_2 = \zeta_+ \otimes \chi_{2\mp} + \zeta_- \otimes \chi_{2\pm}, \tag{2.5}$$

where none of the $d = 8$ spinors can be zero.[2] This decomposition preserves 2 out of the 32 supersymmetries in ten dimensions. Going forward, we find it helpful to define the non-chiral $d = 8$ spinors

$$\chi_1 := \chi_{1+} + \chi_{1-}, \quad \chi_2 := \chi_{2+} + \chi_{2-}. \tag{2.6}$$

From this staring point, in appendix D, we are able to derive necessary and sufficient geometric conditions for a solution of the form (2.1) to preserve minimal supersymmetry. We will now summarise these conditions.

The first thing to appreciate is that totally generic AdS_2 solutions experience an enhancement to AdS_3, unless one imposes that

$$\chi_1^\dagger \gamma_a \chi_1 = \pm \chi_2^\dagger \gamma_a \chi_2, \quad \chi_1^\dagger \hat{\gamma} \chi_1 = \pm \chi_2^\dagger \hat{\gamma} \chi_2, \quad |\chi_1|^2 = |\chi_2|^2 = c e^A, \tag{2.7}$$

where γ_a are a basis of gamma matrices on M_8, $\hat{\gamma}$ the corresponding chirality matrix and c is an integration constant. Thus, if one is interested in true AdS_2 solutions, (2.7) needs to be imposed.[3] Given this, solutions can be defined in terms of the following 0- and 1-form spinor bilinears

$$c e^A \cos\beta := \chi_1^\dagger \hat{\gamma} \chi_1 = \pm \chi_2^\dagger \hat{\gamma} \chi_2, \quad c e^A \sin\beta V := \chi_1^\dagger \gamma_a \chi_1 \mathrm{e}^a = \pm \chi_2^\dagger \gamma_a \chi_2 \mathrm{e}^a, \quad V \cdot V = 1, \tag{2.8}$$

[1] We are assuming a factorization of the $d = 10$ spinors.

[2] See the discussion around (D.4).

[3] Notice that imposing this makes our Ansatz inconsistent with all AdS_d for $d > 2$, not only AdS_3, as they can all be expressed in terms of an AdS_3 factor.

JHEP06(2024)056

for e^a a vielbein basis on M_8 and where $\sin\beta = 0$ is incompatible with $m \neq 0$ (for $m = 0$ AdS_2 blows up to $Mink_2$). In addition to this, we need to define the polyform bilinears[4]

$$\psi := \chi_1 \otimes \chi_2^\dagger, \quad \hat{\psi} := \hat{\gamma}\chi_1 \otimes \chi_2^\dagger, \tag{2.9}$$

with odd/even form degree components $(\psi_\mp, \hat{\psi}_\mp)$. In terms of these, necessary and sufficient conditions for supersymmetry (when $m \neq 0$) are given by

$$e^{2A}H_1 = me^A \sin\beta V - d(e^{2A}\cos\beta), \quad d(e^A \sin\beta V) = 0, \tag{2.10a}$$

$$d_{H_3}(e^{-\Phi}\psi_\pm) = \pm\frac{c}{16}e^A \sin\beta V \wedge f_\pm, \tag{2.10b}$$

$$d_{H_3}(e^{A-\Phi}\hat{\psi}_\mp) - me^{-\Phi}\psi_\pm = \mp\frac{c}{16}e^{2A}(\star_8\lambda f_\pm + \cos\beta f_\pm), \tag{2.10c}$$

$$(\psi_\pm, f_\pm)_8 = \pm\frac{c}{4}e^{-\Phi}\Big(m - \frac{1}{2}e^A \sin\beta\,\iota_V H_1\Big)\mathrm{vol}(M_8). \tag{2.10d}$$

Here the final condition is a pairing constraint where in general the bracket defines the k-dimensional Chevalley-Mukai pairing $(X, Y)_k := X \wedge \lambda(Y)|_k$ (where $_k$ denotes the k-form contribution). In the appendix, three further pairing constraints are presented

$$(\psi_\mp, f_\pm)_7 = 0, \tag{2.11a}$$

$$(\hat{\psi}_\mp, f_\pm)_7 = \pm\frac{1}{8}e^{A-\Phi}c\,\star_8(2dA + \cos\beta H_1), \tag{2.11b}$$

$$(\hat{\psi}_\mp, \star\lambda f_\pm)_7 = \pm\frac{1}{8}e^{A-\Phi}c\,\star_8(2\cos\beta dA + H_1 - 2e^{-A}m\sin\beta V), \tag{2.11c}$$

where it is found that these actually imply (2.10d). However, we establish that (2.11a)–(2.11c) are implied by (2.10a)–(2.10d) during our torsion classes analysis in section 4. The above conditions imply several others, for instance one can derive the following condition independent of $f_\pm$

$$d_{H_3}(e^{A-\Phi}\psi_\mp) = 0, \tag{2.12}$$

which follows from (2.10b) $\wedge V$ and can be useful for extracting necessary conditions.

Of course supersymmetry alone is not sufficient to have a solution. By definition, one must solve the equations of motion of Type II supergravity. One can show that the electric Bianchi identities (2.3) are implied by (2.10a)–(2.10c) when their magnetic cousins (2.2) are assumed to hold. Further (2.10b) implies $V \wedge d_{H_3}f_\pm = 0$ when $dH_3 = 0$. The integrability arguments of [45] then inform us that, when supersymmetry is preserved, this amounts to imposing

$$dH_3 = 0, \quad \iota_V(d_{H_3}f_\pm) = 0, \quad \cos\beta\Big[d(e^{-2\Phi}\star_8 H_1) + \frac{1}{2}(f_\pm, f_\pm)_8\Big] = 0. \tag{2.13}$$

It then follows that the remaining equations of motion of Type II supergravity are implied, though some additional care is required in the presence of sources for the fluxes, i.e. one

[4]Strictly speaking the left-hand side of these expressions is not a polyform, rather it is the components of a polyform whose indices have be contracted with an appropriate number of antisymmetric products of gamma matrices. However such objects can be mapped to forms under the Clifford map, i.e. $\not\psi = \chi_1 \otimes \chi_2^\dagger = \frac{1}{16}\sum_{n=0}^{8}\frac{1}{n!}\chi_2^\dagger\gamma_{\underline{a}_n\cdots\underline{a}_1}\chi_1\gamma^{\underline{a}_1\cdots\underline{a}_n}$ and $\psi = \frac{1}{16}\sum_{n=0}^{8}\frac{1}{n!}\chi_2^\dagger\gamma_{\underline{a}_n\cdots\underline{a}_1}\chi_1 e^{\underline{a}_1\cdots\underline{a}_n}$ are equivalent objects. We are simply suppressing the Dirac slash in the above.

must make sure that they have a supersymmetric embedding and that they come with the appropriate modification of the Bianchi identities.

Let us briefly comment on the Mink_2 limit, $m = 0$: in general, for such solutions, $\sin\beta = 0$ and the conditions (2.7) are not necessary for supersymmetry to hold (without imposing the first of (2.7), one necessarily has an additional uncharged U(1) isometry in M_8), however one can choose to impose these constraints. When one does, (2.10a)–(2.11c) for $m = 0$ provides a set of necessary and sufficient conditions for a subclass of $\mathcal{N} = (1,1)$ Mink_2 solutions; exploring these is outside the scope of this work, but could be interesting.

In the next section we parametrise the bispinors $(\psi_\pm, \hat{\psi}_\pm)$ in terms of a $d = 8$ SU(3)-structure.

2.1 Parametrising the $d = 8$ spinors and G-structures

We expand the $d = 8$ spinors in terms of two unit-norm Majorana-Weyl spinors $\chi_\pm$. Such spinors define a G_2-structure in eight dimensions, spanned by real 1- and 3-forms $(\tilde{V}, \Phi_3)$ in general as

$$\chi_\pm \otimes \chi_\pm^\dagger = \frac{1}{16}\Big(1 \pm \tilde{V}\wedge\Phi_3 - \iota_{\tilde{V}}\star_8\Phi_3 \mp \text{vol}(\text{M}_8)\Big), \quad \chi_\mp \otimes \chi_\pm^\dagger = \frac{1}{16}\Big(\tilde{V} \pm \Phi_3 - \star_8\Phi_3 \mp \iota_{\tilde{V}}\text{vol}(\text{M}_8)\Big). \tag{2.14}$$

We have four Majorana-Weyl spinors that must obey the constraint (D.33). An arbitrary $\pm$ chiral spinor may be decomposed in a basis of $(\chi_\pm, \tilde{U}\chi_\mp)$, with $\tilde{U}$ a 1-form such that $\iota_{\tilde{V}}\tilde{U} = 0$ and $\iota_{\tilde{U}}\tilde{U} = 1$. It is possible to show that we can take

$$\chi_{1+} = \sqrt{c}e^{\frac{1}{2}A}\cos\left(\frac{\beta}{2}\right)\chi_+, \qquad \chi_{1-} = \sqrt{c}e^{\frac{1}{2}A}\sin\left(\frac{\beta}{2}\right)(a\chi_- + b\tilde{U}\chi_+), \tag{2.15}$$

$$\chi_{2-} = \sqrt{c}e^{\frac{1}{2}A}\sin\left(\frac{\beta_\pm}{2}\right)\chi_-, \qquad \chi_{2+} = \sqrt{c}e^{\frac{1}{2}A}\cos\left(\frac{\beta_\pm}{2}\right)(a\chi_+ + b\tilde{U}\chi_-), \tag{2.16}$$

where $a^2 + b^2 = 1$ and

$$\beta_+ = \beta, \quad \beta_- = \beta + \pi, \tag{2.17}$$

without loss of generality. The presence of $\tilde{U}$ means that the G_2-structure generically decomposes in terms of an SU(3)-structure as follows

$$\Phi_3 = -(J\wedge\tilde{U} + \text{Re}\Omega), \quad \iota_{\tilde{V}}\star_8\Phi_3 = \frac{1}{2}J\wedge J - \tilde{U}\wedge\text{Im}\Omega, \quad J\wedge J\wedge J = \frac{3}{4}i\Omega\wedge\overline{\Omega}. \tag{2.18}$$

We find it useful to introduce the following complex 1-form and SU(3)-structure bilinears on the space orthogonal to this:

$$U + iV := (a+ib)(\tilde{U} + i\tilde{V}), \quad \psi_+^{\text{SU(3)}} := (a+ib)e^{-iJ}, \quad \psi_-^{\text{SU(3)}} := \Omega,$$
$$\psi_\pm^{(7)} = \psi_\pm^{\text{SU(3)}} + i\psi_\mp^{\text{SU(3)}}\wedge U. \tag{2.19}$$

The bispinors then take the form

$$\psi_\pm = \frac{e^A c}{16}\text{Re}\Big[\psi_\pm^{(7)} + \cos\beta\,\psi_\mp^{(7)}\wedge V\Big], \qquad \psi_\mp = \frac{e^A c}{16}\sin\beta V\wedge\text{Re}\Big[\psi_\mp^{(7)}\Big], \tag{2.20a}$$

$$\hat{\psi}_\pm = \frac{e^A c}{16}\text{Re}\Big[\psi_\pm^{(7)}\wedge V + \cos\beta\,\psi_\mp^{(7)}\Big], \qquad \hat{\psi}_\mp = \pm\frac{e^A c}{16}\sin\beta\,\text{Re}\Big[\psi_\mp^{(7)}\Big]. \tag{2.20b}$$

We note that when $b = 0$ these bispinors define a G_2-structure, while in the opposite limit, fixing $a = 0$, they define an orthogonal SU(3)-structure. Generically, the bispinors define a $G_2\times G_2$-structure often referred to as an intermediate SU(3)-structure[5] which, when (a, b) are point-dependent, can interpolate between these G-structures as one traverses the internal space.

We note that sufficient spinors solving (2.8) and giving rise to (2.20a)–(2.20b) can be expressed in terms of a pair of unit-norm $d = 7$ Majorana spinors $(\chi_1^{(7)}, \chi_2^{(7)})$ as

$$\chi_1 = \sqrt{e^A c}\begin{pmatrix} \cos\left(\frac{\beta}{2}\right) \\ \sin\left(\frac{\beta}{2}\right)\end{pmatrix} \otimes \chi_1^{(7)}, \quad \chi_2 = \sqrt{e^A c}\begin{pmatrix} \cos\left(\frac{\beta_\mp}{2}\right) \\ \sin\left(\frac{\beta_\mp}{2}\right)\end{pmatrix} \otimes \chi_2^{(7)}, \tag{2.21}$$

where one must decompose the gamma matrices and intertwiner as $\gamma_a = \sigma_2 \otimes \gamma_a^{(7)}$ for $a = 1, \dots, 7$, $\gamma_8 = \sigma_1 \otimes \mathbb{I}$, $\quad B = \mathbb{I} \otimes B^{(7)}$ and take $i\gamma^{(7)}_{1234567} = \mathbb{I}$. We find

$$\psi_+^{(7)} - i\psi_-^{(7)} = \frac{1}{8}\sum_{n=1}^{7}\frac{1}{n!}\chi_2^{(7)\dagger}\gamma^{(7)}_{a_n\dots a_1}\chi_1^{(7)} \mathrm{e}^{a_1\dots a_n}, \tag{2.22}$$

and by decomposing

$$\chi_2^{(7)} = a\chi_1^{(7)} - ibU\chi_1^{(7)} \tag{2.23}$$

we precisely reproduce (2.20a)–(2.20b) and align $V = \mathrm{e}^8$.

The second condition in (2.10a) can be locally solved in general by introducing a local coordinate ρ and a function of this coordinate e^k such that

$$e^A \sin\beta V = e^k d\rho, \tag{2.24}$$

where e^k parametrises diffeomorphisms in ρ so can be set to any convenient non-zero value. The presence of the additional vielbein direction U then means that one can decompose the internal 8-manifold as

$$ds^2(\mathrm{M}_8) = ds^2(\mathrm{M}_6) + U^2 + \frac{e^{-2A+2k}}{\sin^2\beta}d\rho^2, \tag{2.25}$$

where there exist coordinates with respect to which U and M_6 have no legs/components on ρ but can have functional dependence on it[6] and M_6 is spanned by the vielbein directions that make up (J, Ω). The types of M_6 that are compatible with supersymmetry will be explored in section 4 in terms of SU(3)-structure torsion classes.

In the next section we will perform a highly non-trivial check of the conditions we have derived so far by recovering known classes of AdS_3 solution in Type II and $d = 11$ supergravities, modulo duality.

[5]When a G-structure is a product group one often refers to it via the largest subgroup common to both factors in the product.

[6]I.e. $\tilde{k} = e^A \sin\beta V$ defines an almost product structure that is integrable by the second of (2.10a). This ensures we can choose local coordinates like this.

3 Recovering known classes of AdS_3 solutions with duality

There are several existing G-structure classifications that should be recoverable as certain limits of (2.10a)–(2.11c). Clearly, any AdS_2 solution in massless Type IIA supergravity can be lifted to a solution in $d = 11$ supergravity. This can result in two types of solutions: when one has $f_8 = 0$ the lifted solution will contain a round AdS_2 factor, conversely when $f_8 \neq 0$ the $d = 11$ circle becomes fibered over AdS_2, resulting in general in a squashing (and when $f_2 \neq 0$ also a fibering over the internal space) of the Hopf fibration of AdS_3. Another way to arrive at a supersymmetric AdS_2 solution is to take an existing AdS_3 solution[7] and perform T-duality on the Hopf fiber of AdS_3 or SL(2) T-duality on the entire space [27]. Our system (2.10a)–(2.11c) should thus reproduce the necessary geometric conditions for each of these classes of solutions. Establishing that this is indeed the case provides a highly non-trivial check of our results.

There exists a G-structure classification of supersymmetric AdS_2 solutions in $d = 11$ in [34]. The case of squashed and fibered AdS_3 has not received much attention in the literature, but the simpler case of round AdS_3 was considered first in [38] and later in [33, 39]. Finally, supersymmetric AdS_3 solutions of Type II supergravity received the G-structure treatment, in the bispinor approach we utilise for AdS_2, across [40–43]. In this section, as a test of what we have derived, we shall recover the geometric conditions defining all known minimally supersymmetric examples of these classes. The Type IIA reduction of [34] is likewise recoverable but showing this is a more lengthy computation, so we shall not present it here.

In this section we fix the inverse AdS_2 radius $m = 1$ for simplicity.

3.1 AdS_3 solutions in $d = 11$ supergravity

First, we consider supersymmetic AdS_3 solutions of $d = 11$ supergravity presented in [33]. In general the map between the bosonic fields of $d = 11$ supergravity and massless Type IIA supergravity is

$$ds^2_{11} = e^{-\frac{2}{3}\Phi} ds^2 + e^{\frac{4}{3}\Phi}(dz + C_1)^2, \quad dC_1 = F_2, \quad G = F_4 + (dz + C_1) \wedge H, \tag{3.1}$$

where z spans the reduction isometry which is assumed to be 2π periodic and (F_2, F_4, H) are $d = 10$ fluxes of the Type IIA theory. For the case at hand, we want to map to a reduction on the Hopf fiber of AdS_3, i.e.

$$\begin{aligned} ds^2_{11} &= \frac{e^{2\Delta}}{4} ds^2(\text{AdS}_2) + e^{2\Delta}\left(dz + \frac{1}{2}\eta\right)^2 + ds^2(\hat{\text{M}}_8), \quad d\eta = \text{vol}(\text{AdS}_2), \\ G &= \frac{1}{4} e^{2\Delta} \text{vol}(\text{AdS}_2) \wedge \left(dz + \frac{1}{2}\eta\right) \wedge G_1 + G. \end{aligned} \tag{3.2}$$

We must thus constrain the Type IIA fields such that

$$f_+ = f_4 + f_8, \quad H_3 = 0, \quad e^A = \frac{1}{2} e^{\Phi} = \frac{1}{2} e^{\frac{3}{2}\Delta}, \quad ds^2(\text{M}_8) = e^{\Delta} ds^2(\hat{\text{M}}_8), \tag{3.3}$$

which in $d = 10$ (non-democratic) language means we constrain F_4 to be purely magnetic, (F_2, H) to be purely electric and set the remaining fluxes to zero. We already know that

[7] As explained in [27], this process only results in round AdS_2 when starting from AdS_3 solutions with purely magentic NSNS 3-form flux.

JHEP06(2024)056

the internal space of supersymmetric AdS_3 solutions in $d=11$ supports a G_2-structure, hence we can fix $(a=1, b=0)$. The conditions (2.10a)–(2.10c) and the remaining geometric conditions reduce to

$$e^{2A}\star_8\lambda f_8=\frac{1}{2}, \tag{3.4a}$$

$$d(e^{3\Delta}\cos\beta)+e^{3\Delta}H_1-2e^{\frac{3}{2}\Delta}\sin\beta V=0,\quad d(e^{\frac{3}{2}\Delta}\sin\beta V)=0, \tag{3.4b}$$

$$d(\iota_V\star_8\Phi_3-\cos\beta V\wedge\Phi_3)=-e^{\frac{3}{2}\Delta}\sin\beta V\wedge f_4, \tag{3.4c}$$

$$d(e^{\frac{3}{2}\Delta}\sin\beta\Phi_3)-2(\iota_V\star_8\Phi_3-\cos\beta V\wedge\Phi_3)-e^{3\Delta}(\star f_4+\cos\beta f_4)=0, \tag{3.4d}$$

the condition (2.11a) becomes trivial, while (2.11b)–(2.11c) become

$$e^{-3\Delta}\star_8(6d\Delta+2\cos\beta H_1)+e^{-\frac{3}{2}\Delta}\sin\beta\Phi_3\wedge f_4=0, \tag{3.5a}$$

$$e^{-3\Delta}\star_8(6e^{-\frac{3}{2}\Delta}\sin\beta V-6\cos\beta d\Delta-2H_1)-e^{-\frac{3}{2}\Delta}\sin\beta\Phi_3\wedge\star_8 f_4=0. \tag{3.5b}$$

Finally, (2.10d) yields

$$\mathrm{vol}(\mathrm{M}_8)+(\iota_V\star_8\Phi_3-\cos\beta V\wedge\Phi_3)\wedge f_4=8e^{-3\Delta}\left(1-\frac{e^{\frac{3\Delta}{2}}}{4}\sin\beta\iota_V H_1\right)\mathrm{vol}(\mathrm{M}_8). \tag{3.6}$$

The condition (3.4a) simply gives the $d=10$ RR 2-form we expect, $F_2=\frac{1}{2}\mathrm{vol}(\mathrm{AdS}_2)$, while the conditions (3.4b)–(3.5b) precisely reproduce the geometric conditions for supersymmetric AdS_3 presented in (5.3a)-(5.3f) of [33]. One needs to identify[8]

$$\tilde{f}=-\cos\beta,\quad \tilde{\Psi}_3=e^{-\frac{3}{2}\Delta}\sin\beta\Phi_3,\quad \tilde{\Psi}_4=e^{-2\Delta}(\iota_V\star_8\Phi_3-\cos\beta V\wedge\Phi_3),\quad \tilde{F}_1=H_1=G_1$$
$$\tilde{K}=e^{-\frac{1}{2}\Delta}\sin\beta V,\quad \tilde{A}=\Delta,\quad \tilde{\star}_8C_p=(-1)^pe^{-\frac{\Delta}{2}(8-2p)}\star C_p,\quad \tilde{F}_4=f_4=G_1,\quad \tilde{m}=1, \tag{3.7}$$

where we add a tilde to the objects appearing in (5.3a)–(5.3f) of [33]. The condition (3.6) is not quoted in [33], but as (3.5a)–(3.5b) imply this we need not worry. Of course our earlier claim that (2.11a)–(2.11c) are redundant, which we establish it is indeed true in section 4, implies that the 7-form constraints in [33] are actually implied by the rest of the conditions presented there and an 8-form constraint following from (3.6).

3.2 AdS$_3$ solutions in Type II supergravity

In general an AdS_3 solution in Type II supergravity is decomposable in the form

$$ds^2=e^{2A_7}ds^2(\mathrm{AdS}_3)+ds^2(\mathrm{M}_7),$$
$$H^{(10)}=c_0\mathrm{vol}(\mathrm{AdS}_3)+H_7,\quad F_7=f_{7\pm}+e^{3A_7}\mathrm{vol}(\mathrm{AdS}_3)\wedge\star_7\lambda(f_{7\pm}), \tag{3.8}$$

where $(e^{2A_7}, f_{7\pm}, H_7)$ and the dilaton Φ_7 are defined on M_7, c_0 is a constant and the upper/lower signs are taken in Type IIA/IIB. As explained in [27], we are free to T-dualise on

[8]Note that the Hodge duals in Type IIA and $d=11$ are taken with respect to different 8-manifolds, and the conventions for the Hodge dual itself (i.e. whether the components of a form are taken to contract with the leftmost or rightmost indices of the Levi-Civita symbol) are the opposite.

JHEP06(2024)056

the Hopf fiber of AdS_3 and preserve supersymmetry,[9] as long as we fix

$$c_0 = 0 \quad \Rightarrow \quad H^{(10)} = H_7. \tag{3.9}$$

If we start in Type IIB/IIA we will end up with an AdS_2 solution in Type IIA/IIB of the following form[10]

$$\begin{aligned} ds^2 &= \frac{e^{2A_7}}{4} ds^2(\text{AdS}_2) + e^{-2A_7} d\psi^2 + ds^2(\text{M}_7), \quad e^{-\Phi} = e^{-\Phi_7 + A_7}, \\ H &= -\frac{1}{2} d\psi \wedge \text{vol}(\text{AdS}_2) + H_7, \quad F = f_{7\mp} \wedge d\psi \pm \frac{1}{4} e^{3A_7} \text{vol}(\text{AdS}_2) \wedge \star_7 \lambda(f_{7\mp}). \end{aligned} \tag{3.10}$$

Comparing the above and the form of H_1 in (2.10a) it is clear that such solutions should lie within our AdS_2 classification for

$$\begin{aligned} &e^{2A} = \frac{1}{4} e^{2A_7}, \quad e^{-\Phi} = e^{-\Phi_7 + A_7}, \quad V = -e^{-A_7} d\psi, \quad e^{2A} H_1 = -\frac{1}{2} d\psi, \quad H_3 = H_7 \\ &f_\pm = f_{7\mp} \wedge d\psi, \quad \star_8 \lambda(f_\pm) = \pm e^{A_7} \star_7 \lambda(f_{7\pm}), \quad \sin\beta = 1, \end{aligned} \tag{3.11}$$

where we note that the conditions involving $f_\pm$ are indeed consistent, as in our conventions $\star_8 \lambda(f_{7\pm} \wedge V) = \pm \star_7 \lambda(f_{7\pm})$. This makes $(\psi_\pm, \hat{\psi}_\mp)$ strictly orthogonal to V and so, defining

$$\psi_\pm = \pm \frac{1}{4} \psi_{7\pm}, \quad \psi_\mp = -\frac{1}{4} \psi_{7\mp}, \quad \text{vol}(\text{M}_7) = \star_8 V, \tag{3.12}$$

which is consistent as $\cos\beta = 0$, the remaining supersymmetry conditions (2.10b)–(2.10d) reduce to

$$\begin{aligned} &d_{H_7}(e^{A_7 - \Phi_7} \psi_{7\pm}) = 0, \quad d(e^{2A_7 - \Phi_7} \psi_\mp) \pm 2 e^{A_7 - \Phi_7} \psi_{7\pm} = \frac{1}{8} c e^{3A_7} \star_7 \lambda(f_{7\mp}), \\ &\qquad (\psi_{7\pm}, f_{7\mp})_7 = \pm \frac{1}{2} c e^{-\Phi} \text{vol}(\text{M}_7). \end{aligned} \tag{3.13}$$

These conditions precisely reproduce the necessary and sufficient conditions for supersymmetric AdS_3 presented in (B.17) of [43], in the limit that $c_- = 0$ as (3.9) demands, and where the free constant is fixed to $c_+ = 2c$. The only not obviously redundant condition that (2.11a)–(2.11c) produce is

$$(\psi_{7\mp}, f_{7\mp})_6 = \pm \frac{1}{4} e^{A_7 - \Phi_7} \star_7 dA_7, \tag{3.14}$$

where we have made use of the pairing identity $(\psi_\pm, \star_7 \lambda(f_{7\pm})) = \pm(\psi_\mp, f_{7\pm})$. But this condition is just another way to write (B.15) of [43] which was shown to be redundant there by more involved methods. This is consistent with our earlier claim that (2.11a)–(2.11c) are redundant. We have thus established that (2.10a)–(2.10d) are consistent with the Hopf fiber T-dual of AdS_3 solutions providing another non-trivial check of our system.

---

[9] AdS_3 solutions can support two distinct types of supersymmetries of opposite chirality, the duality can preserve all of one chirality and project out those of the opposite chirality. Which chirality is preserved is a choice one is free to make.

[10] With respect to [27] we fix some free signs below for convenience.

4 SU(3)-structure torsion classes analysis

In this section we shall classify the possible internal spaces and physical fields that supersymmetric AdS_2 solutions can support in terms of SU(3)-structure torsion classes.

In section 2.1 we established that the internal manifold M_8 generically supports an SU(3)-structure defined on a submanifold M_6 orthogonal to the vielbein directions (U, V). There is an enhancement to G_2 when $b = 0$, but as the forms of a G_2-structure can be decomposed in terms of those of the SU(3)-structure it is appropriate to perform our general analysis in terms of the torsion classes associated to SU(3).

In general the torsion classes of a $d = 8$ SU(3)-structure are a simple extension of the well known $d = 7$ case [46]. Indeed if one formally fixes one of the 1-forms (U, V) to zero the torsion classes must reproduce this $d = 7$ case; what remains to consider is the portions of $(dJ, d\Omega)$ that have "legs" in $U \wedge V$, that of dV with "legs" in U and vice-versa. These extra terms should be consistent with

$$J \wedge J \wedge J = \frac{3}{4} i\Omega \wedge \overline{\Omega}, \quad J \wedge \Omega = 0 \tag{4.1}$$

under the exterior derivative, which means it is not hard to show that the $d = 8$ torsion class decomposition is

$$\begin{aligned}
dU &= RJ + T_1 + \text{Re}(\iota_{\overline{\mathcal{V}}_1}\Omega) + U \wedge W_0 + V \wedge U_0 + P_0 U \wedge V,\\
dV &= \tilde{R}J + \tilde{T}_1 + \text{Re}(\iota_{\overline{\tilde{\mathcal{V}}}_1}\Omega) + V \wedge \tilde{W}_0 + U \wedge \tilde{U}_0 + \tilde{P}_0 U \wedge V,\\
dJ &= \frac{3}{2}\text{Im}(\overline{W}_1\Omega) + W_3 + W_4 \wedge J + U \wedge \left(\frac{2}{3}\text{Re}E_1 J + T_2 + \text{Re}(\iota_{\overline{\mathcal{V}}_2}\Omega)\right)\\
&\quad + V \wedge \left(\frac{2}{3}\text{Re}E_2 J + T_3 + \text{Re}(\iota_{\overline{\mathcal{V}}_3}\Omega)\right) + 2\text{Im}\mathcal{V}_4 \wedge U \wedge V,\\
d\Omega &= W_1 J \wedge J + W_2 \wedge J + \overline{W}_5 \wedge \Omega + U \wedge \left(E_1\Omega - 2\mathcal{V}_2 \wedge J + S_1\right)\\
&\quad + V \wedge \left(E_2\Omega - 2\mathcal{V}_3 \wedge J + S_2\right) + (\iota_{\overline{\mathcal{V}}_4}\Omega) \wedge U \wedge V.
\end{aligned} \tag{4.2}$$

On the M_6 orthogonal to (U, V) we have: functions $(R, \tilde{R}, P_0, \tilde{P}_0)$ that are real and $E_{1,2}, W_1$ that are complex, 1-forms $U_0, \tilde{U}_0, W_0, \tilde{W}_0, W_4$ that are real and $\mathcal{V}_1, \tilde{\mathcal{V}}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4, W_5$ that are (1,0)-forms, primitive (1,1)-forms[11] $T_1, \tilde{T}_1, T_2, T_3$ that are real and W_2 which is complex. Finally, primitive (2,1)-forms $S_{1,2}$ which are complex and W_3 which is the real part of such a form. These forms transform in irreducible representations of SU(3), and so do not mix with each other.

(J, Ω) obey the following useful relations

$$\frac{1}{n!} \star_6 J^n = \frac{1}{(3-n)!} J^{3-n}, \quad \star_6\Omega = i\Omega, \tag{4.3}$$

[11] A real $d = 6$ (1,1)-form, T, that is primitive obeys $T \wedge J \wedge J = T \wedge \Omega = 0$ by definition.

M_6	Vanishing torsion classes
Complex	$W_1 = W_2 = 0$
Symplectic	$W_1 = W_3 = W_4 = 0$
Half-flat	$\mathrm{Re}W_1 = \mathrm{Re}W_2 = W_4 = W_5 = 0$
Special Hermitian	$W_1 = W_2 = W_4 = W_5 = 0$
Nearly Kähler	$W_2 = W_3 = W_4 = W_5 = 0$
Almost Kähler	$W_1 = W_3 = W_4 = W_5 = 0$
Kähler	$W_1 = W_2 = W_3 = W_4 = 0$
Calabi-Yau	$W_1 = W_2 = W_3 = W_4 = W_5 = 0$

Table 1. A table of well-known SU(3)-structure manifolds in terms of vanishing torsion classes. Note that a manifold may be conformally one of these for certain values of (W_4, W_5).

where in this notation, $J^n = \wedge_{k=1}^n J^k$ for $n > 0$, $J^0 = 1$ for $n = 0$ and $J^n = 0$ for $n < 0$. The torsion classes on the other hand obey

$$
\begin{aligned}
&\mathcal{V}_i\wedge\Omega=0, && \frac{1}{n!}\star_6(\mathcal{V}_i\wedge J^n)=i\frac{1}{(2-n)!}\mathcal{V}_i\wedge J^{2-n}, && \star_6(\overline{\mathcal{V}}_i\wedge\Omega)=i\iota_{\overline{\mathcal{V}}_i}\Omega\\
&\iota_{\overline{\mathcal{V}}_i}\Omega\wedge\Omega=0, && \overline{\iota_{\overline{\mathcal{V}}_i}\Omega}\wedge\Omega=-4\mathcal{V}_i\wedge J\wedge J, && \iota_{\overline{\mathcal{V}}_i}\Omega\wedge J=-i\overline{\mathcal{V}}_i\wedge\Omega\\
&T_i\wedge\Omega=T_i\wedge J\wedge J=0, && \star_6(T_i\wedge J^n)=-T_i\wedge J^{n-1}, &&\\
&S_i\wedge\Omega=S_i\wedge\overline{\Omega}=S_i\wedge J=0, && \star_6 S_i=-iS_i. &&
\end{aligned}
\tag{4.4}
$$

For the case at hand, given that we can locally take

$$
V=\frac{e^k}{e^A\sin\beta}d\rho, \tag{4.5}
$$

we can immediately refine (4.2). In order to do this it is useful to decompose the exterior derivative as

$$
d=\tilde{d}^{(6)}+U\wedge\iota_U d+d\rho\wedge\partial_\rho, \tag{4.6}
$$

where $\tilde{d}^{(6)}$ is a twisted exterior derivative in general.[12] We then have

$$
\tilde{R}=\tilde{T}_1=\tilde{\mathcal{V}}_1=\tilde{U}_0=0,\quad \tilde{W}_0=d^{(6)}\Big(\log(e^A\sin\beta)\Big),\quad \tilde{P}_0=\iota_U d\Big(\frac{1}{e^A\sin\beta}\Big). \tag{4.7}
$$

When one attempts to solve the conditions (2.10a)–(2.11c), it is possible for several of the torsion classes $W_{1,2,3,4,5}$ to become fixed. This gives us information about the type of manifold that can live on M_6, several well-known examples are given in table 1.

[12] I.e. one can introduce a local coordinate y such that $U = e^C(dy + \mathcal{A})$, where $\mathcal{A}$ has no legs in (y, ρ). Then $\tilde{d}^{(6)} = d^{(6)} - \mathcal{A}\wedge\partial_y$, where $d^{(6)}$ is the usual $d = 6$ exterior derivative.

JHEP06(2024)056

To proceed, we further decompose the fluxes as

$$f_\pm = g^1_\pm + U\wedge g^1_\mp + V\wedge g^2_\mp + U\wedge V\wedge g^2_\pm,$$
$$H_3 = H_3^{(6)} + U\wedge H^1_2 + V\wedge H^2_2 + U\wedge V\wedge H_1^{(6)}, \tag{4.8}$$

and then introduce a $d=6$ Hodge dual such that the RR flux, or indeed any form in $d=8$, behaves as

$$\star\lambda(f_\pm) = -\star_6\lambda(g^2_\pm) + U\wedge\star_6\lambda(g^2_\mp) - V\wedge\star_6\lambda(g^1_\mp) + U\wedge V\wedge\star_6\lambda(g^1_\pm),$$
$$\star f_\pm = \star_6 g^2_\pm \pm U\wedge\star_6 g^2_\mp \mp V\wedge\star_6 g^1_\mp + U\wedge V\wedge\star_6 g^1_\pm. \tag{4.9}$$

Note we also have

$$\star_6\star_6 C_k = (-)^k C_k,\quad \lambda(\star_6 C_k) = -(-)^k\star_6\lambda(C_k). \tag{4.10}$$

Examining (2.10a)–(2.10d), at first sight, it appears that these merely serve to fix the RR flux in terms of SU(3)-structure torsion classes and the NSNS 3-form. However, one should appreciate that, as[13] $\cos\beta\neq 1$, (2.10c) can be manipulated to simply define the entire RR flux on its own as

$$e^{2A}\sin^2\beta c f_\pm = \pm 16(\cos\beta - \star_8\lambda)\Big(d_{H_3}(e^{A-\Phi}\hat\psi_\pm) - m e^{-\Phi}\psi_\pm\Big). \tag{4.11}$$

Given this, the remaining conditions then serve to fix the NSNS flux and the possible form of M_8 through restrictions to the SU(3)-structure torsion classes and physical fields. The content of (2.10a) merely defines the electric part of the NSNS flux and defines the local coordinate ρ in (2.24). We introduce the shorthand notation

$$d^{(7)} := \tilde d^{(6)} + U\wedge\iota_U d,\quad f_\pm^{(7)} := g^1_\pm + U\wedge g^1_\mp. \tag{4.12}$$

The content of (2.10b) is simply (2.12), which already only involves the NSNS flux, and a second equation for the part of the RR flux orthogonal to $d\rho$

$$f_\pm^{(7)} = \Big(d^{(7)} - (H_3^{(6)} + U\wedge H^1_2)\wedge\Big)\Big(e^{-\Phi}\cot\beta\,\text{Re}\psi_\mp^{(7)}\Big) \pm e^{-\Phi}\csc\beta(H^2_2 + U\wedge H_1^{(6)})\wedge\text{Re}\psi_\pm^{(7)}$$
$$\mp e^{-k}\partial_\rho\Big(e^{A-\Phi}\text{Re}\psi_\pm^{(7)}\Big). \tag{4.13}$$

Making this consistent with (4.11) leads to further restrictions on the NSNS sector. Finally, one can use the previous expressions to eliminate $(H_1, f_\pm)$ from (2.10d) and to extract further restrictions on the geometry and physical fields.

In the next sections we present all the conditions that the above considerations imply; to derive these we make copious use of the identities in (4.3)–(4.4). Needless to say, this is a long and tedious computation, so we omit the details.

[13] If this were not the case, and one did fix $\cos\beta = 1$, (2.10c) would contain no information about the anti-self-dual 4-form components of f_+ in Type IIA.

4.1 Type IIA

In this section we present the results of our torsion classes analysis in Type IIA supergravity.

Considering first only the differential constants (2.10a)–(2.10c) we find the following conditions on the fields and functions of the spinor Ansatz

$$\tilde{d}^{(6)}(e^{A-\Phi}a) = \iota_U d(e^{A-\Phi}a) = 0, \tag{4.14}$$

which imply $e^{A-\Phi}a$ is a function of ρ in general. The magnetic components of the NSNS flux get fixed in terms of the torsion classes as

$$\begin{aligned}
H_1^{(6)} &= 4\text{Im}(b\mathcal{V}_3 - a\mathcal{U}_1),\\
H_2^1 &= -b\text{Im}W_2 + \frac{2}{3}ae^{-A+\Phi}\iota_U d(e^{A-\Phi}b)J + \text{Re}\iota_{\overline{\mathcal{U}}_2}\Omega,\\
H_2^2 &= H^{(1,1)} + \text{Re}\iota_{\overline{\mathcal{U}}_1}\Omega + \frac{2}{3}a^2\partial_\rho\Big(\frac{b}{a}\Big)e^{A-k}\sin\beta J,\\
H_3^{(6)} &= \text{Re}H^{(2,1)} + \lambda\text{Im}\Omega - \frac{1}{2}e^{-A+\Phi}\iota_U d(e^{A-\Phi}b)\text{Re}\Omega + \Big(e^{-A+\Phi}a\tilde{d}^{(6)}(e^{A-\Phi}b) + 2b\text{Im}\mathcal{V}_1\Big)\wedge J,
\end{aligned}\tag{4.15}$$

where we introduce the function λ, (1,0)-forms $\mathcal{U}_i$ and primitive $(1,1)$ and $(2,1)$ forms $(H^{(1,1)}, H^{(2,1)})$. We find that the following torsion classes are fixed in general

$$\begin{aligned}
T_2 &= -a\text{Im}W_2, \quad \text{Re}\mathcal{V}_2 = a\Big[\frac{1}{2}\Big(W_0 - \tilde{d}^{(6)}(A-\Phi)\Big) - \text{Re}W_5\Big],\\
\text{Im}\mathcal{V}_4 &= -\frac{1}{2}\Big(U_0^D - 4\text{Im}(b\mathcal{U}_1 + a\mathcal{V}_3)\Big) + \frac{1}{2}e^{2A}\sin^2\beta\tilde{d}^{(6)}\Big(\frac{\cos\beta}{e^{2A}\sin^2\beta}\Big),\\
W_4 &= -be^{-A+\Phi}\tilde{d}^{(6)}(e^{A-\Phi}b) + 2a\text{Im}\mathcal{V}_1, \quad \text{Im}E_2 = -\frac{1}{2}e^{2A}\sin^2\beta\iota_U d\Big(\frac{\cos\beta}{e^{2A}\sin^2\beta}\Big),\\
\text{Re}E_2 &= \frac{1}{2}(P_0 - 2me^{-A}\cot\beta) - \frac{1}{2}e^{-2A+2\Phi-k}\partial_\rho(e^{3A-2\Phi}\sin\beta),\\
\text{Re}E_1 &= -\frac{3}{2}\Big(a\text{Im}W_1 + be^{-A+\Phi}\iota_U d(e^{A-\Phi}b)\Big),,
\end{aligned}\tag{4.16}$$

where a superscript D is such that for a complex (p,q)-form ω, $(\text{Re}\omega)^D = \text{Im}\omega$; it follows that for a real 1-form α, $\alpha + i\alpha^D$ is a (1,0)-form. We find additional constraints on the torsion classes, whose solution requires one to make assumptions about the values of (a,b), and thus likely define a branching of possible classes

$$bW_1 = \frac{a}{3}\Big(2\lambda + ie^{-A+\Phi}\iota_U d(e^{A-\Phi}b)\Big), \quad bW_3 = a\text{Re}H^{(2,1)}. \tag{4.17}$$

Moving onto the pairing constraint (2.10d) we find it gives rise to the differential condition

$$\Big[2e^A(2\text{Im}E_1 + 3R + 6a\text{Re}W_1 + 4b\lambda)\sin\beta + m(7+\cos(2\beta))\Big]e^k = 2e^{4A}\sin^4\beta\partial_\rho\Big(\frac{\cos\beta}{e^{2A}\sin^2\beta}\Big). \tag{4.18}$$

This is all that is contained in (2.10a)–(2.10d), that does not serve to fix the form of the RR fluxes. These are as follows:

The components of $g^1_\pm$ are

$$
\begin{aligned}
e^k g^1_0 &= \partial_\rho(e^{A-\Phi}a), \quad e^{2A}\sin\beta g^1_1 = e^{2A-\Phi}b(U^D_0 - W_0\cos\beta) - 4e^{2A-\Phi}U^D_0 + \cot\beta\tilde{d}^{(6)}(e^{2A-\Phi}b\sin\beta),\\
e^{2A+k}\sin\beta g^1_2 &= -e^{2A+k-\Phi}\left[a(H^{(1,1)} + \mathrm{Re}\iota_{\overline{\mathcal{U}}_1}\Omega) + b\cos\beta(T_1 + \mathrm{Re}\iota_{\overline{\mathcal{V}}_1}\Omega) - b(T_3 + \mathrm{Re}\iota_{\overline{\mathcal{V}}_3}\Omega)\right]\\
&\quad -\frac{1}{3}\left[e^{A-\Phi+k}b(2m\cot\beta + 3e^A\cos\beta R - e^A P_0) - e^{3A}\sin^2\beta\partial_\rho\left(\frac{e^{-\Phi}b}{\sin\beta}\right)\right]J,\\
e^{2A+k}\sin\beta g^1_3 &= e^{2A+k-\Phi}\left(\mathrm{Im}S_2 - \cos\beta(b\mathrm{Re}H^{(2,1)} + aW_3 - \mathrm{Re}S_1)\right) + \frac{1}{2}e^{2A+k-\Phi}\iota_U d(\cos\beta)\mathrm{Re}\Omega\\
&\quad -\frac{1}{2}e^{A-\Phi}\left[e^A\partial_\rho(e^A\sin\beta) + e^k\left(e^A P_0 + e^A(2\mathrm{Im}E_1 + 3a\mathrm{Re}W_1 + 2b\lambda)\cos\beta + 2m\cot\beta\right)\right]\mathrm{Im}\Omega\\
&\quad +e^{2A+k-\Phi}\left[2\cos\beta(a\mathrm{Re}W_5 - \mathrm{Im}\mathcal{V}_1) + 2\mathrm{Im}\mathcal{V}_3 - aU^D_0 - a\cos\beta\tilde{d}^{(6)}\log\left(\frac{e^\Phi}{\sin\beta}\right)\right]\wedge J,\\
e^{2A+k}\sin\beta g^1_4 &= e^{2A+k}\left[\sin\beta\tilde{d}^{(6)}(e^{-\Phi}\cot\beta) + e^{-\Phi}\left((-1+2b)U^D_0 + 2\cos\beta(\mathrm{Re}W_5 - a\mathrm{Im}\mathcal{V}_1) + 2a\mathrm{Im}\mathcal{V}_3\right)\right]\wedge\mathrm{Re}\Omega\\
&\quad +e^{2A+k-\Phi}\left(\cos\beta(\mathrm{Re}W_2 + aT_1) - bH^{(1,1)} - aT_3\right)\wedge J\\
&\quad +\frac{1}{6}\left[\csc\beta\partial_\rho\left(e^{3A-\Phi}a\sin^2\beta\right) + 2e^{A+k-\Phi}\left(-ae^A P_0 + 3e^A(aR + \mathrm{Re}W_1)\cos\beta + 2am\cot\beta\right)\right]J^2,\\
\sin\beta g^1_5 &= \frac{1}{2}\left[\sin\beta\cos\beta\tilde{d}^{(6)}\left(\frac{e^{-\Phi}b}{\sin\beta}\right) - e^{-\Phi}\left(bU^D_0 + \cos\beta(-bW_0 + 4\mathrm{Re}\mathcal{U}_2)\right)\right]\wedge J^2,\\
e^{A+k}\sin\beta g^1_6 &= \frac{1}{3!}\left[e^{k-\Phi}\left(-e^A bP_0 + e^A(3bR + 4\lambda)\cos\beta + 2bm\cot\beta\right) + \partial_\rho(e^{2A-\Phi}b\sin\beta)\right]J^3,
\end{aligned}
\tag{4.19}
$$

and for $g^2_\pm$ we find

$$
\begin{aligned}
e^{2A+k}\sin\beta g^2_0 &= e^A\cos\beta\partial_\rho\left(e^{2A-\Phi}b\sin\beta\right) + e^{A+k-\Phi}\left(b(m\sin\beta - e^A\cos\beta P_0 + 2m\cos\beta\cot\beta) + e^A(3bR + 4\lambda)\right),\\
e^{2A}\sin\beta g^2_1 &= -e^{2\Phi}\sin\beta\left(\tilde{d}^{(6)}\left(\frac{e^{2A-3\Phi}b}{\sin\beta}\right)\right)^D + e^{2A-\Phi}b(W^D_0 - 4\mathrm{Im}W_5 - \cos\beta U_0),\\
e^{2A+k}\sin\beta g^2_2 &= e^{2A+k-\Phi}\left(\mathrm{Re}W_2 + aT_1 - \cos\beta(bH^{(1,1)} + aT_3)\right)\\
&\quad +\frac{1}{3}e^{2A}\left(ae^{k-\Phi}(2\mathrm{Im}E_1 - 3R + 2\cos\beta P_0) - \sin\beta\partial_\rho(e^{A-\Phi}a\cos\beta)\right)J\\
&\quad +e^k\left[\frac{1}{2}\mathrm{Im}\iota_{\overline{\mathcal{U}}_3}\Omega + e^{2A-\Phi}\left(\mathrm{Im}\iota_{\overline{W}_5}\Omega + \mathrm{Re}\iota_{(-a\overline{\mathcal{V}}_1 + \cos\beta(\overline{\mathcal{V}}_4 - a\overline{\mathcal{V}}_3 - b\overline{\mathcal{U}}_1))}\Omega\right)\right],\\
e^{2A+k}\sin\beta g^2_3 &= e^{2A+k-\Phi}\left(b\mathrm{Im}H^{(2,1)} + aW^D_3 - \mathrm{Im}S_1 + \cos\beta\mathrm{Re}S_2\right) + \frac{1}{2}e^{-\Phi+k}\sin\beta\iota_U d(e^{2A}\sin\beta)\mathrm{Im}\Omega\\
&\quad +e^{A+k-\Phi}\left[e^A(2(a\mathrm{Im}W_5 + \mathrm{Re}\mathcal{V}_1) + \cos\beta(aU_0 - 2\mathrm{Re}\mathcal{V}_3)) - \frac{e^{A-\Phi}a}{\sin\beta}\left(\tilde{d}^{(6)}(e^\Phi\sin\beta)\right)^D\right]\wedge J\\
&\quad +\frac{1}{2}e^{A-\Phi}\left[e^A\cos\beta\partial_\rho(e^A\sin\beta) + e^k\left(e^A\cos\beta P_0 + e^A(2\mathrm{Im}E_1 + 3a\mathrm{Re}W_1 + 2b\lambda) + 2m\csc\beta\right)\right]\mathrm{Re}\Omega,\\
e^{2A+k}\sin\beta g^2_4 &= e^{2A+k-\Phi}\left(bT_1 + \cos\beta(aH^{(1,1)} - bT_3)\right)\wedge J\\
&\quad +2e^{2A+k-\Phi}\left(b\mathrm{Im}\mathcal{V}_1 + \cos\beta(aU^D_0 - b\mathrm{Im}\mathcal{V}_3)\right)\wedge\mathrm{Re}\Omega\\
&\quad +\frac{1}{12}\left[e^{A+k-\Phi}b\left(e^A(-6R + 2P_0\cos\beta) + m\frac{(-5 + \cos(2\beta))}{\sin\beta}\right) + e^{3A}\sin(2\beta)\sin\beta\partial_\rho\left(\frac{e^{-\Phi}b}{\sin\beta}\right)\right]J^2,\\
e^{2A}\sin\beta g^2_5 &= \frac{1}{2}\left[e^{2A-\Phi}\left(bW^D_0 + \cos\beta(bU_0 - 4\mathrm{Re}\mathcal{U}_1)\right) - \csc\beta\left(\tilde{d}^{(6)}(e^{2A-\Phi}b\sin\beta)\right)^D\right]\wedge J^2,\\
e^{2A+k}g^2_6 &= -\frac{1}{6}\left(e^{2A}\cos\beta\partial_\rho(e^{A-\Phi}a) - e^{A+k-\Phi}am\right)J^3,
\end{aligned}
\tag{4.20}
$$

JHEP06(2024)056

where we have introduced

$$\mathrm{Re}\mathcal{U}_3 := \frac{\tilde{d}^{(6)}(e^{2A-\Phi}\sin\beta)}{\sin\beta}, \tag{4.21}$$

to ease the presentation a little.

Given all the above one finds that (2.11a)–(2.11c) are indeed implied as claimed earlier.

4.2 Type IIB

In this section we present the results of our torsion classes analysis for Type IIB supergravity.

Considering again first (2.10a)–(2.10c), this time we find a single constraint involving only the functions of our Ansatz

$$\tilde{d}^{(6)}(e^{2A-\Phi}a\sin\beta) = 0. \tag{4.22}$$

The magnetic contribution to the NSNS 3-form must decompose in terms of the torsion classes, two functions $\lambda_{1,2}$ and (1,0)-forms $\mathcal{U}_i$ as

$$\begin{aligned}
H_1^{(6)} &= e^{2A}\sin^2\beta\Big(\tilde{d}^{(6)}\Big(\frac{\cos\beta}{e^{2A}\sin^2\beta}\Big)\Big)^D + 4\mathrm{Im}(b\mathcal{V}_3 - a\mathcal{U}_2), \\
H_2^1 &= H_1^{(1,1)} + \lambda_1 J + \mathrm{Re}\iota_{\overline{\mathcal{U}}_1}\Omega, \\
H_2^2 &= H_2^{(1,1)} + \frac{1}{3}e^A\sin\beta\Big[e^A\sin\beta\,\iota_U d\Big(\frac{\cos\beta}{e^{2A}\sin^2\beta}\Big) + 2e^{-k}a^2\partial_\rho\Big(\frac{b}{a}\Big)\Big]J + \mathrm{Re}\iota_{\overline{\mathcal{U}}_2}\Omega, \\
H_3^{(6)} &= \mathrm{Re}H^{(2,1)} + \lambda_2\mathrm{Re}\Omega + \Big[\frac{a}{e^{2A-\Phi}\sin\beta}\tilde{d}^{(6)}\Big(e^{2A-\Phi}b\sin\beta\Big) + 2\mathrm{Re}(a\mathcal{U}_1 - b\mathcal{V}_2)\Big]\wedge J,
\end{aligned} \tag{4.23}$$

while the following torsion classes get fixed in general

$$\begin{aligned}
E_1 &= -\frac{3}{2}a\mathrm{Im}W_1 - \iota_U d(A-\Phi) + b\lambda_2 + \frac{3}{2}iR, \\
E_2 &= \frac{1}{2}P_0 - me^{-A}\cot\beta - \frac{1}{2}e^{-2A-k+2\Phi}\partial_\rho\Big(e^{3A-2\Phi}\sin\beta\Big), \\
\mathrm{Re}W_1 &= -aR, \quad \mathrm{Re}W_2 = -aT_1, \quad \mathrm{Re}W_5 = -\frac{1}{2}\tilde{d}^{(6)}(A-\Phi) + a\mathrm{Im}\mathcal{V}_1, \\
W_4 &= a^2\Big(W_0 + \tilde{d}^{(6)}\log(e^A\sin\beta)\Big) - 2\mathrm{Re}(b\mathcal{U}_1 + a\mathcal{V}_2), \quad \mathrm{Re}S_1 = aW_3 + b\mathrm{Re}H^{(2,1)}, \\
\mathrm{Re}\mathcal{V}_4 &= -\frac{1}{2}U_0 + 2\mathrm{Re}(a\mathcal{V}_3 + b\mathcal{U}_2).
\end{aligned} \tag{4.24}$$

We find the following conditions that define a branching of possible classes of solutions

$$\begin{aligned}
&e^{A-\Phi}bW_0 = \tilde{d}^{(6)}(e^{A-\Phi}b), \quad bR = 0, \quad bT_1 = 0, \quad b\mathcal{V}_1 = 0, \\
&0 = \cos\beta\Big(e^A\sin\beta\partial_\rho(e^{2A-\Phi}a\sin\beta) + (me^{A-\Phi+k}a\cos\beta - 1)\Big),
\end{aligned} \tag{4.25}$$

where we note in particular that the form of U becomes highly constrained when $b \neq 0$, i.e. when we are not in the G_2-structure limit.

We find that the pairing constraint (2.10d) gives rise to the following differential equation involving the torsion classes.

$$\begin{aligned}
&2e^{4A}\sin^3\beta\partial_\rho\Big(\frac{\cos\beta}{e^{2A}\sin^2\beta}\Big) \\
&= e^k\Big[-2e^A\Big(6b\mathrm{Im}W_1 - 2a^2\iota_U d\Big(\frac{b}{a}\Big) + 3\lambda_1 + 4a\lambda_2\Big) + m\csc\beta(7 + \cos(2\beta))\Big].
\end{aligned} \tag{4.26}$$

This just leaves the RR flux to be presented: the components of $g^1_\pm$ are the following

$$
\begin{aligned}
e^{2A+k}g^1_0 &= e^{2A}\partial_\rho(e^{A-\Phi}b) - be^{2A+k-\Phi}P_0\csc\beta + e^{2A+k}\iota_U d\big(e^{-\Phi}a\cot\beta\big),\\
e^{2A}\sin\beta g^1_1 &= e^{2A-\Phi}bU_0 + e^{2A}\sin\beta\tilde{d}^{(6)}\big(e^{-\Phi}a\cot\beta\big),\\
e^{2A+k}\sin\beta g^1_2 &= -e^{2A+k-\Phi}\Big(bH^{(1,1)}_2 + \cos\beta(aH^{(1,1)}_1 - bT_2) + aT_3\Big) - \frac{1}{2}e^{2A+k-\Phi}\mathrm{Re}\iota_{\overline{\mathcal{U}}_4}\Omega\\
&\quad + \frac{1}{3}\Bigg[\frac{e^{2A+k}\tan\beta}{b^2}\iota_U d\big(e^{-\Phi}b^3\cos\beta\cot\beta\big) + e^{2A+k-\Phi}\cos\beta\big(-3a(b\mathrm{Im}W_1+\lambda_1)+2b^2\lambda_2\big)\\
&\quad + 2e^{A+k-\Phi}(ae^AP_0 + am\cot\beta) - e^{3A}\sin^2\beta\partial_\rho\big(e^{-\Phi}a\csc\beta\big)\Bigg]J,\\
e^{2A+k}\sin\beta g^1_3 &= e^{2A+k-\Phi}\Big(\cos\beta(bW_3 - a\mathrm{Re}H^{(2,1)}) - \mathrm{Re}S_2\Big)\\
&\quad + \Big[-e^{2A+k}\sin\beta\tilde{d}^{(6)}(e^{-\Phi}b\cot\beta) + e^{2A+k-\Phi}(aU_0 + 2\mathrm{Re}(\cos\beta\mathcal{U}_1 - \mathcal{V}_3))\Big]\wedge J\\
&\quad + \frac{1}{2}e^{A-\Phi}\big[e^A\partial_\rho(e^A\sin\beta) - e^k\big(e^AP_0 + e^A\cos\beta(3b\mathrm{Im}W_1 + 2a\lambda_2) - 2m\cot\beta\big)\big]\mathrm{Re}\Omega,\\
e^{2A+k}\sin\beta g^1_4 &= e^{2A+k-\Phi}\Big(aH^{(1,1)}_2 - \cos\beta(bH^{(1,1)}_1 + \mathrm{Im}W_2 + aT_2) - bT_3\Big)\wedge J\\
+e^{A+k-\Phi}&\big\{-\cot\beta\tilde{d}^{(6)}(e^A\sin\beta) - 2e^A\mathrm{Re}(a\mathcal{U}_2 - b\mathcal{V}_3) + e^A\cos\beta[W_0 - 2\mathrm{Re}(b\mathcal{U}_1 + a(\mathcal{V}_2 - i\mathcal{V}_1))]\big\}\wedge\mathrm{Im}\Omega\\
&\quad + \frac{1}{6}\Bigg\{\csc\beta\partial_\rho(e^{3A-\Phi}b\sin^2\beta) - \frac{1}{2}e^{A+k-\Phi}\Bigg[\frac{e^{A-\Phi}}{a^2\sin\beta}\iota_U d\big(e^{-\Phi}a^3\sin(2\beta)\big)\\
&\quad + 2b\big(-e^AP_0 + 2e^A\cos\beta(3b\mathrm{Im}W_1 + 3\lambda_1 + 2a\lambda_2)\big) - 4m\cot\beta\Bigg]\Bigg\}J^2,\\
e^{2A+k}\sin\beta g^1_5 &= -\frac{1}{2}e^{2A+k-\Phi}\Big[a\tilde{d}^{(6)}(\cos\beta) + bU_0 - 4\mathrm{Re}\mathcal{U}_2 + 2\cos\beta(aW_0 - 2(\mathrm{Im}\mathcal{V}_1 + \mathrm{Re}\mathcal{V}_2))\Big]\wedge J^2,\\
e^{2A+k}\sin\beta g^1_6 &= \frac{1}{6}\Bigg\{-e^A\partial_\rho(e^{2A-\Phi}a\sin\beta) - \cot\beta e^{A+k-\Phi}\Bigg[e^{A-\Phi}\iota_U d(e^{\Phi}b\csc\beta)\\
&\quad + 2am - e^A\sin\beta\big(3a(b\mathrm{Im}W_1 + \lambda_1) + 2(1+a^2)\lambda_2\big)\Bigg]\Bigg\}J^3,
\end{aligned}
\tag{4.27}
$$

where we have defined

$$
\mathrm{Re}\mathcal{U}_4 := 2\cos\beta(a\mathrm{Re}\mathcal{U}_1 - b\mathrm{Re}\mathcal{V}_2) - 2(b\mathrm{Re}\mathcal{U}_2 + a\mathrm{Re}\mathcal{V}_3) + U_0. \tag{4.28}
$$

The components of $g^2_\pm$ are

$$
\begin{aligned}
e^{2A+k}\sin\beta g^2_0 &= e^A\cos\beta\partial_\rho(e^{2A-\Phi}a\sin\beta) + e^{A+k-\Phi}\Bigg\{\csc\beta e^{A-\Phi}\iota_U d(e^{\Phi}b\sin\beta)\\
&\quad + \frac{1}{2}\big[-6ae^A(b\mathrm{Im}W_1 + \lambda_1) - 4e^A\lambda_2(1+a^2) + am\csc\beta(3+\cos(2\beta))\big]\Bigg\},\\
e^{A+\Phi}\sin\beta g^2_1 &= -e^{-A}\sin\beta a\big(\tilde{d}^{(6)}(e^{2A}\sin\beta)\big)^D - e^A\big(2aW^D_0 + b\cos\beta U^D_0 - 4(\cos\beta\mathrm{Im}\mathcal{U}_2 + \mathrm{Im}\mathcal{V}_2 - \mathrm{Re}\mathcal{V}_1)\big),\\
e^{2A+k}\sin\beta g^2_2 &= e^{2A+k-\Phi}\Big(bH^{(1,1)}_1 + \mathrm{Im}W_2 + aT_2 + \cos\beta(bT_3 - H^{(1,1)}_2)\Big)\\
&\quad + \frac{1}{12}\Bigg\{4\cot\beta\partial_\rho(e^{3A-\Phi}b\sin^2\beta) - \frac{4e^{A+k-2\Phi}}{a^2\sin\beta}\iota_U d(e^{A+\Phi}a^3\sin\beta) + e^{-2A+k-\Phi}a\iota_U d\big(e^{4A}\cos(2\beta)\big)\\
&\quad + 2e^{A+k-\Phi}\big[-4be^A(3(b\mathrm{Im}W_1 + \lambda_1) + 2a\lambda_2) + 2be^AP_0\cos\beta + bm\csc\beta(7+\cos(2\beta))\big]\Bigg\}J\\
&\quad + e^{2A+k-\Phi}\big(a\mathrm{Im}\iota_{\overline{\mathcal{V}}_1}\Omega + \mathrm{Re}\iota_{\overline{\mathcal{U}}_5}\Omega\big),
\end{aligned}
$$

JHEP06(2024)056

$$
\begin{aligned}
e^{2A+k}\sin\beta g_3^2 = &-e^{2A+k-\Phi}\big(a\mathrm{Im}H^{(2,1)}-bW_3^D+\cos\beta\mathrm{Im}S_2\big)\\
&-\frac{1}{2}e^{A-\Phi}\big[e^A\cos\beta\partial_\rho(e^A\sin\beta)-e^k\big(e^A\cos\beta P_0+3e^A b\mathrm{Im}W_1+2e^A a\lambda_2-2m\csc\beta\big)\big]\mathrm{Im}\Omega\\
&-\frac{1}{2}e^k\Bigg[e^A\big(\tilde{d}^{(6)}(e^{A-\Phi}b)\big)^D+\frac{1}{2}e^{6A-\Phi}\sin^4\beta b\bigg(\tilde{d}^{(6)}\bigg(\frac{\cos(2\beta)}{e^{4A}\sin^4\beta}\bigg)\bigg)^D\\
&+e^{2A-\Phi}\big(bW_0^D-2aU_0^D\cos\beta+4\mathrm{Im}(\cos\beta\mathcal{V}_3-\mathcal{U}_1)\big)\Bigg]\wedge J,\\
e^{2A+k}\sin\beta g_4^2 = &-e^{2A+k-\Phi}\Big(aH_1^{(1,1)}-bT_2+\cos\beta(bH_2^{(1,1)}+aT_3)\Big)\wedge J\\
&+e^{2A+k-\Phi}\mathrm{Im}\big(2(-a\mathcal{U}_1+b\mathcal{V}_2)+\cos\beta(b\mathcal{U}_2+a\mathcal{V}_3-iU_0^D)\big)\wedge\mathrm{Re}\Omega\\
&+\frac{1}{12}\sin\beta\Bigg\{e^{3A}\sin(2\beta)\partial_\rho(e^{-\Phi}a\csc\beta)+e^k\csc\beta\bigg[-\frac{2}{b^2}\iota_U d(e^{2A-\Phi}b^3)\\
&+2e^{-\Phi}b\cos(2\beta)\csc\beta\iota_U d(e^{2A}\sin\beta)+6e^{2A-\Phi}a(b\mathrm{Im}W_1+\lambda_1)-4e^{2A-\Phi}b^2\lambda_2\\
&-4e^{2A-\Phi}aP_0\cos\beta+e^{A-\Phi}am\csc\beta(\cos(2\beta)-5)\bigg]\Bigg\}J^2,\\
e^{A+\Phi}\sin\beta g_5^2 = &\,\frac{1}{2}\cos\beta\big(e^A bU_0+e^{3A}a\sin^2\beta\tilde{d}^{(6)}\big(e^{-2A}\cot\beta\csc\beta\big)\big)^D\wedge J^2,\\
e^{2A+k}\sin\beta g_6^2 = &\,\frac{1}{6}\bigg[e^{2A}\sin\beta\cos\beta\partial_\rho\big(e^{A-\Phi}b\big)+e^k\bigg(e^A\iota_U d(e^{A-\Phi}a)\\
&+\frac{1}{4}e^{6A-\Phi}a\sin^4\beta\iota_U d\bigg(\frac{\cos(2\beta)}{e^{4A}\sin^4\beta}\bigg)-e^{A-\Phi}b(\cos\beta e^A P_0+m\sin\beta)\bigg)\bigg]J^3,
\end{aligned}
\tag{4.29}
$$

where we have defined

$$\mathrm{Re}\mathcal{U}_5 := -b\mathrm{Re}\mathcal{V}_1 - a(\cos\beta\mathrm{Re}\mathcal{U}_2 + \mathrm{Re}\mathcal{V}_2) + b\cos\beta\mathrm{Re}\mathcal{V}_3 + \frac{1}{2}W_0 - \frac{1}{2}\tilde{d}^{(6)}\log(e^A\sin\beta). \tag{4.30}$$

Again, as in Type IIA, the conditions (2.11a)–(2.11c) are implied by the above.

5 New classes of AdS_2 solutions

In this section we show the utility of our results by deriving two new interesting classes of AdS_2 solutions, one in each of Type IIA and IIB supergravity.

In section 5.1 we derive a class of $\mathcal{N}=1$ solutions in (massive) Type IIA supergravity for which M_8 decomposes as a foliation of a weak G_2-manifold over an interval and which includes as a special case the $\mathcal{N}=8$ class of [31]. In section 5.2 we derive a broad class of small $\mathcal{N}=4$ solutions for which M_8 decomposes as $\mathrm{S}^2\times\mathrm{CY}_2\times\Sigma_2$. This includes the AdS_3 Hopf fiber T-dual of the CY_2 class of [47], studied in [23], and the double analytic continuation of [23] studied in [24].

In this section we fix the inverse AdS_2 radius $m=1$, as we are free to do without loss of generality.

5.1 $\mathcal{N}=1$ conformal weak G_2-holonomy class

In this section we would like to explore the possibility of solutions with internal space decomposing as a weak G_2-manifold foliated over an interval. We will thus take the internal metric to decompose as

$$ds^2 = e^{2C}ds^2(\mathrm{M}_{\mathrm{WG}_2}) + e^{2k}d\rho^2, \tag{5.1}$$

where (e^C, e^k), the dilaton and the AdS_2 warp factor e^A are functions of ρ alone, which the metric of M_{WG_2}, the weak G_2-holonomy manifold, is independent of. A manifold with weak G_2-holonomy is characterised by the G_2-structure 3-form, Φ_{WG_2} with a single non-vanishing torsion class, namely

$$d\Phi_{WG_2} = 4 \star_{WG_2} \Phi_{WG_2}. \tag{5.2}$$

Well known examples include G_2 cones over nearly-Kähler bases, the known (closed form) examples of such 6-manifolds are (S^6, $S^3 \times S^3$, $\mathbb{CP}^3$, $\mathbb{F}^3$), and one can arrange for compact M_{WG_2} by fixing

$$\begin{aligned}\Phi_{WG_2} &= \sin^2\alpha d\alpha \wedge J_{NK} + \sin^3\alpha \text{Re}(e^{-i\alpha}\Omega_{NK}),\\ \star_{WG_2}\Phi_{WG_2} &= -\frac{1}{2}\sin^4\alpha J_{NK}^2 + \sin^3\alpha d\alpha \wedge \text{Im}(e^{-i\alpha}\Omega_{NK}),\end{aligned} \tag{5.3}$$

where

$$dJ_{NK} = 3\text{Re}\Omega_{NK}, \quad d\text{Im}\Omega_{NK} = -2J_{NK}^2, \quad ds^2(M_{WG_2}) = d\alpha^2 + \sin^2\alpha ds^2(M_{NK}). \tag{5.4}$$

This makes M_{WG_2} a folation of a nearly-Kähler manifold over the interval spanned by α, generically tending to a G_2 cone singularity[14] at $\alpha = 0 \sim 2\pi$. The exception is when the base is taken to be S^6, in which case $M_{WG_2} = S^7$ which is smooth.

Moving forward we shall decompose the bispinors of section 2.1 such that[15]

$$\begin{aligned}&(a, b) = (1, 0), \quad \beta = \beta(\rho), \quad V = e^k d\rho\\ &\psi_+^{(7)} = 1 + e^{4C} \star_{WG_2} \Phi_{WG_2}, \quad \psi_-^{(7)} = -e^{3C}\Phi_{WG_2} - e^{7C}\text{vol}(M_{WG_2}).\end{aligned} \tag{5.5}$$

We shall also assume that the RR and NSNS flux preserve the structure of M_{WG_2}, or in other words that the components of H_3 and $f_\pm$ are non-trivial only along

$$d\rho, \quad \Phi_{WG_2}, \quad \star_{WG_2}\Phi_{WG_2}, \quad \text{vol}(M_{WG_2}) \tag{5.6}$$

and wedge products thereof. In particular this forces $H_3 = 0$ if we want it to obey its Bianchi identity.

Under the above assumptions it is quick to realise that no solutions in Type IIB are possible because the right-hand side of (2.10b) contains a $\star_{WG_2}\Phi_{WG_2}$ that is orthogonal to everything on the left-hand side and cannot be set to zero. In Type IIA things are better. We decompose the magnetic fluxes as

$$f_+ = F_0 + e^k p\Phi_{WG_2} \wedge d\rho + g \star_{WG_2} \Phi_{WG_2} + e^k q\text{vol}(M_{WG_2}) \wedge d\rho, \quad H_3 = 0, \tag{5.7}$$

where (p, q, g) are functions of ρ. The Bianchi identities of the fluxes in regular regions of the space then reduce to

$$dF_0 = 0, \quad \partial_\rho g + 4e^k p = 0. \tag{5.8}$$

[14]Such singularities is believed to be allowed in string theory.

[15]The presence of a weak G_2-manifold does not mean that we must align its associated G_2-structure forms along those of section 2.1; this is an assumption. See section 6.2 of [33] for an example that does not conform to this assumption.

Plugging the above Ansatz into (2.10a)–(2.10d) we find it fixes the functions in the flux as

$$\begin{aligned}
e^{A+k}\sin\beta F_0 &= \partial_\rho(e^{A-\Phi}), \quad e^{A+k}\sin\beta g = \partial_\rho(e^{A+4C-\Phi}) - 4e^{A+3C+k-\Phi}\cos\beta,\\
e^{A+C+k}\sin\beta p &= -\cos\beta\partial_\rho(e^{A+4C-\Phi}) + e^{3C+k-\Phi}(4e^A + e^C\sin\beta),\\
e^{A+k-7C}\sin\beta q &= -\cos\beta\partial_\rho(e^{A-\Phi}) + e^{k-\Phi}\sin\beta
\end{aligned} \tag{5.9}$$

and furnish us with the following ODEs to solve

$$\begin{aligned}
&\partial_\rho(e^{3A+7C-2\Phi}\sin\beta) + 2\cos\beta e^{2A+7C+k-2\Phi}\cos\beta = 0,\\
&\partial_\rho\left(\frac{e^{\frac{7}{2}C-\Phi}\cot\beta}{\sqrt{e^A\sin\beta}}\right) - \frac{e^{\frac{5}{2}C+k-\Phi}}{(e^A\sin\beta)^{\frac{3}{2}}}(3e^C + 14e^A\sin\beta) = 0.
\end{aligned} \tag{5.10}$$

To solve these we find it useful to introduce two functions of ρ, (G, h), defined as

$$e^{3A+7C-2\Phi}\sin\beta = -\frac{L^4h^2}{32c_0}, \quad e^{A+7C-2\Phi}\cot\beta = -\frac{L^2Gh}{\sqrt{8}c_0}, \tag{5.11}$$

where (c_0, L) are constants. We then use a coordinate transformation to fix

$$e^{A+k}\sin\beta = -\frac{L^2}{8\sqrt{2}}, \tag{5.12}$$

and introduce a function $v = v(\rho)$ such that

$$e^C(v-1) = 4\sin\beta e^A. \tag{5.13}$$

This reduces (5.10) to the rather simple conditions

$$G = \frac{1}{c_0}\partial_\rho h, \quad \cos^2\beta = \frac{c_0G^2(1-7v)}{2h\partial_\rho G}, \tag{5.14}$$

which we can use to define $(\cos\beta, G)$ (we take the positive root for $\cos\beta$) thereby solving the necessary conditions for supersymmetry. One can additionally show that all of (2.13) are implied by (5.8), so finding a solutions boils down to solving these constraints.

In summary, we have found a class of solutions with the following NSNS sector

$$\begin{aligned}
\frac{ds^2}{L^2} &= \sqrt{\frac{h}{h''}}\left[\frac{hh''\sqrt{1-7v}}{8\Delta}ds^2(\text{AdS}_2) + \left(\frac{h''}{8h\sqrt{1-7v}}d\rho^2 + \frac{\sqrt{1-7v}}{(v-1)^2}ds^2(\text{M}_{\text{WG}_2})\right)\right],\\
H &= \frac{L^2}{8\sqrt{2}}d\left(\frac{hh'(1-7v)}{\Delta} - \rho\right)\wedge\text{vol}(\text{AdS}_2), \quad e^{-\Phi} = \frac{\sqrt{\Delta}(1-v)^{\frac{7}{2}}}{c_0L^3(1-7v)^{\frac{5}{4}}}\left(\frac{h''}{h}\right)^{\frac{3}{4}},\\
\Delta &= 2hh'' - (1-7v)(h')^2,
\end{aligned} \tag{5.15}$$

where $h' = \partial_\rho h$ and the RR sector is defined as in (5.7), where the functions that appear now take the form

$$\begin{aligned}
F_0 &= \frac{4}{c_0L^4}\partial_\rho\left(\frac{(1-v)^{\frac{7}{2}}h''}{(1-7v)}\right), \qquad g = \frac{4\sqrt{v}}{c_0}\partial_\rho\left(\frac{h\sqrt{v}}{\sqrt{1-v}}\right),\\
e^kp &= -\frac{2v^{\frac{3}{4}}}{c_0(1-v)(1-7v)}\partial_\rho\left((1-v)^{\frac{3}{2}}v^{\frac{1}{4}}h'\right), \quad e^kq = \frac{L^4}{c_0}\left(\partial_\rho\left(\frac{(1-7v)hh'}{(1-v)^{\frac{7}{2}}h''}\right) - \frac{h(3-7v)}{(1-v)^{\frac{7}{2}}}\right).
\end{aligned} \tag{5.16}$$

The Bianchi identities (5.8), away from sources reduce to

$$\partial_\rho^2\left(\frac{(1-v)^{\frac{7}{2}}h''}{(1-7v)}\right)=0,$$
$$\partial_\rho\left(\sqrt{v}\partial_\rho\left(\frac{h\sqrt{v}}{\sqrt{1-v}}\right)\right)-\frac{2v^{\frac{3}{4}}}{(1-v)(1-7v)}\partial_\rho\left((1-v)^{\frac{3}{2}}v^{\frac{1}{4}}h'\right)=0, \tag{5.17}$$

and one has an (at least) $\mathcal{N}=1$ supersymmetric solution whenever these are solved.

The conditions (5.17) appear rather difficult to solve in full generality. One could proceed semi-analytically, i.e. solve the above with a series expansion and then attempt to numerically interpolate; we will not attempt this here. Instead we note that when one fixes $v=v_0$, a constant, (5.17) reduces to

$$h''''=0,\quad v_0(1+5v_0)=0, \tag{5.18}$$

with both $v_0=0$ and $1+5v_0=0$ being valid independent solutions. In each case we have $h'''\propto F_0$, so h is locally a degree three polynomial, though F_0 need not be fixed globally. This fact can potentially be used to glue local patches with different values of F_0 together, leading to infinite classes of solution with D8-branes at the loci where F_0 is discontinuous. Such behaviour has been observed and exploited before in the context of $\text{AdS}_{7,3,2}$ solutions of massive Type IIA [31, 48–50], the last being the most relevant to the case at hand.

When $v_0=0$ the only non-trivial magnetic RR fluxes are f_0 and f_8, and by comparing to [31] it becomes clear we have a generalisation of the class of $\mathcal{N}=8$ $\text{AdS}_2\times\text{S}^7\times\mathcal{I}$ solutions found there, where S^7 can now be any weak G_2-manifold and generically only $\mathcal{N}=1$ is preserved. That such solutions exist is not surprising, as S^7 indeed supports a weak G_2-holonomy.

When $1+5v_0=0$ the metric gets deformed with respect to the $v_0=0$ case, now all of (f_0,f_4,f_8) are non-trivial, but solutions are still governed by the same ODE, $h''''=0$. It would be interesting to explore what solutions lie within this class, but that is beyond the scope of our aims here. We note that the $\mathcal{N}=8$ AdS_2 solutions of [31] can be mapped to the AdS_7 solutions of [48] via double analytic continuation. It would also be interesting to explore the class of deformed (with regard to the fluxes only) AdS_7 solutions that the $1+5v_0=0$ case should map to, and what implications they could have in the context of the $\text{AdS}_7/\text{CFT}_6$ correspondence.

5.2 Small $\mathcal{N}=4$ $\text{AdS}_2\times\text{S}^2\times\text{CY}_2\times\Sigma_2$ class

Two examples of classes of solutions with $\text{AdS}_2\times\text{S}^2\times\text{CY}_2\times\text{S}^1$ foliated over an interval and preserving small $\mathcal{N}=4$ were derived in [23]. The first via AdS_3 Hopf fiber T-dual of an AdS_3 class derived in [47], the second [24] with a double analytic continuation of the first. In this section we will construct a much broader class of solutions containing both these examples as limits.

We seek a solution for which the internal manifold and magnetic NSNS 3-form decompose as

$$ds^2=e^{2C}ds^2(\text{S}^2)+ds^2(\text{M}_4)+V^2+U^2,\quad H_3=e^{2C}\tilde{H}_1\wedge\text{vol}(\text{S}^2)+\tilde{H}_3, \tag{5.19}$$

where $(e^{2C}, \tilde{H}_1, \tilde{H}_3)$ have support on M_4, and where the magnetic component of the RR fluxes decompose in a similar SU(2) invariant fashion. In addition to the SU(2) invariant $\text{vol}(\text{S}^2)$, S^2 supports the following triplets in terms of unit radius embedding coordinates $\mu_{\mathfrak{a}}$, $\mathfrak{a} = 1, 2, 3$,

$$\mu_{\mathfrak{a}}, \quad d\mu_{\mathfrak{a}} \quad \epsilon_{\mathfrak{abc}}\mu_{\mathfrak{b}} d\mu_{\mathfrak{c}}, \quad \mu_{\mathfrak{a}}\text{vol}(\text{S}^2). \tag{5.20}$$

In terms of these we make the following Ansatz[16] for the SU(3)-structure forms and functions appearing in (2.19)

$$J = e^{2C}\text{vol}(\text{S}^2) - \mu_{\mathfrak{a}} j_{\mathfrak{a}}, \quad \Omega = e^{C}(d\mu_{\mathfrak{a}} \wedge j_{\mathfrak{a}} + i\epsilon_{\mathfrak{abc}}\mu_{\mathfrak{b}} d\mu_{\mathfrak{c}} \wedge j_{\mathfrak{c}}), \quad a + ib = e^{-i\alpha}, \tag{5.21}$$

where we will assume that (U, V) have no support on the 2-sphere. Here $j_{\mathfrak{a}}$ are a set of SU(2)-structure 2-forms on M_4 obeying

$$j_{\mathfrak{a}} \wedge j_{\mathfrak{b}} = 2\delta_{\mathfrak{ab}}\text{vol}(\text{M}_4). \tag{5.22}$$

This choice of SU(3)-structure forms ensures that we have $\mathcal{N} = 4$ supersymmetry provided that the fluxes are SU(2) singlets. This is because we are free to rotate $\mu_{\mathfrak{a}}$ by a constant element of SO(3) and change nothing about the physical fields. It is clear from (5.21) that only the charged parts of (5.21) transform under this action so if we demand that the fluxes are SU(2) singlets one can generate further three independent SU(3)-structures in this fashion yielding the claimed $\mathcal{N} = 4$. The necessary conditions for supersymmetry (2.10b)–(2.10c) then decompose into singlet and triplet parts under SU(2). We want the RR flux to preserve this symmetry so only the former may contribute to this. After a long computations one can show that fixing the triplet parts of (2.10b)–(2.10c) to zero amounts to imposing

$$e^{C} + \sin\beta \sin\alpha e^{A} = 0, \tag{5.23a}$$

$$e^{2C}\tilde{H}_1 + e^{A}\sin\beta U + d\Big(e^{2A}\cos\alpha\sin\beta\Big) = 0, \tag{5.23b}$$

$$d(e^{A}\sin\beta V) = 0, \tag{5.23c}$$

$$d(e^{A}\sin\beta U) \wedge j_{\mathfrak{a}} = 0, \tag{5.23d}$$

$$d(e^{2A-\Phi}\sin\alpha\sin\beta j_{\mathfrak{a}}) - e^{2A-\Phi}(\cos\alpha U - \cos\beta\sin\alpha V) \wedge j_{\mathfrak{a}} = 0, \tag{5.23e}$$

$$\tilde{H}_3 \wedge j_{\mathfrak{a}} = \tilde{H}_3 \wedge U \wedge V = 0. \tag{5.23f}$$

The first two of these just define a warp factor and part of the NSNS flux, it is the rest that we must solve for. First, we note that $d(e^{2C}\tilde{H}_1) = 0$ is a necessary condition for the Bianchi identity of the NSNS flux away from the loci of sources, as such in regular regions of a solution we have that $d(e^{A}\sin\beta U) = 0$, which implies (5.23d). We solve this condition and (5.23c) in terms of two local coordinates (x_1, x_2) as

$$U + iV = -\frac{e^{-A}}{\sin\beta}(dx_1 + idx_2). \tag{5.24}$$

Taking the exterior derivative of (5.23e), it then follows that

$$dx_2 \wedge d(e^{-2A}\cot\alpha\csc^2\beta) = dx_1 \wedge d(e^{-2A}\cos\beta\csc^2\beta), \tag{5.25}$$

[16]This Ansatz is by no means general for $\text{AdS}_2 \times \text{S}^2$ solutions, it is merely sufficient to construct the class we seek.

which ensures that the combinations in parentheses are functions of (x_1, x_2) alone, and obey

$$\partial_{x_1}(e^{-2A}\cot\alpha\csc^2\beta) + \partial_{x_2}(e^{-2A}\cos\beta\csc^2\beta) = 0. \tag{5.26}$$

This is an integrability condition which we are free to solve by introducing a local function $u = u(x_1, x_2)$ such that

$$e^{-2A}\cot\alpha\csc^2\beta = -\partial_{x_2}\log u, \quad e^{-2A}\cos\beta\csc^2\beta = \partial_{x_1}\log u. \tag{5.27}$$

It is then possible to recast (5.23d) in the form

$$d\left(\frac{e^{2A-\Phi}\sin\alpha\sin\beta}{u}j_{\mathfrak{a}}\right) = 0, \tag{5.28}$$

which informs us that M_4 is conformally a Calabi-Yau 2-fold, specifically we have

$$ds^2(\text{M}_4) = \frac{e^{-2A+\Phi}u}{\sin\alpha\sin\beta}ds^2(\text{CY}_2). \tag{5.29}$$

Finally, the condition (5.23f) informs us that $\tilde{H}_3$ must decompose in terms of two primitive $(1,1)$-forms on CY_2 as[17]

$$\tilde{H}_3 = dx_1 \wedge X_1^{(1,1)} + dx_2 \wedge X_2^{(1,1)}, \tag{5.30}$$

with $(X_1^{(1,1)}, X_2^{(1,1)})$ anti-self-dual on CY_2 by definition. Having solved the triplet contributions of (2.10b)–(2.10c) we can now extract the RR flux from (2.10c) as in (4.11), and substitute the result into (2.10b) and (2.10c). The first of these results in a single additional condition

$$d(e^{-\Phi}\sin^2\alpha\csc\beta)\wedge dx_1 \wedge dx_2 = 0, \tag{5.31}$$

which informs us that $e^{-\Phi}\sin^2\alpha\csc\beta$ is independent of the coordinates on CY_2. We solve this condition in terms of another local function $h_7 = h_7(x_1, x_2)$ and a constant c_0 as

$$\frac{e^{-\Phi}\sin\alpha}{\sin\beta} = c_0 h_7. \tag{5.32}$$

Due to our judicious redefinitions of the physical fields, (2.10c) then reduces to simply

$$\nabla_2^2 u = 0, \quad \nabla_2^2 := \partial_{x_1}^2 + \partial_{x_2}^2. \tag{5.33}$$

As (2.10a) can be taken to define the electric part of the NSNS flux, the necessary conditions for supersymmetry are now solved. However, we find it useful to introduce one final local function $h_3 = h_3(x_1, x_2, \text{CY}_2)$ as

$$\frac{u^2}{e^{4A}h_7\sin^4\beta} = h_3, \tag{5.34}$$

before we summarise our results and present the explicit form of the RR fluxes.

[17] In this instance a primitive $(1,1)$-form $X^{(1,1)}$ is defined such that it has legs only in the CY_2 directions and satisfies $X^{(1,1)}\wedge j_{\mathfrak{a}} = 0$. In a canonical frame on CY_2 where $(j_3 = \text{e}^{12}+\text{e}^{34},\ j_1 + ij_2 = (\text{e}^1 + i\text{e}^2)\wedge(\text{e}^3 + i\text{e}^4))$ this means that $X^{(1,1)}$ decomposes in a basis of three independent forms $(\text{e}^{12}-\text{e}^{34},\ \text{e}^{13}+\text{e}^{24},\ \text{e}^{14}-\text{e}^{23})$, as such each of $X_{1,2}^{(1,2)}$ can depend on three independent functions of (x_1, x_2, CY_2).

JHEP06(2024)056

In summary, we find a class of solutions with NSNS sector of the form

$$\begin{aligned}
ds^2 &= \frac{u}{\sqrt{h_3h_7}}\bigg(\frac{1}{\Delta_2}ds^2(\text{AdS}_2)+\frac{1}{\Delta_1}ds^2(\text{S}^2)\bigg)+\sqrt{\frac{h_3}{h_7}}ds^2(\text{CY}_2)+\frac{\sqrt{h_3h_7}}{u}(dx_1^2+dx_2^2)\bigg],\\
e^{-\Phi} &= c_0\sqrt{\Delta_1\Delta_2}h_7,\quad H=dB_0\wedge\text{vol}(\text{AdS}_2)+d\tilde{B}_0\wedge\text{vol}(\text{S}^2)+dx_1\wedge X_1^{(1,1)}+dx_2\wedge X_2^{(1,1)},\\
\Delta_1 &= 1+\frac{(\partial_{x_1}u)^2}{h_3h_7},\quad \Delta_2=1-\frac{(\partial_{x_2}u)^2}{h_3h_7},\quad B_0=-x_2-\frac{u\partial_{x_2}u}{h_3h_7\Delta_2},\quad \tilde{B}_0=x_1-\frac{u\partial_{x_1}u}{h_3h_7\Delta_1},
\end{aligned}\tag{5.35}$$

for functions (u,h_7) with support on (x_1,x_2) and h_7 with support on (x_1,x_2,CY_2). Solutions in this class support the following non-trivial magnetic RR fluxes[18]

$$\begin{aligned}
f_1&=c_0\bigg(\star_2 d_2h_7+d\bigg(\frac{\partial_{x_1}u\partial_{x_2}u}{h_3}\bigg)\bigg),\\
f_3&=\tilde{B}_0f_1\wedge\text{vol}(\text{S}^2)-c_0\bigg[\bigg(x_1\star_2 d_2h_7+h_7dx_2-d\bigg(\frac{\partial_{x_2}u(u-x_1\partial_{x_1}u)}{h_3}\bigg)\bigg)\wedge\text{vol}(\text{S}^2)\\
&\quad+h_7\Big(dx_1\wedge X_2^{(1,1)}-dx_2\wedge X_1^{(1,1)}\Big)+\frac{\partial_{x_1}u\partial_{x_2}u}{h_3}\Big(dx_1\wedge X_1^{(1,1)}+dx_2\wedge X_2^{(1,1)}\Big)\bigg],\\
f_5&=\tilde{B}_0f_3\wedge\text{vol}(\text{S}^2)-c_0\bigg[\frac{h_7}{u}\star_4 d_4h_3\wedge\text{vol}_2+\star_2 d_2h_3\wedge\text{vol}_4+d\bigg(\frac{\partial_{x_1}u\partial_{x_2}u}{h_7}\bigg)\wedge\text{vol}_4\\
&\quad+\bigg(-x_1h_7\Big(dx_1\wedge X_2^{(1,1)}-dx_2\wedge X_1^{(1,1)}\Big)+\frac{\partial_{x_2}u(u-x_1\partial_{x_1}u)}{h_3}\Big(dx_1\wedge X_1^{(1,1)}+dx_2\wedge X_2^{(1,1)}\Big)\bigg)\wedge\text{vol}(\text{S}^2)\bigg],\\
f_7&=\tilde{B}_0f_5\wedge\text{vol}(\text{S}^2)+c_0\bigg[-d\bigg(\frac{\partial_{x_2}u(u-x_1\partial_{x_1}u)}{h_7}\bigg)\wedge\text{vol}_4\\
&\quad+x_1\frac{h_7}{u}\star_4 d_4h_3\wedge\text{vol}_2+(x_1\star_2 d_2h_3+h_3dx_2)\wedge\text{vol}_4\bigg]\wedge\text{vol}(\text{S}^2),
\end{aligned}\tag{5.36}$$

where we have decomposed

$$d=d_2+d_4,\quad d_2=dx_i\wedge\partial_{x_i},\tag{5.37}$$

for d_4 the exterior derivative on CY_2 and where $\star_{2,4}$ and $\text{vol}_{2,4}$ are the Hodge duals and volume forms on the unwarped (x_1,x_2) and CY_2 sub-manifolds respectively.

Supersymmetry is solved by simply imposing that globally

$$\nabla_2^2u=0,\tag{5.38}$$

i.e. u must be a harmonic function on (x_1,x_2), and that $(X_1^{(1,1)},\ X_2^{(1,1)})$ are primitive $(1,1)$-forms with "legs" on CY_2 only. Thus, finding solutions becomes a two step process: first one chooses a harmonic function u and specific CY_2 manifold and then one needs to solve (2.13) for this choice. The Bianchi identities of the magnetic fluxes impose that in regular regions of the internal space

$$\begin{aligned}
&d_4X_1^{(1,1)}=d_4X_2^{(1,1)}=0,\quad \partial_{x_2}X_1^{(1,1)}=\partial_{x_1}X_2^{(1,1)},\quad \partial_{x_1}(h_7^2X_1^{(1,1)})=-\partial_{x_2}(h_7^2X_2^{(1,1)}),\\
&\nabla_2^2h_7=0,\quad \frac{h_7}{u}\nabla_4^2h_3+\nabla_2^2h_3+h_7\Big((X_1^{(1,1)})^2+(X_2^{(1,1)})^2\Big)=0,
\end{aligned}\tag{5.39}$$

[18]The electric components are defined in terms of these as in (2.1).

JHEP06(2024)056

where $\nabla^2_{2,4}$ are the Laplacians on the unwarped (x_1, x_2) and CY_2 directions. When these conditions hold, all of (2.13) are implied so one has a solution.

To better understand this class it is instructive to fix $X_1^{(1,1)} = X_2^{(1,1)} = 0$ and $u = 1$ (only $u =$ constant is required but one can then fix $u = 1$ without loss of generality by rescaling coordinates and the other functions). First off, in this limit (5.39) can be solved by fixing both (h_3, h_7) to be constant, we then recover the double T-dual of the D1-D5 near-horizon performed on the Hopf fiber of both its AdS_3 and S^3 factors. If we allow for non-constant (h_3, h_7) then (5.39) reduce to the Bianchi identities on would expect for D3-branes localised within the worldvolume of D7-branes whose relative codimensions are a CY_2 manifold. The metric and dilaton also take the expected form for such branes if they are both extended in $AdS_2\times S^2$, as such, at least in this limit (h_3, h_7) play the role of (D3,D7)-brane warp factors which is the reason for their numerical subscripts. If we now turn on $X_{1,2}^{(1,1)}$ the metric remains unchanged but the D3-brane PDE inherits an additional term from the fluxes. Allowing u to be a more general harmonic function deforms this system and the interpretation of (h_3, h_7) in terms of warp factors of branes is not at all clear, indeed this likely depends on the specific choice of u.

If one imposes that ∂_{x_1} spans a U(1) isometry and additionally fixes $X_1^{(1,1)} = 0$ we recover the class of solutions in [24] obtained by double analytic continuation of the class in [23]. If instead we make ∂_{x_2} a U(1) isometry and $X_2^{(1,1)} = 0$ one recovers the result of T-dualising the $AdS_3\times S^2\times CY_2\times \mathcal{I}$ class of solutions found in [47] on the Hopf fiber of AdS_3, as studied in [23]. One can of course perform this T-duality in the presence of a non-trivial $X_2^{(1,1)}$, in which case AdS_3 becomes fibred over the internal CY_2 generalising the class in [47]. If we additionally impose that ∂_{x_1} is an isometry we further generalise the generalised D1-D5 near-horizon geometries studied in [51] such that both the AdS_3 and S^3 factors are fibered over CY_2.

Specific solutions to the classes in [23] and [24] were considered in the limit that the symmetries of the CY_2 were respected, i.e. with fluxes only depending on the CY_2 through vol_4 and where the warp factors only depend on (x_1, x_2). In this limit, the classes of [23, 24] can be embedded into a classification of $AdS_2 \times S^2 \times CY_2 \times \Sigma_2$ in [15, 16], which likewise restricts dependence on CY_2. It would be interesting to see how the $X_1^{(1,1)} = X_2^{(1,1)} = 0$ and $h_3 = h_3(x_1, x_2)$ limit of the class we present here fits within this existing classification, and whether it is actually equivalent. We stress that as our class has non-trivial $(X_1^{(1,1)}, X_2^{(1,1)})$ and $h_3 = h_3(x_1, x_2, CY_2)$ it lies beyond the results of [15, 16].

In the limit $u = 1$ we note that AdS_2 and S^2 share the same warping, so this class could potentially be fruitful in the study of four-dimensional black hole near-horizon geometries. We leave this interesting possibility to be studied elsewhere.

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JHEP06(2024)056

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A Type II supergravity and conventions

We work with the "democratic" conventions of [44] for Type II supergravities. The bosonic fields split into two sectors: the NSNS sector containing the metric, dilaton and NSNS flux, respectively (g, Φ, H), and the RR sector containing the RR polyform F. The RR flux is subject to the self-duality constraint

$$F = \star\lambda(F), \tag{A.1}$$

where for a k-form C_k — to which we will make frequent reference in this section — $\lambda(C_k) := (-)^{\lfloor \frac{k}{2} \rfloor} C_k$, and we define Hodge dualisation in all dimensions and signatures as

$$\star \mathrm{e}^{\underline{M}_1 \cdots \underline{M}_k} := \frac{1}{(d-k)!} \epsilon_{\underline{M}_{k+1} \cdots \underline{M}_{d-k}}{}^{\underline{M}_1 \cdots \underline{M}_k} \; \mathrm{e}^{\underline{M}_{k+1} \cdots \underline{M}_{d-k}}, \tag{A.2}$$

for $\mathrm{e}^{\underline{M}}$ a vielbein (underlined indices are flat spacetime indices).

A solution to Type II supergravity must obey the following equations of motion and Bianchi identities away from the loci of sources

$$\begin{aligned} &d_H F = 0, \quad dH = 0, \quad d(e^{-2\Phi} \star H) = \frac{1}{2}(F, F)_8, \\ &2R - H^2 - 8e^{\Phi}\nabla^2 e^{-\Phi} = 0, \quad R_{MN} + 2\nabla_M \nabla_N \Phi - \frac{1}{2} H^2_{MN} - \frac{e^{2\Phi}}{4}(F)^2_{MN} = 0, \end{aligned} \tag{A.3}$$

where $(F, F)_8$ is the 8-form part of $F \wedge \lambda(F)$, and

$$\begin{aligned} (C_k)_M &:= \iota_{dx^M} C_k, \quad (C_k)^2 := \sum_k \frac{1}{k!}(C_k)_{M_1 \ldots M_k}(C_k)^{M_1 \ldots M_k}, \\ C^2_{MN} &:= \sum_k \frac{1}{(k-1)!}(C_k)_{M M_1 \ldots M_{k-1}}(C_k)_N{}^{M_1 \ldots M_{k-1}}. \end{aligned} \tag{A.4}$$

Such a solution preserves supersymmetry if it supports two Majorana-Weyl spinors $\epsilon_{1,2}$ that satisfy the constraints (here and elsewhere the upper/lower signs are taken in Type IIA/IIB supergravity)

$$\left(\nabla_M - \frac{1}{4} H_M\right)\epsilon_1 + \frac{e^{\Phi}}{16} F \Gamma_M \epsilon_2, \quad \left(\nabla_M + \frac{1}{4} H_M\right)\epsilon_2 \pm \frac{e^{\Phi}}{16}\lambda(F)\Gamma_M \epsilon^1 = 0, \tag{A.5a}$$

$$\left(\nabla - \frac{1}{4} H - d\Phi\right)\epsilon_1 = 0, \quad \left(\nabla + \frac{1}{4} H - d\Phi\right)\epsilon_2 = 0, \tag{A.5b}$$

where here and throughout this work when a form acts on a spinor it does so through the Clifford map, i.e.

$$C_k \epsilon = \not{C}_k \epsilon := (C_k)^{M_1 \ldots M_k} \Gamma_{M_1 \ldots M_k} \epsilon. \tag{A.6}$$

More generally the Clifford map states that the following are equivalent

$$C_k = (C_k)_{\underline{M}_1\cdots\underline{M}_k}\mathrm{e}^{\underline{M}_1\cdots\underline{M}_k} \equiv \not{C}_k = (C_k)_{\underline{M}_1\cdots\underline{M}_k}\Gamma^{\underline{M}_1\cdots\underline{M}_k}, \tag{A.7}$$

where $\Gamma^{\underline{M}}$ is a basis of flat spacetime gamma matrices. This implies the following equivalence for spinor bilinears in d even dimensions

$$\epsilon_1\otimes\epsilon_2^\dagger = \frac{1}{2^{\lfloor\frac{d}{2}\rfloor}}\sum_{k=1}^{d}\frac{1}{k!}\epsilon_2^\dagger\Gamma_{\underline{M}_k\cdots\underline{M}_1}\epsilon_1\Gamma^{\underline{M}_1\cdots\underline{M}_k} \equiv \frac{1}{2^{\lfloor\frac{d}{2}\rfloor}}\sum_{k=1}^{d}\frac{1}{k!}\epsilon_2^\dagger\Gamma_{\underline{M}_k\cdots\underline{M}_1}\epsilon_1\mathrm{e}^{\underline{M}_1\cdots\underline{M}_k}, \tag{A.8}$$

in odd dimensions the right-hand side of the above equivalence contains the left-hand side twice so one needs to add a $\frac{1}{2}$ to account for this. The action of gamma matrices on forms can then be viewed through the following map

$$\Gamma^{\underline{M}}C_k = (\mathrm{e}^{\underline{M}}\wedge + \iota_{\mathrm{e}^{\underline{M}}})C_k, \quad (-)^k C_k\Gamma^{\underline{M}} = (\mathrm{e}^{\underline{M}}\wedge - \iota_{\mathrm{e}^{\underline{M}}})C_k, \tag{A.9}$$

where strictly speaking these expressions hold inside a slash as in (A.6), but we shall suppress this.

The basis of gamma matrices appearing here is such that, for an intertwiner $B^{(10)}$, defining Majorana conjugation as $\epsilon^c := B^{(10)}\epsilon^*$ we have

$$(B^{(10)})^{-1}\Gamma_M B^{(10)} = \Gamma_M^*, \quad B^{(10)}(B^{(10)})^* = \mathbb{I}. \tag{A.10}$$

We define the chirality matrix in ten dimensions to be

$$\hat{\Gamma} := \Gamma^{\underline{0}}\ldots\Gamma^{\underline{9}} \tag{A.11}$$

and the spinors obey the following under it

$$\hat{\Gamma}\epsilon_1 = \epsilon_1, \quad \hat{\Gamma}\epsilon_2 = \mp\epsilon_2. \tag{A.12}$$

Our conventions thus far imply another useful relationship

$$\hat{\Gamma}C_k = \star\lambda(C_k), \tag{A.13}$$

where again we leave the slash implicit.

B Killing spinors and Killing spinor bilinears on AdS_2

On AdS_2 of inverse radius m, there are Killing spinors of $\pm$ chirality $\zeta_\pm$ that obey the Killing spinor equation

$$\nabla_\mu\zeta_\pm = \frac{m}{2}\gamma_\mu^{(2)}\zeta_\mp. \tag{B.1}$$

We can take $\gamma_\mu^{(2)}$ to be real, so that $\zeta_\pm$ are also real. One can then show that the spinors give rise to the following bilinears under the Clifford map

$$\zeta_\pm\otimes\overline{\zeta}_\pm = \frac{1}{2}(v\pm u), \quad \zeta_\pm\otimes\overline{\zeta}_\mp = \frac{1}{2}f(\pm 1 - \mathrm{vol}(\mathrm{AdS}_2)), \tag{B.2}$$

JHEP06(2024)056

where $\bar{\zeta} := (\gamma^{(2)}_{\underline{0}}\zeta)^\dagger$, and these objects obey the following differential relations

$$df = -mu, \quad dv = -2mf\text{vol}(\text{AdS}_2), \quad \nabla_{(\mu}v_{\nu)} = 0, \quad \nabla_{(\mu}u_{\nu)} = -mfg_{\mu\nu}. \tag{B.3}$$

Thus $v^\mu\partial_{x^\mu}$ is a Killing vector, while $u^\mu\partial_{x^\mu}$ is not, rather it is a conformal Killing vector. The forms also obey the following conditions

$$\begin{aligned} &|v|^2 = -f^2, && |u|^2 = f^2, && u\cdot v = 0,\\ &\iota_u\text{vol}(\text{AdS}_2) = v, && \iota_v\text{vol}(\text{AdS}_2) = u, && f^2\text{vol}(\text{AdS}_2) = -v\wedge u. \end{aligned} \tag{B.4}$$

For the pairing constraints some additional conditions will be useful; one can show that

$$v\zeta_\pm = \pm f\zeta_\mp, \quad u\zeta_\pm = -f\zeta_\mp. \tag{B.5}$$

Thus $v^\pm = \frac{1}{2}(v\pm u)$ is such that

$$v^\pm\zeta_\pm = 0, \quad v^\mp\zeta_\pm = \pm f\zeta_\mp, \quad \overline{\zeta}_\pm v^\mp\zeta_\pm = 2v^\mp\cdot v^\pm = -f^2, \quad v^\pm\cdot v^\pm = 0. \tag{B.6}$$

The identities presented in this section are most easily derived in terms of a specific parameterisation of AdS_2, for instance one can take

$$ds^2(\text{AdS}_2) = -e^{2mr}dt^2 + dr^2. \tag{B.7}$$

Taking the flat spacetime gamma matrices to be $\gamma^{(2)}_{\underline{\mu}} = (i\sigma_2,\sigma_1)_{\underline{\mu}}$, and with respect to the obvious vielbein suggested by the diagonal metric above, we have a t-independent solution to (B.1) given by:

$$\zeta_+ = e^{mr}\begin{pmatrix}1\\0\end{pmatrix}, \quad \zeta_- = e^{mr}\begin{pmatrix}0\\1\end{pmatrix}. \tag{B.8}$$

In terms of these we then have

$$v = e^{2mr}dt, \quad u = -e^{mr}dr, \quad f = e^{mr}. \tag{B.9}$$

Of course there is a second solution to (B.1), dual to conformal supercharges rather than space-time ones under the AdS/CFT. One could choose to align $\zeta_\pm$ along a linear combination of these and what appears in (B.8). This will lead to a different form of (f,u,v), but they will still obey the identities presented earlier.

C Refining the pairing constraints for the time-like Killing vector case

Following [44], a supersymmetric solution of Type II supergravity must obey the following differential conditions

$$d\tilde{K} = \iota_K H, \quad d_H(e^{-\Phi}\Psi) = -(\iota_K + \tilde{K}\wedge)F, \quad \nabla_{(M}K_{M)} = 0, \tag{C.1}$$

where (H,F) are the NSNS and RR fluxes, Φ is the dilaton and the remaining objects can be defined in terms of the two Majorana-Weyl Killing spinors $\epsilon_{1,2}$ that Type II solutions can support as

$$K := \frac{1}{2}(K_1+K_2), \quad \tilde{K} := \frac{1}{2}(K_1-K_2), \quad K_i := \frac{1}{32}\overline{\epsilon}_i\Gamma_M\epsilon_i e^M, \quad \Psi := \epsilon_1\otimes\overline{\epsilon}_2, \tag{C.2}$$

where $\bar{\epsilon} := (\Gamma_{\underline{0}}\epsilon)^\dagger$, $K_{1,2}$ are null and obey $K_i\epsilon_i = 0$ and where $K^M\partial_M$ is a Killing vector with respect to all bosonic supergravity fields and with respect to which $\epsilon_{1,2}$ are singlets.

The above conditions are not however sufficient to imply supersymmetry in general; for that one must solve the rather more cumbersome pairing constraints, namely

$$\left(\mathrm{e}_{1+}\Psi\mathrm{e}_{2+},\ \Gamma^{MN}\left[(-1)^{\deg(\Psi)}d_H\left(e^{-\Phi}\Psi\mathrm{e}_{+2}\right)+\frac{e^{\Phi}}{2}\star d(e^{-2\Phi}\star\mathrm{e}_{+2})\Psi-F\right]\right)=0, \tag{C.3a}$$

$$\left(\mathrm{e}_{1+}\Psi\mathrm{e}_{2+},\ \left[d_H\left(e^{-\Phi}\mathrm{e}_{+1}\Psi\right)-\frac{e^{\Phi}}{2}\star d(e^{-2\Phi}\star\mathrm{e}_{+1})\Psi-F\right]\Gamma^{MN}\right)=0, \tag{C.3b}$$

where $\mathrm{e}_{1,2+}$ are non-trivial null vectors/1-forms that must obey

$$\mathrm{e}_{1+}\cdot K_1=\mathrm{e}_{2+}\cdot K_2=\frac{1}{2}, \tag{C.4}$$

but one is otherwise free to choose the precise form of. The pairing itself as a geometric object is simply defined as $(X,Y)_d := X\wedge\lambda(Y)|_d$, however it also has equivalent representations in terms of a spinor bilinear and trace, i.e. in dimension d we have

$$\begin{aligned}\frac{1}{2^{[\frac{d}{2}]}}\mathrm{Tr}(\star XY)&=(-1)^{\deg(X)}(X,Y),\\ (X\Psi Y,C)&=-\frac{1}{32}(-1)^{\deg(\Psi)}\bar{\epsilon}_1XCY\epsilon_2\mathrm{vol}_d,\\ \bar{\epsilon}_1XCY\epsilon_2&=-(-1)^{\deg(\Psi)}\mathrm{Tr}(Y\lambda(\Psi)XC),\end{aligned} \tag{C.5}$$

which we make extensive use of in this and the following appendix. The conditions (C.1) and (C.3a)–(C.3b) are necessary and sufficient for supersymmetry. Let us now simplify these conditions under the assumption that $K^M\partial_M$ is a time-like Killing vector.

The first thing we need to do is to make a choice for $\mathrm{e}_{1,2+}$. Given that

$$K_1\cdot K_2=2K^2\neq 0, \tag{C.6}$$

in the timelike case we can simply take

$$\mathrm{e}_{1+}=\frac{1}{4K^2}K_2,\quad \mathrm{e}_{2+}=\frac{1}{4K^2}K_1. \tag{C.7}$$

It is then simple to show that three of the objects appearing in the pairing constraints can be written as

$$\mathrm{e}_{1+}\Psi\mathrm{e}_{2+}\propto K\Psi K,\quad \Psi\mathrm{e}_{2+}=\frac{1}{2K^2}\Psi K,\quad \mathrm{e}_{1+}\Psi=\frac{1}{2K^2}K\Psi \tag{C.8}$$

Moving forward, a useful condition was already derived in [45] namely for time-like,

$$d(e^{-2\Phi}\star K_{1,2})=0, \tag{C.9}$$

importantly these conditions are implied by (C.1). From this it follows that

$$\frac{e^{\Phi}}{2}\star d(e^{-2\Phi}\star\mathrm{e}_{2+})=-\frac{e^{\Phi}}{2}\star d(e^{-2\Phi}\star\mathrm{e}_{1+})=\frac{e^{-\Phi}}{8}\mathcal{L}_{\tilde{K}}K^{-2}. \tag{C.10}$$

Next given that one necessarily has $\iota_K\Psi = -\tilde{K}\wedge\Psi$ and $\mathcal{L}_K\Psi = 0$ it is possible to show that

$$(-1)^{\deg(\Psi)}d_H\left(\frac{e^{-\Phi}}{2K^2}\Psi K\right) = \frac{1}{2K^2}(-1)^{\deg(\Psi)}d_H\left(e^{-\Phi}\Psi\right)K + e^{-\Phi}d\left(\frac{\tilde{K}+K}{2K^2}\right)\wedge\Psi,$$
$$d_H\left(\frac{e^{-\Phi}}{2K^2}K\Psi\right) = -\frac{1}{2K^2}Kd_H\left(e^{-\Phi}\Psi\right) - e^{-\Phi}d\left(\frac{\tilde{K}-K}{2K^2}\right)\wedge\Psi, \tag{C.11}$$

allowing one to commute K past d_H. Inside the pairings these terms simplify further due to

$$K_1K_2+K_2K_1\propto\mathbb{I},\quad d_H(e^{-\Phi}\Psi)=-(\iota_K+\tilde{K}\wedge)F=2(K_1F-(-1)^{\deg\Psi}FK_2),\quad F\epsilon_2=\overline{\epsilon}_1F=0. \tag{C.12}$$

This ensures that inside the pairings the $d_H(e^{-\Phi}\Psi)K$ terms vanish as

$$(K_1F-(-1)^{\deg\Psi}FK_2)KK_1\epsilon_2 \propto (K_1F-(-1)^{\deg\Psi}FK_2)\epsilon_2 = 0 \tag{C.13}$$

and the $Kd_H(e^{-\Phi}\Psi)$ term as

$$\overline{\epsilon}_1K_2K(K_1F-(-1)^{\deg\Psi}FK_2) = 0. \tag{C.14}$$

The pairing conditions for the time-like case can thus be expressed as

$$\left(K\Psi K,\ \Gamma^{MN}\left[d\left(\frac{\tilde{K}+K}{2K^2}\right)\wedge\Psi + \frac{1}{8}\mathcal{L}_{\tilde{K}}\left(\frac{1}{K^2}\right)\Psi - e^{\Phi}F\right]\right) = 0, \tag{C.15a}$$
$$\left(K\Psi K,\ \left[d\left(\frac{\tilde{K}-K}{2K^2}\right)\wedge\Psi - \frac{1}{8}\mathcal{L}_{\tilde{K}}\left(\frac{1}{K^2}\right)\Psi + e^{\Phi}F\right]\Gamma^{MN}\right) = 0, \tag{C.15b}$$

in full generality.

D Derivation of conditions for $\mathcal{N}=1$ AdS_2 in Type II supergravity

We take the most general Ansatz for a ten-dimensional solution with a warped AdS_2 factor, namely

$$ds^2 = e^{2A}ds^2(AdS_2) + ds^2(M_8), \tag{D.1}$$
$$F = f_\pm + e^{2A}\text{vol}(AdS_2)\wedge\star_8\lambda(f_\pm),\quad H = e^{2A}\text{vol}(AdS_2)\wedge H_1 + H_3 \tag{D.2}$$

with the dilaton Φ, warp factor A and the forms $f_\pm$, H_1 and H_3 depending on M_8 directions only; here and elsewhere the upper signs are in Type IIA and the lower in Type IIB supergravity. The $d=10$ Majorana-Weyl Killing spinors are

$$\epsilon_1 = \zeta_+\otimes\chi_{1+} + \zeta_-\otimes\chi_{1-},\quad \epsilon_2 = \zeta_+\otimes\chi_{2\mp} + \zeta_-\otimes\chi_{2\pm}, \tag{D.3}$$

where every spinor here is Majorana, (ζ_+,ζ_-) are chiral Killing spinors on AdS_2 obeying (B.1) and (χ_{i+},χ_{i-}) are chiral spinors on M_8. None of these latter spinors can vanish when $m\neq 0$; to see this one can consider the necessary spinorial supersymmetry conditions

$$(\nabla - \frac{1}{4}H - d\Phi)\epsilon_1 = \left(\nabla + \frac{1}{4}H - d\Phi\right)\epsilon_2 = 0, \tag{D.4}$$

given the AdS_2 Killing spinor equations maps between ζ_+ and ζ_- under ∇, we conclude that the only way to fix one of (χ_{i+}, χ_{i-}) to zero is to also fix the inverse AdS_2 radius $m = 0$, resulting in Mink_2. We decompose the $d = 10$ gamma matrices as

$$\Gamma_\mu = e^A \gamma^{(2)}_\mu \otimes \hat{\gamma}, \quad \Gamma_a = \mathbb{I} \otimes \gamma_a, \quad B^{(10)} = \mathbb{I} \otimes B. \tag{D.5}$$

From this we can immediately compute K, $\tilde{K}$

$$K = \frac{1}{64}\Big(e^A(|\chi_1|^2 + |\chi_2|^2)v + e^A(\chi_1^\dagger \hat{\gamma}\chi_1 \mp \chi_2^\dagger \hat{\gamma}\chi_2)u\Big) - \frac{1}{32} f k, \tag{D.6}$$

$$\tilde{K} = \frac{1}{64}\Big(e^A(|\chi_1|^2 - |\chi_2|^2)v + e^A(\chi_1^\dagger \hat{\gamma}\chi_1 \pm \chi_2^\dagger \hat{\gamma}\chi_2)u\Big) - \frac{1}{32} f \tilde{k} \tag{D.7}$$

where (f, u, v) are objects on AdS_2 defined in appendix B, we introduce $\chi_{1,2} := \chi_{1,2+} + \chi_{1,2-}$, and

$$k_a := \frac{1}{2}(\chi_1^\dagger \gamma_a \chi_1 \mp \chi_2^\dagger \gamma_a \chi_2), \quad \tilde{k}_a := \frac{1}{2}(\chi_1^\dagger \gamma_a \chi_1 \pm \chi_2^\dagger \gamma_a \chi_2). \tag{D.8}$$

The next step is to use these bilinears to reduce (C.1) to conditions on M_8 only and, in the process, we will establish that (2.7) is necessary for solutions for which AdS_2 does not only appear as a factor within a higher-dimensional AdS space.

First off $\nabla_{(M}K_{N)} = 0$ imposes

$$\begin{aligned}
\nabla_{(a}k_{b)} &= 0,\\
\mathcal{L}_k A + \frac{m}{2} e^{-A}(\chi_1^\dagger \hat{\gamma}\chi_1 \mp \chi_2^\dagger \hat{\gamma}\chi_2) &= 0,\\
d(e^{-A}(|\chi_1|^2 + |\chi_2|^2)) &= 0,\\
d(e^{-A}(\chi_1^\dagger \hat{\gamma}\chi_1 \mp \chi_2^\dagger \hat{\gamma}\chi_2)) + 2m e^{-2A} k &= 0.
\end{aligned} \tag{D.9}$$

These together imply that if k is non-trivial for AdS_2 (i.e. $m \neq 0$), then it's an isometry of the internal metric but not of the warp factor. Indeed one can introduce a local coordinate ρ such that

$$-e^{-A}(\chi_1^\dagger \hat{\gamma}\chi_1 \mp \chi_2^\dagger \hat{\gamma}\chi_2) = h(\rho), \quad k = \frac{1}{2m} h' e^{2A} d\rho. \tag{D.10}$$

Clearly ρ spans this isometry, and h parametrises diffeomorphism invariance in this direction. We can thus parametrise

$$k^a \partial_{x^a} = \partial_\rho, \quad \Rightarrow \quad |k|^2 = \frac{1}{2m} h' e^{2A}, \tag{D.11}$$

allowing us to define the vielbein direction

$$\mathrm{e}^k := \frac{k}{|k|} = \sqrt{\frac{h'}{2m}} e^A d\rho, \quad ds^2(\mathrm{M}_8) = (\mathrm{e}^k)^2 + ds^2(\mathrm{M}_7). \tag{D.12}$$

We demand that locally the warp factor is independent of ρ since M_8 respects the isometry, which leads to

$$e^A = \sqrt{\frac{2m}{h'}} e^{A_7}, \quad \partial_\rho A_7 = 0. \tag{D.13}$$

JHEP06(2024)056

Substituting this into the Lie derivative conditions then yields, without loss of generality,

$$h=\frac{2}{m}\tanh(\rho) \tag{D.14}$$

and so the metric becomes

$$ds^2=e^{2A_7}\Big[m^2\cosh^2\rho ds^2(\text{AdS}_2)+d\rho^2\Big]+ds^2(\text{M}_7), \tag{D.15}$$

which is AdS_3 rather than AdS_2.

Second, from $d\tilde{K}=\iota_K H$ we get

$$\begin{aligned}
&d\tilde{k}=\iota_k H_3, && \text{(D.16)}\\
&\iota_k(e^{2A}H_1)=me^A(|\chi_1|^2-|\chi_2|^2), && \text{(D.17)}\\
&e^{-A}(\chi_1^\dagger\hat{\gamma}\chi^1\mp\chi_2^\dagger\hat{\gamma}\chi_2)(e^{2A}H_1)+d(e^A(|\chi_1|^2-|\chi_2|^2))=0, && \text{(D.18)}\\
&e^{-A}(|\chi_1|^2+|\chi_2|^2)(e^{2A}H_1)+d(e^A(\chi_1^\dagger\hat{\gamma}\chi^1\pm\chi_2^\dagger\hat{\gamma}\chi_2))=2m\tilde{k}. && \text{(D.19)}
\end{aligned}$$

Proceeding with the local coordinate ρ again, the fact that away from source $d(e^{2A}H_1)=0$ and given (D.17)–(D.18) we have

$$e^A(|\chi_1|^2-|\chi_2|^2)=g(\rho),\quad m\partial_\rho\log g=h\quad\Rightarrow\quad e^{2A}H_1=bm\cosh^2\rho d\rho,\quad db=0, \tag{D.20}$$

then (D.19) implies

$$d\tilde{k}=0,\quad\Rightarrow\quad\iota_k H_3=0,\quad\Rightarrow\quad\mathcal{L}_k H_3=\iota_k dH_3 \tag{D.21}$$

so when $dH_3=0$, i.e. away from sources, the NSNS flux also respects the isometries of AdS_3 if $k\neq 0$ (at least locally away from sources for $d(e^{2A}H_1)$).

Third, one can show that Ψ decomposes as

$$\Psi=\pm\frac{1}{2}f\Big(\psi_\pm-e^{2A}\text{vol}(\text{AdS}_2)\wedge\hat{\psi}_\pm\Big)\mp\frac{e^A}{2}\Big(v\wedge\psi_\mp+u\wedge\hat{\psi}_\mp\Big), \tag{D.22}$$

where we define the $d=8$ bispinors

$$\psi:=\chi_1\otimes\chi_2^\dagger,\quad\hat{\psi}:=\hat{\gamma}\chi_1\otimes\chi_2^\dagger, \tag{D.23}$$

and the $\pm$ subscripts refer to the even/odd form degree parts of these. On the other hand we find

$$\begin{aligned}
(\iota_K+\tilde{K}\wedge)f_\pm=\frac{1}{16}\Big[&(\zeta_+\otimes\overline{\zeta}_-)_2\wedge(\iota_k+\tilde{k}\wedge)\star_8(e^{2A}\lambda f_\pm)-(\zeta_+\otimes\overline{\zeta}_-)_0(\iota_k+\tilde{k}\wedge)f_\pm\\
&+\frac{1}{4}v\wedge\Big(e^{-A}(\chi_1^\dagger\hat{\gamma}\chi^1\mp\chi_2^\dagger\hat{\gamma}\chi_2)\star_8(e^{2A}\lambda f_\pm)+e^A(|\chi_1|^2-|\chi_2|^2)f_\pm\Big)\\
&+\frac{1}{4}u\wedge\Big(e^{-A}(|\chi_1|^2+|\chi_2|^2)\star_8\lambda(e^{2A}f_\pm)+e^A(\chi_1^\dagger\hat{\gamma}\chi_1\pm\chi_2^\dagger\hat{\gamma}\chi_2)f_\pm\Big)\Big].
\end{aligned} \tag{D.24}$$

So we find

$$d_{H_3}(e^{-\Phi}\psi_\pm)=\pm\frac{1}{16}(\iota_k+\tilde{k}\wedge)f_\pm, \tag{D.25}$$

$$d_{H_3}(e^{A-\Phi}\psi_\mp)=\mp\frac{1}{32}\Big(e^{-A}(\chi_1^\dagger\hat{\gamma}\chi^1\mp\chi_2^\dagger\hat{\gamma}\chi_2)\star_8\lambda(e^{2A}f_\pm)+e^{A-\Phi}(|\chi_1|^2-|\chi_2|^2)f_\pm\Big), \tag{D.26}$$

$$d_{H_3}(e^{A-\Phi}\hat{\psi}_\mp)-me^{-\Phi}\psi_\pm=\mp\frac{1}{32}\Big(e^{-A}(|\chi_1|^2+|\chi_2|^2)\star_8\lambda(e^{2A}f_\pm)+e^{A}(\chi_1^\dagger\hat{\gamma}\chi_1\pm\chi_2^\dagger\hat{\gamma}\chi_2)f_\pm\Big), \tag{D.27}$$

$$d_{H_3}(e^{2A-\Phi}\hat{\psi}_\pm)+e^{2A-\Phi}H_1\wedge\psi_\pm-2me^{A-\Phi}\psi_\mp=\mp\frac{1}{16}(\iota_k+\tilde{k}\wedge)\star_8\lambda(e^{2A}f_\pm). \tag{D.28}$$

Returning to the local coordinate ρ, and given that away from sources the RR flux should obey

$$d_{H_3}f_\pm=d_{H_3}(e^{2A}\star_8\lambda f_\pm)-e^{2A}H_1\wedge f_\pm=0, \tag{D.29}$$

taking d_{H_3}(D.26) yields

$$d_{H_3}(-h(e^{2A}\star_8\lambda f_\pm)+gf_\pm)=-h'd\rho\wedge(e^{2A}\star_8\lambda f_\pm)=0, \tag{D.30}$$

i.e. $f_\pm$ is orthogonal to k, so what has been derived thus far implies

$$\mathcal{L}_kf_\pm=0, \tag{D.31}$$

so the RR fluxes also respect the isometries of AdS_3 away from possible sources along ρ. We conclude that in any regular region of a solution

$$k\neq 0\quad\Rightarrow\quad\text{warped AdS}_3. \tag{D.32}$$

Note that a similar result was found for AdS_3 solutions preserving $\mathcal{N}=(1,1)$ supersymmetry in [43]. This argument of course breaks down if $m=0$, the Mink_2 limit: for this we generically have $\mathcal{L}_kA=0$ and one can show that, when non-zero, $k^a\partial_{x^a}$ is actually a Killing vector with respect to the entire solution under which the spinors are uncharged. There is obviously the potential for Mink_2 to get enhanced to Mink_3 in this case, but this is no longer guaranteed as k can appear fibered over the rest of the internal space. One is still free to fix $k=0$ when $m=0$, the point is that it is no longer general to do so — for $m\neq 0$ there is no such problem. Thus if we are interested in AdS_2 solutions, we should fix

$$\begin{aligned}k&=0,\qquad\chi_1^\dagger\hat{\gamma}\chi_1=\pm\chi_2^\dagger\hat{\gamma}\chi_2,\quad|\chi_1|^2=|\chi_2|^2=ce^A,\\ d\tilde{k}&=0,\quad 2ce^{2A}H_1=2m\tilde{k}-d(e^A(\chi_1^\dagger\hat{\gamma}\chi_1\pm\chi_2^\dagger\hat{\gamma}\chi_2)),\end{aligned} \tag{D.33}$$

for $c>0$ some constant, to solve all these conditions.

This reduces the $d=10$ 1-forms K to

$$K=\frac{1}{32}e^{2A}cv,\quad\Rightarrow\quad(K)^\mu=\frac{c}{32}v^\mu,\quad K^2=-\Big(\frac{c}{32}\Big)^2e^{2A}f^2, \tag{D.34}$$

so the ten-dimensional Killing vector is time-like for all AdS_2 solutions that are not merely the embedding of AdS_2 into AdS_3. The bispinor conditions then truncate to

$$d_{H_3}(e^{-\Phi}\psi_\pm) = \pm\frac{1}{16}\tilde{k}\wedge f_\pm, \tag{D.35}$$

$$d_{H_3}(e^{A-\Phi}\psi_\mp) = 0, \tag{D.36}$$

$$d_{H_3}(e^{A-\Phi}\hat{\psi}_\mp) - me^{-\Phi}\psi_\pm = \mp\frac{c}{16}e^{2A}\star_8\lambda f_\pm \mp\frac{1}{32}e^A(\chi_1^\dagger\hat{\gamma}\chi_1 \pm \chi_2^\dagger\hat{\gamma}\chi_2)f_\pm, \tag{D.37}$$

$$d_{H_3}(e^{2A-\Phi}\hat{\psi}_\pm) + e^{2A-\Phi}H_1\wedge\psi_\pm - 2me^{A-\Phi}\psi_\mp = \mp\frac{1}{16}\tilde{k}\wedge\star_8 e^{2A}\lambda f_\pm. \tag{D.38}$$

Note that the Bianchi identities of H_3 and $f_\pm$ imply that of $e^{2A}\star_8\lambda(f_\pm)$ by (D.35) and d_{H_3}(D.37). Further a supersymmetric solution must obey $(\tilde{K}\wedge + \iota_K)\Psi = (K\wedge - \iota_{\tilde{K}})\Psi = 0$, which in terms of eight-dimensional conditions becomes

$$\begin{aligned} &e^A c\psi_\mp = \tilde{k}\wedge\psi_\pm, && e^A c\mathcal{G}\psi_\mp = \tilde{k}\wedge\hat{\psi}_\pm, && e^A c\mathcal{G}\psi_\pm = \tilde{k}\wedge\hat{\psi}_\mp + e^A c\hat{\psi}_\pm,\\ &e^A c\hat{\psi}_\mp = -\iota_{\tilde{k}}\hat{\psi}_\pm, && e^A c\mathcal{G}\hat{\psi}_\mp = -\iota_{\tilde{k}}\psi_\pm, && e^A c\mathcal{G}\hat{\psi}_\pm = -\iota_{\tilde{k}}\psi_\mp + e^A c\psi_\pm \end{aligned} \tag{D.39}$$

where we employ the short-hand $e^A c\mathcal{G} = \chi_1^\dagger\hat{\gamma}\chi_1 = \pm\chi_2^\dagger\hat{\gamma}\chi_2$. The first line of which, along with (D.33), can be used to prove the redundancy of (D.36) and (D.38).

What remains is to reduce the pairing constraints to $d = 8$ conditions. Going forward, and in the main text we define

$$\mathcal{G} = \cos\beta, \quad \tilde{k} = ce^A\sin\beta V, \tag{D.40}$$

for V a unit norm 1-form, as we are free to do without loss of generality. Note that it is not possible to fix $\sin\beta = 0$ as that would set some of the chiral $d = 8$ spinors to zero and result in $Mink_2$ as explained around (D.4). Taking (C.15a)–(C.15b) as our starting point, for the case at hand we have

$$K = \frac{c}{32}e^{2A}f^2 dt, \quad K^2 = -\left(\frac{c}{32}\right)^2 e^{2A}f^2, \quad \tilde{K} = \frac{cfe^A}{32}\Big(-\cos\beta \mathrm{e}^r + \sin\beta V\Big), \tag{D.41}$$

where $\mathrm{e}^t = e^{mr}dt$, $\mathrm{e}^r = dr$, from which it follows that

$$\begin{aligned} d\left(\frac{\tilde{K}\pm K}{2K^2}\right)\wedge\Psi &= \frac{1}{2}\left[\frac{1}{K^2}(\iota_K H)\wedge\Psi - d\left(\frac{1}{K^2}\right)\wedge\iota_K\Psi\right]\\ &= \frac{16}{e^A fc}(\mathrm{e}^r\wedge H_1\wedge\Psi - 2(dA + me^{-A}\mathrm{e}^r)\wedge\iota_{\mathrm{e}^t}\Psi) \end{aligned} \tag{D.42}$$

where we have used that $\tilde{K}\wedge\Psi = -\iota_K\Psi$. We also have that

$$K\Psi K \propto \Gamma^0\Psi\Gamma^0, \quad \frac{1}{8}\mathcal{L}_{\tilde{K}}\left(\frac{1}{K^2}\right) = -\frac{8}{ce^A f}(\cos\beta me^{-A} + \sin\beta\mathcal{L}_V dA). \tag{D.43}$$

We are now ready to simplify (D.36)–(D.38) component by component, we find it useful to split the $d = 10$ index as $M = (t, r, a)$, i.e. the two directions along AdS_2 and along M_8. In what follows we make frequent use of (C.5).

The tr components are both equal and give rise to

$$(\psi_\pm, f_\pm)_8 = \pm\frac{c}{4}e^{-\Phi}\Big(m - \frac{1}{2}e^A \sin\beta\iota_V H_1\Big)\text{vol}(\text{M}_8). \tag{D.44}$$

The ta components yield

$$(\psi_\mp, \star\lambda f_\pm)_7 = 0, \quad (\hat{\psi}_\mp, f_\pm)_7 = \pm\frac{1}{8}e^{A-\Phi}c\star_8(2dA + \cos\beta H_1). \tag{D.45}$$

The ra components yield

$$(\psi_\mp, f_\pm)_7 = 0, \quad (\hat{\psi}_\mp, \star\lambda f_\pm)_7 = \pm\frac{1}{8}e^{A-\Phi}c\star_8(2\cos\beta dA + H_1 - 2e^{-A}m\sin\beta V), \tag{D.46}$$

by wedging these conditions with V one can form a linear combination that implies $\sin\beta$(D.44) $= 0$, but one cannot fix $\sin\beta = 0$ hence the 8-form pairing is implied by the $d = 7$ ones. The ab components are more complicated, but through a lengthy computation, extracting all the conditions that the 7-form pairings impose on the flux, it is possible to show that they too are implied. By manipulating $(\hat{\Psi}_\pm,$ (D.35)$)_7$ or $(\Psi_\mp,$ (D.37)$)_7$ one extracts

$$(\psi_\mp, \star\lambda f_\pm + \cos\beta f_\pm)_7 = 0, \tag{D.47}$$

from which it follows that $(\psi_\mp, \star\lambda f_\pm)_7 = 0$ is implied by $(\psi_\mp, f_\pm)_7 = 0$.

We have now extracted necessary and sufficient $d = 8$ geometric conditions for minimally supersymmetric AdS_2 solutions of Type II supergravity, our results are summarised in section 2.

E Comment on [34] and $\mathcal{N} = 1$ AdS_2 in $d = 11$ supergravity

In [34] a class of supersymmetic AdS_2 solutions in $d = 11$ supergravity was derived under the assumption that the internal 9-manifold supports an SU(4)-structure. In this appendix we show that it is actually general, as solutions supporting any other structure are merely the embeddings of AdS_2 into supersymmetic AdS_3 solutions.

In [52], geometric conditions that are necessary for supersymmetry in $d = 11$ supergravity were derived; they are defined in terms of the following bilinears

$$K^{(11)} := \epsilon^\dagger\Gamma_0\Gamma_{\underline{M}}\epsilon\,\text{e}^{\underline{M}}, \quad \Omega^{(11)} := \frac{1}{2}\epsilon^\dagger\Gamma_0\Gamma_{\underline{MN}}\epsilon\,\text{e}^{\underline{MN}}, \quad \Sigma^{(11)} := \frac{1}{5!}\epsilon^\dagger\Gamma_0\Gamma_{\underline{M}_1\cdots\underline{M}_5}\epsilon\,\text{e}^{\underline{M}_1\cdots\underline{M}_5}, \tag{E.1}$$

where ϵ is the Majorana Killing spinor of $d = 11$ supergravity, $\Gamma_{\underline{M}}$ are flat spacetime gamma matrices in $d = 11$ and $\text{e}^{\underline{M}}$ a corresponding vielbein. The conditions these objects must satisfy are

$$\begin{aligned} d\Omega^{(11)} &= \iota_{K^{(11)}}G, \quad \nabla_{(M}K^{(11)}_{N)} = 0,\\ d\Sigma^{(11)} &= \iota_{K^{(11)}}\star_{11}G - \Omega^{(11)}\wedge G,\\ \star_{11}dK^{(11)} &= \frac{2}{3}\Omega^{(11)}\wedge\star_{11}G - \frac{1}{3}\Sigma^{(11)}\wedge G. \end{aligned} \tag{E.2}$$

Clearly this makes $(K^{(11)})^M\partial_M$ a Killing vector, and it turns out that when this is taken to be timelike the above system is also sufficient for supersymmetry.

We would now like to use (E.2) to establish that, for AdS_2 solutions, it is necessary for the internal space to support an SU(4)-structure. To this end we decompose the metric and fluxes as

$$ds^2 = e^{2\Delta}ds^2(\text{AdS}_2) + ds^2(\hat{\text{M}}_9), \quad G = e^{2\Delta}\text{vol}(\text{AdS}_2)\wedge G_2 + G_4, \tag{E.3}$$

and take our $d = 11$ gamma matrices to decompose as

$$\Gamma_\mu = e^{\Delta}\gamma^{(2)}_\mu \otimes \mathbb{I}, \quad \Gamma_a = \sigma_3\otimes\gamma^{(9)}_a, \quad a = 1,\dots,9. \tag{E.4}$$

We will define the $d = 9$ gamma matrices in terms of the $d = 8$ ones in (D.5) as $\gamma^{(9)}_a = \gamma_a$ for $a = 1,\dots,8$ and $\gamma^{(9)}_9 = \gamma_{1\dots8}$, in this way the intertwiner defining $d = 11$ Majorana conjugation is the same as it was in $d = 10$ and we can take the $d = 11$ spinor to be

$$\epsilon = \epsilon_1 + \epsilon_2, \tag{E.5}$$

where $\epsilon_{1,2}$ are defined as in (D.3) (for the upper signs specifically). We now compute the 1-form in (E.1) and find

$$\begin{aligned}K^{(11)} &= -\Big(e^{\Delta}(|\chi_1|^2+|\chi_2|^2)v + e^{\Delta}(\chi_1^\dagger\gamma^{(9)}_9\chi_1 - \chi_2^\dagger\gamma^{(9)}_9\chi_2)u - f\xi\Big),\\ \xi &= -(\chi^\dagger_{1+}\gamma^{(9)}_a\chi_{2-} + \chi^\dagger_{1+}\gamma^{(9)}_a\chi_{2-} + 2(\chi^\dagger_{1+}\gamma^{(9)}_a\chi_{2+} + \chi^\dagger_{2-}\gamma^{(9)}_a\chi_{1-}))\text{e}^a\end{aligned} \tag{E.6}$$

where (f, u, v) are the scalar and two 1-forms on AdS_2 defined in appendix B. We note that this has precisely the same structure as (D.6), so imposing $\nabla_{(M}K^{(11)}_{N)} = 0$ leads to a repeat of the argument between (D.8) and (D.15); jumping to the punch line: generic AdS_2 solutions experience an enhancement to AdS_3, for true AdS_2 solutions we must impose

$$d(e^{-\Delta}(|\chi_1|^2+|\chi_2|^2)) = 0, \quad \chi_1^\dagger\gamma^{(9)}_9\chi_1 = \chi_2^\dagger\gamma^{(9)}_9\chi_2, \quad \xi = 0, \tag{E.7}$$

which in turn makes $(K^{(11)})^M\partial_M$ necessarily timelike, just as the analogue was for Type II supergravity. Solving these conditions amounts to imposing that

$$\chi_{1+} = \chi_{2+} = 0, \quad |\chi_1|^2 = |\chi_2|^2 = \frac{1}{2}e^{\Delta} \tag{E.8}$$

without loss of generality. Then upon defining a real vector $V^{(4)}$ and SU(4)-structure forms $(J^{(4)}, \Omega^{(4)})$ as

$$e^{\Delta}V^{(4)} = \chi^\dagger\gamma^{(9)}_{\underline{a}}\chi\,\text{e}^{\underline{a}}, \quad e^{\Delta}J^{(4)} = -\frac{i}{2}\chi^\dagger\gamma^{(9)}_{\underline{ab}}\chi\,\text{e}^{\underline{ab}}, \quad e^{\Delta}\Omega^{(4)} = \frac{1}{4!}\chi^{c\dagger}\gamma^{(9)}_{\underline{abcd}}\chi\,\text{e}^{\underline{abcd}}, \tag{E.9}$$

for

$$\chi := i(\chi_1 + i\chi_2). \tag{E.10}$$

We find that the forms in (E.1) become

$$\begin{aligned}&K^{(11)} = -e^{2\Delta}v, \quad \Omega^{(11)} = e^{\Delta}(-u\wedge V^{(4)} - fJ^{(4)}),\\ &\Sigma^{(11)} = e^{\Delta}\Big(-e^{2\Delta}f\text{vol}(\text{AdS}_2)\wedge J^{(4)}\wedge V^{(4)} + \frac{1}{2}e^{\Delta}v\wedge J^{(4)}\wedge J^{(4)} - e^{\Delta}u\wedge\text{Re}\Omega^{(4)} + fV^{(4)}\wedge\text{Im}\Omega^{(4)}\Big),\end{aligned} \tag{E.11}$$

which are clearly spanned by forms on AdS_2 and those defining an SU(4)-structure in $d = 9$. These precisely reproduce the equivalent forms of [34], so we have proved that the SU(4)-structure "assumption" of that paper, is actually no assumption at all but rather a necessary condition for true supersymmetic AdS_2 solutions in $d = 11$ supergravity. Fixing $m = 1$ as [34] does, for completeness, we quote the necessary and sufficient geometric conditions for supersymmetry in $d = 11$

$$d(e^{\Delta} J^{(4)}) = 0, \tag{E.12a}$$

$$d(e^{2\Delta} V^{(4)}) + e^{\Delta} J^{(4)} + e^{2\Delta} G_2 = 0, \tag{E.12b}$$

$$d(e^{\Delta} V^{(4)} \wedge \text{Im}\Omega^{(4)}) - e^{\Delta} J^{(4)} \wedge G_4 = 0, \tag{E.12c}$$

$$d(e^{2\Delta} \text{Re}\Omega^{(4)}) - e^{\Delta} V \wedge \text{Im}\Omega^{(4)} + e^{2\Delta}(\star_9 G_4 - V^{(4)} \wedge G_4) = 0, \tag{E.12d}$$

$$\star_9(2V^{(4)} \wedge \star_9 G_2 + \text{Re}\Omega^{(4)} \wedge G_4) + 6 d\Delta = 0, \tag{E.12e}$$

$$J^{(4)} \wedge J^{(4)} \wedge G_4 = 0, \tag{E.12f}$$

$$e^{\Delta}(2J^{(4)} \wedge \star_9 G_2 - V \wedge \text{Im}\Omega^{(4)} \wedge G_4) = 6\text{Vol}(\text{M}_9). \tag{E.12g}$$

In [34] the condition

$$V^{(4)} \wedge (\text{Im}\Omega^{(4)} \wedge G_2 + J^{(4)} \wedge G_4) = 0, \tag{E.13}$$

is also quoted, but this is implied by (E.12b) and (E.12c).

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