SCUOLA DI DOTTORATO
UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

Dipartimento di / Department of

Economia, Metodi Quantitativi e Strategie d'Impresa

Dottorato di Ricerca in / PhD program Economics and Statistics Ciclo / Cycle XXXIV

Curriculum in (se presente / if it is) Economics

Essays on empirical DSGE models

Cognome / Surname Massari Nome / Name Giorgio

Matricola / Registration number 763327

Tutore / Tutor: Prof. Patrizio Tirelli

Supervisor: Prof. Patrizio Tirelli
(se presente / if there is one)

Coordinatore / Coordinator: Prof. Matteo Manera

ANNO ACCADEMICO / ACADEMIC YEAR 2021-2022

Essays on empirical DSGE models

University of Milano-Bicocca

Giorgio Massari

Supervisor: Patrizio Tirelli

January 9, 2023

Abstract

This thesis is composed of two independent essays on empirical DSGE models. While they aim to answer different research questions, the two chapters share a common thread in the methodology, *i.e.* the building of an empirical model and its subsequent estimation via Bayesian techniques.

The first chapter, **Whither Liquidity Shocks?**, studies the empirical relevance of (flight-to-)liquidity shocks in the context of DSGE models characterized by a non-trivial financial sector, focusing on U.S. data. We also investigate how our findings affect the estimated dynamics and determinants of the natural interest rate, and, consequently, the interpretation of the monetary policy stance. First, we shed light on the strongly counterfactual implications that popular models of liquidity shocks have for asset returns and the composition of firms' liabilities, including the return spread between bank deposits and T-bills and the share of bank loans on corporate debt. Further, the implied estimate of the natural interest rate entails that the interest rate gap rose during recessions and fell thereafter. Then, by including the relevant financial variables as observables in our empirical model, we are able to show that liquidity shocks played a negligible role and became virtually irrelevant after 2010. We also find that the slowdown in productivity growth, not liquidity shocks, caused the post-2010 fall in the natural rate.

The second chapter, **Who Killed Business Dynamism in the U.S.?**, deals with one of the salient trends that characterized the U.S. economy in the latest decades. In particular, we offer a new interpretation of the long-term dynamics in the firm entry rate. According to our findings, its decline was the consequence of a persistent combination of adverse(favorable) productivity shocks to potential entrants(incumbents), while the long-term increase in price markups did not play a significant role. In spite of the "Schumpeterian" structure of our model, not all recessions had a "cleansing" effect, because the combination of shocks associated with the specific episodes had markedly different effects on the dispersion of firms' efficiency. Finally, our model allows us to estimate and distinguish between average firm efficiency and total factor productivity. Whilst the former's growth rate is relatively stable, we rationalize the procyclical pattern of TFP growth and its long-term decline with adjustments in the extensive margin, *i.e.* variations in the mass of firms.

Contents

Chapter 1

Whither Liquidity Shocks?

Joint with Patrizio Tirelli

Abstract We show that popular models of (flight-to-)liquidity shocks have strongly counterfactual implications for asset returns and the composition of firms' liabilities, including the return spread between bank deposits and T-bills and the share of bank loans on corporate debt. Further, the implied estimate of the natural interest rate entails that the interest rate gap rose during recessions and fell thereafter. By including the relevant financial variables as observables in our empirical model, we can show that liquidity shocks played a negligible role and became virtually irrelevant after 2010. We also find that the slowdown in productivity growth, not liquidity shocks, caused the post-2010 fall in the natural rate.

1.1 Introduction

This paper highlights the counterfactual implications of popular modelizations of liquidity shocks, interpreted as flight-to-quality episodes. We investigate the implications that these findings have for the interpretation of the long-term interest rates' decline and for our understanding of business cycles.

Interest rates have been gradually falling for a long time in many advanced economies, including the U.S. The Great Recession episode saw a further acceleration in this downward trend, leading to an unprecedented period of policy rates at (or even below) the zero lower bound (ZLB) for several quarters. This spurred renewed attention to the concept of the natural interest rate (defined by Wicksell, 1898 as the equilibrium interest rate compatible with price and economic stability) and to the causes and consequences of its decline, with obvious implications for the scope of monetary policy and for the analysis of business cycles. While some authors, most notably Summers (2014), pointed to long-term factors that explain this declining trend and that may have brought about an era of "secular stagnation", others emphasized the special role played by the financial crisis (e.g. Borio, 2014).

Regardless of the longer- or shorter-run perspective, safety and liquidity have been pivotal subjects of this debate. Among the explanations behind the natural rate decline stand an increasing propensity to save, in particular through safe assets. This view, related to the global saving glut hypothesis by Bernanke (2005), argues that a scarcity of safe assets and their rising attractiveness led to a secular decline in their yields with respect to less safe instruments. On the other hand, the financial crisis revealed liquidity and safety issues in markets that were previously regarded as (close to) risk-free, as showed by Kacperczyk and Schnabl (2013) for money market funds. The associated decoupling between policy rates and returns on assets characterized by different liquidity and safety attributes paved the way for a reconsideration of monetary policy transmission mechanisms (see for instance Benigno and Benigno, 2022).

A number of papers have found liquidity shocks to be an important driver of the U.S. business cycle. Christiano et al. (2015) identify a "consumption wedge" shock, specified as a preference for safe and liquid assets, as a fundamental driver behind the Great Recession. In a model that considers endogenous growth, Anzoategui et al. (2019) show that liquidity shocks have been crucial in driving the U.S. economy to a lower productivity trend.

One approach to modeling liquidity shocks (Kiyotaki and Moore, 2012; Jermann and Quadrini, 2012) emphasizes the limited resaleability of firms' equity when entrepreneurs are subject to a borrowing constraint. Fisher (2015) offers an alternative microfoundation within a standard New Keynesian framework, showing how the Smets and Wouters (2007) risk premium shock is formally equivalent to a (time-varying) preference for holding risk-free assets. Building on the work of Krishnamurthy and Vissing-Jorgensen (2012), such preference is justified with the liquidity and safety attributes that characterize government bonds, so that a positive realization of the risk premium shock assumes the interpretation of a flight to quality. By simply turning bond holdings into an argument of the household's utility function, the microfoundation of liquidity shocks proposed by Fisher (2015) can be easily incorporated into DSGE models characterized by complex financial markets. Del Negro et al. (2017) (DGGT henceforth) build a model with financial frictions and further develop the specification of flight-to-quality shocks, distinguishing between safety and liquidity; their results ascribe to these two components a fundamental role in determining U.S. business cycle fluctuations.

The literature on liquidity shocks has important implications for the identification of the natural rate, r^*, *i.e.* the flexible-price rate on a risk-free asset. Barsky et al. (2014) and Gerali and Neri (2019) use frictionless DSGE models to estimate r^*. Both find large fluctuations in the natural rate that are due to risk premium shocks, interpreted as exogenous reductions in the required return on savings. DGGT study the determinants of r^* using both time-series and DSGE models: their analysis establishes a link between the persistence of liquidity shocks and the long-term decline in the natural interest rate. Another strand of literature employs semi-structural models in the spirit of Laubach and Williams (2003) and emphasizes the role played by the productivity-growth slowdown in the long-term decline of r^* (see for instance Laubach and Williams, 2016).[1] Eggertsson et al. (2019), with a quantitative life-cycle model, similarly find that productivity contributed to dragging the natural rate down,

[1] Specifically, the model developed by Laubach and Williams (2003), and its updated estimates, attribute a little more than 50% of the r^* decline since 1998 to the slowdown in trend growth, while the remaining fraction is imputed to other unspecified drivers.

but also point out the importance of the demographic shift, in the form of reductions in fertility and mortality (analogous findings are shared by other works in the OLG framework, *e.g.* Gagnon et al., 2021, and Jones, 2022).[2]

A simple and straightforward consideration motivates our contribution. By their nature, flight-to-quality shocks affect complex financial markets where different intermediaries channel funds from households to entrepreneurs. To begin with, a liquidity shock leads households to a portfolio reallocation towards "safe" assets. This, in turn, implies that return spreads across assets must adjust. Thus, a *prima-facie* external validation for the relevance of liquidity shocks should be found in the observed patterns of these spreads. Consider for instance the characterization of liquidity shocks exploited in DGGT. According to the flight-to-quality interpretation, households shift their desired portfolio composition from bank deposits to Treasuries, and in equilibrium the deposit rate must rise relative to the return on T-bills. In fact, the liquidity shock requires a divergence between deposit and policy rates which is apparently unobservable over the sample period 1964:III-2019:IV. Panel (a) of Figure 1.1 plots the fed funds rate against the secondary market rate on U.S. 3-month certificates of deposits (CDs). The two series show a strong co-movement and both decline during recessions. According to Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), and Krishnamurthy and Li (2022), observed spreads between returns on bank deposits and T-Bills are sufficient to establish that these two assets are imperfect substitutes in terms of their liquidity attributes. Still, these spreads do not show a tendency to increase around recession periods and are notably smaller than the spread implied by liquidity shocks in DGGT (see panel (b) of Figure 1.1).

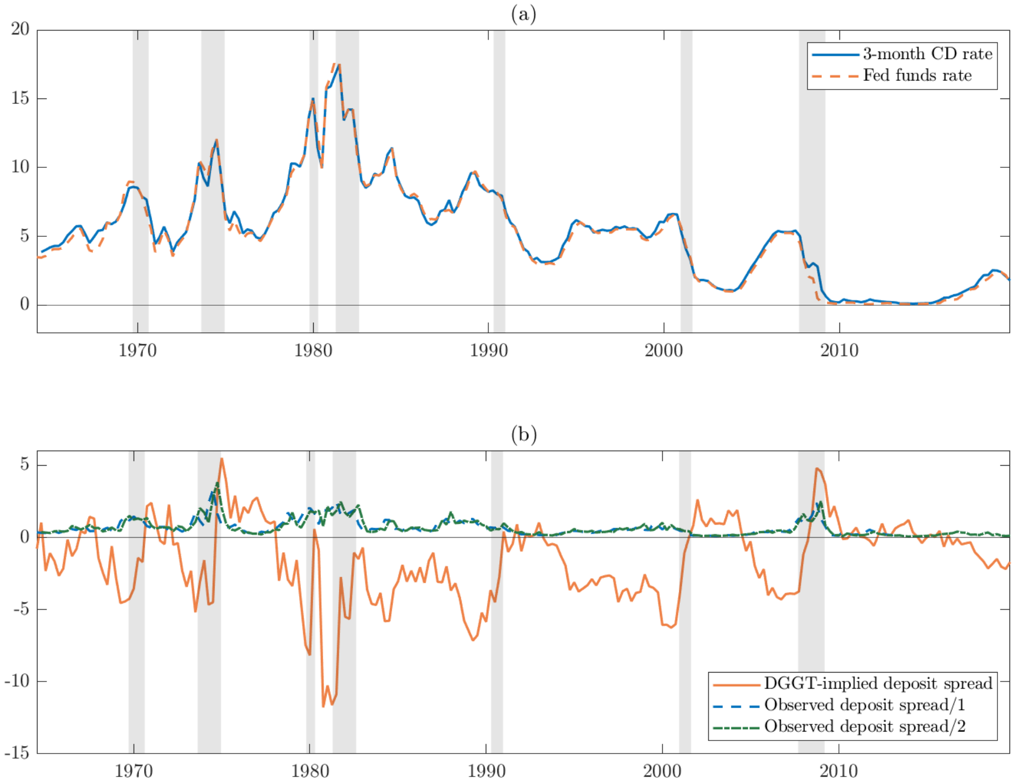
In our view, empirical models should test the ability of liquidity shocks to predict return spreads and portfolio adjustments that are consistent with observed patterns. As pointed out in Shi (2015), this concern is justified because asset prices and returns are central to the transmission mechanism of liquidity shocks. Further, flight-to-quality shocks imply that excesses of savings are a persistent feature of business cycle fluctuations and implicitly call for a policy response that should target such asset prices and returns.

We build a business cycle model that accounts for two potential transmission mechanisms that characterize liquidity shocks. The first one follows DGGT, where a flight to quality implies that households shift out of bank deposits. The second one is based on a richer financial market structure where firms, in addition to equity capital, obtain funds via bank loans and corporate bonds. Here we depart from a longstanding tradition in business cycle modeling, where financial frictions are typically associated with the existence of a single type of financial intermediary. In fact, there is evidence that firms with access to the corporate debt market also borrow from banks, and typically substitute between bank loans and non-bank financing over the business cycle (Rauh and Sufi, 2010; Adrian et al., 2013; Becker and Ivashina, 2014).

In the second version of our model, commercial banks collect funds through liquid deposits and lend to entrepreneurs; non-bank financial intermediaries (NBFIs henceforth) issue deposits that are subject to liquidity shocks, and invest in corporate bonds. In this framework,

[2]The impact demographic factors exert on r^* works (also) through shifts in saving and investment behaviors. Bean et al. (2015) distinguish these into shifts in propensity to save, propensity to invest, and demand for safe assets.

Figure 1.1: Fed funds rate, deposit rate, and deposit spreads



Note: Panel (a): data. Panel (b): the solid line is the posterior mode of the smoothed deposit rate/fed funds rate spread (obtained with our re-estimation of DGGT); "Observed deposit spread/1": 6-month CD rate - 6-month T-bill rate (Krishnamurthy and Vissing-Jorgensen, 2012); "Observed deposit spread/2": 3-month CD rate - 3-month T-bill rate (Nagel, 2016; the same measure is considered in a robustness test by Krishnamurthy and Li, 2022). 1964:III-2019:IV.

flight-to-quality shocks imply a household-portfolio reallocation out of non-bank deposits, toward bank deposits *and* T-Bills. One novel feature brought about by this assumption is that, following an adverse liquidity shock, the shift in households' portfolios toward bank deposits might favor a symmetrical change in firms' liabilities towards bank loans.[3] Our approach differs from Kiyotaki and Moore (2012), where flight-to-quality shocks hit firms' ability to raise funds and induce households to shift their portfolios towards assets that provide liquidity services. In fact, our characterization is consistent with U.S. capital markets, where bond financing to the nonfinancial corporate sector accounts for about three-fifths of

[3]The idea that households turn to bank deposits in times of market stress is consistent with the evidence in Lin (2020). First, looking at financial assets owned by U.S. households (and non-profit organizations), he finds that the shares of deposits and corporate equities tend to move in opposite directions. Second, households increase demand for deposits during stock market crashes, and investor sentiment negatively affects deposits growth above the effect of stock market returns. Lastly, variations in households' deposit holdings directly affect banks' loan supply. This last finding corroborates the transmission mechanism of liquidity shocks in our enriched model.

total funds (De Fiore and Uhlig, 2011).

The distinctive feature of our empirical analysis is the inclusion among the observables of those financial variables that the model identifies as central to the transmission of liquidity shocks. More in detail, we first estimate an augmented version of the DGGT model that accounts for the observed time series of returns on bank deposits. Conversely, the alternative specification of our model implies that liquidity shocks impact the portfolio composition of entrepreneurs' liabilities and the return spread between deposits at banks and NBFIs. The model is therefore estimated with the addition of two observables, *i.e.* a proxy for the interest rate on non-bank deposits and the ratio between bank loans and total credit to the business sector.

The inclusion of financial variables is decisive. We cannot find a significant impact of liquidity shocks on the business cycle. The reason why this happens is indeed simple: in both versions of our model, liquidity shocks imply adjustments in asset prices and returns that are at odds with the corresponding observables added to the model. When the DGGT model is not constrained to match the observed bank deposit rate, liquidity shocks are prominent because the predicted volatility of the return spread between bank deposits and Treasuries is far larger than in the data. When the NBFI model is not constrained to match financial variables, liquidity shocks are important because the model mispredicts the observed dynamics of both the bank loans' share and the return spread between bank and non-bank deposits.

Furthermore, our results challenge the view that liquidity shocks caused the persistent decline in the natural interest rate. We find a more pronounced role for the productivity slowdown via persistent technology shocks, in line with the narrative in Laubach and Williams (2016). Relative to DGGT, we also obtain significantly higher estimates of the natural interest rate during the ZLB period, consistently with Wu's (2017) discussion of DGGT.[4] As a consequence of the higher estimated natural rate, our results call for a reconsideration of the Fed interest rate policy: in contrast with common wisdom (see for instance Cúrdia, 2015, and Gerali and Neri, 2019), our estimates of the interest rate gap suggest that the interest rate policy was indeed expansionary during the last quarters of the ZLB period.

Finally, we discuss a novel result concerning the Fed monetary policy stance as measured by the gap between the real policy and natural rates. The DGGT-estimated r^* has the questionable implication that the policy rate gap systematically increased in recession periods since 1960. In other words, the real fed funds rate turned from expansionary, at the onset, to contractionary, at the end of every recession. On the contrary, our estimates of r^* imply that the interest rate gap was indeed procyclical during most recession episodes. We obtain this result because, relative to DGGT, our estimates imply a smaller fall of the natural rate in the occurrence of recessions.

From a modeling perspective, we contribute to the DSGE literature that incorporates financial frictions. We combine the building blocks of the seminal works by Christiano et al. (2014) and Gertler and Karadi (2011), and we depart from the assumption of a single financial intermediary. Hirakata et al. (2011) and Suh and Walker (2016) also integrate financial frictions both at the entrepreneur and at the banking level, but they consider a unique financial intermediary. Somewhat closer to our NBFI specification, Durdu and Zhong (2021) build and estimate a model with bank and non-bank intermediaries; differently from our

[4]We refer in particular to her comments on the "implausibly negative nominal r^*" obtained by DGGT.

approach, their model features two types of entrepreneurs who distinctly borrow from banks or non-banks. In fact, our framework gives importance to the distinction between bank and non-bank credit (to the same firm), whose macroeconomic relevance has been underlined by De Fiore and Uhlig (2011), Becker and Ivashina (2014), and Herman et al. (2017) among the others.

We also contribute to the literature that investigates the counterfactual implications of liquidity shocks. Shi (2015) shows that an adverse liquidity shock in the spirit of Kiyotaki and Moore (2012) generates a counterfactual increase in equity prices. Attempts at addressing this issue include Cui and Radde (2019), who introduce costly financial intermediation, and Guerron-Quintana and Jinnai (2019), who eliminate the implausible rise in equity prices by adding to the model an endogenous growth mechanism that generates a persistent fall in dividends following the liquidity shock. The counterfactual implications of liquidity shocks uncovered in this paper cannot be solved in a similar way, simply because return spreads are not a by-product, but rather a necessary driver for the propagation of these shocks.

Finally, our finding that the post-2010 natural rate was consistently higher than in DGGT is in line with results in Kiley (2015) and Juselius et al. (2017), who estimate a higher r^* with respect to the Laubach and Williams (2003) benchmark, due to the inclusion of the financial cycle in the model and in the data used for estimation.[5]

The remainder of the paper is organized as follows. Section 1.2 describes the theoretical model that encompasses the DGGT and NBFI specifications. Section 1.3 introduces the estimation details and presents the empirical results. Section 1.4 concludes.

1.2 Model

We build on the New York Fed DSGE model, which accounts for variable capacity utilization, indexation to past inflation in the price and wage Phillips curves,[6] and a time-varying inflation target in the Central Bank's monetary policy rule. Exogenous TFP dynamics incorporate both a stochastic trend and a trend-stationary component. Entrepreneurs borrow funds from financial intermediaries and their ability to turn raw physical capital into efficient capital units is subject to idiosyncratic efficiency (risk) shocks (see Christiano et al., 2014). Less efficient entrepreneurs will go bankrupt and the lenders will repossess the proceedings of the loan upon payment of a monitoring cost.[7] Expected returns from loans are therefore lower than the contractual lending rate.

The key innovation is that we allow for two alternative characterizations of the financial sector whose contribution to the model economy depends on the value assigned to the dummy α^{b^L}. When $\alpha^{b^L} = 0$, the model replicates DGGT, *i.e* liquidity shocks drive a wedge

[5] Cukierman (2016) and Taylor and Wieland (2016) show theoretically how the omission of relevant variables, such as those characterizing the financial cycle, may cause a downward bias in the estimate of r^*. In a nutshell, by extending the Laubach and Williams (2003) model with additional variables, the output gap does not depend uniquely on the wedge between the actual and the natural rates. Hence, a strongly negative output gap does not necessarily have to correspond to a strongly negative r^*. While both papers formally make this argument for a semi-structural model, Taylor and Wieland (2016) suggest that the same mechanism should apply to a DSGE model.

[6] The model adopts the Kimball aggregator in the intermediate goods and labor markets.

[7] Entrepreneurs are also constrained to use both loans and their own funds to buy physical capital.

between the riskless rate on T-bills and the bank deposit rate. When $\alpha^{b^L} = 1$ ($NBFI$ model henceforth), bank deposits and T-Bills are perfect substitutes in households' portfolios, *i.e* they equally provide liquidity services, and the model allows for non-bank financial intermediaries, broadly interpreted as investment funds that buy corporate bonds. In equilibrium, entrepreneurs treat bank and non-bank intermediaries as suppliers of homogeneous funds. By contrast, the liabilities of bank and non-bank intermediaries are imperfect substitutes in households' portfolios, because the latter do not provide liquidity services. Liquidity shocks, therefore, affect the spread between the two deposit rates. In what follows we provide the details of these alternative characterizations, whereas the full set of equilibrium conditions is described in Appendix A.1.

1.2.1 Household portfolio choices

Household ι's expected lifetime utility is based on preferences defined over consumption, c_t, labor supply, L_t^h, and real holdings of a bundle of liquid assets, b_t^L:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{(c_t(\iota) - \xi c_{t-1})^{1-\sigma_c}}{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1+\nu_l} L_t^h(\iota)^{1+\nu_l} \right) + v_t U\left(b_t^L(\iota)\right) \right\}, \tag{1.1}$$

and v_t is a shock to the desire for liquidity.[8]

We posit that

$$b_t^L(\iota) = \frac{B_t(\iota)}{P_t} + \left(1 + \frac{D_t^b(\iota)}{P_t}\right)^{\alpha^{b^L}} - 1,$$

where P_t is the price of consumption goods, B_t and D_t^b respectively define one-period nominal government bonds[9] and nominal bank deposits, and $\alpha^{b^L} = \{0, 1\}$ identifies the (il)liquid status of bank deposits. Bank deposits and government bonds respectively yield the nominal rates $R_t^{d,b}$ and R_t, where the latter also is the *nominally* risk-free rate set by the Central Bank.

The flow budget constraint is

$$c_t(\iota) + \frac{B_t(\iota)}{R_t P_t} + \frac{D_t^b(\iota)}{R_t^{d,b} P_t} + \alpha^{b^L} \frac{D_t^{NBFI}(\iota)}{R_t^{d,NBFI} P_t} \leq \left(\frac{B_{t-1}(\iota)}{P_{t-1}} + \frac{D_{t-1}^b(\iota)}{P_{t-1}} + \alpha^{b^L} \frac{D_{t-1}^{NBFI}(\iota)}{P_{t-1}} \right) \frac{1}{\pi_t} + \\ + w_t L_t^h(\iota) - T_t(\iota) + \Pi_t(\iota), \tag{1.2}$$

where π_t is the inflation rate, w_t is the real wage, Π_t and T_t define the consumption value of dividends and lump-sum taxes, respectively. $D_t^{NBFI}(\iota)$ denote deposits held at a non-bank financial intermediary, which yields the nominal rate $R_t^{d,NBFI}$.

In the symmetrical equilibrium, the FOCs relevant for our analysis are:

$$\lambda_t = U'(c_t),$$

$$\lambda_t = v_t U'\left(b_t^L\right) + \beta E_t \left[\lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{1.3}$$

[8] The treatment of the preference for liquidity strictly follows Fisher (2015), and we obviously assume that $U(\bullet)$ is positive, increasing, and concave.

[9] We will use "government bonds" interchangeably with "Treasuries" or "T-bills" to refer to B_t.

$$\lambda_t = \alpha^{b^L} \left[v_t U'\left(b_t^L\right)\right] \left(1 + \frac{D_t^b}{P_t}\right)^{\alpha^{b^L}-1} + \beta E_t\left[\lambda_{t+1}\frac{R_t^{d,b}}{\pi_{t+1}}\right]. \tag{1.4}$$

When $\alpha^{b^L} = 0$, log-linearization yields

$$\hat{\lambda}_t = \hat{\varepsilon}_t^l + \hat{R}_t - E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\hat{\lambda}_{t+1}\right], \tag{1.5}$$

$$\hat{\lambda}_t = \hat{R}_t^{d,b} - E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\hat{\lambda}_{t+1}\right], \tag{1.6}$$

where $\hat{\varepsilon}_t^l = \lambda^{-1}U'(b^L)v_t$ is the normalized liquidity shock, which is assumed to follow an AR(1) process.[10] Note that:

$$\hat{R}_t^{d,b} = \hat{R}_t + \hat{\varepsilon}_t^l. \tag{1.7}$$

Thus, a positive liquidity shock inevitably raises the return on bank deposits above the monetary policy rate. Whilst this mechanism is left implicit in DGGT, a shock with an analogous transmission is central to the theoretical model proposed by Benigno and Nisticò (2017).[11]

By contrast, when $\alpha^{b^L} = 1$ both assets provide liquidity services and

$$\hat{R}_t^{d,b} = \hat{R}_t.$$

Demand for D_t^{NBFI} is by assumption nil when $\alpha^{b^L} = 0$, whereas when $\alpha^{b^L} = 1$ it is driven by the standard Euler equation:

$$\lambda_t = \beta E_t\left[\lambda_{t+1}\frac{R_t^{d,NBFI}}{\pi_{t+1}}\right].$$

In this case, the liquidity shock drives a wedge between the return on deposits at the NBFI and the return on the two liquid assets, $\hat{R}_t$:

$$\hat{R}_t^{d,NBFI} = \hat{R}_t + \hat{\varepsilon}_t^l.$$

1.2.2 Entrepreneurs

In addition to his own resources, N_t^e, the representative entrepreneur borrows from financial intermediaries the funds, L_t, necessary to purchase from capital goods producers the physical capital, $\bar{k}_t$, at the market price Q_t:

$$Q_{t-1}\bar{k}_{t-1} = N_{t-1}^e + L_{t-1}.$$

Following Christiano et al. (2014), $\bar{k}_t$ is then transformed into effective capital conditionally to an idiosyncratic efficiency shock ω_t, and rented to intermediate goods producers at the

[10] Variables without the time subscript denote the respective steady-state values.

[11] Their model explicitly departs from the literature, that for the most part studies shocks to credit spread between deposit and lending rates, to focus on shocks that raise the wedge between risk-free and deposit rates, the latter being less liquid than money. Our work can be regarded as an empirical test of the theoretical transmission mechanism proposed by Benigno and Nisticò (2017).

nominal rental rate R_t^k. At the end of the period, the undepreciated capital is sold back to capital goods producers. Entrepreneurs' profits are

$$\Pi_t^e = \tilde{R}_t^k \omega_t Q_{t-1} \bar{k}_{t-1} - L_{t-1} R_t^{c,L}, \tag{1.8}$$

where $\tilde{R}_t^k$ is the gross nominal return to capital (that includes proceedings from selling undepreciated capital), and $R_t^{c,L}$ is the contractual lending rate. The combination of a predetermined lending rate with idiosyncratic productivity shocks exposes entrepreneurs to bankruptcy risk. In every period the productivity threshold $\overline{\omega}_t$ identifies the zero-profit condition that determines the fraction of bankrupt entrepreneurs, $F_t\left(\omega_t < \overline{\omega}_t\right)$. Note that banks repossess the assets of bankrupt entrepreneurs at the monitoring cost μ. In equilibrium, the following condition must hold:

$$\left[1 - F_t\left(\omega_t < \overline{\omega}_t\right)\right] L_{t-1} R_t^{c,L} + (1-\mu) \int_0^{\overline{\omega}_t} \tilde{R}_t^k \omega_t Q_{t-1} \bar{k}_{t-1} dF_t\left(\omega\right) = L_{t-1} R_t^L,$$

where R_t^L is the average return on loans.

1.2.3 Financial intermediaries

When $\alpha^{b^L} = 0$, we strictly follow DGGT, and perfectly competitive banks turn deposits into loans that earn the rate R_t^L. In this case,

$$R_t^{d,b} = R_t^L.$$

When $\alpha^{b^L} = 1$, both banks and non-bank intermediaries supply loans:

$$L_t = L_t^b + L_t^{NBFI}.$$

The structure of non-bank financial intermediaries is very simple. The representative $NBFI$ is subject to the following technology:[12]

$$L_t^{NBFI} = \left(D_t^{NBFI}\right)^{\alpha_{NBFI}}, \ \alpha_{NBFI} < 1. \tag{1.9}$$

For any given average market return on loans, R_{t+1}^L, profit maximization yields the following supply condition for $NBFI$ loans:

$$L_t^{NBFI} = \left(\frac{R_{t+1}^L}{R_t^{d,NBFI}}\right)^{\frac{\alpha_{NBFI}}{1-\alpha_{NBFI}}}. \tag{1.10}$$

Our modeling strategy for the banking sector follows Gertler and Karadi (2011). This financial friction implies that the endogenous spread on bank deposits is countercyclical, dampening the implausible surge in the supply of bank loans that would otherwise occur

[12]In Mehra et al. (2011), returns to scale in the financial intermediation technology are constant, and the intermediation cost is a fixed proportion of loans. This, in turn, generates a fixed loan rate spread. Following their strategy here, where two financial intermediaries supply loans at the market rate, leads to model indeterminacy.

when liquidity shocks raise bank deposits. In a sense, this choice "stacks the cards" against our conjecture that liquidity shocks imply a counterfactual adjustment in the composition of firms' liabilities.

We posit that bankers may divert a fraction Λ of deposits. This, in turn, requires that bankers put skin in the game by accumulating their own net worth. Bankers exit the financial sector and become workers with probability $(1-\theta)$. Therefore, individual banking activity is expected to last $(1-\theta)^{-1}$ periods.[13] Exiting bankers transfer their net worth to households, who provide new bankers with an initial endowment corresponding to a fraction Ω of last-period loans, L^b_{t-1}. Bank loans amount to

$$L^b_t = \alpha^{b^L} N^b_t + D^b_t, \tag{1.11}$$

where N^b_t defines bankers' net worth. Bank profit maximization yields the following FOCs:

$$\nu_t = (1-\theta)\beta\frac{\lambda_{t+1}}{\lambda}\left(R^L_{t+1} - R^{d,b}_t\right) + \beta\theta\frac{\lambda_{t+1}}{\lambda_t}m_{t+1}\nu_{t+1}, \tag{1.12}$$

$$\eta_t = (1-\theta)\beta\frac{\lambda_{t+1}}{\lambda_t}R^{d,b}_t + \theta\beta\frac{\lambda_{t+1}}{\lambda_t}\zeta_{t+1}\eta_{t+1}, \tag{1.13}$$

$$\phi^b_t = \frac{\eta_t}{\Lambda - \nu_t}, \tag{1.14}$$

$$\zeta_t = (R^L_t - R^{d,b}_{t-1})\phi^b_{t-1} + R^{d,b}_{t-1}, \tag{1.15}$$

$$m_t = N^b_t/N^b_{t-1} = \frac{\phi^b_t}{\phi^b_{t-1}}\zeta_t, \tag{1.16}$$

$$N^b_t = \theta\zeta_t\varepsilon^{Nb}_t N^b_{t-1} + \Omega L^b_{t-1}. \tag{1.17}$$

ν_t and η_t respectively define the value to the banker of one additional unit of loans and net worth, ϕ^b_t is bank leverage, ζ_t is the growth rate of bank loans, and m_t is the growth rate of surviving bankers' net worth. ε^{Nb}_t is a shock that hits net worth accumulation as in Gertler and Karadi (2011) and follows an AR(1) process.

1.3 Empirics

The first step in our empirical analysis is a straightforward replication of the estimates obtained in DGGT (*i.e.* $\alpha^{b^L} = 0$), which we take as a benchmark model, in order to discuss the implications of liquidity shocks for the smoothed series of the deposit rate. The second step is the estimation of the same empirical model *augmented* by the inclusion of one additional observable, *i.e.* a proxy for the commercial bank deposit rate. Finally, we depart from the DGGT benchmark and estimate the NBFI version of our model (*i.e.* $\alpha^{b^L} = 1$). As explained in detail below, our third estimation features two additional observables with respect to DGGT, specifically a measure of the non-bank deposit rate and the share of bank over total loans.

[13]This assumption is typically made to prevent bankers from accumulating net worth up to the point where they would no longer need deposits to supply loans.

1.3.1 DGGT model

Following DGGT, the set of observables includes: real GDP and GDI growth for output, core PCE and GDP deflator for inflation, real consumption and investment growth, real wage growth, TFP growth, hours worked, fed funds rate, fed funds rate expectations up to 6 quarters ahead to account for the ZLB and forward guidance, 10-year Treasury yield, 10-year inflation expectations to account for a time-varying inflation target, spread between Aaa-rated corporate bonds and 20-year Treasury yields, spread between Baa-rated corporate bonds and 20-year Treasury yields. The estimation is carried out over the period 1960:I-2019:IV.[14]

The model features a standard set of shocks and a few measurement errors.[15] The issue posed by the ZLB constraint is addressed by augmenting the monetary policy rule with anticipated (or news) shocks. This is combined with the inclusion of fed funds rate forecasts in the set of observables, so that the model's expectations for the policy rate match the market expectations. The liquidity shock is made up of a liquidity and a safety component, each of which is the sum of two AR(1) processes, meant to pick up highly persistent (with autoregressive coefficient fixed at 0.99) and transitory flight-to-quality episodes. In other words, there are actually four such shocks possibly hitting the economy: permanent liquidity, transitory liquidity, permanent safety, and transitory safety.

The presence of the two spreads among the observables is crucial to identify liquidity and safety shocks. Given their importance in this context, we report here the two related measurement equations while presenting the remaining ones in Appendix A.3:

$$\text{Aaa - 20-year Treasury spread} = 100\ln(\varepsilon^{liq}) + E_t\left[\frac{1}{80}\sum_{j=0}^{79}\hat{\varepsilon}^{liq}_{t+j}\right] + e^{Aaa}_t, \tag{1.18}$$

$$\text{Baa - 20-year Treasury spread} = 100\ln\left(\varepsilon^{liq}\varepsilon^{safe}SP_*\right) + E_t\left[\frac{1}{80}\sum_{j=0}^{79}(\hat{\tilde{R}}^k_{t+j+1} - \hat{R}_{t+j})\right] + e^{Baa}_t. \tag{1.19}$$

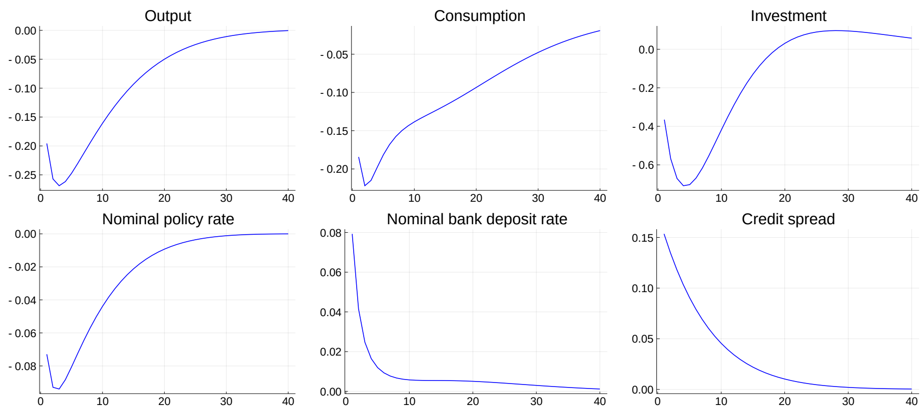
The difference between Aaa-rated corporate bonds and Treasury yields is assumed to represent a liquidity premium. Specifically, it is mapped to the model as the sum of the steady-state liquidity premium ε^{liq}, the (expectations of future) liquidity shocks $\hat{\varepsilon}^{liq}_{t+j}$, and measurement error e^{Aaa}_t. Conversely, the spread between Baa-rated bonds and Treasury yields accounts for both safety and liquidity components, in addition to the default risk of entrepreneurs. The term in square brackets in (1.19) is the endogenous spread between the return on capital and the risk-free rate, where bank leverage, safety and liquidity shocks, and entrepreneur risk shocks enter. The observation equation is augmented with safety (ε^{safe}) and liquidity steady-state premia, an additional estimated spread SP_*, and measurement error e^{Baa}_t.

As the measurement equations show, the distinction between liquidity and safety shocks is purely "empirical", meaning that there is no endogenous difference between the two before

[14]Notice that we extend the estimation sample with respect to DGGT, who consider the period 1960:I-2016:III. All our findings hold irrespective of which of the two samples we use.

[15]The full list of shocks and measurement errors is presented in Appendix A.2.

Figure 1.2: IRFs to an adverse transitory liquidity shock (DGGT)



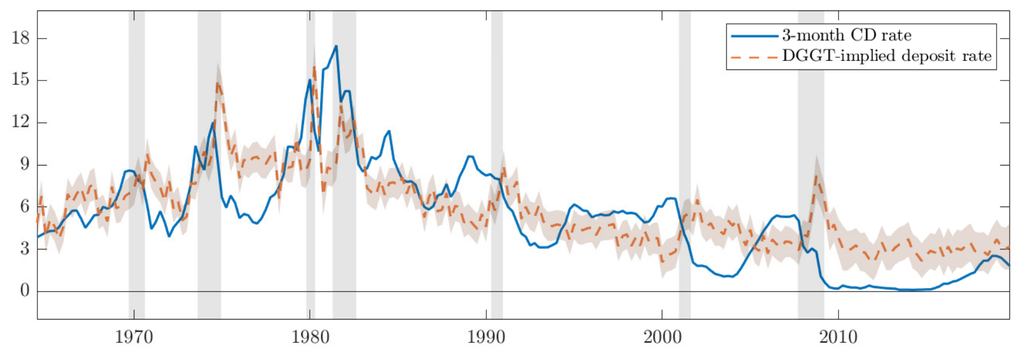
Note: Estimated impulse response functions at the posterior mode.

taking the model to the data. Indeed, conditional on their standard deviation and autocorrelation being equal, liquidity and safety shocks will produce the same effects in all the model specifications we consider.

Figure 1.2 shows the estimated impulse response functions (IRFs henceforth) to a transitory liquidity shock. According to the flight-to-quality interpretation, households raise their demand for Treasuries, the deposit rate rises and so does the interest rate on loans, leading to a positive credit spread (*i.e.* a positive differential between lending and policy rates). The liquidity shock causes a slump in both consumption and investment.

Figure 1.3 plots the observed deposit rate against the smoothed series obtained from the benchmark estimation, which identifies liquidity shocks as one of the main drivers of business cycle fluctuations. As discussed in the introduction, the liquidity shock requires a divergence between deposit and policy rates which is unobservable over the sample period. In fact, the DGGT-implied deposit rate appears to be considerably more volatile than its proxy in the

Figure 1.3: Observed and model-implied bank deposit rate (DGGT)



Note: The dashed line is the posterior mean and the shaded area shows the 68% posterior coverage interval of the smoothed bank deposit rate (DGGT estimation). The solid line is the observed 3-month CD interest rate. 1964:III-2019:IV.

data.[16]

1.3.2 Augmented DGGT model

The augmented DGGT model (Augmented henceforth) is constrained to match one additional observable, *i.e.* a proxy for the banking sector deposit rate. Our series of choice is the secondary market rate on 3-month CDs, assets that match the 3-month maturity of T-bills whose return in the empirical model is proxied by the fed funds rate. The same measure is used by Hirakata et al. (2011) in the estimation of a DSGE model.[17]

The measurement equation for the deposit rate clarifies how this additional observable is going to discipline the estimate of liquidity shocks:

$$\begin{aligned}\text{3-month CD rate} &= 100(R^{d,b} - 1) + \hat{R}^{d,b}_t \\ &= 100(R\varepsilon^{liq}\varepsilon^{safe} - 1) + \hat{R}_t + \hat{\varepsilon}^{liq}_t + \hat{\varepsilon}^{safe}_t,\end{aligned} \tag{1.20}$$

where the second equality makes use of the deposit spread defined in (1.7), and $R^{d,b} = R\varepsilon^{liq}\varepsilon^{safe}$ is the implied steady-state deposit rate. Liquidity and safety shock now directly affect (and are identified by) the two corporate spreads and the wedge between the deposit and policy rates.

The CD rate time series is available from 1964:III to the end of the estimation sample and displays an average positive differential of 14 basis points over the fed funds rate. This raises the issue of matching the calibration of steady-state premiums chosen by DGGT. In their estimates, the deposit rate is an unobserved variable and they calibrate positive steady-state premiums for liquidity and safety that allow matching the average convenience yields found by Krishnamurthy and Vissing-Jorgensen (2012) for the Aaa- and the Baa-Treasury spreads. This calibration implicitly grants a steady-state spread for the deposit rate of 73 basis points over the risk-free policy rate and entails that it lies between the corresponding yields on Aaa and Baa corporate bonds. This is hard to reconcile with the observed average CD rate. To solve the problem, we have therefore chosen to demean both corporate-Treasury spreads and to remove steady-state safety and liquidity premiums ($\varepsilon^{liq} = \varepsilon^{safe} = 0$).

Existing alternatives to our proxy of the deposit rate would be characterized by lower average returns. For instance, Angeloni and Faia (2013) and Bekiros et al. (2018) consider the M2 own rate, a weighted average of the rates received on the interest-bearing assets included in M2. The M2 aggregate bundles assets of different maturities and its rate of return is characterized, on average, by a substantially *negative* differential with respect to the fed funds rate (around -2% between 1960:I and 2019:II, when the M2 own rate series was discontinued). An alternative measure is provided by Drechsler et al. (2017), who construct the average rate paid by commercial banks on savings deposits using U.S. Call Reports data from 1986:I to 2013:IV. Over this period, the savings deposit rate is on average 128 basis

[16] Absent liquidity shocks, the model-implied deposit rate would be equal to the policy rate, and thus very close to its observed counterpart (see Figure 1.1, panel (a)).

[17] Pesaran and Xu (2016) and Hollander and Liu (2016) consider the average between 1-, 3-, and 6-month secondary market CD rates. The differences between this average and the 3-month rate are in the order of basis-point decimals.

points lower than the fed funds rate.[18]

A full description of the prior distributions and the posterior estimates is left for the Appendix (Tables A1 and A2). With respect to the original DGGT specification, in all our estimations we impose a looser prior on the standard deviation of permanent safety and liquidity shocks. We discuss here the most significant differences between the Augmented and the DGGT posterior estimates.

The inverse of the elasticity of intertemporal substitution, σ_c, increases from 0.90 to 1.30. Given the non-separable preferences in consumption and leisure, this is sufficient to determine a shift from substitutability to complementarity between consumption and labor. As shown by Furlanetto and Seneca (2014), complementarity in consumption and labor is important to obtain procyclical consumption responses to MEI shocks.[19] Regarding the parameters determining internal persistence, the elasticity of investment adjustment costs, S'', is considerably larger and shifts from the lower to the upper end of the prior distribution, whereas the consumption habits parameter, h, falls from 0.49 to 0.21. In spite of the apparent anomaly relative to the benchmark, this is an intriguing result. In fact, empirical DSGE models obtain estimated values for the habit parameter that are at odds with microeconometric evidence (see Havranek et al., 2017). As per the parameters of the exogenous processes, *transitory* liquidity and safety shocks are less persistent and have a smaller standard deviation with respect to DGGT. On the other hand, the standard deviation of permanent flight-to-quality shocks remains fairly stable.[20]

The estimated IRFs to a liquidity shock are very close to those in Figure 1.2,[21] but the smoothed series obtained for the deposit rate obviously matches the corresponding observable we use to estimate the model. This, in turn, suggests that liquidity shocks might have a negligible impact on business cycle fluctuations.

Figure 1.4 shows the historical decomposition of GDP growth in the benchmark (panel (a)) and in the Augmented estimation (panel (b)) over the last twenty years of the sample. We focus on liquidity, productivity, MEI, risk, and monetary policy shocks. According to the DGGT model, the role of liquidity shocks was particularly pronounced in the last two recessions, whereas risk and MEI shocks played a lesser role. By contrast, the Augmented model proposes a quite different narrative: the importance of liquidity shocks is eroded in favor of MEI and productivity shocks. Similar conclusions hold for the growth rates of consumption and investment, with the former (latter) most impacted by technology (MEI) shocks.[22]

[18] In Appendix B, we discuss the estimates obtained when the M2 own rate and the savings deposit rate are chosen to proxy the model-implied bank deposit rate. We consider the alternative of imposing the DGGT steady-state calibration of the deposit rate, which implies that the estimated shocks are forced to match the gap with the observed average return on deposits. Our results are fully confirmed.

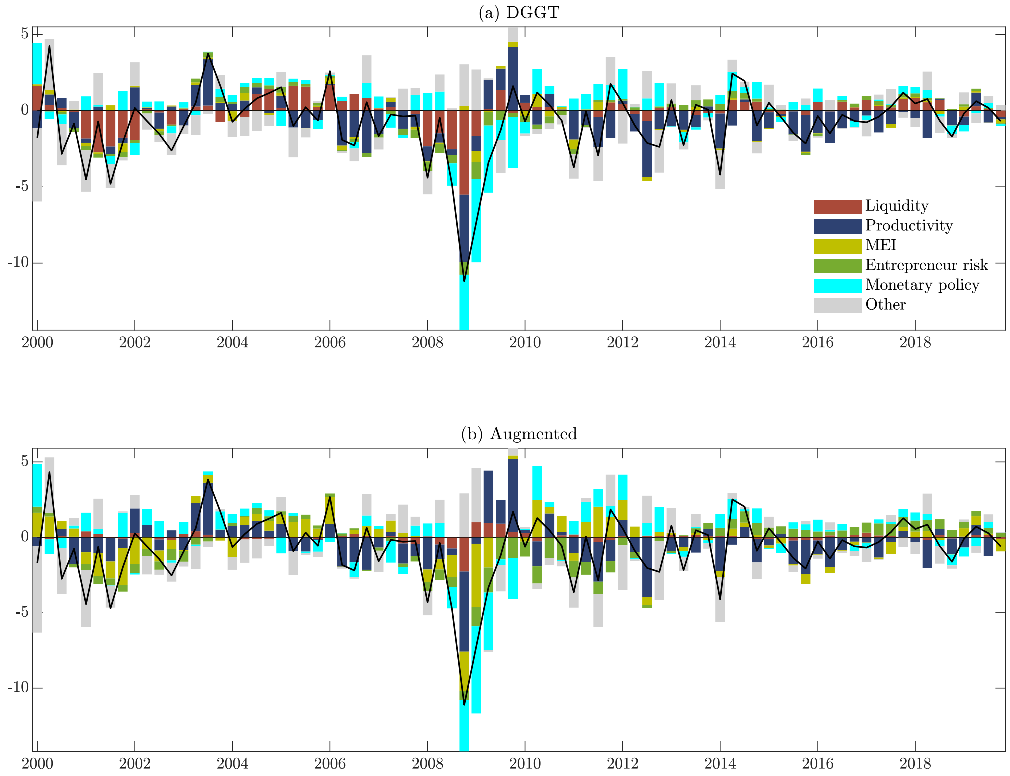
[19] Our posterior estimates do not violate the assumptions that consumption and leisure are non-inferior goods and that the utility function is concave (see Bilbiie, 2009, and Bilbiie, 2011).

[20] We note in passing that our re-estimation of DGGT led to a substantial change in the MEI shock autocorrelation coefficient, which shifted from 0.96 in the original estimates to 0.24 in our results.

[21] Results available upon request.

[22] See Table E1 in the Appendix for a variance decomposition analysis over the full sample and considering all shocks. The findings described above are confirmed. Table E1 also shows how DGGT attributes the largest part of the real policy rate's variation to liquidity shocks. Their role is nearly irrelevant in Augmented, and it is quite evenly replaced by technology, MEI, risk, and monetary policy shocks.

Figure 1.4: GDP growth historical shock decomposition (DGGT vs Augmented)



Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": sum of contemporaneous and anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values. 2000:I-2019:IV.

Finally, it is interesting to note how the estimation handles the simultaneous observation of the two corporate spreads and the deposit rate, considering their relevance in identifying liquidity shocks. As evident from the Aaa-Treasury spread equation (1.18), the latter is determined only by liquidity shocks and measurement error: if the estimated standard deviation of liquidity shocks declines when adding the deposit rate, there is no other endogenous or exogenous variable that can make up for some of their effects on the Aaa-Treasury spread, beyond the measurement error itself. Indeed, the variance explained by the measurement error increases from 6% to 26% in the DGGT and Augmented estimations, respectively. Differently, the Baa-Treasury spread observation equation (1.19) contains a fully endogenous component in the excess return on capital, which is influenced by all structural shocks. In this respect, our Augmented estimation ends up relying less heavily on liquidity shocks, as expected, but also on measurement errors (whose contribution to the variance decomposition drops from 48% to 26% when the deposit rate is observed; see Table E1 in the Appendix).

1.3.3 NBFI model

The NBFI model is built on the assumption that the return spread between bank deposits and T-bills is always nil, whereas liquidity shocks affect the spread between NBFI and commercial bank deposits. This, in turn, impacts on the share of bank loans out of total loans to firms. In this respect, liquidity shocks play an additional role that is absent in DGGT, which makes us consider the use of *two* supplementary observables in order to test their empirical plausibility.

Relative to the Augmented model, we replace our proxy for the banking sector deposit rate with a proxy for the deposit rate at NBFIs. Drawing from Krishnamurthy and Li (2022),[23] this is identified in the interest rate on 90-day P1-rated commercial paper (P1CP), whose data are available starting 1971:II. Since we do not take a stand on the precise nature of the non-bank intermediary, we insert a measurement error in the observation equation of the NBFI deposit rate, which otherwise mimics the bank deposit rate measurement equation (1.20). The set of observables also includes the growth rate of the ratio between commercial bank loans and total credit to the non-financial business sector. We construct this observable as in Becker and Ivashina (2014).[24] Following condition (1.17), we also estimate a shock to bankers' net worth accumulation, ε_t^{Nb}. As the P1CP rate is on average extremely close to the fed funds rate (the spread between the two series amounts to 1 basis point between 1971:II and 2019:IV), we follow the same strategy adopted in the Augmented case, *i.e.* we set steady-state safety and liquidity premiums at zero and demean the corporate spreads.

Before turning to the estimates, we briefly discuss the calibration of the parameters and steady-state values absent in the DGGT framework. Steady-state bank leverage, ϕ^b, and bankers' survival probability, θ, are set at 4 and 0.97156 respectively, as in Gertler and Karadi (2011). We calibrate the steady-state share of bank over total credit at 0.4, corresponding to the average ratio in the data used for estimation and consistent with De Fiore and Uhlig (2011). Lastly, we choose a value of 0.99 for the returns-to-scale coefficient in the NBFI intermediation technology, α_{NBFI}.[25]

Relative to the benchmark DGGT model, posterior estimates exhibit the following features. The (inverse) elasticity of intertemporal substitution, σ_c, is well above unity (1.42), the degree of consumption habits, h, is rather small (0.31), and S'', the elasticity of investment adjustment costs, is larger and close to its prior mean (4.06). These three estimates go in the same direction as in the Augmented case. Likewise, both (transitory) safety and liquidity shocks are less persistent and smaller in magnitude than in DGGT. Finally, wage markup shocks assume a high autocorrelation coefficient (0.88), as do shocks to bankers' net worth (0.95).

Impulse response functions to a liquidity shock are qualitatively similar to the ones estimated with the benchmark DGGT model, but the relative volatility induced by the shock is unambiguously limited in the NBFI model (see Figure 1.5).[26] In fact, the liquidity shock

[23]Somewhat similarly to the present work, Krishnamurthy and Li (2022) consider a model with money (bank deposits), near-money (non-bank deposits), and Treasury bonds, where the three assets are allowed to differ in terms of substitutability and liquidity attributes.

[24]See Appendix A.4 for a detailed description of how this and the other observables are constructed.

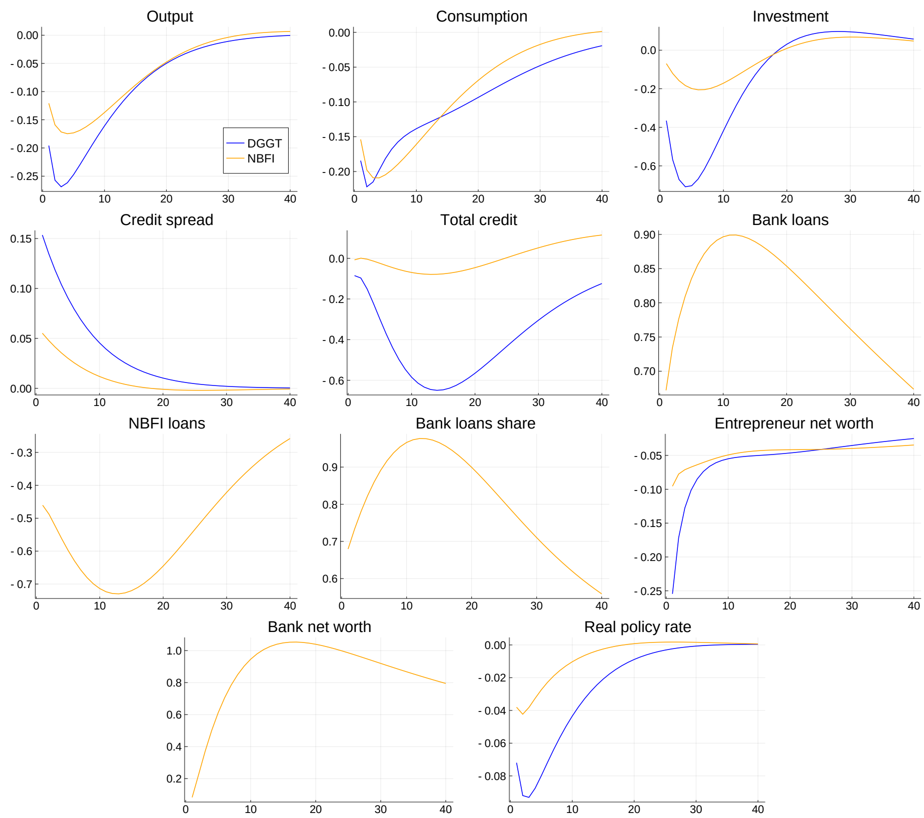
[25]We examine the calibration of α_{NBFI} and its implications in Appendix C.

[26]For the sake of comparison, Figure 1.5 reproduces IRFs to a shock of the same standard deviation and persistence. Specifically, we choose the DGGT estimates.

induces households to accept a lower return on bank deposits, raising the bankers' continuation value. This, in turn, limits the credit spread and dampens the investment contraction. The IRFs also depict a marked and persistent increase in the share of bank loans and in banks' net worth. This casts doubts on the possibility that liquidity shocks play a major role in driving business cycle fluctuations: considering the whole sample, the correlation between the growth rates of the bank-credit share and GDP (investment) is 0.08 (0.23), similarly to Durdu and Zhong (2021), who find a weakly positive correlation of both bank and non-bank credit growth with real activity.[27]

In contrast with liquidity shocks, an adverse bank net worth shock produces a simultaneous fall in output and in the share of bank loans.[28] Following the shock, entrepreneurs turn to NBFIs, whose credit rises but not enough to offset the bank credit crunch, so that total lending persistently decreases. The drop in the real policy rate, though not immediate, insulates households' consumption that responds countercyclically. Importantly, the magnitude of the response is substantially larger for financial than for macroeconomic variables. This translates into a limited influence of bank net worth shocks in the shock decomposition of the main macro aggregates.

Figure 1.5: IRFs to an adverse transitory liquidity shock (DGGT vs NBFI)



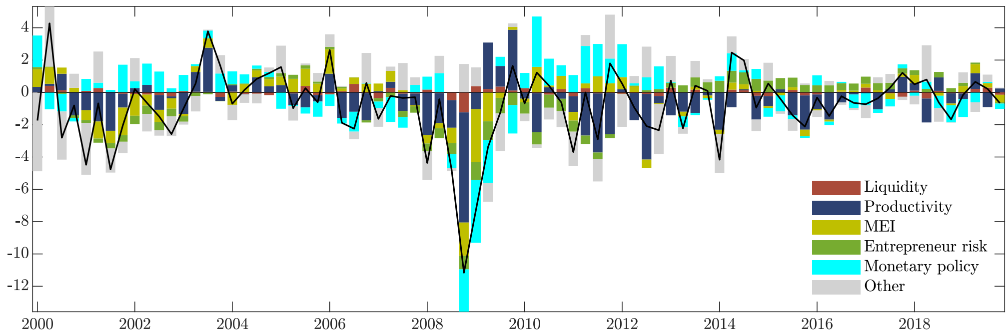
Note: Estimated impulse response functions at the posterior mode.

[27]Durdu and Zhong (2021) consider a different sample (1987:I-2015:I) and a different dataset, which they build tracing credit through intermediation chains that depart from non-financial corporate borrowers.

[28]See Figure E1 in the Appendix.

IRFs to the remaining shocks are similar across the DGGT and NBFI estimates, with one important exception. The response of consumption to a MEI shock turns from countercyclical to procyclical in the NBFI model,[29] thanks to the shift in the estimated elasticity of intertemporal substitution that grants complementarity between consumption and labor. Conversely, consumption still reacts positively to an adverse entrepreneur risk shock in NBFI,[30] thus replicating the pattern observed for the bank net worth shock. In fact, both MEI and financial-friction shocks have often been recognized in the literature for not being able to generate a positive co-movement between investment and consumption (see for instance Furlanetto and Seneca, 2014, and Suh and Walker, 2016).

Figure 1.6: GDP growth historical shock decomposition (NBFI)



Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": sum of contemporaneous and anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values. 2000:I-2019:IV.

Figure 1.6 reports GDP growth historical decomposition according to the NBFI model. Even during the GFC episode, liquidity shocks are barely noticeable and play no role in the other periods. MEI shocks assume higher relevance especially between 2008 and 2010, confirming the results obtained with the Augmented estimation.[31] As for the observed credit spreads, our NBFI model does a poorer job of matching the Aaa- and Baa-Treasury spreads, whose variance explained by measurement errors significantly increases with respect to both the DGGT and Augmented estimates (as shown in Table E1 in the Appendix).

To conclude this discussion, we describe how the estimates of NBFI change when we remove the P1CP rate from the set of observables. This exercise is helpful for understanding where the "constraints" on the potential role of liquidity shocks come from. Under this alternative specification, the liquidity shock makes a comeback (albeit less pervasively than in DGGT) because the empirical model is not forced to replicate the dynamics observed for

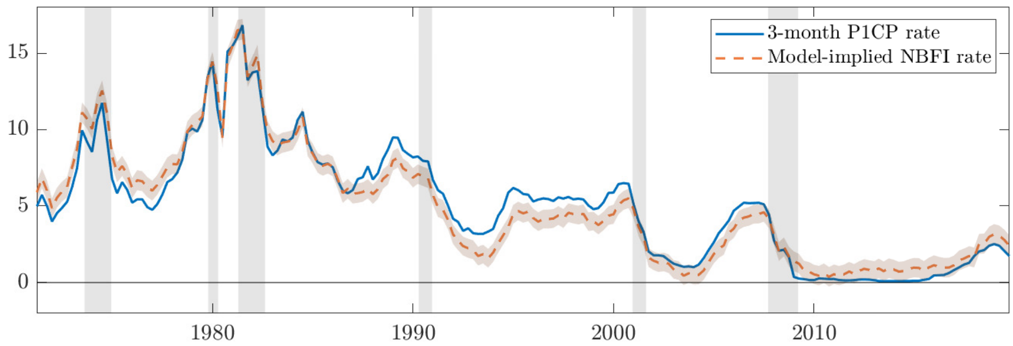
[29]See Figure E2 in the Appendix.

[30]See Figure E3 in the Appendix.

[31]Table E1 in the Appendix further demonstrates the consistency between Augmented and NBFI estimates in terms of variance decomposition. The two models draw a slightly different picture for the determinants of the real policy rate: according to NBFI, this was less affected by MEI and risk shocks, and more influenced by productivity and monetary policy.

the NBFI deposit rate.[32] In fact, a gap opens up between the observed and the model-implied NBFI deposit rate (see Figure 1.7). Interestingly, the mismatch of the non-bank deposit rate is much less pronounced with respect to the bank deposit rate in DGGT. Including the bank-loans share in the observables apparently alleviates the counterfactual implications of liquidity shocks.

Figure 1.7: Observed and model-implied NBFI deposit rate (NBFI, P1CP rate not observed)



Note: The dashed line is the posterior mean and the shaded area shows the 68% posterior coverage interval from the alternative NBFI estimation (with bank-loans-share growth as the unique additional observable). The solid line is the observed 3-month P1CP interest rate. 1971:II-2019:IV

1.3.4 The natural rate and the Fed monetary policy stance. A reinterpretation

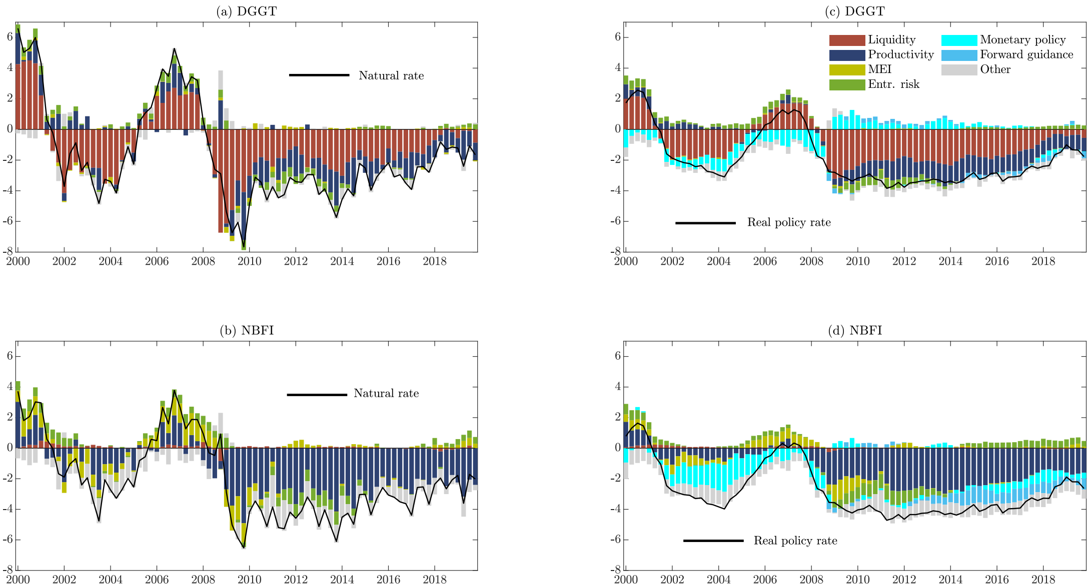
There are some interesting insights from the historical decomposition of the natural interest rate, *i.e.* the risk-free rate estimated when prices and nominal wages are flexible and markup shocks are assumed away. Liquidity shocks have a one-for-one impact on the natural rate: to fully absorb an adverse liquidity shock, the real interest rate should adjust by a magnitude such that the households' incentive to turn to liquid assets is neutralized. This is exactly what happens under flexible prices, explaining why r^* falls one-for-one.

Panels (a) and (b) of Figure 1.8 report the historical decomposition of the natural rate estimates obtained from the DGGT and NBFI models. According to the DGGT model, liquidity shocks are responsible for the post-2000 persistent fall in r^*. The NBFI estimates tell a quite different story: liquidity shocks did not matter, and the natural rate decline is instead mainly attributed to a slowdown in productivity growth.

Panels (c) and (d) show the historical decomposition of the observed real policy rates. These are obviously co-determined by nominal frictions and react to the full set of shocks. The two decompositions "inherit" the drastically different role of liquidity shocks in determining the two r^* estimates. Further, the DGGT and NBFI estimates crucially differ in the role assigned to monetary policy shocks. According to DGGT, policy shocks were mildly expansionary between 2000 and 2009 and turned mildly contractionary thereafter. According to NBFI estimates, monetary shocks played an important role in depressing the policy

[32]See Figure D1, panel (a), in the Appendix for the historical decomposition of GDP growth. Appendix D reports a detailed discussion of this alternative NBFI estimation.

Figure 1.8: Real policy and natural rates historical shock decomposition (DGGT vs NBFI)



Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": contemporaneous monetary policy shocks; "Forward guidance": sum of anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values. 2000:I-2019:IV.

rate between 2002 and 2006, in line with the Great Deviation view (Taylor, 2007, 2011), and were also markedly expansionary after 2013. Over this period, most of the monetary easing comes from forward guidance shocks, while these latter effects are virtually absent in DGGT estimates.

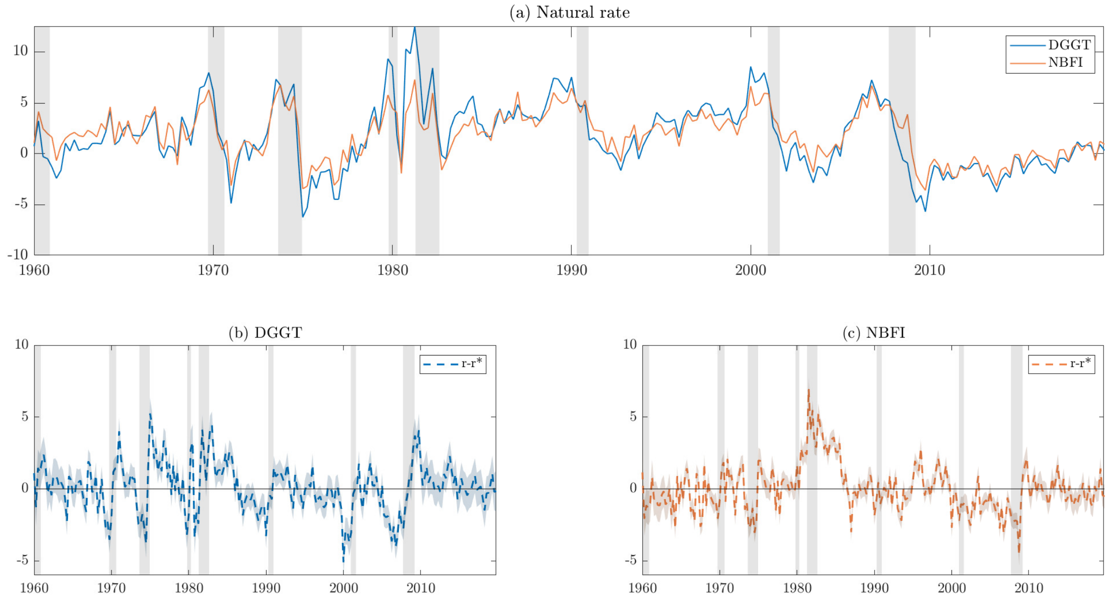
The two models have different implications for the estimated volatilities of r^*. Figure 1.9, panel (a), shows that DGGT predicts a larger volatility of the natural rate: at the posterior mode, the standard deviation of r^* in NBFI is 28% smaller than in DGGT, and this is almost entirely due to the different contribution of estimated liquidity shocks.

Both models predict that r^* is relatively high towards the end of an expansionary phase before abruptly falling during the recession, but the DGGT model systematically estimates a stronger contraction with respect to NBFI. Panels (b) and (c) of Figure 1.9 report the estimated cyclical patterns of the interest rate gap. From a Wicksellian perspective, it is the gap between the actual and the natural interest rate that matters for determining the former's stabilizing role, rather than the value of the policy rate itself.

According to DGGT, a countercyclical gap exists between the policy rate and r^*: this gap typically reaches a minimum before the onset of every recession and is consistently increasing thereafter, suggesting strong swings in monetary policy that turns from accommodative to restrictive during the recessionary episode. By contrast, NBFI estimates suggest that, at least since 1981, the gap reaches the maximum at the onset of the recession, and falls afterward.

In this regard, focusing on the $r - r^*$ gaps estimated for the GFC episode is illuminating:

Figure 1.9: Natural rates and policy rate gaps (DGGT vs NBFI)



Note: Panel (a): smoothed estimate of r^* at the posterior mode. Panels (b) and (c): smoothed estimate of the interest rate gap (*i.e.* real policy rate minus natural rate); the dashed line is the posterior mean and the shaded area shows the 68% posterior coverage interval. 1960:I-2019:IV.

according to DGGT, the gap was strongly negative in 2007, then gradually increased to become strongly positive in 2009, when the recovery began. According to NBFI, the monetary stance was almost neutral in 2007 and turned strongly expansionary during the crisis period. Crucially, given that the real policy rate is an observed variable, the opposite patterns in the $r - r^*$ gaps are entirely determined by the different estimates the two models generate for r^*.

1.4 Conclusions

We build a business cycle model that encompasses two alternative characterizations of financial markets: a standard one, with a single financial intermediary; and a more complex one, where bank and non-bank intermediaries coexist. In both cases, we shed light on the counterfactual implications liquidity shocks have for deposit rates and for the portfolio composition of firms' liabilities. Once we extend the standard set of observables with the relevant financial variables, we find that liquidity shocks did not play a significant role in the U.S. business cycle.

Further, our estimates are less pessimistic about the fall of the natural interest rate and do not support the popular view that it is explained by flight-to-liquidity shocks. In this regard, we identify the slowdown in productivity growth as the main responsible. We also find that the interest rate gap (*i.e.* the wedge between the real policy rate and r^*) was procyclical in occasion of most recession episodes, thus indicating an expansionary monetary policy stance during these periods.

References

Adrian, T., Colla, P., and Song Shin, H. (2013). Which financial frictions? Parsing the evidence from the financial crisis of 2007 to 2009. *NBER Macroeconomics Annual*, 27:159–214.

Angeloni, I. and Faia, E. (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, 60(3):311–324.

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and R&D as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Barsky, R., Justiniano, A., and Melosi, L. (2014). The natural rate of interest and its usefulness for monetary policy. *American Economic Review: Papers and Proceedings*, 104(5):37–43.

Bean, C. R., Broda, C., Ito, T., and Kroszner, R. S. (2015). Low for long? Causes and consequences of persistently low interest rates. Geneva Reports on the World Economy 17, CEPR-ICMB.

Becker, B. and Ivashina, V. (2014). Cyclicality of credit supply: Firm level evidence. *Journal of Monetary Economics*, 62:76–93.

Bekiros, S., Nilavongse, R., and Uddin, G. S. (2018). Bank capital shocks and countercyclical requirements: Implications for banking stability and welfare. *Journal of Economic Dynamics and Control*, 93:315–331.

Benigno, G. and Benigno, P. (2022). Managing monetary policy normalization. Staff Reports 1015, Federal Reserve Bank of New York.

Benigno, P. and Nisticò, S. (2017). Safe assets, liquidity, and monetary policy. *American Economic Journal: Macroeconomics*, 9(2):182–227.

Bernanke, B. S. (2005). The global saving glut and the U.S. current account deficit. Speech 77, Board of Governors of the Federal Reserve System (U.S.).

Bilbiie, F. O. (2009). Non-separable preferences, fiscal policy puzzles, and inferior goods. *Journal of Money, Credit and Banking*, 41(2-3):443–450.

Bilbiie, F. O. (2011). Non-separable preferences, Frisch labor supply and the consumption multiplier of government spending: One solution to a fiscal policy puzzle. *Journal of Money, Credit and Banking*, 43(1):221–251.

Borio, C. (2014). The financial cycle and macroeconomics: What have we learnt? *Journal of Banking & Finance*, 45:182–198.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the Great Recession. *American Economic Journal: Macroeconomics*, 7(1):110–167.

Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk shocks. *American Economic Review*, 104(1):27–65.

Cui, W. and Radde, S. (2019). Search-based endogenous asset liquidity and the macroeconomy. *Journal of the European Economic Association*, 18(5):2221–2269.

Cukierman, A. (2016). Reflections on the natural rate of interest, its measurement, monetary policy and the zero lower bound. CEPR Discussion Papers 11467, Centre for Economic Policy Research.

Cúrdia, V. (2015). Why so slow? A gradual return for interest rates. *FRBSF Economic Letter*, 2015-32.

De Fiore, F. and Uhlig, H. (2011). Bank finance versus bond finance. *Journal of Money, Credit and Banking*, 43(7):1399–1421.

Del Negro, M., Giannone, D., Giannoni, M. P., and Tambalotti, A. (2017). Safety, liquidity, and the natural rate of interest. *Brookings Papers on Economic Activity*, 2017(Spring):231–316.

Drechsler, I., Savov, A., and Schnabl, P. (2017). The deposits channel of monetary policy. *Quarterly Journal of Economics*, 132(4):1819–1876.

Durdu, C. B. and Zhong, M. (2021). Understanding bank and nonbank credit cycles: A structural exploration. BIS Working Papers 919, Bank for International Settlements. Forthcoming in the Journal of Money, Credit and Banking.

Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2019). A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics*, 11(1):1–48.

Fernald, J. G. (2014). A quarterly, utilization-adjusted series on total factor productivity. Working Paper Series 2012-19, Federal Reserve Bank of San Francisco.

Fisher, J. D. (2015). On the structural interpretation of the Smets-Wouters "Risk Premium" shock. *Journal of Money, Credit and Banking*, 47(2-3):511–516.

Furlanetto, F. and Seneca, M. (2014). Investment shocks and consumption. *European Economic Review*, 66:111–126.

Gagnon, E., Johannsen, B. K., and López-Salido, D. (2021). Understanding the new normal: The role of demographics. *IMF Economic Review*, 69(2):357–390.

Gerali, A. and Neri, S. (2019). Natural rates across the atlantic. *Journal of Macroeconomics*, 62:103019.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.

Guerron-Quintana, P. A. and Jinnai, R. (2019). On liquidity shocks and asset prices. Bank of Japan Working Paper Series 19-E-4, Bank of Japan. Forthcoming in the Journal of Money, Credit and Banking.

Havranek, T., Rusnak, M., and Sokolova, A. (2017). Habit formation in consumption: A meta-analysis. *European Economic Review*, 95:142–167.

Herman, A., Igan, D., and Solé, J. (2017). The macroeconomic relevance of bank and nonbank credit: An exploration of U.S. data. *Journal of Financial Stability*, 32:124–141.

Hirakata, N., Sudo, N., and Ueda, K. (2011). Do banking shocks matter for the U.S. economy? *Journal of Economic Dynamics and Control*, 35(12):2042–2063.

Hollander, H. and Liu, G. (2016). The equity price channel in a New-Keynesian DSGE model with financial frictions and banking. *Economic Modelling*, 52:375–389.

Jermann, U. and Quadrini, V. (2012). Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–71.

Jones, C. (2022). Aging, secular stagnation and the business cycle. *The Review of Economics and Statistics*, pages 1–46.

Juselius, M., Borio, C., Disyatat, P., and Drehmann, M. (2017). Monetary policy, the financial cycle, and ultra-low interest rates. *International Journal of Central Banking*, 13:55–89.

Kacperczyk, M. and Schnabl, P. (2013). How safe are money market funds? *The Quarterly Journal of Economics*, 128(3):1073–1122.

Kiley, M. T. (2015). What can the data tell us about the equilibrium real interest rate? Finance and Economics Discussion Series 2015-77, Board of Governors of the Federal Reserve System (U.S.).

Kiyotaki, N. and Moore, J. (2012). Liquidity, business cycles, and monetary policy. NBER Working Paper 17934, National Bureau of Economic Research.

Krishnamurthy, A. and Li, W. (2022). The demand for money, near-money, and Treasury bonds. NBER Working Paper 30051, National Bureau of Economic Research.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). The aggregate demand for Treasury debt. *Journal of Political Economy*, 120(2):233–267.

Laubach, T. and Williams, J. C. (2003). Measuring the natural rate of interest. *The Review of Economics and Statistics*, 85(4):1063–1070.

Laubach, T. and Williams, J. C. (2016). Measuring the natural rate of interest redux. *Business Economics*, 51(2):57–67.

Lin, L. (2020). Bank deposits and the stock market. *The Review of Financial Studies*, 33(6):2622–2658.

Mehra, R., Piguillem, F., and Prescott, E. C. (2011). Costly financial intermediation in neoclassical growth theory. *Quantitative Economics*, 2(1):1–36.

Nagel, S. (2016). The liquidity premium of near-money assets. *Quarterly Journal of Economics*, 131:1927–1971.

Pesaran, M. H. and Xu, T. (2016). Business cycle effects of credit shocks in a DSGE model with firm defaults. Mimeo.

Rauh, J. D. and Sufi, A. (2010). Capital structure and debt structure. *The Review of Financial Studies*, 23(12):4242–4280.

Shi, S. (2015). Liquidity, assets and business cycles. *Journal of Monetary Economics*, 70:116–132.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Suh, H. and Walker, T. B. (2016). Taking financial frictions to the data. *Journal of Economic Dynamics and Control*, 64:39–65.

Summers, L. H. (2014). U.S. economic prospects: Secular stagnation, hysteresis, and the zero lower bound. *Business Economics*, 49(2):65–73.

Taylor, J. B. (2007). Housing and monetary policy. NBER Working Paper 13682, National Bureau of Economic Research.

Taylor, J. B. (2011). *Macroeconomic Lessons from the Great Deviation*, pages 387–395. University of Chicago Press.

Taylor, J. B. and Wieland, V. (2016). Finding the equilibrium real interest rate in a fog of policy deviations. *Business Economics*, 51:147–154.

Wicksell, K. (1898). *Interest and Prices: A Study of the Causes Regulating the Value of Money*. MacMillan, London. Translated by R.F. Kahn (1936).

Wu, J. C. (2017). Comment on "Safety, liquidity, and the natural rate of interest". *Brookings Papers on Economic Activity*, 2017(Spring):303–310.

A Empirical model

A.1 Model description

As mentioned in Section 1.2, we estimate the DGGT version of the New York Fed DSGE model and an enhanced version of it that adds non-bank financial intermediaries and Gertler and Karadi (2011) frictions to the banking sector. We refer the reader to DGGT online appendix for a comprehensive description of their model, while the present section is limited to the specification of growth in the model economy and to a list of the log-linearized equilibrium conditions.

Growth is exogenous and driven by the technology process Z_t^*, defined as

$$Z_t^* = e^{\frac{1}{1-\alpha}\tilde{z}_t} Z_t^p e^{\gamma t},$$

which includes a stochastic trend (Z_t^p), a deterministic trend (γ), and a stationary component ($\tilde{z}_t$). $\tilde{z}_t$ and the growth rate of Z_t^p follow an AR(1) process.

Equations common to DGGT and NBFI[33]

- Household marginal utility of consumption:

$$\hat{c}_t = he^{-z^*}(\hat{c}_{t-1} - \hat{z}_t^*) - \frac{1 - he^{-z^*}}{\sigma_c}\hat{\tilde{\lambda}}_t + \frac{\sigma_c - 1}{\sigma_c}\frac{wL^h}{c}\hat{L}_t^h, \tag{1.A1}$$

  where $z_t^* = \frac{Z_t^*}{Z_{t-1}^*}$ is the stochastic growth rate of the economy and $\tilde{\lambda}_t$ is transformed marginal utility of consumption ($\lambda_t = \tilde{\lambda}_t\,(Z_t^*)^{-\sigma_c}$).

- Household liquid asset Euler equation:

$$\hat{\tilde{\lambda}}_t = \hat{\tilde{\lambda}}_{t+1} + \hat{R}_t - E_t\left[\hat{\pi}_{t+1}\right] + \hat{\varepsilon}_t^l - \sigma_c E_t[\hat{z}_{t+1}^*]. \tag{1.A2}$$

- Optimal investment decision:

$$\hat{i}_t = \frac{\hat{q}_t}{S'' e^{2z^*}(1+\tilde{\beta})} + \frac{1}{1+\tilde{\beta}}(\hat{i}_{t-1} - \hat{z}_t^*) + \frac{\tilde{\beta}}{1+\tilde{\beta}}E_t[\hat{i}_{t+1} + \hat{z}_{t+1}^*] + \hat{\mu}_t, \tag{1.A3}$$

  where $\tilde{\beta} = \beta e^{(1-\sigma_c)z^*}$ and $\hat{\mu}_t$ is a shock to the marginal efficiency of investment (MEI) that follows an AR(1) process.

- Optimal rate of capital utilization:

$$\hat{u}_t = \frac{1-\psi}{\psi}\hat{r}_t^k. \tag{1.A4}$$

[33] We use the following notation: for a given variable x_t, $\hat{x}_t$ and x respectively represent its log-deviation from the steady state and its steady-state value.

- Effective capital:

$$\hat{k}_t = \hat{\bar{k}}_{t-1} + \hat{u}_t - \hat{z}_t^*. \tag{1.A5}$$

- Entrepreneur nominal return to capital:

$$\hat{\tilde{R}}_t^k = \frac{r^k}{r^k + 1 - \delta} \hat{r}_t^k + \frac{1-\delta}{r^k + 1 - \delta} \hat{q}_t - \hat{q}_{t-1} + \hat{\pi}_t. \tag{1.A6}$$

- Entrepreneur excess return on capital (*i.e.* spread between expected return on capital and borrowing rate for entrepreneurs):

$$E_t[\hat{\tilde{R}}_{t+1}^k - \hat{R}_t^L] = \zeta_{sp,b}(\hat{q}_t + \hat{\bar{k}}_t - \hat{n}_t^e) + \hat{\tilde{\sigma}}_{\omega,t}, \tag{1.A7}$$

where $\hat{\tilde{\sigma}}_{\omega,t}$ is a shock to the riskiness of entrepreneurs (risk) that follows an AR(1) process.

- Entrepreneur net worth evolution:

$$\begin{aligned} \hat{n}_t^e = \zeta_{n^e,\tilde{R}^k}\left(\hat{\tilde{R}}_t^k - \hat{\pi}_t\right) - \zeta_{n^e,R}\left(\hat{R}_{t-1}^L - \hat{\pi}_t\right) + \zeta_{n^e,qK}(\hat{q}_{t-1} + \hat{\bar{k}}_{t-1}) + \\ + \zeta_{n^e,n^e}\hat{n}_{t-1}^e - \gamma_* \frac{v}{n^e}\hat{z}_t^* - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\hat{\tilde{\sigma}}_{\omega,t-1}, \end{aligned} \tag{1.A8}$$

where the ζ terms are steady-state elasticities, v is steady-state entrepreneur equity, and γ_* is the fraction of surviving entrepreneurs.

- Production function:

$$\hat{y}_t = \Phi\left[\alpha \hat{k}_t + (1-\alpha)\hat{L}_t^h\right]. \tag{1.A9}$$

- Capital evolution:

$$\hat{\bar{k}}_t = \left(1 - \frac{i}{\bar{k}}\right)\left(\hat{\bar{k}}_{t-1} - \hat{z}_t^*\right) + \frac{i}{\bar{k}}\hat{i}_t + \frac{i}{\bar{k}} S'' e^{2z^*}(1+\tilde{\beta})\hat{\mu}_t. \tag{1.A10}$$

- Real rental rate of capital:

$$\hat{r}_t^k = \hat{L}_t^h + \hat{w}_t - \hat{k}_t. \tag{1.A11}$$

- Real marginal costs:

$$\widehat{mc}_t = \hat{w}_t + \alpha\left(\hat{L}_t^h - \hat{k}_t\right). \tag{1.A12}$$

- Marginal rate of substitution between consumption and labor:

$$\hat{\mu}_{w,t} = \hat{w}_t - \nu_l \hat{L}_t^h - \frac{1}{1 - he^{-z^*}}\hat{c}_t + \frac{he^{-z^*}}{1 - he^{-z^*}}\left(\hat{c}_{t-1} - \hat{z}_t^*\right). \tag{1.A13}$$

- Price Phillips curve:

$$\hat{\pi}_t = \frac{\iota_p}{1 + \iota_p\tilde{\beta}}\hat{\pi}_{t-1} + \frac{\tilde{\beta}}{1 + \iota_p\tilde{\beta}}E_t[\hat{\pi}_{t+1}] + \frac{(1 - \zeta_p\tilde{\beta})(1 - \zeta_p)}{\zeta_p[(\Phi - 1)\epsilon_p + 1](1 + \iota_p\tilde{\beta})}\widehat{mc}_t + \hat{\lambda}_{p,t}, \tag{1.A14}$$

where $\hat{\lambda}_{p,t}$ is a price markup shock that follows an ARMA(1,1) process.

- Wage Phillips curve:

$$w_t = -\frac{(1-\zeta_w\tilde{\beta})(1-\zeta_w)}{\zeta_w\left[(\lambda_w-1)\epsilon_w+1\right](1+\tilde{\beta})}\hat{\mu}_{w,t} + \frac{1}{1+\tilde{\beta}}(\hat{w}_{t-1} - \hat{z}_t^* + \iota_w\hat{\pi}_{t-1}) - \frac{1+\iota_w\tilde{\beta}}{1+\tilde{\beta}}\hat{\pi}_t + \\ + \frac{\tilde{\beta}}{1+\tilde{\beta}}E_t[\hat{w}_{t+1} + \hat{\pi}_{t+1} + \hat{z}_{t+1}^*] + \hat{\lambda}_{w,t}, \tag{1.A15}$$

where $\hat{\lambda}_{w,t}$ is a wage markup shock that follows an ARMA(1,1) process.

- Aggregate resource constraint:

$$\hat{y}_t = \frac{c}{i}\hat{c}_t + \frac{i}{y}\hat{i}_t + r^k\frac{k}{y}\hat{u}_t + g_*\hat{g}_t, \tag{1.A16}$$

where exogenous government spending is defined as $\hat{g}_t = log\left(\frac{G_t}{Z_t^* y g_*}\right)$ and follows an AR(1) process (the shock is allowed to be correlated with stationary technology innovations).

- Monetary policy rule:

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\psi_1(\hat{\pi}_t - \pi_t^*) + \psi_2(\hat{y}_t - \hat{y}_t^f)\right] + \psi_3\left[(\hat{y}_t - \hat{y}_t^f) - (\hat{y}_{t-1} - \hat{y}_{t-1}^f)\right] + \hat{r}_t^m, \tag{1.A17}$$

where $\hat{y}_t^f$ is output in the flexible-price economy, $\hat{\pi}_t^*$ is a stochastic inflation target (that follows an AR(1) process), and $\hat{r}_t^m$ is a monetary policy shock. The latter evolves as follows:

$$\hat{r}_t^m = \rho_{r^m}\hat{r}_{t-1}^m + \epsilon_t^R + \sum_{k=1}^{K}\epsilon_{k,t-k}^R, \tag{1.A18}$$

where ϵ_t^R is the standard contemporaneous shock, whereas $\epsilon_{k,t-k}^R$ is a shock that is known to agents at time $t-k$ but takes effect in time t: thus, it can be interpreted as a forward guidance shock in that it anticipates future policy decisions by k quarters.

- The DGGT model is closed by the following condition (that derives from the household illiquid asset Euler equation):

$$\hat{R}_t^L = \hat{R}_t^{d,b} = \hat{R}_t + \hat{\varepsilon}_t^l. \tag{1.A19}$$

Equations specific to NBFI

- Equation (1.A19) is replaced by

$$\hat{R}_t^{d,b} = \hat{R}_t, \tag{1.A20}$$

$$\hat{R}_t^{d,NBFI} = \hat{R}_t + \hat{\varepsilon}_t^l. \tag{1.A21}$$

- NBFI (and bank) lending rate:

$$\hat{R}_t^L = \hat{R}_t^{d,NBFI} + (1-\alpha_{NBFI})\hat{d}_t^{NBFI}. \tag{1.A22}$$

- NBFI loan supply:

$$\hat{l}_t^{NBFI} = \alpha_{NBFI}\hat{d}_t^{NBFI}. \tag{1.A23}$$

- Bank loan supply:

$$\hat{l}_t^b = \hat{\phi}_t^b + \hat{n}_t^b. \tag{1.A24}$$

- Value of bank capital:

$$\hat{\nu}_t = \tilde{\beta}\left[\frac{1-\theta}{\nu}(R^L - R) + \theta m\right] E_t\left[\hat{\tilde{\lambda}}_{t+1} - \hat{\tilde{\lambda}}_t - \sigma_c \hat{z}_{t+1}^*\right] + \frac{1-\theta}{\nu}\tilde{\beta}E_t\left[R^L\hat{R}_{t+1}^L - R\hat{R}_t^{d,b}\right] + \\ +\theta\tilde{\beta}mE_t\left[\hat{m}_{t+1} + \hat{\nu}_{t+1}\right]. \tag{1.A25}$$

- Value of bank net worth:

$$\hat{\eta}_t = E_t\left[\hat{\tilde{\lambda}}_{t+1} - \sigma_c\hat{z}_{t+1}^*\right] - \hat{\tilde{\lambda}}_t + \left(1-\theta\tilde{\beta}\zeta\right)\hat{R}_t^{d,b} + \theta\tilde{\beta}\zeta E_t\left[\hat{\zeta}_{t+1} + \hat{\eta}_{t+1}\right]. \tag{1.A26}$$

- Bank optimal leverage:

$$\hat{\phi}_t^b = \hat{\eta}_t + \frac{\nu}{\Lambda - \nu}\hat{\nu}_t. \tag{1.A27}$$

- Growth rate of bank capital:

$$\zeta\hat{\zeta}_t = R^L\phi^b\hat{R}_t^L + (R^L - R)\phi^b\hat{\phi}_{t-1}^b + R(1-\phi^b)\hat{R}_{t-1}^d. \tag{1.A28}$$

- Growth rate of bank net worth:

$$\hat{m}_t = \hat{\zeta}_t + \hat{\phi}_t^b - \hat{\phi}_{t-1}^b. \tag{1.A29}$$

- Bank net worth evolution:

$$\hat{n}_t^b = \theta\zeta e^{-z^*}\left(\hat{\zeta}_t - \hat{z}_t^* + \hat{n}_{t-1}^b + \hat{\varepsilon}_t^{Nb}\right) + \Omega\phi^b\hat{l}_t^b. \tag{1.A30}$$

- Total loan supply:

$$\hat{l}_t = \frac{l^b}{l}\hat{l}_t^b + \frac{1-l^b}{l}\hat{l}_t^{NBFI}. \tag{1.A31}$$

- Entrepreneur balance sheet:

$$\hat{q}_t + \hat{\bar{k}}_t = \frac{1-n^e}{\bar{k}}\hat{l}_t + \frac{n^e}{\bar{k}}\hat{n}_t^e. \tag{1.A32}$$

- Share of bank loans over total credit:

$$\hat{l}_t^{b,share} = \hat{l}_t^b - \hat{l}_t. \tag{1.A33}$$

A.2 Shocks and measurement errors

The model is characterized by the following structural shocks: stationary technology shock and shock to the growth rate of technology; liquidity shock; MEI shock; risk shock; wage and price markup shock; government spending shock; inflation target shock; contemporaneous monetary policy shock; anticipated monetary policy shocks up to six quarters ahead; bank net worth shock (only for the NBFI specification).

The flight-to-quality shock appearing in equilibrium conditions (1.A2), (1.A19), and (1.A21), $\hat{\varepsilon}_t^l$, is defined as the sum of safety and liquidity shocks:

$$\hat{\varepsilon}_t^l = \hat{\varepsilon}_t^{safe} + \hat{\varepsilon}_t^{liq},$$

where the first term is the sum of transitory and permanent safety shocks, namely $\hat{\varepsilon}_t^{safe,T}$ and $\hat{\varepsilon}_t^{safe,P}$, and the second is the sum of transitory and permanent liquidity shocks, $\hat{\varepsilon}_t^{liq,T}$ and $\hat{\varepsilon}_t^{liq,P}$. All four shocks follow AR(1) processes.

In addition to the structural shocks, measurement errors, denoted by e_t, are assumed for a subset of the observables. These include output growth (with two distinct errors on GDP and GDI growth), inflation (with two distinct errors on core PCE inflation and GDP deflator inflation), the 10-year Treasury yield, TFP growth, the Aaa- and Baa-Treasury spreads, and the NBFI deposit rate (only for the NBFI specification).

A.3 Measurement equations

$$\begin{aligned}
\text{GDP growth} &= 100\gamma + (\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t^*) + e_t^{gdp} - e_{t-1}^{gdp} \\
\text{GDI growth} &= 100\gamma + (\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t^*) + e_t^{gdi} - e_{t-1}^{gdi} \\
\text{Consumption growth} &= 100\gamma + (\hat{c}_t - \hat{c}_{t-1} + \hat{z}_t^*) \\
\text{Investment growth} &= 100\gamma + (\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t^*) \\
\text{Real wage growth} &= 100\gamma + (\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t^*) \\
\text{Hours} &= \bar{L} + \hat{L}_t^h \\
\text{Core PCE inflation} &= 100(\pi - 1) + \hat{\pi}_t + e_t^{pce} \\
\text{GDP deflator inflation} &= 100(\pi - 1) + \delta_{gdpdef} + \gamma_{gdpdef}\hat{\pi}_t + e_t^{gdpdef} \\
\text{Fed funds rate} &= 100(R - 1) + \hat{R}_t \\
\text{Fed funds rate expectations} &= 100(R - 1) + E_t\left[\frac{1}{40}\hat{R}_{t+j}\right], \quad j = 1, ..., 6 \\
\text{10-year Treasury yield} &= 100(R - 1) + E_t\left[\frac{1}{40}\sum_{j=0}^{39}\hat{R}_{t+j}\right] + e_t^{10y} \\
\text{10-year inflation expectations} &= 100(\pi - 1) + E_t\left[\frac{1}{40}\sum_{j=0}^{39}\hat{\pi}_{t+j}\right] \\
\text{TFP growth, demeaned} &= \hat{z}_t^* + \frac{\alpha}{1-\alpha}(\hat{u}_t - \hat{u}_{t-1}) + e_t^{tfp}
\end{aligned}$$

$$\text{Aaa - 20-year Treasury spread} = 100 \ln(\varepsilon^{liq}) + E_t\left[\frac{1}{80}\sum_{j=0}^{79}\hat{\varepsilon}^{liq}_{t+j}\right] + e^{Aaa}_t$$

$$\text{Baa - 20-year Treasury spread} = 100 \ln\left(\varepsilon^{liq}\varepsilon^{safe}SP_*\right) + E_t\left[\frac{1}{80}\sum_{j=0}^{79}(\hat{\tilde{R}}^k_{t+j+1} - \hat{R}_{t+j})\right] + e^{Baa}_t \quad \text{(DGGT, Augmented)}$$

$$= 100 \ln\left(\varepsilon^{liq}\varepsilon^{safe}SP_*\frac{d^{NBFI}}{l^{NBFI}}\right) + E_t\left[\frac{1}{80}\sum_{j=0}^{79}(\hat{\tilde{R}}^k_{t+j+1} - \hat{R}_{t+j})\right] + e^{Baa}_t \quad \text{(NBFI)}$$

$$\text{3-month CD rate} = 100(R\varepsilon^{liq}\varepsilon^{safe} - 1) + \hat{R}^{d,b}_t \quad \text{(Augmented)}$$

$$\text{3-month P1CP rate} = 100(R\varepsilon^{liq}\varepsilon^{safe} - 1) + \hat{R}^{d,NBFI}_t + e^{R^{NBFI}}_t \quad \text{(NBFI)}$$

$$\text{Bank-loans-share growth} = \hat{l}^{b,share}_t - \hat{l}^{b,share}_{t-1} \quad \text{(NBFI)}$$

$\bar{L}$ is the mean level of hours worked, which is estimated. δ_{gdpdef} and γ_{gdpdef} allow for a different "matching function" between the model concept of inflation and the GDP deflator measure with respect to core PCE inflation. SP_* is an estimated interest rate spread. $\frac{d^{NBFI}}{l^{NBFI}}$ is an additional steady-state spread component arising from the decreasing returns to scale in the technology of NBFIs.

A.4 Data

Sources

Data construction for the observables used in the benchmark estimation strictly follows DGGT. Data on nominal GDP [GDP], nominal GDI [GDI], the GDP deflator [GDPDEF], core PCE inflation [PCEPILFE], nominal personal consumption expenditures [PCE], and nominal fixed private investment [FPI] are produced at a quarterly frequency by the Bureau of Economic Analysis (BEA) and are included in the National Income and Product Accounts (NIPA). Average weekly hours of production and nonsupervisory employees for total private industries [AWHNONAG], civilian employment [CE16OV], and the civilian non-institutional population [CNP16OV] are produced by the Bureau of Labor Statistics (BLS) at a monthly frequency. The first of these series is obtained from the Establishment Survey, and the remaining from the Household Survey. Both surveys are released in the BLS Employment Situation Summary. We take quarterly averages of the monthly data. Compensation per hour for the non-farm business sector [COMPNFB] is obtained from the Labor Productivity and Costs release and is produced by the BLS at a quarterly frequency. The federal funds rate [DFF] is obtained from the Federal Reserve Board's H.15 release at a business day frequency. The 10-year Treasury yield (zero-coupon, continuously compounded) series [SVENY10] is made available by the Board of Governors of the Federal Reserve System at a business day frequency. Corporate-Treasury spreads are computed as the difference between the Moody's seasoned Baa (Aaa) corporate bond yield [BAA] ([AAA]) and the yield on U.S. Treasury securities at 20-year constant maturity [GS20], also at a business day frequency (all data obtained from the Federal Reserve Board's H.15 release). We take quarterly averages of the annualized daily data. Quarterly data on 10-year CPI inflation expectations are made available by the Federal Reserve Bank of Philadelphia: these combine the series from the Survey of Professional Forecasters [INFCPI10YR] since 1991:IV and data from the Blue Chip Economic Indicators from 1979:IV to 1991:I. The measure for TFP growth is from Fernald (2014),[34] whose updated series (unadjusted for utilization) is made available by the Federal Reserve Bank of San Francisco at a quarterly frequency [dtfp; alpha]. Lastly, data on interest rate expectations, from 1 to 6 quarters ahead, are taken directly from the DGGT dataset,[35] since they use internal data from the Federal Reserve Board on the implied federal funds rate derived from OIS quotes.

Turning to the Augmented estimation, our baseline measure for the deposit rate is the secondary market rate on 3-month CDs, which comes from the OECD Main Economic Indicators database [IR3TCD01USQ156N] and is available at a quarterly frequency. The alternative series for the 1- [CD1M] and 6-month secondary market CD rates [CD6M], both discontinued in July 2013, are obtained from the Federal Reserve Board's H.15 release (monthly averages of business-day data). The M2 own rate series [M2OWN], discontinued in July 2019, was made available by the Board of Governors of the Federal Reserve System at a monthly frequency. Our last measure is the savings deposit rate from Drechsler et al. (2017), who gather monthly bank data from U.S. Call reports and compute the average interest rate paid on different forms of deposits by U.S. commercial banks. We take quarterly averages of the monthly data.

Our proxy for the interest rate on non-bank deposits is constructed by combining two sources: the interest rate on 3-month prime commercial paper [WCP3M], available from 1971:II to 1996:IV (Federal Reserve Board's H.15 release), and the 90-day AA nonfinancial commercial paper interest rate [RIFSPPNAAD90NB], available from 1997:I (Board of Governors of the Federal Reserve System). The former is available at a weekly frequency, the latter at a business day frequency, and we take quarterly averages of both. Data on bank and non-bank loans are from Table B.103 of the Financial Accounts of the United States Z.1 release, *i.e.* we consider the liabilities of the Nonfinancial Corporate Business sector: bank credit is the sum of Other Loans and Advances [OLALBSNNCB] and Bank Loans Not Elsewhere Classified [BLNECLBSNNCB], while non-bank credit is the sum of Commercial Paper [CPLBSNNCB] and Corporate Bonds [CBLBSNNCB], all available at a quarterly frequency.

Transformations

Following DGGT, civilian population data are treated with a Hodrick-Prescott filter. The resulting series is used to transform GDP, GDI, consumption, investment, and hours worked in per-capita terms. GDP, GDI, consumption, investment, and wages are also set in real terms by dividing them by the GDP deflator. The fed funds rate, the 10-year Treasury yield, the corporate-Treasury spreads, 10-year inflation expectations, and both bank and non-bank deposit rates are divided by 4 to express them in quarterly terms. 10-year inflation is further

[34] Available at `https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/`.

[35] Available at `https://github.com/FRBNY-DSGE/rstarBrookings2017`.

adjusted for the average differential between CPI and GDP deflator inflation. Finally, the TFP growth series is demeaned, divided by 4 (to convert it into quarterly growth rates), and divided by Fernald (2014) estimate of the labor share to express it in labor-augmenting terms.

$$
\begin{aligned}
\text{Output growth} &= 100\Delta\ln[(GDP/GDPDEF)/CNP16OV] \\
&= 100\Delta\ln[(GDI/GDPDEF)/CNP16OV] \\
\text{Consumption growth} &= 100\Delta\ln[(PCEC/GDPDEF)/CNP16OV] \\
\text{Investment growth} &= 100\Delta\ln[(FPI/GDPDEF)/CNP16OV] \\
\text{Wage growth} &= 100\Delta\ln(COMPNFB/GDPDEF) \\
\text{Hours worked} &= 100\ln[(AWHNONAG)(CE16OV/100)/CN16OV] \\
\text{Core PCE inflation} &= 100\Delta\ln(PCEPILFE) \\
\text{GDP deflator inflation} &= 100\Delta\ln(GDPDEF) \\
\text{Fed funds rate} &= DFF/4 \\
\text{Fed funds rate expectations} &= OIS/4 \\
\text{10-year bond yield} &= SVENY10/4 \\
\text{10-year inflation expectations} &= (INFCPI10YR - 0.5)/4 \\
\text{TFP growth, demeaned} &= (dtfp, demeaned)/[4(1 - alpha)] \\
\text{Aaa - 20-year Treasury spread} &= (AAA - GS20)/4 \\
\text{Baa - 20-year Treasury spread} &= (BAA - GS20)/4 \\
\text{3-month CD rate} &= IR3TCD01USQ156N/4 \\
\text{3-month P1CP rate} &= RIFSPPNAAD90NB/4 \\
\text{Bank-loans-share growth} &= 100\Delta\ln\left[\frac{OLALBS + BLNECLBS}{OLALBS + BLNECLBS + CPLBS + CBLBS}\right]
\end{aligned}
$$

A.5 Parameters

Description and priors

Table A1: Parameters' description and priors

Parameter	Description	Type	Prior Mean	SD
	Steady State			
100γ	Technology growth rate	N	0.400	0.100
α	Capital share	N	0.300	0.050
$100(\beta^{-1} - 1)$	Discount rate	G	0.250	0.100
σ_c	Inverse EIS	N	1.500	0.370
h	Consumption habits	B	0.700	0.100
ν_l	Inverse Frisch elasticity	N	2.000	0.750

Table A1: Parameters' description and priors

Parameter	Description	Type	Prior Mean	SD
δ	Capital depreciation rate	-	0.025	-
Φ_p	Production fixed costs	-	1.000	-
S''	Investment adj. costs	N	4.000	1.500
ψ	Utilization costs	B	0.500	0.150
$\bar{L}$	Mean level of hours	N	-45.000	5.000
λ_w	SS wage markup	-	1.500	-
π_*	SS inflation	-	0.500	-
g_*	SS govt. spending	-	0.180	-
	Nominal Rigidities			
ζ_p	Calvo price stickiness	B	0.500	0.100
ζ_w	Calvo wage stickiness	B	0.500	0.100
ι_p	Price indexation	B	0.500	0.150
ι_w	Wage indexation	B	0.500	0.150
ϵ_p	Price Kimball curvature	-	10.000	-
ϵ_w	Wage Kimball curvature	-	10.000	-
	Policy			
ψ_1	Weight on inflation	N	1.500	0.250
ψ_2	Weight on output gap	N	0.120	0.050
ψ_3	Weight on output gap growth	N	0.120	0.050
ρ_R	Interest rate smoothing	B	0.750	0.100
	Financial Frictions			
$F(\bar{\omega})$	Entrepreneur SS prob. of default	-	0.030	-
SP_*	SS spread	G	1.000	0.100
$\zeta_{sp,b}$	Spread elasticity to leverage	B	0.050	0.005
γ_*	Entrepreneur survival prob.	-	0.990	-
α_{NBFI}	NBFI returns to scale (*)	-	0.990	-
ϑ	Banker survival prob. (*)	-	0.972	-
ε^{safe}	SS safety premium	-	0.000 (0.065)	-
ε^{liq}	SS liquidity premium	-	0.000 (0.117)	-
	Exogenous Processes			
ρ_g	Govt. spending a.c.	B	0.500	0.200
ρ_μ	MEI a.c.	B	0.500	0.200
ρ_{z^p}	Permanent technology a.c.	-	0.990	-
ρ_z	Stationary technology a.c.	B	0.500	0.200
$\rho_{liq,P}$	Permanent liquidity a.c.	-	0.990	-
$\rho_{liq,T}$	Transitory liquidity a.c.	B	0.500	0.200
$\rho_{safe,P}$	Permanent safety a.c.	-	0.990	-
$\rho_{safe,T}$	Transitory safety a.c.	B	0.500	0.200
ρ_{σ_ω}	Entrepreneur risk a.c.	B	0.750	0.150
ρ_{π_*}	Inflation target a.c.	-	0.990	-
ρ_{λ_f}	Price markup a.c.	B	0.500	0.200
ρ_{λ_w}	Wage markup a.c.	B	0.500	0.200

Table A1: Parameters' description and priors

Parameter	Description	Type	Prior Mean	SD
η_{λ_f}	Price markup MA coeff.	B	0.500	0.200
η_{λ_w}	Wage markup MA coeff.	B	0.500	0.200
ρ_{r^m}	Monetary policy a.c.	B	0.500	0.200
ρ_{Nb}	Banker net worth a.c. (*)	B	0.500	0.200
η_{gz}	Govt. spending/technology corr.	B	0.500	0.200
σ_g	Govt. spending s.d.	IG	0.100	2.000
σ_μ	MEI s.d.	IG	0.100	2.000
σ_{z^p}	Permanent technology s.d.	IG	0.100	2.000
σ_z	Stationary technology s.d.	IG	0.100	2.000
$\sigma_{liq,P}$	Permanent liquidity s.d.	IG	0.030	6.000
$\sigma_{liq,T}$	Transitory liquidity s.d.	IG	0.100	2.000
$\sigma_{safe,P}$	Permanent safety s.d.	IG	0.030	6.000
$\sigma_{safe,T}$	Transitory safety s.d.	IG	0.100	2.000
σ_{σ_ω}	Entrepreneur risk s.d.	IG	0.050	4.000
σ_{π_*}	Inflation target s.d.	IG	0.030	6.000
σ_{λ_f}	Price markup s.d.	IG	0.100	2.000
σ_{λ_w}	Wage markup s.d.	IG	0.100	2.000
σ_{r^m}	Monetary policy s.d.	IG	0.100	2.000
σ_{Nb}	Banker net worth s.d. (*)	IG	0.100	2.000
$\sigma_{1,r}$	1-quarter-ahead mon. policy s.d.	IG	0.200	4.000
$\sigma_{2,r}$	2-quarter-ahead mon. policy s.d.	IG	0.200	4.000
$\sigma_{3,r}$	3-quarter-ahead mon. policy s.d.	IG	0.200	4.000
$\sigma_{4,r}$	4-quarter-ahead mon. policy s.d.	IG	0.200	4.000
$\sigma_{5,r}$	5-quarter-ahead mon. policy s.d.	IG	0.200	4.000
$\sigma_{6,r}$	6-quarter-ahead mon. policy s.d.	IG	0.200	4.000
	Measurement			
δ_{gdpdef}	GDP deflator param. 1	N	0.000	2.000
γ_{gdpdef}	GDP deflator param. 2	N	1.000	2.000
ρ_{gdp}	GDP growth a.c.	N	0.000	0.200
ρ_{gdi}	GDI growth a.c.	N	0.000	0.200
ϱ_{gdp}	GDP/GDI corr.	N	0.000	0.400
ρ_{gdpdef}	GDP deflator a.c.	B	0.500	0.200
ρ_{pce}	PCE inflation a.c.	B	0.500	0.200
ρ_{AAA}	Aaa-Treasury spread a.c.	B	0.500	0.100
ρ_{BBB}	Baa-Treasury spread a.c.	B	0.500	0.100
ρ_{10y}	10-year yield a.c.	B	0.500	0.200
ρ_{tfp}	TFP growth a.c.	B	0.500	0.200
$\rho_{R^{NBFI}}$	NBFI deposit rate a.c. (*)	B	0.500	0.200
σ_{gdp}	GDP growth s.d.	IG	0.100	2.000
σ_{gdi}	GDI growth s.d.	IG	0.100	2.000
σ_{gdpdef}	GDP deflator s.d.	IG	0.100	2.000
σ_{pce}	PCE inflation s.d.	IG	0.100	2.000
σ_{AAA}	Aaa-Treasury s.d.	IG	0.100	2.000

Table A1: Parameters' description and priors

Parameter	Description	Type	Prior Mean	SD
σ_{BBB}	Baa-Treasury s.d.	IG	0.100	2.000
σ_{10y}	10-year yield s.d.	IG	0.750	2.000
σ_{tfp}	TFP growth s.d.	IG	0.100	2.000
$\sigma_{R^{NBFI}}$	NBFI deposit rate s.d. (*)	IG	0.100	2.000

Note: For Inverse Gamma (IG) prior mean and standard deviation, τ and ν reported. When the type and the standard deviation of a prior are not specified, the parameter of interest is fixed. Terms in round brackets refer to the prior specifications used in the estimation of DGGT, when different from the baseline Augmented and NBFI priors. Parameters denoted with (*) are specific to the NBFI-model estimation.

Posterior estimates

Table A2: Parameters' Estimates

Parameter	DGGT Posterior Mean	Augmented Posterior Mean	Augmented 90.0% L.	Augmented 90.0% U.	NBFI Posterior Mean	NBFI 90.0% L.	NBFI 90.0% U.
			Steady State				
100γ	0.434	0.418	0.342	0.495	0.429	0.344	0.510
α	0.187	0.225	0.206	0.244	0.118	0.104	0.132
$100(\beta^{-1}-1)$	0.287	0.135	0.065	0.206	0.114	0.043	0.183
σ_c	0.903	1.303	1.172	1.426	1.420	1.148	1.696
h	0.495	0.208	0.159	0.257	0.310	0.216	0.397
ν_l	2.482	2.618	1.728	3.486	2.815	2.072	3.527
δ	0.025	0.025	-	-	0.025	-	-
Φ_p	1.000	1.000	-	-	1.000	-	-
S''	1.466	4.818	3.315	6.292	4.056	3.281	4.751
ψ	0.615	0.683	0.574	0.801	0.714	0.561	0.867
δ_{gdpdef}	0.001	0.008	-0.028	0.044	0.013	-0.030	0.055
$\bar{L}$	-47.413	-47.538	-49.924	-45.276	-46.009	-48.320	-43.700
λ_w	1.500	1.500	-	-	1.500	-	-
π_*	0.500	0.500	-	-	0.500	-	-
g_*	0.180	0.180	-	-	0.180	-	-
			Nominal Rigidities				
ζ_p	0.957	0.952	0.943	0.961	0.951	0.940	0.961
ζ_w	0.967	0.962	0.956	0.968	0.971	0.967	0.976
ι_p	0.210	0.253	0.110	0.385	0.261	0.124	0.405
ι_w	0.821	0.809	0.713	0.913	0.897	0.835	0.958
ϵ_p	10.000	10.000	-	-	10.000	-	-
ϵ_w	10.000	10.000	-	-	10.000	-	-
			Policy				
ψ_1	1.797	1.649	1.367	1.929	1.221	1.062	1.375

Table A2: Parameters' Estimates

Parameter	DGGT Posterior Mean	Augmented Posterior Mean	Augmented Posterior 90.0% L.	Augmented Posterior 90.0% U.	NBFI Posterior Mean	NBFI Posterior 90.0% L.	NBFI Posterior 90.0% U.
ψ_2	0.277	0.191	0.147	0.236	0.088	0.058	0.121
ψ_3	0.335	0.371	0.320	0.426	0.284	0.231	0.338
ρ_R	0.856	0.819	0.777	0.861	0.670	0.603	0.736
			Financial Frictions				
$F(\bar{\omega})$	0.030	0.030	-	-	0.030	-	-
SP_*	1.047	0.970	0.807	1.137	1.222	1.064	1.373
$\zeta_{sp,b}$	0.045	0.049	0.042	0.055	0.042	0.037	0.048
γ_*	0.990	0.990	-	-	0.990	-	-
α_{NBFI}	-	-	-	-	0.990	-	-
ϑ	-	-	-	-	0.972	-	-
ε^{safe}	0.065	0.000	-	-	0.000	-	-
ε^{liq}	0.117	0.000	-	-	0.000	-	-
			Exogenous Processes				
ρ_g	0.991	0.992	0.986	0.998	0.989	0.985	0.994
ρ_μ	0.237	0.340	0.254	0.428	0.388	0.223	0.547
ρ_{z^p}	0.990	0.990	-	-	0.990	-	-
ρ_z	0.935	0.963	0.944	0.975	0.967	0.956	0.980
$\rho_{liq,P}$	0.990	0.990	-	-	0.990	-	-
$\rho_{liq,T}$	0.870	0.489	0.252	0.730	0.459	0.192	0.721
$\rho_{safe,P}$	0.990	0.990	-	-	0.990	-	-
$\rho_{safe,T}$	0.626	0.593	0.381	0.809	0.489	0.207	0.760
ρ_{σ_ω}	0.987	0.957	0.938	0.976	0.963	0.934	0.989
ρ_{π_*}	0.990	0.990	-	-	0.990	-	-
ρ_{λ_f}	0.807	0.850	0.782	0.917	0.824	0.737	0.911
ρ_{λ_w}	0.324	0.342	0.099	0.560	0.884	0.878	0.892
η_{λ_f}	0.596	0.677	0.543	0.841	0.685	0.549	0.836
η_{λ_w}	0.402	0.412	0.207	0.604	0.891	0.885	0.898
η_{gz}	0.500	0.410	0.130	0.687	0.531	0.237	0.820
ρ_{r^m}	0.191	0.110	0.038	0.182	0.215	0.101	0.332
ρ_{Nb}	-	-	-	-	0.946	0.914	0.981
σ_g	2.240	2.350	2.142	2.553	2.268	2.070	2.456
σ_μ	0.422	0.665	0.584	0.748	0.602	0.521	0.684
σ_{z^p}	0.050	0.073	0.062	0.083	0.061	0.047	0.074
σ_z	0.522	0.556	0.506	0.608	0.556	0.507	0.604
$\sigma_{liq,P}$	0.019	0.021	0.018	0.023	0.012	0.011	0.012
$\sigma_{liq,T}$	0.067	0.014	0.013	0.015	0.036	0.031	0.040
$\sigma_{safe,P}$	0.013	0.014	0.013	0.015	0.029	0.026	0.032
$\sigma_{safe,T}$	0.155	0.031	0.028	0.034	0.035	0.030	0.041
σ_{σ_ω}	0.067	0.138	0.097	0.177	0.149	0.080	0.227
σ_{π_*}	0.056	0.063	0.049	0.076	0.030	0.020	0.039
σ_{λ_f}	0.067	0.060	0.043	0.076	0.068	0.052	0.084
σ_{λ_w}	0.395	0.399	0.359	0.445	0.378	0.344	0.414
σ_{r^m}	0.225	0.238	0.218	0.256	0.224	0.205	0.244

Table A2: Parameters' Estimates

Parameter	DGGT Posterior Mean	Augmented Posterior Mean	Augmented 90.0% L.	Augmented 90.0% U.	NBFI Posterior Mean	NBFI 90.0% L.	NBFI 90.0% U.
σ_{Nb}	-	-	-	-	1.316	1.041	1.575
$\sigma_{1,r}$	0.092	0.100	0.075	0.118	0.098	0.079	0.124
$\sigma_{2,r}$	0.090	0.087	0.066	0.105	0.081	0.065	0.095
$\sigma_{3,r}$	0.088	0.084	0.070	0.097	0.083	0.065	0.100
$\sigma_{4,r}$	0.092	0.095	0.072	0.111	0.085	0.068	0.099
$\sigma_{5,r}$	0.090	0.095	0.073	0.128	0.088	0.072	0.105
$\sigma_{6,r}$	0.086	0.088	0.072	0.103	0.094	0.078	0.111
			Measurement				
δ_{gdpdef}	0.001	0.008	-0.028	0.044	0.013	-0.030	0.055
γ_{gdpdef}	1.047	1.031	0.962	1.101	1.027	0.954	1.099
ρ_{gdp}	0.017	0.023	-0.213	0.242	-0.009	-0.229	0.211
ρ_{gdi}	0.947	0.941	0.907	0.975	0.940	0.902	0.973
ϱ_{gdp}	-0.154	-0.183	-0.809	0.402	-0.178	-0.806	0.414
ρ_{gdpdef}	0.412	0.401	0.266	0.538	0.416	0.284	0.548
ρ_{pce}	0.222	0.254	0.072	0.421	0.233	0.060	0.389
ρ_{AAA}	0.639	0.722	0.613	0.837	0.750	0.667	0.832
ρ_{BBB}	0.913	0.954	0.939	0.969	0.892	0.856	0.926
ρ_{10y}	0.951	0.946	0.919	0.979	0.944	0.914	0.976
ρ_{tfp}	0.271	0.272	0.157	0.380	0.287	0.177	0.394
$\rho_{R^{NBFI}}$	-	-	-	-	0.465	0.183	0.736
σ_{gdp}	0.243	0.241	0.200	0.285	0.244	0.200	0.289
σ_{gdi}	0.311	0.308	0.275	0.347	0.312	0.277	0.346
σ_{gdpdef}	0.171	0.173	0.159	0.186	0.175	0.160	0.188
σ_{pce}	0.116	0.119	0.103	0.134	0.118	0.101	0.135
σ_{AAA}	0.022	0.027	0.024	0.031	0.035	0.033	0.038
σ_{BBB}	0.050	0.059	0.054	0.064	0.063	0.060	0.067
σ_{10y}	0.123	0.121	0.112	0.130	0.127	0.115	0.136
σ_{tfp}	0.667	0.619	0.554	0.685	0.712	0.650	0.774
$\sigma_{R^{NBFI}}$	-	-	-	-	0.037	0.032	0.042

B Robustness checks on the Augmented estimation

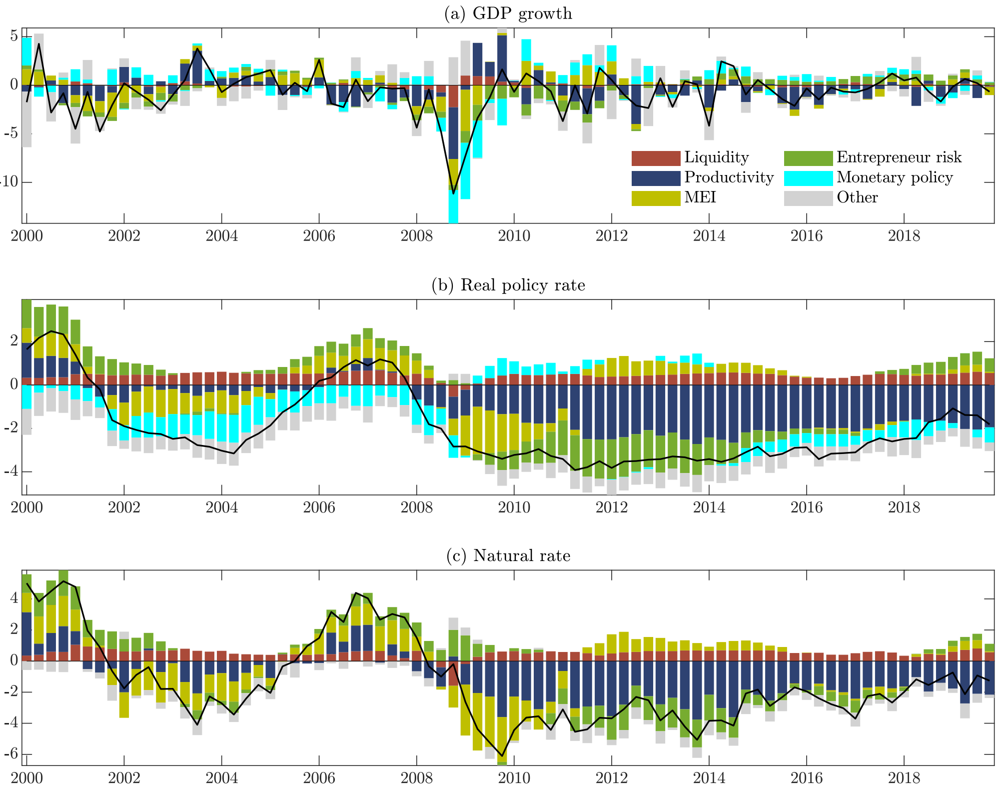
This section presents the main results from a set of robustness exercises on the Augmented estimation. We first re-estimate the model imposing the steady-state calibration of DGGT concerning safety and liquidity premiums. Then, we test the validity of our baseline results using two alternative proxies for the bank deposit rate.

B.1 Estimation with alternative calibration

In the baseline Augmented estimation, we demeaned the corporate spreads to make the steady-state calibration consistent with the observed CD rate. We now reintroduce the "original" Baa- and Aaa-Treasury spreads as observables (not demeaned), and fix the liquidity and safety premiums as in DGGT. As a consequence, the steady-state deposit rate is assumed to be 73 basis points higher than the fed funds rate.

Parameter estimates are virtually unchanged with respect to our baseline specification.[36] Results in terms of historical decomposition also replicate our baseline findings, as reported in Figure B1, except for slight differences in the real policy and the natural rates. There is a small and nearly constant positive contribution of liquidity shocks to the real policy rate, which is reflected in the estimate of the natural interest rate. This is directly linked to the different calibration, and to the aforementioned steady-state premium of the deposit rate over the policy rate. Because the CD rate in the data is closer to the fed funds rate (on average) than implied by the calibration, the model identifies a sequence of negative liquidity shocks that raises the policy rate and is able to cancel the model-imposed wedge.

Figure B1: Historical shock decomposition (Augmented, alternative calibration)



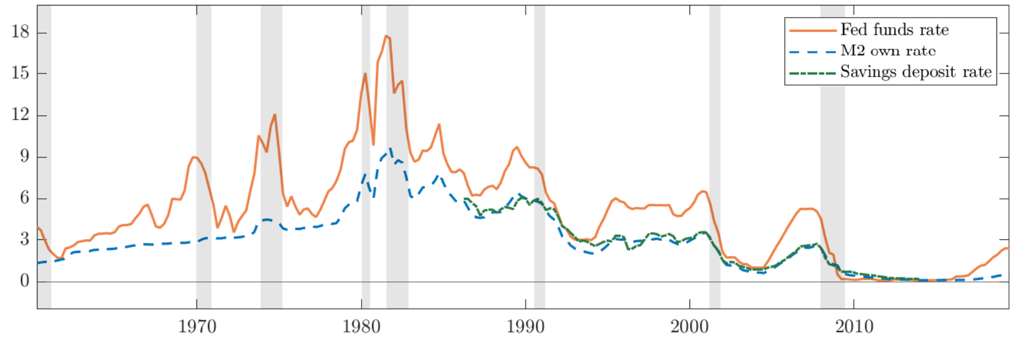
Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": sum of contemporaneous and anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values. 2000:I-2019:IV.

[36]A full description of the posterior estimates of this and of the following robustness exercises is available upon request.

B.2 Estimation with alternative data

The alternative deposit rate measures we use as observables are the M2 own rate and the savings deposit rate. The M2 own rate is available up to 2019:II, whereas the savings deposit rate series spans from 1986:I to 2013:IV. Figure B2 shows the two series as opposed to the fed funds rate.

Figure B2: Fed funds rate vs alternative deposit rate measures



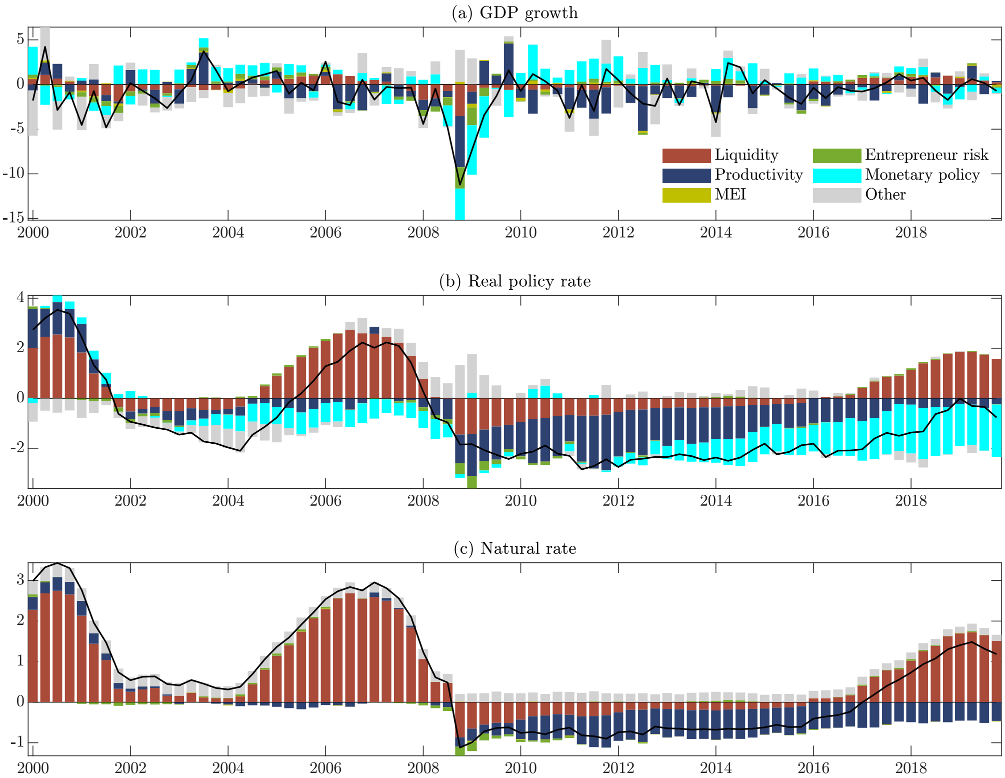
Note: The fed funds rate and the M2 own rate are plotted from 1960:I to 2019:II. The savings deposit rate is from Drechsler et al. (2017), and is displayed over the period 1986:I-2013:IV.

Both series display a substantial and negative differential with respect to the fed funds rate. In fact, the savings deposit is a more liquid saving instrument than the certificate of deposit. The M2 aggregate similarly comprises several assets that are more liquid than CDs (including savings deposits). We see that the two series, when jointly available, are fairly close in terms of level and co-movement. Indeed, estimates turn out to be similar using the two proxies (this appendix reports the results obtained with the M2 own rate only).

When we estimate the model matching the M2 own rate, we observe meaningful variations in the posterior of some crucial parameters. In particular, the estimated autocorrelation of *transitory* liquidity (and safety) shocks is around 0.9, implying that the model requires all flight-to-quality shocks to be highly persistent in order to match the deviations between the fed funds rate and the deposit rate. Moreover, the posterior mean of the inverse EIS σ_c is extremely small (0.41) and, combined with the estimate of β (0.999), entails a low steady-state real interest rate of 1.04%.

Figure B3 shows the historical decomposition of GDP growth, the real policy rate, and the natural rate. The role of liquidity shocks in explaining GDP growth (top panel) is dominated by other disturbances, namely technology and monetary policy shocks, confirming our baseline results. Turning to the interest rates, there is a strong contribution by liquidity shocks (panels (b) and (c)). However, the largest part of their impact consists in *raising* the policy rate, at least up to 2008. Even thereafter, their contribution to dragging the real fed funds rate down is small compared to technology shocks; after 2017, they explain the departure from the ZLB. This result stems from the low steady-state real interest rate: the model estimates this value to be closer to the average deposit rate than to the fed funds rate, and then attributes the rise of the policy rate with respect to its steady state to a series of negative liquidity shocks. Since these shocks are persistent, the natural interest rate is estimated to be exceptionally less volatile and closer to the actual real rate than in DGGT (see Figure E4).

Figure B3: Historical shock decomposition (Augmented, M2 own rate observed)



Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": sum of contemporaneous and anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values. 2000:I-2019:IV.

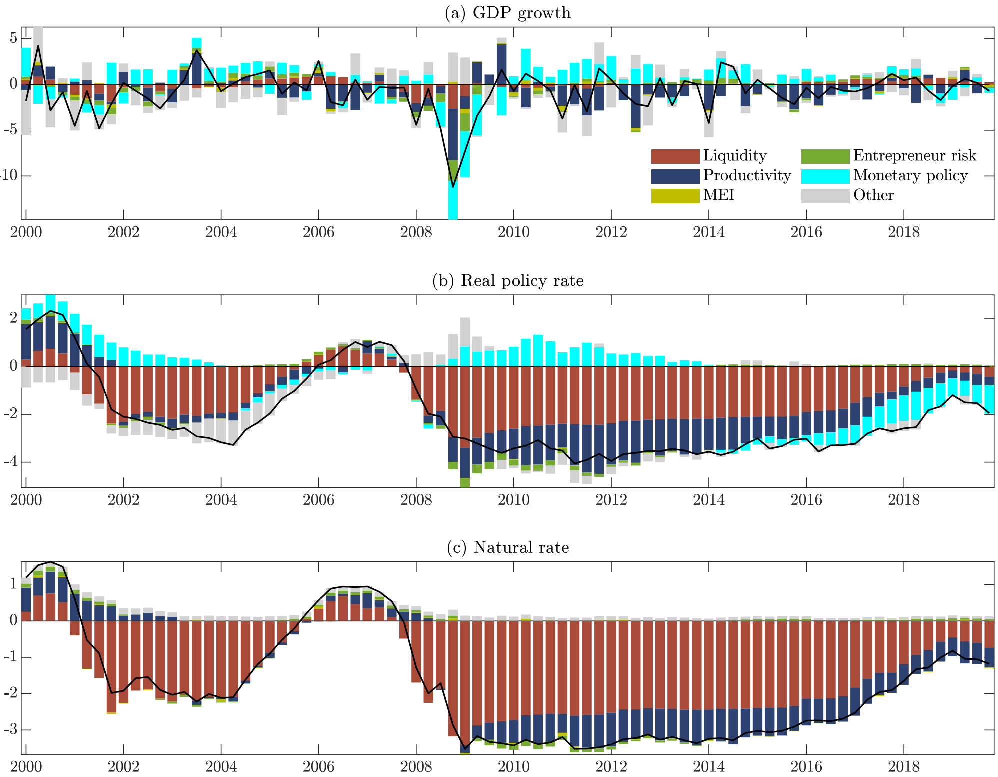
We conduct a final robustness check by shifting the M2 own rate upwards to have the same sample mean as the 3-month CD rate over the period 1964:III-2016:III, and by estimating the model with this new measure. We are so able to remove the observed negative differential between the M2 own rate and the fed funds rate, which is at odds with the assumptions of the benchmark model. Though evidently artificial, this adjustment may be rationalized with a premium for the transaction services provided by the assets included in M2. Such a premium is not modeled and thus is cleared away from the data.

This new estimation yields parameter estimates that are close to the version with the unadjusted M2 own rate, *i.e.* a low σ_c (0.57) and high autocorrelation coefficient for all liquidity shocks (larger than 0.86).[37] The shock decomposition of GDP growth is also virtually unaffected by the adjustment of the M2 own rate (see panel (a) of Figure B4). On the other

[37]The discount rate β is instead estimated to be closer to the baseline, as is the steady state real interest rate. This follows from the upward shift of the deposit rate proxy.

hand, the picture for the real policy rate is similar to the benchmark estimation without the observed deposit rate, with liquidity shocks pushing down interest rates following the financial crisis. Still, the estimated path of the natural rate is strikingly different with respect to the one found by DGGT, as Figure E4 shows.

Figure B4: Historical shock decomposition (Augmented, adjusted M2 own rate observed)



Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": sum of contemporaneous and anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values. 2000:I-2019:IV

In conclusion, even though the use of alternative deposit rate proxies allows for a larger influence of liquidity shocks with respect to our baseline, these are overshadowed by other shocks (mainly to productivity) and are effectively irrelevant for GDP growth after 2010. Additionally, the inclusion of a deposit rate measure that is farther from the fed funds rate, forces the model to explain the deposit spread with highly persistent liquidity shocks. This has profound implications for the dynamics of the natural interest rate that gets remarkably closer to the real policy rate.

C α_{NBFI} calibration

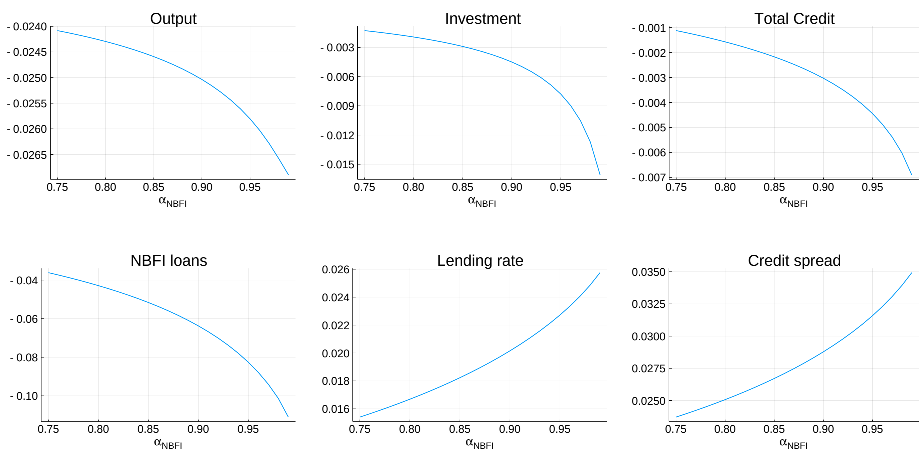
The parameter α_{NBFI} determines the degree of returns to scale in the NBFI-transformation of deposits into loans. As mentioned in Section 1.2.3, we do not impose constant returns to scale because this would lead to an indeterminate solution of our model. In fact, if we assumed $\alpha_{NBFI} = 1$, bank and non-bank credit would not be separately identified. The goal of our calibration strategy is therefore to choose a value for α_{NBFI} so as to maintain the transmission mechanism of liquidity shocks close to DGGT. As we show below, a relatively high value of α_{NBFI} safeguards liquidity shocks from having counterfactual implications for the co-movement between output, on the one hand, and investment and credit, on the other.

In order to understand the impact of α_{NBFI}, combine the log-linearized equilibrium conditions (1.A20) and (1.A21) to get an expression for the spread between lending and policy rates:

$$\hat{R}^L_t - \hat{R}_t = \hat{\varepsilon}^l_t + (1 - \alpha_{NBFI})\hat{d}^{NBFI}_t. \tag{1.C1}$$

When an adverse liquidity shock hits (*i.e.* a positive realization of $\hat{\varepsilon}^l_t$), the spread widens, but there is a simultaneous force that pulls in the opposite direction. Consistently with a flight to quality, the amount of deposits held at NBFIs, $\hat{d}^{NBFI}_t$, decreases and reduces the spread. As is clear from (1.C1), the larger α_{NBFI}, the smaller the dampening effect of NBFI deposits. Further, looking at (1.A23), as α_{NBFI} increases, the reduction in NBFI loans is larger following a liquidity shock.

Figure C1: Impact responses to an adverse transitory liquidity shock (NBFI)



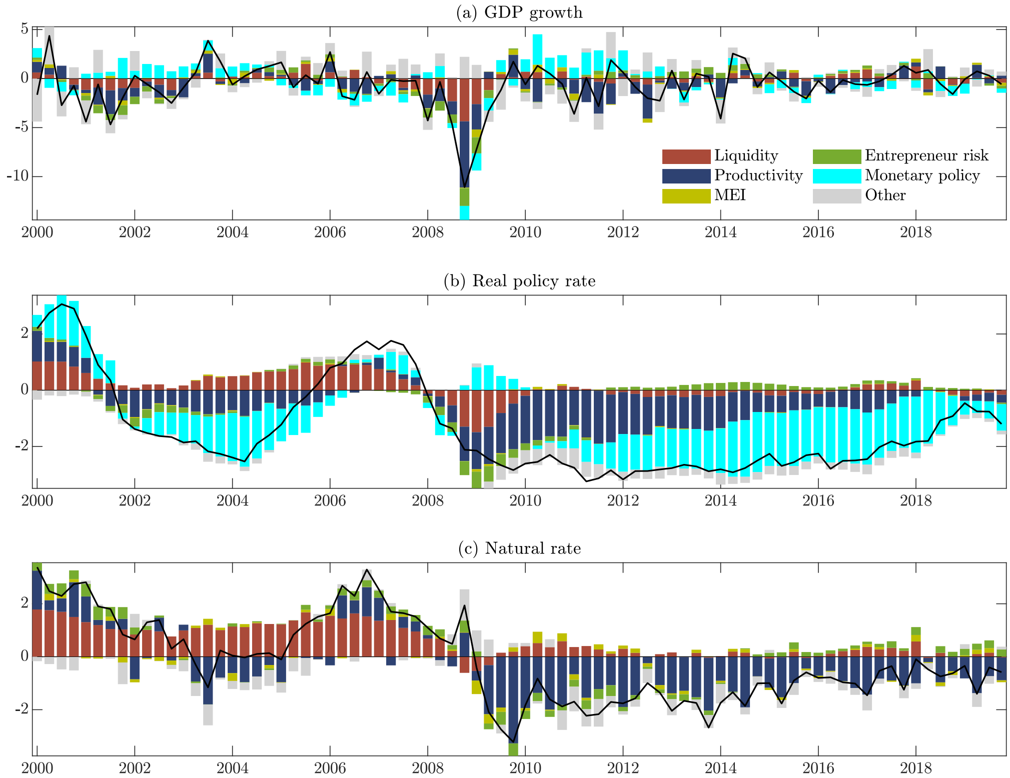
Note: Estimated impact responses at the posterior mode.

Figure C1 shows the estimated response of selected variables on impact, after an adverse liquidity shock, for a range of α_{NBFI} values that goes from 0.75 to 0.99. In line with the previous discussion, as we move to the left of the α_{NBFI}-grid, lending rates and the credit spread increase by less, while NBFI loans decrease by less. This prompts a smaller reduction in credit, and eventually in investment. For smaller values of α_{NBFI} and/or a different autocorrelation coefficient of the shock, the response of credit and investment may turn countercyclical (notice how the impact response of output is relatively more stable). In this sense, the choice of $\alpha_{NBFI} = 0.99$ is "conservative" and allows credit to co-move with output, consistently with the behavior induced by liquidity shocks in DGGT (see Figure 1.5).

D Alternative NBFI estimation

We conduct an alternative estimation of the NBFI model, where the bank-loans-share growth rate is the only additional observable with respect to DGGT. The model is thus not constrained to match the P1CP rate as a measure of the non-bank deposit rate. As shown in Figure 1.7, this estimation leads to a mismatch between the model-implied NBFI deposit rate and the observed return on P1CP. Though apparently small, this gap is sufficient to generate a quite relevant role for liquidity shocks as a driver of GDP growth. However, as evident in panel (a) of Figure D1, the predominance of flight-to-quality shocks is limited to the Great Recession period.

Figure D1: Historical shock decomposition (NBFI, P1CP rate not observed)



Note: Historical shock decomposition at the posterior mode. "Liquidity": sum of transitory liquidity, transitory safety, permanent liquidity, and permanent safety shocks; "Productivity": sum of stationary and permanent technology shocks; "Monetary policy": sum of contemporaneous and anticipated monetary policy shocks; "Other": sum of all other shocks and measurement errors, and initial values.

As shown in panel (b), there is a widespread contribution of monetary policy shocks to the real fed funds rate dynamics. This channel is stronger than in our baseline NBFI estimation, especially between 2002 and 2006, and after 2012. The natural rate is exceptionally less volatile than in DGGT, and liquidity shocks play virtually no role in explaining its decline (panel (c)). Surprisingly enough, the estimated pattern of r^* since the mid-80s is roughly in line with those predicted by the alternative Augmented estimates (*i.e.* with the M2 own rate as bank deposit rate; see Figure E4). If anything, these alternative estimates suggest that monetary policy was even more expansionary than implied by our baseline, at least after the GFC.

E Additional tables and figures

E.1 Tables

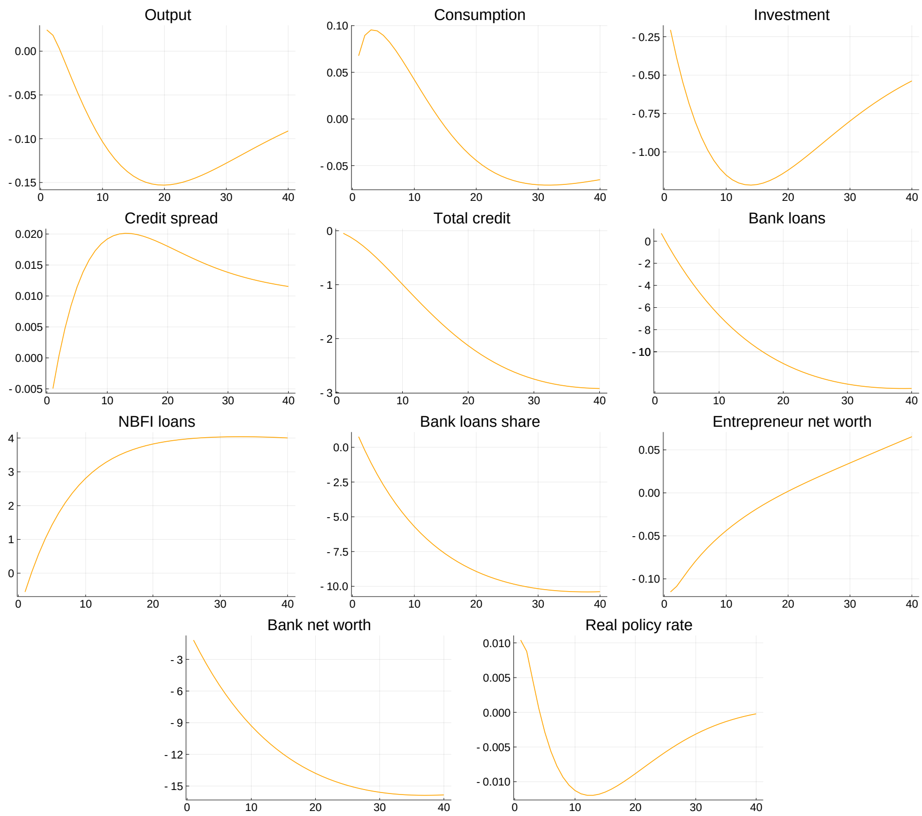
Table E1: Variance decomposition (full sample)

	Liquidity	Tech.	MEI	Risk	Govt.	Monetary	Markup	Bank n.w.	m.e.
GDP growth									
DGGT	24%	25%	3%	3%	11%	17%	3%	-	14%
Augmented	3%	36%	14%	5%	8%	17%	2%	-	15%
NBFI	3%	39%	8%	3%	10%	18%	2%	2%	15%
Consumption growth									
DGGT	30%	24%	3%	6%	14%	19%	4%	-	-
Augmented	4%	42%	3%	7%	18%	22%	3%	-	-
NBFI	5%	49%	3%	3%	11%	24%	2%	3%	-
Investment growth									
DGGT	24%	13%	9%	29%	1%	19%	5%	-	-
Augmented	1%	8%	56%	24%	1%	6%	2%	-	-
NBFI	2%	9%	54%	16%	2%	9%	4%	6%	-
Real policy rate									
DGGT	69%	12%	1%	2%	2%	11%	3%	-	-
Augmented	5%	29%	15%	18%	2%	27%	4%	-	-
NBFI	6%	35%	7%	9%	1%	35%	4%	3%	-
Aaa-Treasury spread									
DGGT	94%	-	-	-	-	-	-	-	6%
Augmented	74%	-	-	-	-	-	-	-	26%
NBFI	58%	-	-	-	-	-	-	-	42%
Baa-Treasury spread									
DGGT	32%	4%	2%	3%	4%	3%	3%	-	48%
Augmented	10%	17%	9%	9%	11%	9%	10%	-	26%
NBFI	5%	10%	2%	10%	0%	3%	1%	2%	67%

Note: Variance decomposition at the posterior mode. The percentage contribution is given by the variance of a variable accounted for by each of the listed (groups of) shocks divided by the summed variances of the same variable accounted for by all (groups of) shocks.

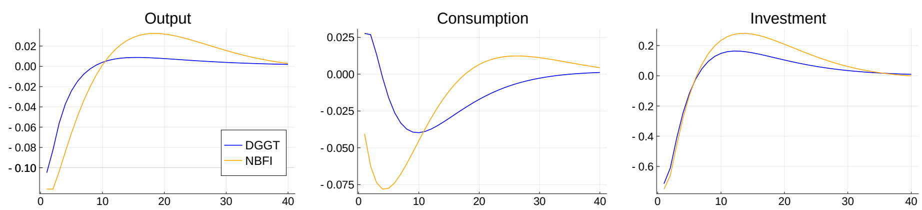
E.2 Figures

Figure E1: IRFs to an adverse bank net worth shock (NBFI)



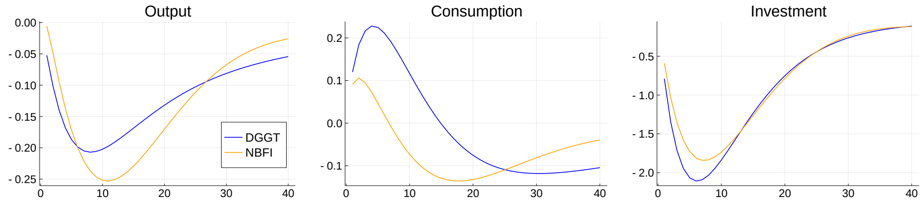
Note: Estimated impulse response functions at the posterior mode.

Figure E2: IRFs to an adverse MEI shock, selected macro aggregates (DGGT vs NBFI)



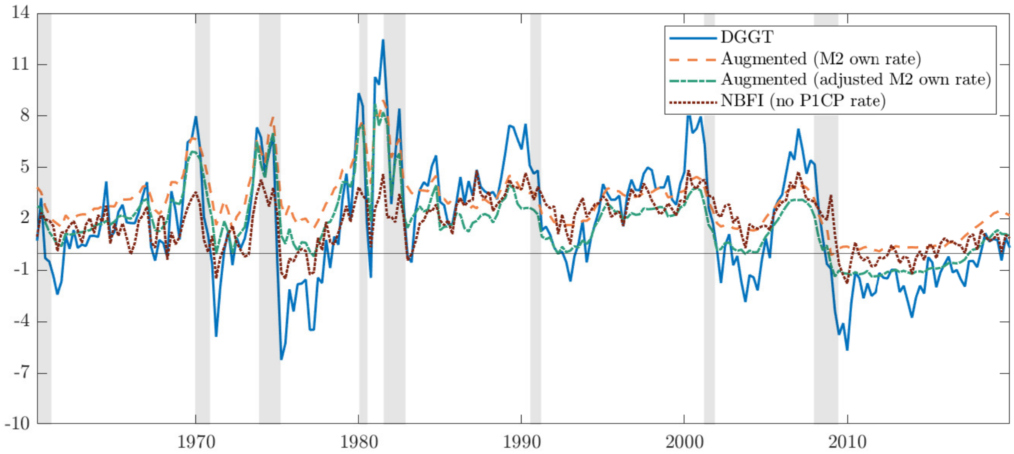
Note: Estimated impulse response functions at the posterior mode. The autocorrelation coefficient and the standard deviation of the shock are fixed at the DGGT-posterior-mode estimates.

Figure E3: IRFs to an adverse entrepreneur risk shock, selected macro aggregates (DGGT vs NBFI)



Note: Estimated impulse response functions at the posterior mode. The autocorrelation coefficient and the standard deviation of the shock are fixed at the DGGT-posterior-mode estimates.

Figure E4: Alternative r^* estimates



Note: Smoothed estimates of r^* at the posterior mode. 1960:I-2019:IV.

Chapter 2

Who Killed Business Dynamism in the U.S.?

Joint with Bianca Barbaro and Patrizio Tirelli

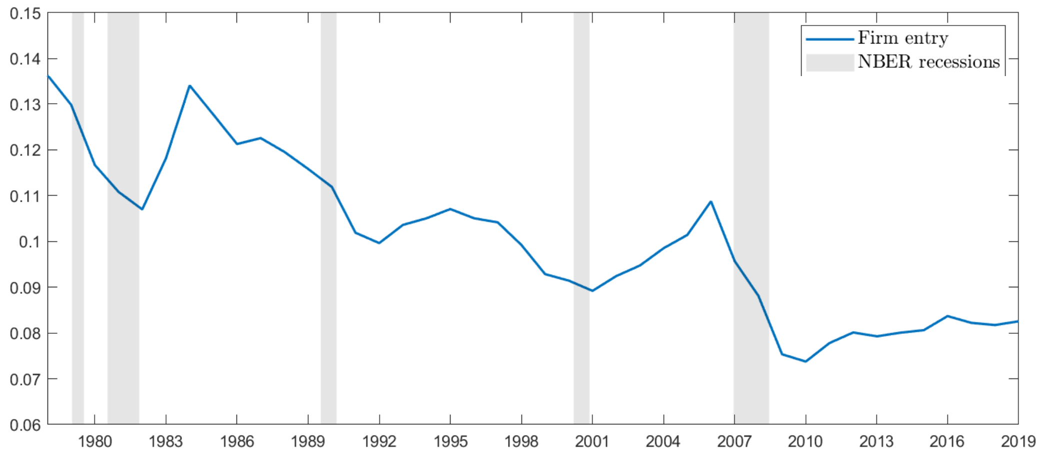
Abstract We offer a new interpretation of the long-term dynamics in the U.S. firm entry rate. Its decline was the consequence of a persistent combination of adverse(favorable) productivity shocks to potential entrants(incumbents), while the long-term increase in price markups did not play a significant role. In spite of the "Schumpeterian" structure of our model, not all recessions had a "cleansing" effect, because the combination of shocks associated with the specific episodes had markedly different effects on the dispersion of firms' efficiency. Finally, the extensive margin allows to rationalize the procyclical pattern of TFP growth and its long-term decline.

2.1 Introduction

We offer a new interpretation of the long-term dynamics in the U.S. firm entry rate, documented in Figure 2.1: it exhibits a procyclical pattern and gradually falls up to 2010. Since then, it is essentially flat. In fact, a consistent reduction in the number of young firms (Decker et al., 2014) and a slowdown in productivity growth (Fernald, 2014; Boppart and Li, 2021) characterized the U.S. economy over the last decades.

In this regard, the literature on endogenous firm dynamics sees new businesses as the principal source of innovation in the economy (Asturias et al., 2017; Alon et al., 2018), and treats the falling entry rate as the key factor behind the productivity slowdown. For instance, Gourio et al. (2016) document that entry shocks cause a 1-1.5 percent increase in GDP, lasting over ten years. Increasing concentration and greater market power, reported in studies such as Autor et al. (2020) and Grullon et al. (2019), are identified as the main culprits, leading to the rise in markups documented in De Loecker et al. (2020), Gutiérrez and Philippon (2016), and Eggertsson et al. (2021).

Figure 2.1: Firm entry rate, U.S., 1978-2019



Note: Annual data (Business Dynamics Statistics)

By contrast, other studies emphasize the important contribution of older firms to innovation and productivity growth. According to Hsieh and Klenow (2018), incumbents account for the lion's share of innovation through improvements on their own products, whereas at most one-quarter of U.S. productivity growth is ascribable to creative destruction and inputs reallocation towards relatively new firms. Fort et al. (2018) document the limited effect of firm entry and exit on the overall decline in U.S. manufacturing employment between 1977 and 2012, and conclude that incumbents might have been successful in raising their productivity relative to new entrants. Garcia-Macia et al. (2019) find that most growth comes from incumbents' contribution, with the role of entrants and creative destruction fading over the latest decades. Similarly, Klenow and Li (2021) show how fluctuations in U.S. productivity growth have been mostly driven by variations in incumbents' ability to innovate.

We build a new model that encompasses these alternative views. We incorporate endogenous firm dynamics, as in Hopenhayn (1992), Asturias et al. (2017), and Piersanti and Tirelli (2020), into a stochastic growth model where technology improvements are determined by the different shocks that hit potential new entrants and incumbent firms. Long-term stochastic growth and business cycle dynamics are interpreted on the grounds of a set of shocks that hit the economy at low and high frequencies. In our model, the entry rate falls either if the internal productivity growth of potential entrants is subject to an adverse shock, or if favorable productivity shocks hit incumbents, or if price markups increase.[1] We estimate the model and let the data speak to the relative importance of these alternative mechanisms for the decline in firm entry.

Our results are summarized as follows. The estimated model predicts a declining entry rate even if we exclude entry data from the set of observables. This is a very important preliminary result, suggesting that we are not forcing the model to rationalize long-run entry data "artificially" included in the set of observables. A persistent combination of ad-

[1]Further, the entry rate falls if adverse demand shocks lower the value of the entry decision.

verse(favorable) productivity shocks to potential entrants(incumbents) causes the long-run decline in the model-predicted entry rate. This pattern is fully confirmed when we add the entry rate to the set of observed variables. The model predicts a long-term increase in price markups that is consistent with documented evidence, but we cannot find a persistent effect of markup shocks on the entry rate decline.

Our model allows us to estimate the dynamics of both TFP and average firm efficiency. The growth of firm efficiency is relatively stable and acyclical. By contrast, TFP growth is subject to a gradual slowdown and has an unambiguous procyclical pattern. Our original contribution is that adjustments in the extensive margin, *i.e.* the mass of firms, drive these findings. Non-technology shocks explain the cyclical pattern of TFP growth, via their effect on the extensive margin. We also find that recessions induced by adverse demand shocks do not have a cleansing effect, *i.e.* they do not reduce the dispersion of firms' idiosyncratic efficiency.

Our interpretation of the entry rate decline, which points to the importance of productivity dynamics, is complementary to contributions that emphasize the importance of demographic factors, such as Hopenhayn et al. (2018), Pugsley and Şahin (2018), Karahan et al. (2019), and Peters and Walsh (2021).

We also contribute to a growing literature on business cycle models of endogenous firm dynamics. Lewis and Poilly (2012) and Lewis and Stevens (2015) estimate models in the tradition of Bilbiie et al. (2007) and Bilbiie et al. (2012), but their focus is different as they neglect the analysis of long-term entry dynamics and the implications for long-run growth. Just like us, Clementi and Palazzo (2016) build upon Hopenhayn's (1992) model, but they treat entry and exit as an exogenous amplification mechanism of productivity shocks that symmetrically hit all firms. To the best of our knowledge, this is the first contribution that incorporates the effect of asymmetric productivity shocks on the entry rate.

Finally, we contribute to the literature that investigates the procyclical pattern of TFP and the sluggish recoveries following major crises. Anzoategui et al. (2019) focus on the role of R&D; Qiu and Ríos-Rull (2022) link firms' TFP to the number of varieties each firm is able to sell. Other studies obtain procyclical TFP either in consequence of sectoral productivity changes (Swanson, 2006) or through a combination of increasing returns and increased utilization of the production factors (Gottfries et al., 2021). In our framework, instead, the extensive margin of goods production drives TFP dynamics.

The paper is organized as follows: section 2 describes the model, section 3 provides information on the estimation procedure, results are presented in section 4, section 5 concludes.

2.2 Model[2]

Households demand a bundle of differentiated retail goods

$$C_t = \left(\int_0^1 C_t(r)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} dr \right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}, \tag{2.1}$$

[2]See Appendix G for the full set of equilibrium conditions and for the derivation of key equations.

supply capital services, k_t, to firms in the intermediate-goods producing sector (INT henceforth), and sell the services of a differentiated labor type ι to the competitive labor packers who assemble the labor bundle

$$l_t = \left(\int_0^1 l_t(\iota)^{\frac{\epsilon^w - 1}{\epsilon^w}} dh \right)^{\frac{\epsilon^w}{\epsilon^w - 1}} \tag{2.2}$$

that enters the production of INT-goods. INT-goods are sold to retailers.

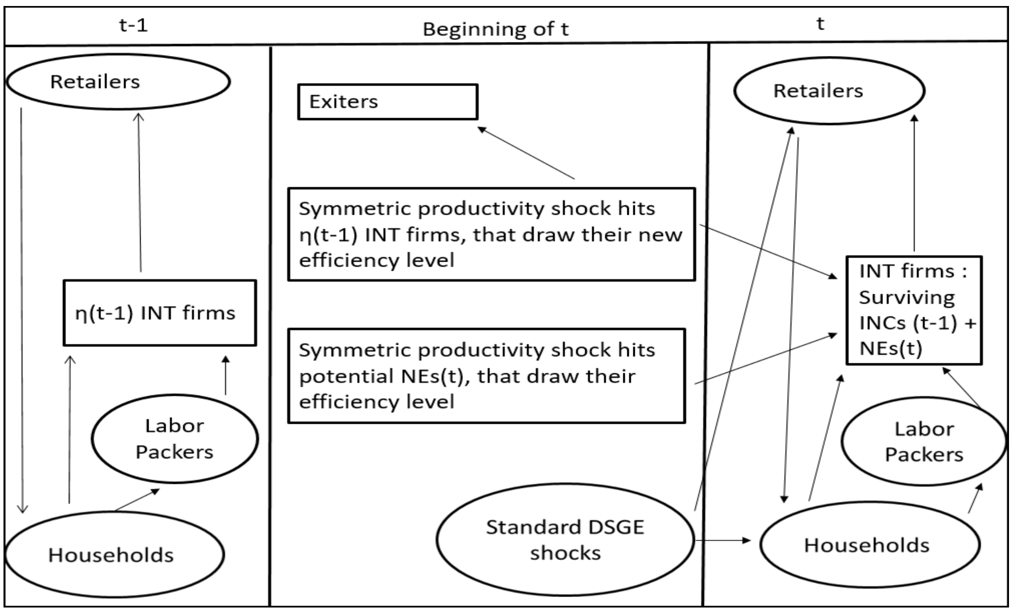
The perfectly competitive INT-firms have mass η_t, distributed between new entrants, NE_t, and incumbents, INC_t, who survived out of the η_{t-1} firms active at time $t-1$:

$$\eta_t = NE_t + INC_t. \tag{2.3}$$

Both NEs and $INCs$ group heterogeneous firms that are subject to idiosyncratic productivity shocks.

At the beginning of each period, two sets of shocks hit the economy. The first one is a set of demand and supply shocks that characterize standard DSGE models, *i.e.* marginal efficiency of investment, retail price markups and labor supply, monetary and fiscal policy. The second one includes two independent productivity shocks that symmetrically affect the idiosyncratic efficiency distribution of NEs and $INCs$ respectively. The sequence of events unfolds as in Figure 2.2.

Figure 2.2: Model sequence of events



2.2.1 INT-sector

The production function of a generic firm f is:

$$y_t^{f,j} = A_t^{f,j}\left(Z_t^{f,j}\right)^{\gamma}, \tag{2.4}$$

$$Z_t^{f,j} = \left[(k_t^{f,j})^{\alpha}(l_t^{f,j})^{(1-\alpha)}\right], \tag{2.5}$$

where $j = NE, INC$. $A_t^{f,j}$ defines the firm-specific level of productivity, $\gamma < 1$ is the degree of decreasing return to scale, $Z_t^{f,j}$ is a Cobb-Douglas bundle of factor inputs. Firm dividends are

$$d_t^{f,j} = p_t y_t^{f,j} - r_t^k k_t^{f,j} - w_t l_t^{f,j} - w_t \phi^j , \tag{2.6}$$

where p_t is the consumption price of INT-goods, r_t^k is the real rental rate of capital, w_t is the consumption real wage and ϕ^j is the exogenous fixed production cost defined in labor units. Factor demands are:

$$k_t^{f,j} = \alpha\gamma\frac{p_t y_t^{f,j}}{r_t^k}, \tag{2.7}$$

$$l_t^{f,j} = (1-\alpha)\gamma\frac{p_t y_t^{f,j}}{w_t}, \tag{2.8}$$

and

$$p_t^z = \left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)} \tag{2.9}$$

is the consumption price of Z_t. Note that the capital intensity of the input bundle $Z_t^{f,j}$ does not vary across firms, but its scale obviously grows with firm efficiency:

$$Z_t^{f,j} = \left[\frac{p_t}{p_t^z}A_t^{f,j}\gamma\right]^{\frac{1}{1-\gamma}}. \tag{2.10}$$

The firm supply function therefore is

$$y_t^{f,j} = \left(A_t^{f,j}\right)^{\frac{1}{1-\gamma}}\left[\gamma\frac{p_t}{p_t^z}\right]^{\frac{\gamma}{1-\gamma}}. \tag{2.11}$$

From (2.6) and (2.11), the firm's value can be written recursively as

$$V_t\left(A_t^{f,j}\right) = (1-\gamma)\left[A_t^{f,j}\frac{p_t\gamma^{\gamma}}{(p_t^z)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_t\phi^j + E_t\left\{\Lambda_{t+1}V_{t+1}\left(A_{t+1}^{f,j}\right)\right\} , \tag{2.12}$$

where ϕ^j allows to identify the cutoff values $\hat{A}_t^j$ that define the entry and exit productivity thresholds

$$V_t\left(\hat{A}_t^j\right) = 0. \tag{2.13}$$

Right from the outset, note that these thresholds react to current economic conditions, *i.e.* an increase in p_t unambiguously raises the firm value and lowers the idiosyncratic efficiency level that meets the profitability condition, whereas an increase in the price of inputs or in the fixed cost would work in the opposite direction. Future valuation of the firm also matters, and firms may operate under temporarily negative profitability.

New entrants

At the beginning of period t, potential NEs draw their productivity level $A_t^{f,NE}$ from the Pareto distribution

$$f_t\left(A_t^{NE}\right) = \int_{z_t}^{+\infty} \frac{\xi\left(z_t\right)^{\xi}}{\left(A_t^{f,NE}\right)^{\xi+1}} d\left(A_t^{f,NE}\right) = 1, \tag{2.14}$$

where

$$z_t = z_{t-1} g_t^z \tag{2.15}$$

defines the technology frontier, g_t^z is the stochastic exogenous firm productivity driver in the long run

$$\ln(g_t^z) = (1-\rho^z)\ln(g^z) + \rho^z \ln(g_{t-1}^z) + \varepsilon_t^z;\ \varepsilon_t^z \sim N\left(0, \sigma^z\right) \tag{2.16}$$

and g^z defines the deterministic productivity growth trend. The mass of new entrants is

$$NE_t = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi\left(z_t\right)^{\xi}}{\left(A_t^{f,NE}\right)^{\xi+1}} d\left(A_t^{f,NE}\right) = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi}, \tag{2.17}$$

where $\hat{A}_t^{NE}$ defines the productivity threshold such that $V_t\left(\hat{A}_t^{NE}\right) = 0$.

Incumbents

At the beginning of period t, the η_{t-1} firms draw their idiosyncratic productivity from the following distribution:

$$f_t(\hat{A}_t^{INC}) = \int_{\hat{A}_{t-1}^{INC} g^z (1-\delta^{INC})\Psi_t}^{+\infty} \frac{\xi\left[\hat{A}_{t-1}^{INC} g^z\left(1-\delta^{INC}\right)\Psi_t\right]^{\xi}}{\left(A_t^{f,INC}\right)^{\xi+1}} d(A_t^{f,INC}), \tag{2.18}$$

where $\hat{A}_{t-1}^{INC}$, $V_t\left(\hat{A}_{t-1}^{INC}\right) = 0$, defines the productivity threshold that characterized the distribution of INC_{t-1} firms; by setting $g^z\left(1-\delta^{INC}\right) < 1$ we assume that, on average, the η_{t-1} firms deplete their knowledge capital.[3] Finally,

$$\ln\left(\Psi_t\right) = \rho^{\Psi} \ln\left(\Psi_{t-1}\right) + \varepsilon_t^{\Psi};\ \varepsilon_t^{\Psi} \sim N\left(0, \sigma^{\Psi}\right) \tag{2.19}$$

denotes the equivalent of a standard productivity shock. The mass of incumbents is

$$INC_t = \eta_{t-1} H_t, \tag{2.20}$$

where

$$H_t = \int_{\hat{A}_t^{INC}}^{+\infty} \frac{\xi\left[\hat{A}_{t-1}^{INC} g^z\left(1-\delta^{INC}\right)\Psi_t\right]^{\xi}}{\left(A_t^{f,INC}\right)^{\xi+1}} d(A_t^{f,INC}) = \left(\frac{\hat{A}_{t-1}^{INC} g^z\left(1-\delta\right)\Psi_t}{\hat{A}_t^{INC}}\right)^{\xi} \tag{2.21}$$

[3] This is akin to Liu et al. (2020) and the literature cited therein.

is the endogenous survival probability for the η_{t-1} firms. The expected efficiency of the η_{t-1} firms is

$$E_{t-1}\left\{A_t^{\eta_{t-1}}\right\} = \frac{\xi}{\xi-1}E_{t-1}\left\{\hat{A}_{t-1}^{INC}g^z\left(1-\delta\right)\Psi_t\right\}\ . \tag{2.22}$$

The mass of exiting firms is

$$EX_t = \eta_{t-1}\left(1-H_t\right) = \eta_{t-1}\left(\frac{\hat{A}_t^{INC} - \hat{A}_{t-1}^{INC}g^z\left(1-\delta\right)\Psi_t}{\hat{A}_t^{INC}}\right)^{\xi}\ . \tag{2.23}$$

Thresholds

We now derive the efficiency thresholds associated with the intertemporal zero profit condition (2.13). To begin with, from condition (2.22) notice that firms operative in t are confronted with the same present value of future dividends

$$E_t\left\{V\left(A_{t+1}^{f,j}\right)\right\} = \int_{\widehat{A}_{t+1}^{INC}}^{+\infty} V_{t+1}\left(A_{t+1}^{f,INC}\right)\frac{\xi\left(\widehat{A}_{t+1}^{INC}\right)^{\xi}}{\left(A_{t+1}^{f,INC}\right)^{\xi+1}}d\left(A_{t+1}^{f,INC}\right) = E_t\left\{H_{t+1}V_{t+1}^{av}\right\}, \tag{2.24}$$

where V_{t+1}^{av} defines the continuation value of the η_t firms conditional to survival in $t+1$. In recursive form,

$$V_{t+1}^{av} = \frac{\xi\left(1-\gamma\right)}{\xi\left(1-\gamma\right)-1}\left[\frac{\left(1-\gamma\right)^{1-\gamma}}{\gamma^{\gamma}}\frac{p_{t+1}}{\left(p_{t+1}^z\right)^{\gamma}}\hat{A}_{t+1}^{INC}\right]^{\frac{1}{1-\gamma}} - w_{t+1}\phi^{INC} + E_{t+1}\left\{\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right\}. \tag{2.25}$$

Given the shape of the Pareto distribution, the condition

$$\xi\left(1-\gamma\right) > 1$$

is necessary to ensure that $E_t\left\{V\left(A_{t+1}^{f,j}\right)\right\}$ converges to finite value.

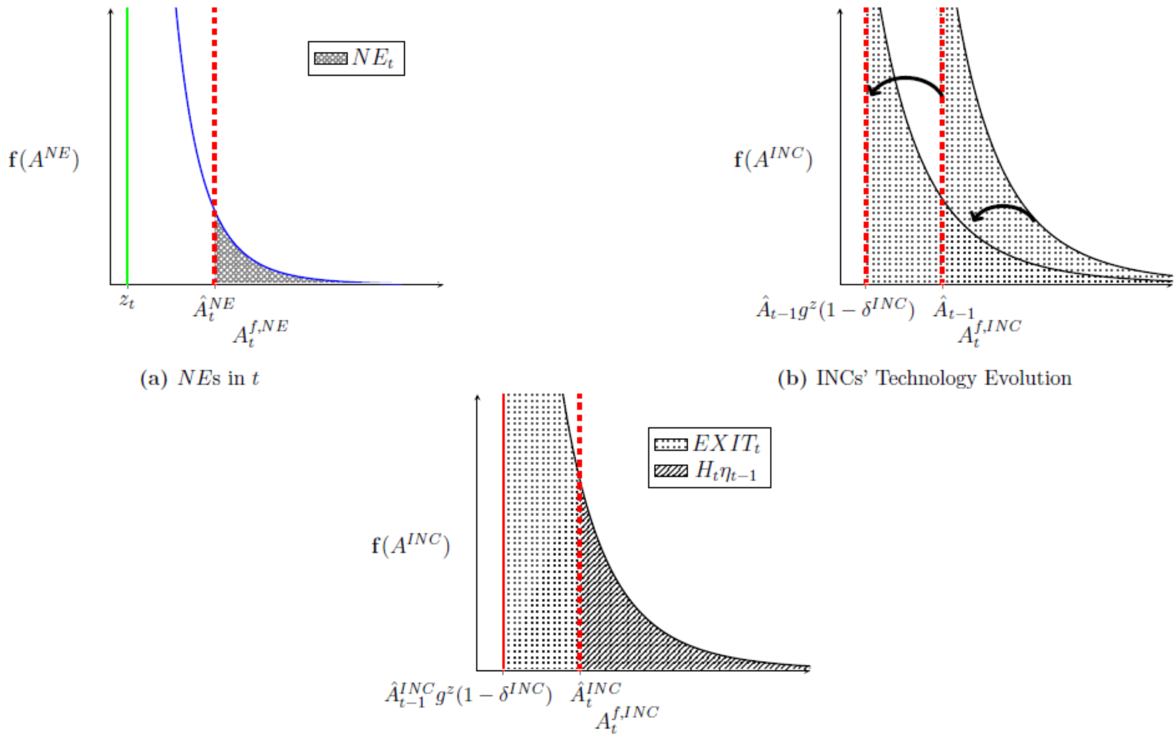
Using (2.13) and (2.25), the following condition identifies the thresholds for INC_t and NE_t firms:

$$\hat{A}_t^j = \left[\frac{w_t\phi^j - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}\left(1-\gamma\right)}\right]^{1-\gamma}\frac{\left(p_t^z\right)^{\gamma}}{p_t}. \tag{2.26}$$

Increases in the participation cost $w_t\phi^j$ and in the price of the input bundle p_t^z raise the productivity threshold, whereas increases in current or discounted future profitability, respectively determined by p_t and $\Lambda_{t+1}H_{t+1}V_{t+1}^{av}$, allow relatively less efficient firms to operate in the market.

Figure 2.3 provides a graphical representation of how NEs and $INCs$ are distributed. Panel (a) identifies the fraction of potential entrants that choose to operate in t. In Panel (b) we represent the distribution of the depreciated knowledge capital inherited by the η_{t-1} firms. Finally, in Panel (c) $\hat{A}_t^{INC}$ splits the support between exiting and surviving η_{t-1} firms.

Figure 2.3: Model firm dynamics



(a) NEs in t

(b) INCs' Technology Evolution

(c) Exit in t

The following condition highlights the impact of productivity shocks on firm dynamics:

$$\eta_t = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi} + \eta_{t-1}\left(\frac{\hat{A}_{t-1}^{INC} g^z \left(1-\delta\right)\Psi_t}{\hat{A}_t^{INC}}\right)^{\xi}. \tag{2.27}$$

From (2.15) it is easy to see that shocks to z_t have permanent effects on the support of the NEs distribution. Using (2.26), we get

$$\eta_t = \left[\begin{array}{c} \left(\frac{z_t}{\left[w_t\phi^{NE} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}\right]^{1-\gamma}}\right)^{\xi} + \\ \eta_{t-1}\left(\frac{\hat{A}_{t-1}^{INC} g^z (1-\delta)\Psi_t}{\left[w_t\phi^{INC} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right\}\right]^{1-\gamma}}\right)^{\xi} \end{array}\right] \left(\frac{\gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma} p_t}{\left(p_t^z\right)^{\gamma}}\right)^{\xi}. \tag{2.28}$$

Thus, a positive shock to z_t creates a supply congestion effect that lowers p_t and raises the productivity thresholds. Our estimates will show that this is associated with an increase in both entry and exit rates. This mechanism, akin to a Schumpeterian cleansing effect, is enriched by the role of discounted future profitability: ceteris paribus, the larger $\hat{A}_t^{INC}$ also raises the firm survival probability in the next period, H_{t+1}, and causes a persistent downward pressure on the price of INT-goods. The initial ε_t^z shock permanently raises the expected z values in the NEs distribution support. This, combined with the $\hat{A}_t^{INC}$ increase, generates a sequence of falling prices and endogenously increasing firm productivity.[4]

The Ψ_t shock raises the survival probability of the η_{t-1} firms and triggers a fall in p_t. In this case, both entry and exit rates fall. Finally, demand shocks also matter for firms' productivity and entry/exit flows. In fact, any change in demand for INT-goods that raises $\frac{p_t}{p_t^z}$ will lower the productivity thresholds, raising both $INCs$ and NEs.

INT-sector aggregation

Production of INT-goods is

$$Y_t^{INT} = Y_t^{NE} + Y_t^{INC} \ , \tag{2.29}$$

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{f,NE} \left[\left(k_t^{f,NE} \right)^{\alpha} \left(l_t^{f,NE} \right)^{1-\alpha} \right]^{\gamma} dF \left(A_t^{f,NE} \right) \ , \tag{2.30}$$

$$Y_t^{INC} = \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{f,INC} \left[\left(k_t^{f,INC} \right)^{\alpha} \left(l_t^{f,INC} \right)^{1-\alpha} \right]^{\gamma} dF \left(A_t^{f,INC} \right) \ . \tag{2.31}$$

Straightforward manipulations yield the supply function

$$Y_t^{INT} = \frac{\xi\,(1-\gamma)}{\xi(1-\gamma)-1} \left[NE_t \left(\hat{A}_t^{NE} \right)^{\frac{1}{1-\gamma}} + INC_t \left(\hat{A}_t^{INC} \right)^{\frac{1}{1-\gamma}} \right] \left(\frac{\gamma p_t}{p_t^z} \right)^{\frac{\gamma}{1-\gamma}} \ , \tag{2.32}$$

where

$$\frac{\xi\,(1-\gamma)}{\xi(1-\gamma)-1} \left(\hat{A}_t^{j} \right)^{\frac{1}{1-\gamma}} \left(\frac{\gamma p_t}{p_t^z} \right)^{\frac{\gamma}{1-\gamma}}$$

denotes the average production of j-type firms.

Note that an increase in p_t has manifold effects. First, it increases the price/cost margin, $\frac{p_t}{p_t^z}$. Second, it raises the mass of j-firms (see conditions 2.17, 2.21 and 2.26). Third, by loosening the zero-profit condition (2.26), it reduces the average firm efficiency $\hat{A}_t^j$. The supply elasticity is

$$\frac{\partial Y_t^{INT}}{\partial p_t} \frac{p_t}{Y_t^{INT}} = \xi - 1 \ .$$

From conditions (2.7) and (2.8), factor-inputs demands are:

$$K_t^{INT} = \alpha\gamma \frac{p_t Y_t^{INT}}{r_t^k} \ , \tag{2.33}$$

$$L_t^{INT} = (1-\alpha)\gamma \frac{p_t Y_t^{INT}}{w_t} + \phi^{NE} NE_t + \phi^{INC} INC_t \ . \tag{2.34}$$

[4] See Piersanti and Tirelli (2020) for a detailed discussion.

2.2.2 Retailers

There is a continuum of monopolistic retailers $r \in (0,1)$, and final output is a CES bundle of differentiated goods:

$$Y_t = \left(\int_0^1 Y_t\left(r\right)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} dr \right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}} , \tag{2.35}$$

where

$$\ln(\epsilon_t^p) = (1-\rho^p)\ln(\epsilon^p) + \rho^p \ln(\epsilon_{t-1}^p) + \varepsilon_t^p - \eta^p \varepsilon_{t-1}^p;\ \varepsilon_t^p \sim N\left(0, \sigma^p\right) \tag{2.36}$$

allows the identification of standard price markup shocks.[5]

Retailers face Calvo rigidities and either re-optimize with probability $1-\Gamma_p$ or follow the simple indexation rule

$$P_t(r) = \left(\pi_{t-1}^{\mu_p} \overline{\pi}_{ss}^{1-\mu_p}\right) P_{t-1} \ . \tag{2.37}$$

Their price is a combination of steady-state and past inflation indexed by the parameter μ_p.

The solution to the retailers' pricing problem is:

$$P_t^{1-\epsilon_t^p} = \left(1-\Gamma_p\right)\left(P_t^*\right)^{1-\epsilon_t^p} + \Gamma_p\left(\pi_{t-1}^{\mu_p}\overline{\pi}_{ss}^{1-\mu_p}P_{t-1}\right)^{1-\epsilon_t^p} , \tag{2.38}$$

where P_t^* is the optimal price level and P_t is the retail price index. Aggregating across individual retailers, we obtain

$$Y_t = \frac{Y_t^{INT}}{\xi_t^p} , \tag{2.39}$$

where ξ_t^p is the standard measure of price dispersion under Calvo pricing.

2.2.3 Households

The representative household ι, $\iota \in (0,1)$, maximizes

$$E_t \sum_{i=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \left(C_{t+i} - hC_{t+i-1}\right)^{1-\sigma} \right] \exp\left(\psi \frac{\sigma - 1}{1+\varphi} \zeta_{t+i}^l l_{t+i}\left(\iota\right)^{1+\varphi} \right), \tag{2.40}$$

where ζ_t^l is a labor supply shock

$$\ln(\zeta_t^l) = \rho^l \ln(\zeta_{t-1}^l) + \varepsilon_t^l;\ \varepsilon_t^l \sim N\left(0, \sigma^l\right) \ , \tag{2.41}$$

subject to:[6]

$$C_t + I_t + \frac{B_t}{P_t} = w_t\left(\iota\right) l_t\left(\iota\right) + \left(r_t^k - a_t^u\right) U_t K_t + R_{t-1}^n \frac{B_{t-1}}{P_t} + D_t^F \ . \tag{2.42}$$

C_t is consumption of the retail goods bundle, D_t^F are firm dividends, B_t is a one-period nominally riskless bond with gross remuneration R_t^n, U_t denotes variable capacity utilization, and $a_t^u = \gamma_1\left(U_t - 1\right) + \frac{\gamma_2}{2}\left(U_t - 1\right)^2$ defines its adjustment cost.

[5] We follow Smets and Wouters (2007) in modeling the price markup shock as an ARMA(1,1) process. This allows us to catch high-frequency fluctuations in inflation.

[6] We implicitly assume that risk-sharing schemes insulate individual consumption from idiosyncratic shocks to the household wage bill.

The capital stock evolves as follows:

$$K_{t+1} = \mu_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t + (1-\delta)K_t \ , \tag{2.43}$$

where δ is the depreciation rate, $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ defines investment adjustment costs, and μ_t,

$$\ln(\mu_t) = \rho^{\mu} \ln(\mu_{t-1}) + \varepsilon_t^{\mu};\ \varepsilon_t^{\mu} \sim N\left(0, \sigma^{\mu}\right) \ , \tag{2.44}$$

is a shock to the marginal efficiency of investment (MEI shock).

Households face the downward-sloping demand function

$$l_t\left(\iota\right) = \left(\frac{w_t\left(\iota\right)}{w_t}\right)^{-\epsilon^w} l_t \ , \tag{2.45}$$

and Calvo rigidities affect wage-setting decisions: each household either optimizes with probability Γ_w or follows the indexation rule

$$w_t\left(\iota\right) = \frac{\pi_{t-1}^{\mu_w} \overline{\pi}_{ss}^{1-\mu_w} w_{t-1}\left(\iota\right)}{\pi_t}. \tag{2.46}$$

Wage dynamics are

$$w_t^{1-\epsilon^w} = \left(1-\Gamma_w\right)\left(w_t^*\right)^{1-\epsilon^w} + \Gamma_w \left(\frac{\pi_{t-1}^{\mu_w} \overline{\pi}_{ss}^{1-\mu_w} w_{t-1}}{\pi_t}\right)^{1-\epsilon^w} \ , \tag{2.47}$$

where w_t^* is the wage set by re-optimizing households.

2.2.4 Monetary Policy

We opt for a very simple Taylor rule,[7]

$$\frac{R_t^n}{R_{ss}^n} = \left(\frac{R_{t-1}^n}{R_{ss}^n}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\overline{\pi}_t}\right)^{\kappa^{\pi}} \left(\frac{Y_t}{Y_{t-1}}\right)^{\kappa^y}\right]^{1-\rho_i} \zeta_t^r \ , \tag{2.48}$$

where R_{ss}^n is the steady-state nominal interest rate, ζ_t^r is a monetary policy shock

$$\ln(\zeta_t^r) = \rho^r \ln(\zeta_{t-1}^r) + \varepsilon_t^r;\ \varepsilon_t^r \sim N\left(0, \sigma^r\right) \tag{2.49}$$

and

$$\ln\left(\overline{\pi}_t\right) = \left(1-\rho^{\pi}\right)\left(\overline{\pi}_{ss}\right) + \rho^{\pi} \ln\left(\overline{\pi}_{t-1}\right) + \varepsilon_t^{\pi};\ \varepsilon_t^{\pi} \sim N\left(0, \sigma^{\pi}\right) \tag{2.50}$$

is the stochastic inflation target.

[7] We also experimented with the complex rule in Smets and Wouters (2007). Our results were confirmed.

2.2.5 Market clearing

Market clearing requires:[8]

$$L_t = L_t^{INT} \ , \tag{2.51}$$

$$K_t = K_t^{INT} \ , \tag{2.52}$$

$$Y_t = C_t + I_t + G_t \ . \tag{2.53}$$

where $G_t = g_t^s Y_{ss}$ denotes public consumption as a fraction of steady-state output and

$$\ln(g_t^s) = (1-\rho^{g^s})\ln(g^s) + \rho^{g^s}\ln(g_{t-1}^s) + \varepsilon_t^{g^s};\ \varepsilon_t^{g^s} \sim N\left(0, \sigma^{g^s}\right) \tag{2.54}$$

denotes a public consumption shock as in Smets and Wouters (2007).

2.2.6 Shocks and the endogenous persistence of efficiency thresholds

To support intuition, we discuss here the IRFs to productivity, MEI, and markup shocks (Figures 2.4).[9] The choice of this specific subset of shocks is motivated by their relative importance for the subsequent analysis of observed entry rate dynamics.

Our purpose is to clarify the endogenous propagation mechanism that drives the response of firms' productivity to exogenous shocks. Right from the outset, note that technology shocks to $NE(INC)$ firms adversely affect the valuation of the other group of firms, and therefore impact on exit(entry) rates. Further, demand and markup shocks affect entry/exit rates through the price/cost margin of INT-firms. This, in turn, matters for average firm productivity that unambiguously falls in the occurrence of INT-sector demand-driven booms and vice-versa.

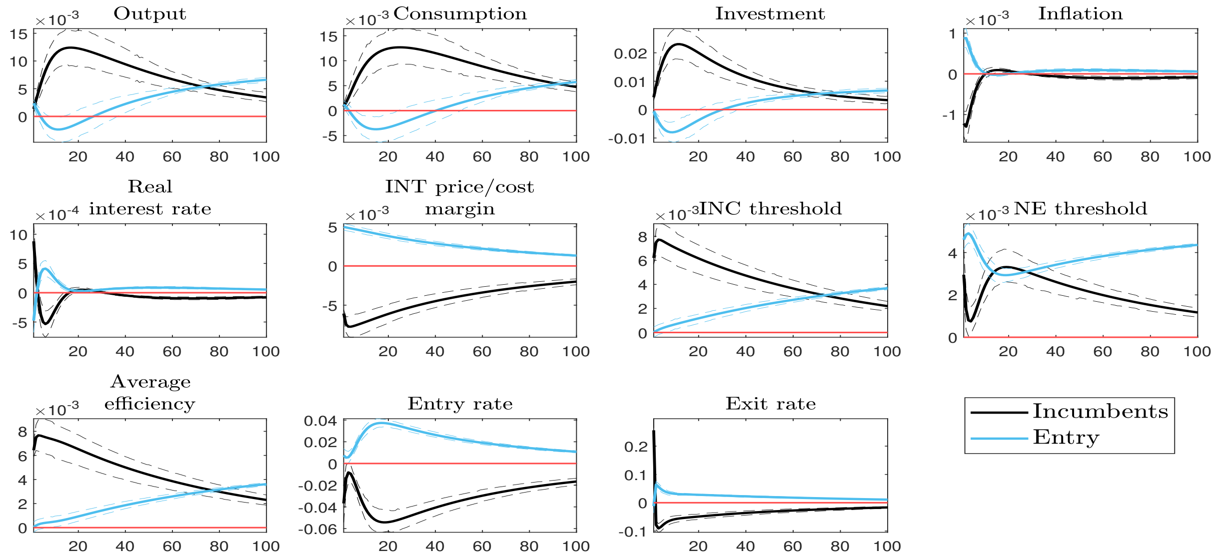
Consider first a white noise entry shock, ε_t^z. From condition (2.16) it is easy to see that the shock entails a permanent increase in the new entrants' productivity shifter z_t. There are permanent effects on consumption and investment that materialize at very low frequencies. By looking at the dynamics of the productivity thresholds for both NEs and $INCs$, one can gauge the persistence of the endogenous amplification mechanism, that turns the shock into a permanent increase in average firm productivity which is as large as the initial increase in z_t. Even if ε_t^z has no persistence, increased supply from NEs immediately raises the productivity threshold for INC_t firms. This, in turn, triggers a gradual and very persistent upward shift in the support of the $H_t\eta_{t-1}$ mass of surviving incumbents (see condition 2.21), causing a twofold effect. On the one hand, the incumbents' expected survival probability falls. On the other hand, the increase in $\hat{A}_t^{INC}$ drives the long-term response of output. The short-run transitions require a careful discussion. The increased competition from NEs lowers the present value of potential incumbents and raises the exit rate. In fact, the shock is associated with an episode of "creative destruction". This effect is so strong that the initial surge in the exit rate reduces the extensive margin pushing up the consumption price of INT-goods and the marginal cost for firms in the retail sector. This, in turn, causes a persistent increase in

[8]The model is solved up to first order. We, therefore, neglect price and nominal wage dispersion.

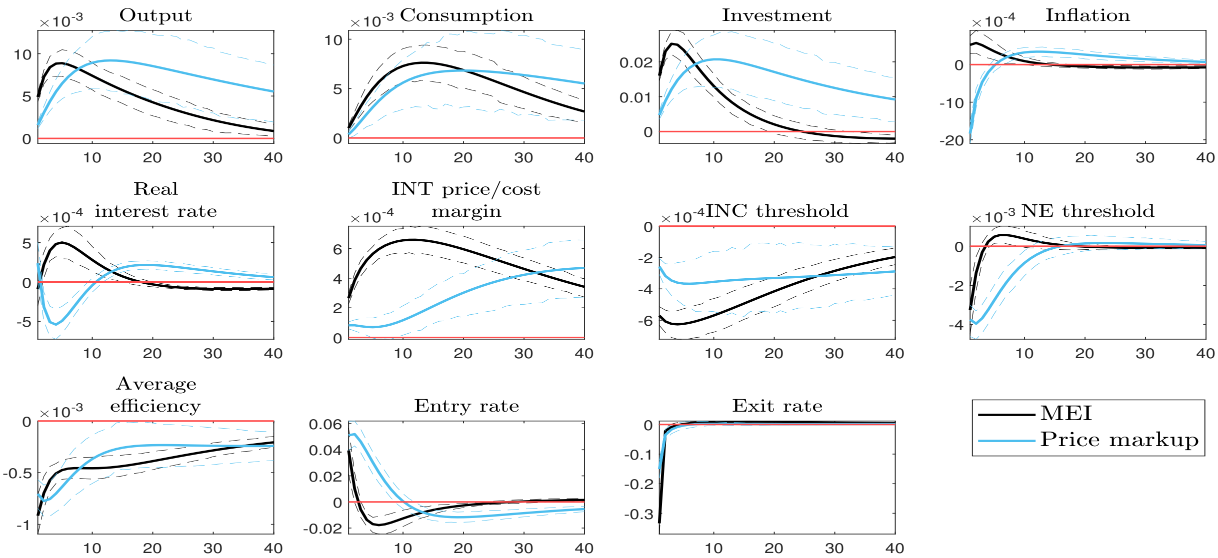
[9]Parameters are calibrated at the posterior-mean values obtained for our baseline model (see section 2.4 below).

Figure 2.4: Estimated IRFs

(a)



(b)



Note: Quarterly estimated mean impulse responses (solid lines) with 90% HPD intervals (dashed lines). Panel (a): one-standard-deviation entry and incumbents' productivity shocks. Panel (b): one-standard-deviation MEI and price markup shocks.

inflation and in real interest rates that initially lowers both consumption and investment. Finally, note that the shock raises the price/cost margin of INT-firms.

The incumbents' productivity shock, ε_t^{Ψ}, is by assumption temporary and has an estimated autoregressive coefficient $\rho^{\Psi} = 0.247$ at the posterior mean. The shock affects the bulk of the η_{t-1} firms and therefore has a large effect on the supply of INT-goods. The decrease in $\frac{p_t}{(p_t^z)^\gamma}$ causes a persistent fall in the entry rate, and it is initially so strong that the exit rate increases too. Note that ε_t^{Ψ} increases the density of firms characterized by $A_t^{f,INC} > \hat{A}_t^{INC}$. For this reason, after a few quarters the exit rate falls below steady state and the number of incumbents picks up again. The initial reduction in the number of incumbents is associated with shifts to the support of the $H_t\eta_{t-1}$ mass of surviving incumbents (see condition 2.21), raising the average efficiency of these firms. Due to the persistent fall in $\frac{p_t}{(p_t^z)^\gamma}$, both productivity thresholds remain above steady state for a prolonged period. In line with standard productivity shocks, the increased supply of INT-goods has a deflationary effect that triggers an expansionary monetary policy, stimulating both consumption and investment.

The MEI shock drives a standard boom in demand. The increase in $\frac{p_t}{(p_t^z)^\gamma}$ raises(lowers) the entry(exit) rate. This has non-negligible implications for the productivity thresholds and for average firm efficiency that persistently fall. Finally, a negative markup shock has an unambiguously expansionary effect. The shock pulls up $\frac{p_t}{(p_t^z)^\gamma}$, lowering productivity thresholds and increasing(reducing) the entry(exit) rate. The average efficiency of INT-firms falls. The expansionary monetary policy response to the shock supports the growth of both consumption and investment.

2.3 Bayesian estimation

We estimate the model on U.S. data spanning from 1966:I to 2019:IV (with a presample of four quarters starting in 1965:I). The dataset consists of the *yearly* firm entry rate measured by the Business Dynamics Statistics (BDS), and of seven standard macroeconomic variables observed at a quarterly frequency: worked hours, the Fed funds rate, the inflation rate (GDP deflator), and the growth rates of GDP, investment, consumption, and wages in real terms. The macroeconomic observables and the initial date are the same considered by Smets and Wouters (2007), whose results we take as a benchmark reference for business cycle analysis.[10]

As regards our measure of firm entry, we choose the BDS database, which gathers information on the entire universe of U.S. firms.[11] This source has been widely employed to study various features of business dynamism. Examples include Hathaway and Litan (2014), who analyze the geographical aspects of the decline in U.S. business dynamism, Decker et al. (2014), who study the role played by entrepreneurship (in the form of startup rates) in U.S. job creation, Gourio et al. (2016), who use a VAR to estimate the effects of a shock to the number of startups, and Karahan et al. (2019), who link the fall in firm entry to the slow-

[10]Appendix F.1 contains a detailed discussion of data sources, definitions, and transformations.

[11]The BDS data are aggregated starting from the Longitudinal Business Database (LBD), that tracks single establishments and firms since 1976. These micro-data are used, among the others, by Decker et al. (2020) to discriminate between possible reasons behind the firm entry decline.

ing pace of labor supply growth. An alternative source for data on startups is provided by the Bureau of Labor Statistics (BLS), whose records are available at a quarterly frequency. However, the BLS data are characterized by two crucial features that make the BDS more suitable for our purposes. First, its sample starts in 1992 and does not allow us to study the persistent decline in the entry rate. Second, the BLS provides data on *establishment* entry rates: while these might be more useful to investigate job creation and destruction, they are arguably less relevant for new business formation, as new business establishments do not necessarily represent new firms. Conversely, the BDS database does distinguish between firms and establishments.[12]

In order to deal with the mixed-frequency nature of our dataset, we construct an annualized model-implied measure of firm entry.[13] In particular, we take the sum of new entrants over the periods $t : t-3$ divided by the average of total firms over the periods $t : t-7$, where each period t denotes a quarter. This is consistent with the BDS measure of firm entry, which is defined as the number of firm births in each year divided by the average number of firms in that and in the previous year. Then, our model-implied variable is matched with the observed BDS data only in the final quarter of each year. The values for the remaining quarters are treated as missing observations and inferred by the Kalman filter.

Hirose and Inoue (2016) suggest that ZLB periods may bias the estimates of some parameters and shocks. To check for this, we estimated the model over the subsample 1966:I-2007:III. Further, we estimated the model over the full sample after replacing the Fed funds rate with the shadow rate, obtained by Wu and Xia (2016), from 1990:I up to the end of our sample.

Another important issue concerns the analysis of unconventional monetary policies, which are not considered in our model. To some extent, these policy actions might be captured by MEI shocks, which may be interpreted as disturbances that affect the financial system ability to turn savings into capital (Justiniano et al., 2011).[14]

2.3.1 Calibration and priors

Following common practice, we calibrate some parameters that are hard to identify (Table 2.1). These include the capital depreciation rate, $\delta = 0.025$, corresponding to a 10% depreciation rate per year; the capital share $\alpha = 0.33$, corresponding to a steady-state share of capital income roughly equal to 30%; the labor disutility parameter ψ is calibrated to pin down the steady-state level of worked hours at 0.33; the steady-state product and labor market elasticities, ϵ^p and ϵ^w, are set at 6 and 21, implying steady-state markups of 20% and 5% respectively, as in Christiano et al. (2014). The share of government spending in aggregate output, $g^s = 0.18$, and the AR(1) parameter in (2.50), $\rho^\pi = 0.99$, are borrowed from Del Negro et al. (2015).

We set firms' return, $\gamma = 0.9$, in the range of Basu and Fernald (1997) estimates, and the tail index of the Pareto distribution, $\xi = 15$, following Asturias et al. (2017).

[12]BLS data are used by Casares et al. (2020) to estimate a model with endogenous entry and exit. Differently from our long-term perspective, their focus is on the period following the financial crisis and on the relationship between U.S. business cycle fluctuations and the extensive margin.

[13]See Pfeifer (2013) for references on methodology.

[14]We also experimented with an additional risk-premium shock that did not play any significant role in our estimates.

Table 2.1: Calibrated parameters

Parameter	L_{ss}	g^s	ϵ^p	ϵ^w	α	γ	δ	ξ	$entry$	$w_{ss}\phi^{INC}$	ϕ^{ratio}	ρ^{π}
Value	0.33	0.18	6	21	0.33	0.9	0.025	15	0.025	0.05	0.7	0.99

We set the de-trended support of the NEs distribution, z, the depreciation rate of firms efficiency, δ^{INC}, the de-trended and wage-adjusted fixed production costs, $w_{ss}\phi^j$, to calibrate some steady-state variables that characterize firm dynamics and the structure of the INT-goods sector. We set the firm entry rate, $\frac{NE}{\eta} = 2.5\%$, to match the 10% average yearly entry rate observed over the period 1978-2019. The steady-state number of firms, η, is normalized at 1. The fixed costs of production in labor units, $w\left(\phi^{NE} + \phi^{INC}\right)$, amount to $13,8\%$ of total GDP (Bilbiie et al., 2012; Colciago and Etro, 2010). The relative production size of NEs, which ultimately depends on the ratio of fixed costs, is 0.7, close to the value reported in Clementi and Palazzo (2016). The remaining parameters are estimated with Bayesian techniques. Priors are in line with those adopted in previous empirical DSGE models. In particular, most prior distributions are borrowed from Smets and Wouters (2007) with few minor differences. We slightly reduce the prior standard deviation of φ, the inverse Frisch elasticity parameter. The prior for $\bar{\pi}_{ss}$ is looser and centered on a higher mean, following Del Negro et al. (2015). Finally, the Taylor rule response to GDP growth and the two Calvo parameters are assigned a higher prior mean, closer to Christiano et al. (2014) and Justiniano et al. (2011).

2.4 Results

Table 2.2 describes parameters and shock processes and reports our posterior estimates for the baseline, full sample model.[15] Consumption habits are in line with Justiniano et al. (2011), whereas both Calvo parameters are close to the values reported by Del Negro et al. (2015) and are substantially smaller than in Del Negro et al. (2017) and Casares et al. (2020). The elasticity of capital utilization costs is slightly higher than in Casares et al. (2020) and Justiniano et al. (2011) who find a value of 0.84. Lastly, investment adjustment costs are close to Lewis and Stevens (2015) and below the estimate obtained by Christiano et al. (2014).

Table 2.2: Estimated parameters and structural shocks

		Prior			Posterior			
	Description	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	Inverse EIS	norm	1.500	0.3750	1.173	0.0415	1.1048	1.2389
φ	Inverse Frisch elasticity	norm	2.000	0.5000	2.354	0.4387	1.6174	3.0601
h	Consumption habits	beta	0.700	0.1000	0.840	0.0331	0.7860	0.8940
$100(\beta^{-1}-1)$	Discount factor	gamm	0.250	0.2000	0.169	0.0798	0.0332	0.2887
$\bar{\pi}_{ss}$	SS inflation rate	gamm	0.750	0.4000	0.716	0.3398	0.1743	1.2297
$100(g^z-1)$	Deterministic trend	norm	0.400	0.1000	0.350	0.0437	0.2791	0.4230

(Continued on next page)

[15]See Appendix F.2 for a detailed discussion of parameters' identification and convergence.

Table 2.2: (continued)

	Description	Prior			Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ_π	Taylor rule coeff. on π	norm	1.500	0.2500	1.736	0.1767	1.4447	2.0237
κ_y	Taylor rule coeff. on y	norm	0.200	0.0500	0.239	0.0453	0.1647	0.3136
ρ_i	Policy rate per.	beta	0.750	0.1000	0.764	0.0274	0.7196	0.8094
Γ_p	Price rigidity	beta	0.650	0.1000	0.834	0.0215	0.7982	0.8688
μ_p	Price indexation	beta	0.500	0.1500	0.247	0.0971	0.0923	0.4000
Γ_w	Wage rigidity	beta	0.650	0.1000	0.815	0.0493	0.7345	0.8988
μ_w	Wage indexation	beta	0.500	0.1500	0.315	0.1457	0.0924	0.5217
γ_I	Investment adjustment costs	norm	4.000	1.5000	9.415	1.1286	7.5649	11.2810
σ_a	Capital utilization elasticity	beta	0.500	0.1500	0.884	0.0483	0.8092	0.9601
ρ^μ	MEI shock per.	beta	0.500	0.2000	0.556	0.0683	0.4554	0.6648
ρ^r	Monetary shock per.	beta	0.500	0.2000	0.305	0.0617	0.2064	0.4099
ρ^p	Price markup shock per.	beta	0.500	0.2000	0.984	0.0064	0.9742	0.9943
η^p	Price markup shock MA par.	beta	0.500	0.2000	0.363	0.0682	0.2513	0.4757
ρ^l	Labor supply shock per.	beta	0.500	0.2000	0.172	0.0636	0.0666	0.2739
ρ^Ψ	Incumbents' prod. shock per.	beta	0.500	0.2000	0.247	0.0667	0.1378	0.3566
ρ^{g^s}	Gov. spending shock per.	beta	0.500	0.2000	0.961	0.0106	0.9442	0.9779
σ^z	Entry shock s.d.	gamm	0.100	0.0500	0.005	0.0003	0.0047	0.0055
σ^μ	MEI shock s.d.	gamm	0.100	0.0500	0.090	0.0154	0.0655	0.1145
σ^r	Monetary policy shock s.d.	gamm	0.100	0.0500	0.002	0.0001	0.0022	0.0026
σ^p	Price markup shock s.d.	gamm	0.100	0.0500	0.079	0.0104	0.0625	0.0959
σ^l	Labor supply shock s.d.	gamm	0.100	0.0500	0.158	0.0363	0.0995	0.2146
σ^Ψ	Incumbents' prod. shock s.d.	gamm	0.100	0.0500	0.006	0.0003	0.0053	0.0063
σ^{g^s}	Gov. spending shock s.d.	gamm	0.100	0.0500	0.033	0.0017	0.0306	0.0360
σ^π	Inflation target shock s.d.	gamm	0.100	0.0500	0.001	0.0002	0.0008	0.0013

Note: The last two columns report the lower (HPD inf) and the upper bound (HPD sup) of the parameter's 90% highest posterior density interval.

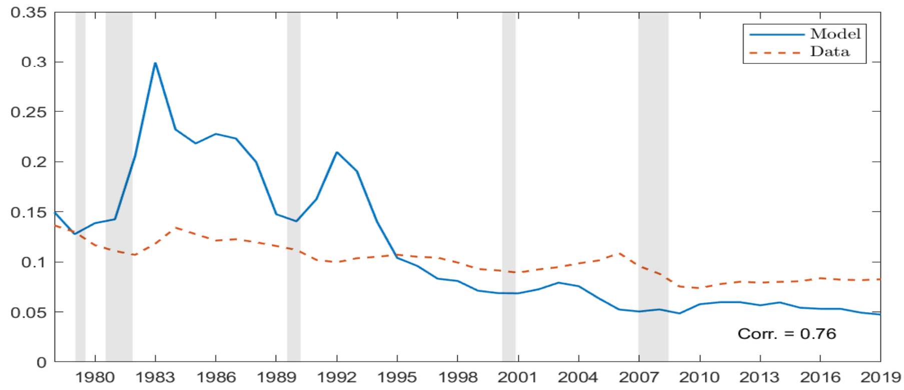
In Appendix H, we draw a comparison with the posterior estimates coming from a standard New Keynesian (NK) model, estimated on the same sample and with the same observables (excluding the entry rate). We also benchmark the interpretation of the U.S. business cycle provided by our model against the established narrative based on the standard NK model.

2.4.1 Drivers of the entry rate decline

A preliminary step in our analysis is a discussion of the model-implied entry rate obtained in an "uninformed" estimation that excludes the entry rate series from the set observables.[16] Via this exercise, we can test the model's ex-ante ability to capture entry dynamics, without

[16]Removing firm entry from the observables does not produce significant variations in most parameters' posterior estimates with a few exceptions. The uninformed estimation generates a higher degree of wage indexation, while the elasticity of investment adjustment costs and the MEI-shock autoregressive coefficient are respectively smaller and larger when the model is not constrained to explain entry rate dynamics.

Figure 2.5: Model-implied entry rate, 1978:I-2019:IV (unobserved-entry estimation)



Note: The solid line shows the annualized smoothed estimate of the model-implied entry rate at the posterior mean (unobserved-entry estimation). The dashed line shows the annual firm entry rate in the data (BDS).

forcing it to rationalize long-run entry data. As apparent from Figure 2.5, the model-implied entry rate grossly overpredicts the response of the observed one to post-recession recoveries up to the mid-90s, before shifting mildly below its empirical counterpart. Nevertheless, information coming from the standard set of observed macroeconomic variables and the need to match the U.S. business cycle data is sufficient for the model to predict a long-term decline in firm entry, and its correlation with the observed series is rather large (0.76). The model also predicts a flattening of the entry rate following the Global Financial Crisis (GFC), consistent with the data.

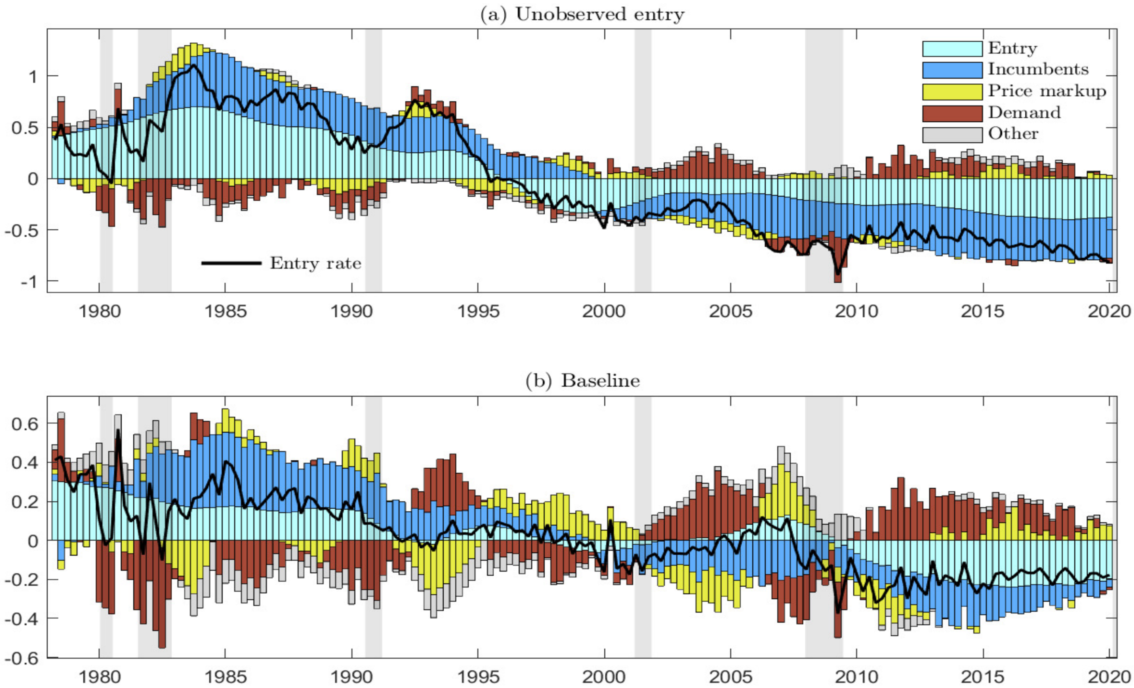
It is also interesting to look at the historical decomposition of the model-implied entry rate (Figure 2.6, Panel (a)), where the estimated technology shocks that persistently lower(raise) the productivity of NEs($INCs$) also imply the prediction of a long-term decline in the entry rate.

Figure 2.6, Panel (b), presents the historical decomposition of the observed entry obtained in our baseline model. Relative to the uninformed estimation, the contribution of technology shocks is virtually unchanged, confirming that productivity growth of potential NEs gradually declined over the sample and that NEs were crowded out by the technology shocks that raised $INCs$ productivity. Price-markup and other, mainly demand, shocks contribute to predicting the observed entry rate, essentially compensating the gap between the model-implied and observed series. These shocks bring down the entry rate when it is relatively high (between 1978 and the mid-90s), and tend to raise it thereafter. These results are fully confirmed when we implement robustness checks either restricting the sample to the pre-GFC period, *i.e.* the estimation sample is truncated at 2007:III, or substituting the shadow rate for the observed interest rate.[17]

As a final remark, Table 2.3 summarizes the contribution of different shocks to GDP growth according to the baseline and unobserved-entry estimates since 1978, *i.e.* the first year when the baseline model is "constrained" to match the entry rate. Perhaps surprisingly,

[17]The posterior estimates of these alternative specifications are presented in Table J1.

Figure 2.6: Historical shock decomposition: model-implied vs observed entry rate, 1978:I-2019:IV



Note: The solid line is firm entry in log-deviations from its steady state (quarterly estimate at the posterior mean). The colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values). Panel (a): unobserved-entry estimation. Panel (b): baseline estimation (firm entry coincides with the observed one).

technology shocks play a lesser role when we match firm entry and are replaced by demand shocks. Furthermore, the two models agree about the relative contributions of the technology shocks that affect NEs and $INCs$.

Table 2.3: Historical shock decomposition "summary": GDP growth, 1978:I-2019:IV

	Shocks				
	Incumbents	Entry	Price markup	Demand	Other
Unobserved entry	16.3%	11.9%	11.3%	48.0%	12.4%
Baseline	10.2%	7.1%	9.8%	60.7%	12.2%

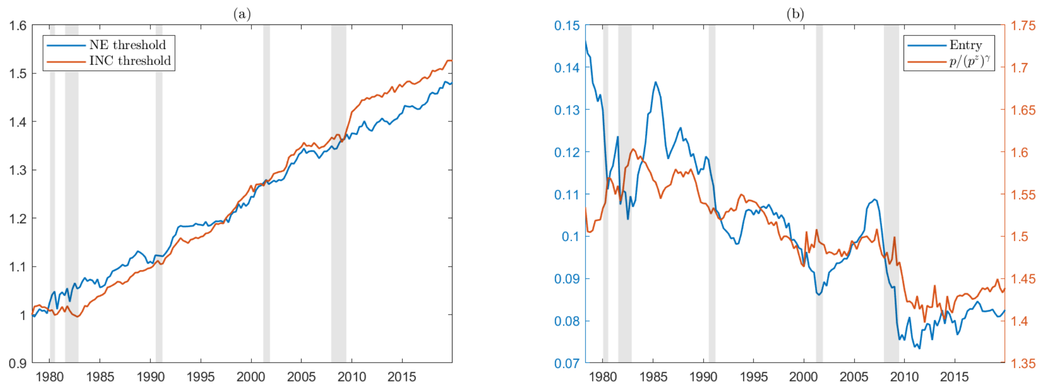
Note: For each shock group, the percentage terms refer to its average contribution to GDP growth, as obtained from the historical shock decomposition (at the posterior mean), over the period 1978:I-2019:IV.

Discussion

The minor role of markup shocks, mostly relegated to cyclical dynamics in entry, is in contrast with Gutiérrez and Philippon (2017), who argue that the increase in price markups might have been at the root of the observed secular decline in entry rates. Our model does predict a long-term increase in price markups and in "pure" profits (in line with evidence reported in Traina, 2018), and highlights the persistent decline in the demand elasticity of retail goods, ϵ_t^p, as the main driver of long-run markup dynamics.[18] However, this latter effect has only short-lived implications for the entry rate.

Conversely, we identify a reversal in technology shocks as the main reason for the decline in the long-run entry rate. As shocks for $NEs(INCs)$ turned less(more) favorable, the productivity threshold for potential entrants increased reducing their probability to effectively enter the economy. This pattern, displayed in our historical decomposition of entry, is indeed reflected in a divergence between the productivity thresholds of new entrants and incumbents (see Panel (a) of Figure 2.7). The efficiency gap between incumbents and new entrants has gradually increased since the mid-1980s. This is consistent with the evidence that incumbents accounted for the lion's share of innovation in the latest decades (Hsieh and Klenow, 2018; Garcia-Macia et al., 2019). The declining growth rate of new entrants' productivity may instead be reminiscent of the findings in Bloom et al. (2020), who document a fall in research productivity during the past 15 years, arguing that ideas are getting increasingly harder to find.

Figure 2.7: Model predictions for productivity thresholds and the price/cost margin



Note: All lines depict quarterly smoothed estimates at the posterior mean (baseline estimation). Panel (a): NE productivity threshold $\hat{A}_t^{NE}$ (blue) and INC productivity threshold $\hat{A}_t^{INC}$ (orange); both series are normalized at 1 in 1978:I. Panel (b): price/cost margin of INT-firms (orange) and entry rate (blue). 1978:I-2019:IV.

We also estimate a strong correlation (78%) between the entry rate and the price/cost margin index $\frac{p_t}{(p_t^z)}$ (see Panel (b) of Figure 2.7). The historical decomposition of $\frac{p_t}{(p_t^z)}$ is essentially driven by technology shocks, whereas those demand shocks that matter for the entry rate bear nearly symmetrical effects on the prices of the intermediate goods and of the

[18] See Figure J1 in the Appendix.

Z bundle.[19] This is a novel result, whereby $\frac{p_t}{(p_t^z)}$ can be interpreted as a summary statistics capturing the occurrence of technology shocks that determine the dynamics of firm entry.

To conclude this discussion, we highlight the reasons why technology shocks may have similar implications for the entry rate and for the price/cost margin in the INT-sector. Consider first the case of an adverse entry shock (Figure 2.4).[20] In this case, the short-run effect of the shock temporarily raises output and demand for labor and capital but depresses the consumption price of intermediate goods and lowers $\frac{p_t}{(p_t^z)}$. Then, consider a favorable shock to the productivity of incumbents. For reasons already discussed in section 2.5 above, the entry rate drops, but the sustained output expansion raises both the wage rate and the rate of return on capital. As a result, p_t^z increases and the price/cost margin of INT-firms inevitably falls.

The following sections present additional results regarding the implications of our model for total factor productivity and other measures of business dynamism.

2.4.2 Firm efficiency and total factor productivity in the long run

We define TFP, average firm efficiency, and efficiency dispersion respectively as

$$TFP_t = \int_{\hat{A}_t^{NE}}^{\infty} A_t^{f,NE} + \int_{\hat{A}_t^{INC}}^{\infty} A_t^{f,INC} = \frac{\xi}{\xi - 1}\left(NE_t \hat{A}_t^{NE} + INC_t \hat{A}_t^{INC}\right) , \tag{2.55}$$

$$\hat{A}_t^{av} = \frac{TFP_t}{\eta_t} , \tag{2.56}$$

$$\Sigma_t^A = \frac{\xi}{(\xi - 2)\,\eta_t}\left[NE_t\left(\hat{A}_t^{NE}\right)^2 + INC_t\left(\hat{A}_t^{INC}\right)^2\right] . \tag{2.57}$$

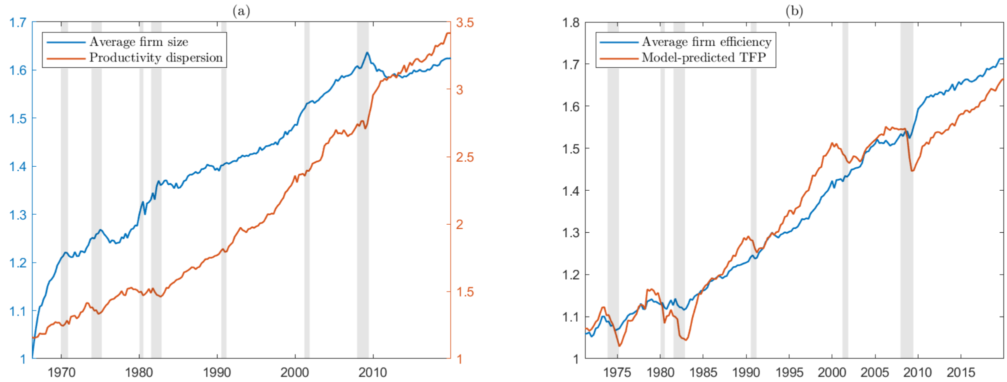
Both technology and non-technology shocks explain the volatility of the growth rates of TFP_t, $\hat{A}_t^{av}$, and Σ_t^A.[21] Shocks to incumbents' productivity determine about 92% of average efficiency growth volatility, while the contribution of non-technology shocks is less than 8%. We observe a similar variance decomposition for the growth rate of efficiency dispersion. By contrast, demand shocks have predominant effects on TFP growth through their impact on the extensive margin.

The left panel of Figure 2.8 plots the post-1966 series of firm size, proxied by the average size of the Z_t bundle, and firm efficiency dispersion. Hopenhayn et al. (2018) find that average firm size, measured by the number of employees, rose by 20% between 1977 and 2014. Our measure, which also accounts for capital accumulation, predicts a 28% increase over the same period. As for productivity dispersion, Kehrig (2015) reports that it "doubled" over the period 1972-2009. Our estimation implies that in 2009 the productivity-dispersion measure was about 2.3-times larger than in 1972.

Kehrig (2015) shows that the dispersion of firms' total factor productivity in U.S. manufacturing is greater in recessions than in booms. He builds on this result to discriminate

[19]See Figure J2 in the Appendix.
[20]Note that the IRFs in Figure 2.4 display the effects of a *positive* entry shock.
[21]See Table J2 in the Appendix for the unconditional variance decomposition.

Figure 2.8: Model predictions for TFP, firm efficiency and firm size



Note: All lines depict quarterly smoothed estimates at the posterior mean (baseline estimation). Panel (a): Average firm size (blue), proxied by the ratio of the factor-inputs bundle (Z_t) over the number of firms (η_t), and productivity dispersion (orange), *i.e.* weighted average of the productivity dispersion of NEs and $INCs$ (dispersion of INC (NE) firm productivities is computed as the variance of the left-truncated Pareto distribution INC (NE) firms draw efficiency from; the truncation is given by the respective productivity threshold that varies over time, while the shape of the distribution is constant); 1966:I-2019:IV (the smoothed series of Z_t and η_t are normalized at 1 in 1966:I). Panel (b): average firm efficiency $\hat{A}_t^{av}$ (blue) and model-predicted TFP $\eta_t \hat{A}_t^{av}$ (orange); 1971:I-2019:IV ($\hat{A}_t^{av}$ is normalized at 1 in 1966:I).

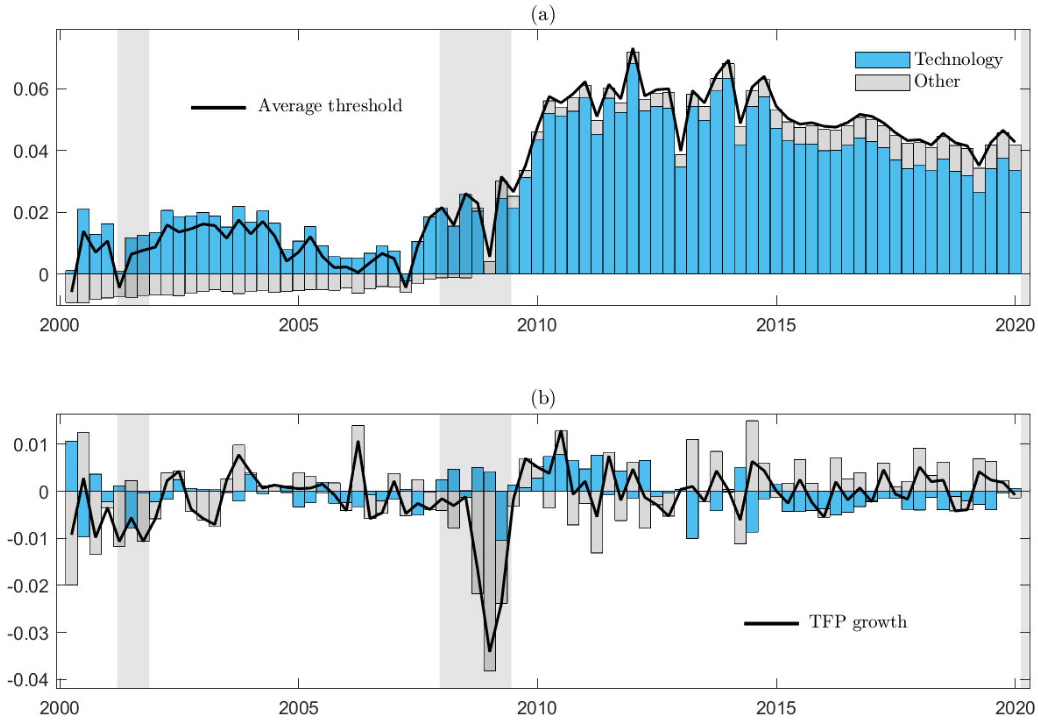
between "Schumpeterian" models that unambiguously praise the cleansing effect of recessions, and the "sullying view" supported by models in the tradition of Melitz (2003), where the procyclical pattern of input costs generates opposite effects of efficiency dispersion. Even if our INT-firm sector is inherently "Schumpeterian", we find that the estimated pattern of efficiency dispersion in recessions is ambiguous. According to our estimates (Figure 2.9, top panel), during recession episodes in 2001 and 2007, the productivity thresholds are almost entirely determined by technology shocks, but these shocks did not play a key role in determining the recession (see Figure H1, top panel, for the historical decomposition of GDP growth).[22]

Extensive margin dynamics drive our estimated TFP measure, as shown in Panel (b) of Figure 2.8 where the wedge between TFP and average efficiency is entirely accounted for by variations in the mass of firms. Consistently with previous evidence (Field, 2010, 2011), TFP growth is strongly procyclical, it has declined since 2005, but the GFC apparently marked a watershed, as pointed out in studies such as Anzoategui et al. (2019) and Bianchi et al. (2019).[23] Differently from these studies, our estimates interpret the TFP slowdown during the GFC as the consequence of adverse non-technology shocks that mainly operated through the extensive margin (Figure 2.9, bottom panel).

[22]To rationalize this result, consider that, because of the predominant presence of incumbents in the market, $\hat{A}_t^{av}$ is mainly affected by the $INCs$' threshold and the latter is strongly sensitive to the price/cost margin, which does not necessarily decrease during recessions (see Panel (b) of Figure 2.7). We further discuss the determinants of productivity thresholds in Appendix G.3.

[23]By contrast, Fernald (2014) points out that productivity behaved similarly to previous episodes of severe recession, but recovered strongly once the recession ended.

Figure 2.9: Historical shock decomposition: average threshold (de-trended) and TFP growth rate, 1978:I-2019:IV



Note: The colored bars are the contributions of the grouped shocks ("Other" includes labor supply shocks and contribution from initial values). Panel (a): The solid line is the de-trended average productivity threshold in log-deviations from its steady state (quarterly estimate at the posterior mean). Panel (b): The solid line is TFP growth in log deviations from its steady state (quarterly estimate at the posterior mean).

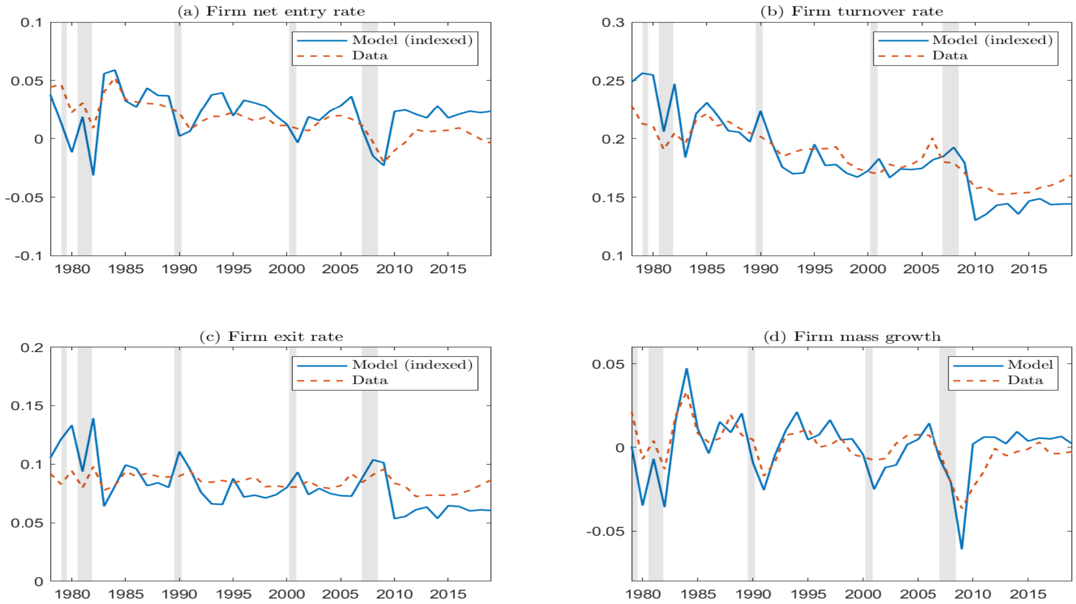
2.4.3 Other measures of business dynamism

In addition to the entry rate, our model bears predictions for other measures of business dynamism, such as net entry and turnover, respectively $\frac{NE_t - EX_t}{\eta_t}$ and $\frac{NE_t + EX_t}{\eta_t}$. The model does a reasonably good job in predicting either variable (see Panels (a) and (b) of Figure 2.10), but there is a tendency to predict a pro(counter)cyclical pattern that is difficult to detect in the observed series for net entry(turnover). In fact, these results are driven by the gap between the model-predicted and the observed exit rate series, as shown in Panel (c).[24]

Due to the overestimated countercyclical pattern of the exit rate, our model also exaggerates the procyclicality of $\Delta\eta_t$ (see Panel (d) of Figure 2.10). This effect is particularly strong in occasion of the recessions that marked the beginning and the end of the Great Moderation period. This suggests that our results concerning the deep TFP fall during the GFC should be taken with some caution.

[24]Since exit flows contribute to determining the number of firms, one might wonder whether this bias could have implications for the interpretation of the entry-rate drivers. We discuss this issue in Appendix I.

Figure 2.10: Model predictions for other measures of business dynamism, 1978-2019



Note: Solid lines show annualized smoothed estimates at the posterior mean; dashed lines display the respective counterparts in the data (BDS). The model-implied series in panels (a), (b), and (c) are indexed at the 2000:I observed values. Both series in Panel (d) are expressed in percentage deviations from their sample mean.

2.5 Conclusions

The paper establishes a strong connection between the long-term decline in the entry rate and the asymmetric technology shocks that persistently hit new entrants and incumbent firms. By contrast, the model-implied cumulative increase in price markups did not contribute to the concurrent fall in the entry rate. Importantly, these results are confirmed even if we exclude the entry rate from the observed variables.

Our results emphasize the importance of the extensive margin in determining the long-term slowdown in TFP growth. The extensive margin also introduces a hitherto unexplored channel for the transmission of non-technology shocks to the cyclical component of aggregate TFP.

The model challenges popular wisdom on the "cleansing" effect of recessions: demand-driven recessions do not necessarily generate survival of the fittest.

Finally, we highlight the reduction in price/cost margins of INT-firms as a single statistic that captures the effects of technology shocks on entry decisions. Micro-econometric analysis should investigate the responsiveness of the entry rate to price/cost margins. We leave this for future work.

References

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Mihoubi, F., Mutschler, W., Pfeifer, J., Ratto, M., Rion, N., and Villemot, S. (2022). Dynare: Reference manual version 5. Dynare Working Papers 72, CEPREMAP.

Alon, T., Berger, D., Dent, R., and Pugsley, B. (2018). Older and slower: The startup deficit's lasting effects on aggregate productivity growth. *Journal of Monetary Economics*, 93:68–85.

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and R&D as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Asturias, J., Hur, S., Kehoe, T. J., and Ruhl, K. J. (2017). Firm entry and exit and aggregate growth. Working Paper 23202, National Bureau of Economic Research.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., and Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135(2):645–709.

Basu, S. and Fernald, J. G. (1997). Returns to scale in U.S. production: Estimates and implications. *Journal of Political Economy*, 105(2):249–283.

Bianchi, F., Kung, H., and Morales, G. (2019). Growth, slowdowns, and recoveries. *Journal of Monetary Economics*, 101:47–63.

Bilbiie, F. O., Ghironi, F., and Melitz, M. J. (2007). Monetary policy and business cycles with endogenous entry and product variety. *NBER Macroeconomics Annual*, 22:299–353.

Bilbiie, F. O., Ghironi, F., and Melitz, M. J. (2012). Endogenous entry, product variety, and business cycles. *Journal of Political Economy*, 120(2):304–345.

Bloom, N., Jones, C. I., Van Reenen, J., and Webb, M. (2020). Are ideas getting harder to find? *American Economic Review*, 110(4):1104–44.

Boppart, T. and Li, H. (2021). Productivity slowdown: reducing the measure of our ignorance. CEPR Discussion Papers 16478, C.E.P.R. Discussion Papers.

Casares, M., Khan, H., and Poutineau, J.-C. (2020). The extensive margin and US aggregate fluctuations: A quantitative assessment. *Journal of Economic Dynamics and Control*, 120:103997.

Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk shocks. *American Economic Review*, 104(1):27–65.

Clementi, G. L. and Palazzo, B. (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics*, 8(3):1–41.

Colciago, A. and Etro, F. (2010). Real business cycles with Cournot competition and endogenous entry. *Journal of Macroeconomics*, 32(4):1101–1117.

De Loecker, J., Eeckhout, J., and Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, 135(2):561–644.

Decker, R. A., Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2014). The role of entrepreneurship in US job creation and economic dynamism. *Journal of Economic Perspectives*, 28(3):3–24.

Decker, R. A., Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2020). Changing business dynamism and productivity: Shocks versus responsiveness. *American Economic Review*, 110(12):3952–90.

Del Negro, M., Giannone, D., Giannoni, M. P., and Tambalotti, A. (2017). Safety, liquidity, and the natural rate of interest. *Brookings Papers on Economic Activity*, 2017(Spring):231–316.

Del Negro, M., Giannoni, M. P., and Schorfheide, F. (2015). Inflation in the Great Recession and New Keynesian models. *American Economic Journal: Macroeconomics*, 7(1):168–96.

Eggertsson, G. B., Robbins, J. A., and Wold, E. G. (2021). Kaldor and Piketty's facts: The rise of monopoly power in the United States. *Journal of Monetary Economics*, 124:S19–S38.

Fernald, J. G. (2014). Productivity and potential output before, during, and after the Great Recession. In *NBER Macroeconomics Annual 2014, Volume 29*, NBER Chapters, pages 1–51. National Bureau of Economic Research, Inc.

Field, A. J. (2010). The procyclical behavior of total factor productivity in the united states, 1890-2004. *The Journal of Economic History*, 70(2):326–350.

Field, A. J. (2011). *A Great Leap Forward: 1930s Depression and U.S. Economic Growth*. Yale University Press.

Fort, T. C., Pierce, J. R., and Schott, P. K. (2018). New perspectives on the decline of US manufacturing employment. *Journal of Economic Perspectives*, 32(2):47–72.

Garcia-Macia, D., Hsieh, C.-T., and Klenow, P. J. (2019). How destructive is innovation? *Econometrica*, 87(5):1507–1541.

Gottfries, N., Mickelsson, G., and Stadin, K. (2021). Deep Dynamics. CESifo Working Paper Series 8873, CESifo.

Gourio, F., Messer, T., and Siemer, M. (2016). Firm entry and macroeconomic dynamics: A state-level analysis. *American Economic Review*, 106(5):214–18.

Grullon, G., Larkin, Y., and Michaely, R. (2019). Are US industries becoming more concentrated? *Review of Finance*, 23(4):697–743.

Gutiérrez, G. and Philippon, T. (2016). Investment-less growth: An empirical investigation. Working Paper 22897, National Bureau of Economic Research.

Gutiérrez, G. and Philippon, T. (2017). Declining competition and investment in the U.S. Working Paper 23583, National Bureau of Economic Research.

Hathaway, I. and Litan, R. (2014). Declining business dynamism in the United States: A look at states and metros. *Brookings Economic Studies*.

Hirose, Y. and Inoue, A. (2016). The zero lower bound and parameter bias in an estimated DSGE model. *Journal of Applied Econometrics*, 31(4):630–651.

Hopenhayn, H., Neira, J., and Singhania, R. (2018). From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share. Working Paper 25382, National Bureau of Economic Research.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, 60(5):1127–1150.

Hsieh, C.-T. and Klenow, P. J. (2018). The Reallocation Myth. Working Papers 18-19, Center for Economic Studies, U.S. Census Bureau.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2011). Investment shocks and the relative price of investment. *Review of Economic Dynamics*, 14(1):102–121. Special issue: Sources of Business Cycles.

Karahan, F., Pugsley, B., and Şahin, A. (2019). Demographic origins of the startup deficit. Working Paper 25874, National Bureau of Economic Research.

Kehrig, M. (2015). The cyclical nature of the productivity distribution. *Working Paper*.

Klenow, P. J. and Li, H. (2021). Innovative growth accounting. *NBER Macroeconomics Annual*, 35:245–295.

Lewis, V. and Poilly, C. (2012). Firm entry, markups and the monetary transmission mechanism. *Journal of Monetary Economics*, 59(7):670–685.

Lewis, V. and Stevens, A. (2015). Entry and markup dynamics in an estimated business cycle model. *European Economic Review*, 74:14–35.

Liu, J., Grubler, A., Ma, T., and Kogler, D. (2020). Identifying the technological knowledge depreciation rate using patent citation data: a case study of the solar photovoltaic industry. *Scientometrics*, 126.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Peters, M. and Walsh, C. (2021). Population growth and firm dynamics. Working Paper 29424, National Bureau of Economic Research.

Pfeifer, J. (2013). A guide to specifying observation equations for the estimation of DSGE models. Working paper, University of Mannheim.

Piersanti, F. M. and Tirelli, P. (2020). Endogenous productivity dynamics in a two-sector business cycle model. Working Papers 434, University of Milano-Bicocca, Department of Economics.

Pugsley, B. W. and Şahin, A. (2018). Grown-up business cycles. *The Review of Financial Studies*, 32(3):1102–1147.

Qiu, Z. and Ríos-Rull, J.-V. (2022). Procyclical productivity in new keynesian models. Working Paper 29769, National Bureau of Economic Research.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Swanson, E. T. (2006). The Relative Price and Relative Productivity Channels for Aggregate Fluctuations. *The B.E. Journal of Macroeconomics*, 6(1):1–39.

Traina, J. (2018). Is aggregate market power increasing? Production trends using financial statements. Working Paper 17, Stigler Center for the Study of the Economy and the State.

Vinci, F. and Licandro, O. (2021). Switching-track after the Great Recession. Working Paper Series 2596, European Central Bank.

Wu, J. C. and Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, 48(2-3):253–291.

F Estimation

F.1 Data

Data on real GDP [GDPC1], the GDP deflator [GDPDEF], nominal personal consumption expenditures [PCEC], and nominal fixed private investment [FPI] are produced at a quarterly frequency by the Bureau of Economic Analysis and are included in the National Income and Product Accounts (NIPA). Average weekly hours in the nonfarm business sector [PRS85006023] and hourly compensation in the nonfarm business sector [PRS85006103] are produced by the Bureau of Labor Statistics (BLS) at a quarterly frequency. The civilian employment level [CE16OV] and the civilian non-institutional population [CNP16OV] are also produced by the BLS at a monthly frequency. We take quarterly averages of the monthly data. The federal funds rate [FEDFUNDS] is obtained from the Federal Reserve Board's H.15 release at a business day frequency. We take quarterly averages of the annualized daily data. All these data are collected from FRED (except for hourly wages, retrieved from the BLS database), and are transformed following Smets and Wouters (2007). Data on total firms and firm births (defined as firms born during the last 12 months) are produced by the Census Bureau, within the Business Dynamics Statistics (BDS) survey, at an annual frequency. In a robustness estimation, we use shadow rate data from Wu and Xia (2016).[25]

Table F1: Data Transformation

Data	Transformation
Output growth	$100\Delta \ln\left(\frac{GDPC1}{CNP16OV_{index}}\right)$
Investment growth	$100\Delta \ln\left[\left(\frac{FPI}{GDPDEF}\right)\frac{1}{CNP16OV_{index}}\right]$
Consumption growth	$100\Delta \ln\left[\left(\frac{PCEC}{GDPDEF}\right)\frac{1}{CNP16OV_{index}}\right]$
Wages growth	$100\Delta \ln\left(\frac{PRS85006103}{GDPDEF}\right)$
Hours worked	$100 \ln\left[\left(\frac{PRS85006023}{CNP16OV_{index}}\right)\left(\frac{CE16OV_{index}}{100}\right)\right]$
Inflation	$100\Delta \ln(GDPDEF)$
Nominal interest rate	$FEDFUNDS/4 \quad [SHADOWRATE/4]$
Entry rate	$100 \ln\left(1 + \frac{BIRTHS_y}{(FIRMS_y + FIRMS_{y-1})/2}\right)$

F.2 Parameters' identification and convergence

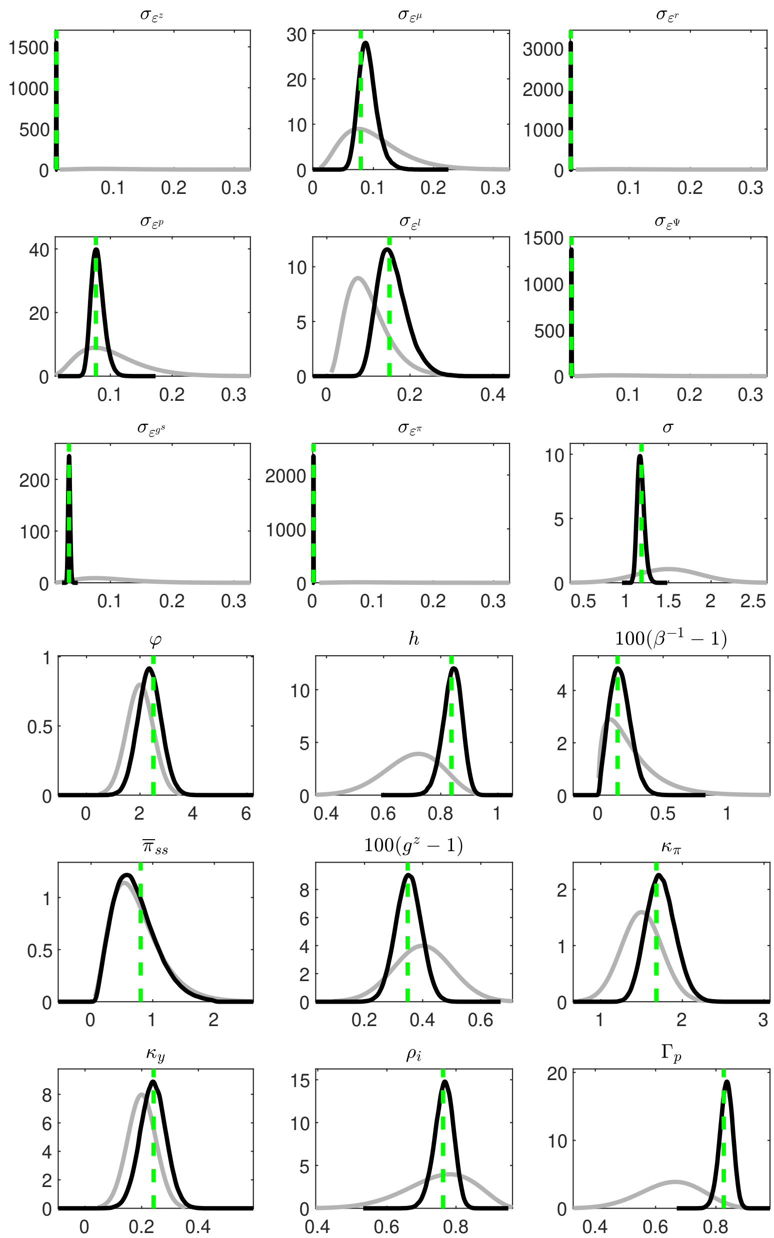
The model is solved using a first-order approximation around the deterministic steady state and is estimated using Dynare 4.6.2 (Adjemian et al., 2022). The baseline estimation is run with a single Markov Chain of 2 million draws, of which we discard the first 400 thousand. The overall acceptance ratio of the Metropolis-Hastings algorithm is close to 26%. Estimation results are virtually identical if we run four chains of 500 thousand draws each.

[25] `https://sites.google.com/view/jingcynthiawu/shadow-rates`

Prior-posterior plots

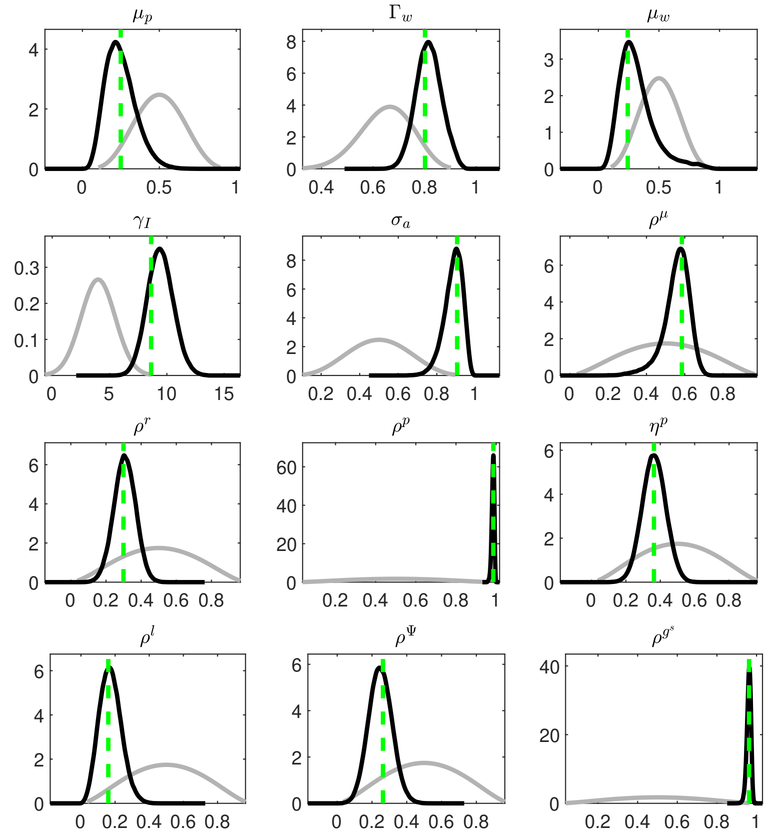
All posterior distributions are well-shaped and tighter than the respective priors, with the exception of the steady-state inflation rate whose prior and posterior distributions almost overlap, indicating a weak identification for this parameter. We do not consider this a major weakness of our estimation, since the posterior mean is close to the average inflation rate in the data and to the estimates in the DSGE literature.

Figure F1: Prior-posterior plots



(Continued on next page)

Figure F1: (continued)



Convergence

We consider two convergence diagnostics tests. The Geweke test uses a χ-square test to compare the means of draws from 400,000 to 720,000 and from 1,200,000 to 2,000,000. The null hypothesis is that the two sample means are equal, suggesting that draws from the two samples come from the same distribution, and thus that the chain has converged. In order to tackle the impact of draws' correlation on the estimates, a Newey and West (1987)-type estimator is used that tapers spectral density. The Raftery and Lewis test identifies the number of burn-in and the number of draws after burn-in required to estimate the q=0.025 percentile (corresponding to a 95% HPDI) with a precision of 0.5% with 95% certainty. If the number of burn-in and required draws is below the number of draws considered in the estimation, we can conclude that the chain has converged.

Looking at the p-values accounting for serial correlation (with taper), the null hypothesis for equality of means of the Geweke test (Table F2) is not rejected for all parameters but κ_π and ρ^l, at a 5% significance level. On the contrary, the Raftery and Lewis test (Table F3) delivers a maximum number of required draws well below the 2 million used in our estimation. In order to further examine the convergence of κ_π and ρ^l, we look at the trace plots of the two parameters (see Figures F2 and F3): in neither case do we spot evident drifts or jumps to other modes. Therefore, we are led to conclude that the Markov Chain has converged to the ergodic distribution.

Table F2: Geweke (1992) Convergence Tests, based on means of draws 400000 to 720000 vs 1200000 to 2000000. p-values are for χ^2-test for equality of means.

	Posterior		p-values			
Parameter	*Mean*	*Std*	*No Taper*	4% *Taper*	8% *Taper*	15% *Taper*
σ_{ε^z}	0.0051	0.0003	0.3410	0.9308	0.9232	0.9069
σ_{ε^μ}	0.0906	0.0154	0.0000	0.5877	0.5327	0.5003
σ_{ε^r}	0.0024	0.0001	0.0000	0.1742	0.1723	0.1592
σ_{ε^p}	0.0795	0.0105	0.0006	0.8070	0.7835	0.7506
σ_{ε^l}	0.1584	0.0370	0.0000	0.3895	0.3909	0.3583
$\sigma_{\varepsilon^\Psi}$	0.0058	0.0003	0.0000	0.1566	0.1995	0.2521
$\sigma_{\varepsilon^{g^s}}$	0.0333	0.0017	0.0000	0.1229	0.0868	0.0659
σ_{ε^π}	0.0011	0.0002	0.0000	0.3065	0.2557	0.2323
σ	1.1723	0.0418	0.0000	0.4398	0.4220	0.3782
φ	2.3524	0.4382	0.0000	0.4047	0.3570	0.3561
h	0.8402	0.0331	0.0000	0.7524	0.7516	0.7310
$100(\beta^{-1}-1)$	0.1687	0.0797	0.0000	0.1027	0.1037	0.0663
$\bar{\pi}_{ss}$	0.7145	0.3385	0.0000	0.1459	0.1240	0.1242
$100(g^z-1)$	0.3508	0.0437	0.8886	0.9892	0.9877	0.9866
κ_π	1.7367	0.1770	0.0000	0.0301	0.0155	0.0081
κ_y	0.2391	0.0453	0.0000	0.3856	0.3896	0.4003
ρ_i	0.7642	0.0274	0.0000	0.2307	0.3054	0.3418
Γ_p	0.8337	0.0215	0.0002	0.8269	0.8024	0.7629
μ_p	0.2471	0.0975	0.0000	0.0961	0.0838	0.0568
Γ_w	0.8153	0.0493	0.0000	0.4520	0.4308	0.4062
μ_w	0.3161	0.1458	0.0000	0.1583	0.1291	0.1166
γ_I	9.4144	1.1290	0.9057	0.9924	0.9915	0.9904
σ_a	0.8844	0.0481	0.0000	0.5240	0.5560	0.5497
ρ^μ	0.5555	0.0686	0.0000	0.3768	0.3432	0.3348
ρ^r	0.3059	0.0617	0.0000	0.7046	0.7270	0.7181
ρ^p	0.9841	0.0064	0.0000	0.4358	0.4603	0.4485
η^p	0.3625	0.0686	0.0000	0.5989	0.5790	0.5659
ρ^l	0.1722	0.0636	0.0000	0.0203	0.0084	0.0003
ρ^Ψ	0.2472	0.0666	0.0000	0.5467	0.5078	0.4864
ρ^{g^s}	0.9609	0.0105	0.1021	0.9080	0.8946	0.8667

Table F3: Raftery/Lewis (1992) Convergence Diagnostics, based on quantile q=0.025 with precision r=0.005 with probability s=0.950.

$Variables$	$M(burn-in)$	$N(req.draws)$	$N+M(totaldraws)$	$k(thinning)$
σ_{ε^z}	56	60622	60678	1
σ_{ε^μ}	68	73755	73823	1
σ_{ε^r}	90	95568	95658	11
σ_{ε^p}	72	77938	78010	1
σ_{ε^l}	253	293418	293671	18
$\sigma_{\varepsilon^\Psi}$	65	70697	70762	1
$\sigma_{\varepsilon^{g^s}}$	83	85280	85363	8
σ_{ε^π}	91	98169	98260	1
σ	1040	1123633	1124673	43
φ	77	83752	83829	1
h	236	251685	251921	15
$100(\beta^{-1}-1)$	46	50096	50142	1
$\bar{\pi}_{ss}$	37	39767	39804	1
$100(g^z-1)$	68	73880	73948	1
κ_π	54	58891	58945	1
κ_y	87	94815	94902	1
ρ_i	111	121169	121280	1
Γ_p	106	115155	115261	1
μ_p	53	57460	57513	1
Γ_w	77	83610	83687	1
μ_w	59	63990	64049	1
γ_I	105	109813	109918	11
σ_a	180	195344	195524	1
ρ^μ	375	411367	411742	1
ρ^r	81	87327	87408	1
ρ^p	109	118552	118661	1
η^p	64	69629	69693	1
ρ^l	53	56834	56887	1
ρ^Ψ	78	84465	84543	1
ρ^{g^s}	173	189324	189497	1
$Maximum$	1040	1123633	1124673	43

Figure F2: Trace plot for parameter κ_π

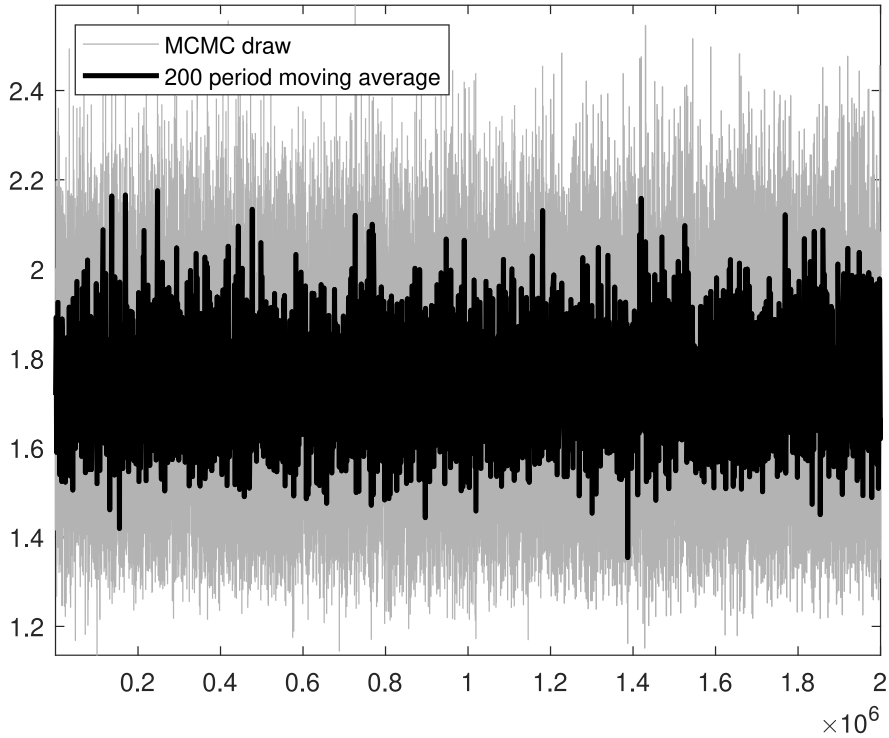
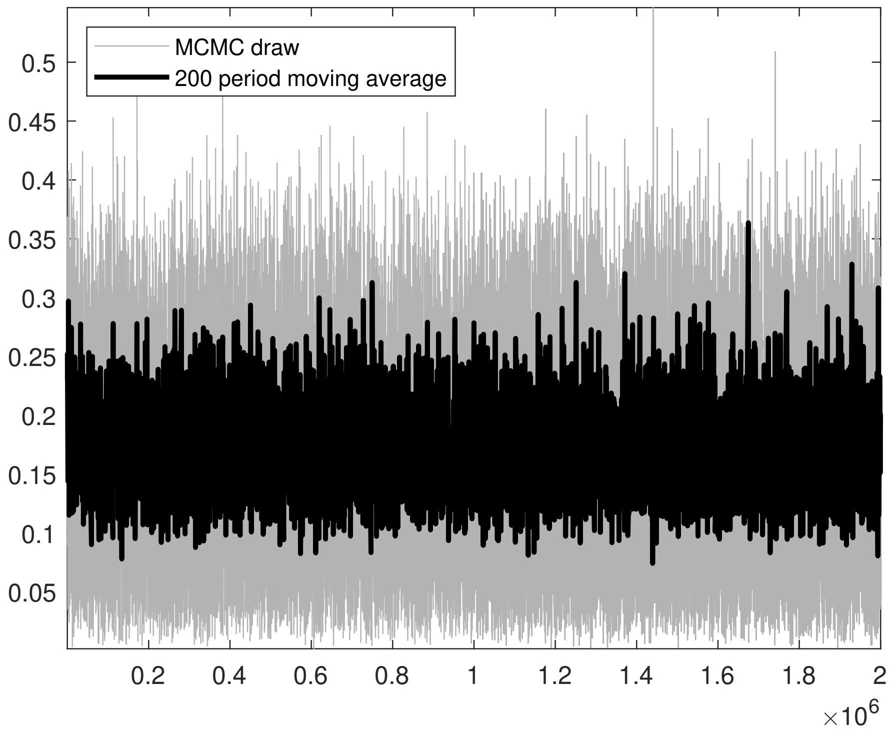


Figure F3: Trace plot for parameter ρ^l



G Theoretical DSGE model

G.1 Set of dynamic equations

Tables from (G1) to (G3) summarize the system of dynamic equations:

Table G1: List of dynamic equations/1

	Descriptions	Equations
1)	Marginal utility of consumption	$\lambda_t = (C_t - hC_{t-1})^{-\sigma} \exp\left(\frac{\sigma-1}{1+\varphi}\zeta_t^l L_t^{1+\varphi}\right),$
2)	Marginal rate of substitution	$MRS_t = (C_t - hC_{t-1})\left(\psi\zeta_t^l L_t^{\varphi}\right),$
3)	Euler equation from capital	$\frac{\lambda_t}{E_t\{\lambda_{t+1}\}} = \beta\left[E_t\frac{\{r_{t+1}^k\}}{Q_t} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right],$
4)	Euler equation	$i_t = (\pi_{t+1})\ \frac{\lambda_t}{\lambda_{t+1}\beta},$
5)	FOC variable capital utilization	$r_t^k = \gamma_1 + \gamma_2\left(U_t - 1\right),$
6)	Euler equation for investments	$1 = Q_t\mu_t^i\left[1 - \left(S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}}\right)\right)\right] + \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}\mu_{t+1}^i S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right],$
7)	Capital law of motion	$K_{t+1} = \mu_t^i\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t + (1-\delta)K_t,$
8)	Production bundle cost	$p_t^z = \left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)},$
9)	Incumbents' productivity threshold	$\hat{A}_t^{INC} = \left[\frac{w_t\phi^{INC} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(p_t^z)^{\gamma}}{p_t},$
10)	New entrants' productivity threshold	$\hat{A}_t^{NE} = \left[\frac{w_t\phi^{NE} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(p_t^z)^{\gamma}}{p_t},$
11)	Discounted value of future dividends	$V_{t+1}^{av} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\hat{A}_{t+1}^{INC}\gamma}{\left(p_{t+1}^z\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_{t+1}\phi^{INC} + E_{t+1}\left[\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right],$
12)	Survival probability	$H_t = \left(\frac{\hat{A}_{t-1}^{INC}g^z\left(1-\delta^{INC}\right)\Psi_t}{\hat{A}_t^{INC}}\right)^{\xi},$

Table G2: List of dynamic equations/2

	Descriptions	Equations
13)	New entrants	$NE_t = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi},$
14)	Incumbents	$INC_t = \eta_{t-1} H_t,$
15)	Exit	$EX_t = \eta_{t-1}\left(1 - H_t\right),$
16)	Active firms	$\eta_t = NE_t + INC_t,$
17)	Share of entry	$entry_t = \frac{NE_t}{\eta_t},$
18)	Share of exit	$exit_t = \frac{EX_t}{\eta_t},$
19)	INT-output	$Y_t^{INT} = \frac{\xi\left\{\left[w_t\phi^{NE} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]\right]NE_t + \left[w_t\phi^{INC} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]\right]INC_t\right\}}{p_t[\xi(1-\gamma)-1]},$
20)	Capital demand	$K_t = K_t^{INT} = \frac{\alpha\gamma}{r_t^k} p_t Y_t^{INT},$
21)	Labor demand	$L_t = L_t^{INT} = \frac{(1-\alpha)\gamma}{w_t} p_t Y_t^{INT} + NE_t\phi^{NE} + INC_t\phi^{INC},$
22)	TFP	$TFP_t = \frac{\xi}{\xi-1}\left(NE_t\hat{A}_t^{NE} + INC_t\hat{A}_t^{I}\right),$
23)	Average firms' efficiency	$\hat{A}_t^{av} = \frac{TFP_t}{\eta_t}$
24)	Efficiency dispersion	$\Sigma_t^A = \frac{\xi}{(\xi-2)\eta_t}\left[NE_t\left(\hat{A}_t^{NE}\right)^2 + INC_t\left(\hat{A}_t^{INC}\right)^2\right]$
25)	Solow Residual	$SR_t = \frac{Y_t}{\left[(K_t)^{\alpha}(L_t)^{1-\alpha}\right]^{\gamma}},$
26)	Set of Calvo price eq. (1)	$a_{1,t} = Y_t\left(\Pi_t^{C*}\right) + \beta\,\Gamma_p\,\frac{\Pi_t^{C*}}{\Pi_{t+1}^{C*}}\left(\frac{\pi_t^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}}\right)^{1-\epsilon_t^p}\frac{\lambda_{t+1}}{\lambda_t}\,a_{1,t+1},$
27)	Set of Calvo price eq. (2)	$a_{2,t} = \tilde{P}_t\,Y_t + \frac{\lambda_{t+1}}{\lambda_t}\,\beta\,\Gamma_p\left(\frac{\pi_t^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}}\right)^{\left(-\epsilon_t^p\right)} a_{2,t+1},$
28)	Set of Calvo price eq. (3)	$a_{1,t} = \frac{\epsilon_t^p\,a_{2,t}}{\epsilon_t^p - 1},$
29)	Set of Calvo price eq. (4)	$1 = \left(1 - \Gamma_p\right)\left(\Pi_t^{C*}\right)^{1-\epsilon_t^p} + \Gamma_p\left(\frac{\pi_{t-1}^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t}\right)^{1-\epsilon_t^p},$
30)	Set of Calvo price eq. (5)	$\xi_t^p = \left(1 - \Gamma_p\right)\left(\Pi_t^{C*}\right)^{\left(-\epsilon_t^p\right)} + \Gamma_p\left(\frac{\pi_{t-1}^{\mu_p}\bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t}\right)^{\left(-\,\epsilon_t^p\right)} \xi_{t-1}^p,$

Table G3: List of dynamic equations/3

	Descriptions	Equations
31)	Set of Calvo wages eq. (1)	$a^w_{1,t} = \lambda_t w_t^{\epsilon^w} L_t + \beta\Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{\epsilon^w-1} a^w_{1,t+1},$
32)	Set of Calvo wages eq. (2)	$a^w_{2,t} = \varphi w_t^{(1+\theta)\epsilon^w} L_t^{1+\theta} + \beta\ \Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{(1+\theta)\epsilon^w} a^w_{2,t+1},$
33)	Set of Calvo wages eq. (3)	$(w_t^{\#})^{1+\epsilon^w\theta} = \frac{\epsilon^w}{\epsilon^w-1}\frac{a^w_{2,t}}{a^w_{1,t}},$
34)	Set of Calvo wages eq. (4)	$w_t^{1-\epsilon^w} = (1-\Gamma_w)\left(w_t^{\#}\right)^{1-\ \epsilon^w} + \Gamma_w\left(w_{t-1}\frac{\pi_{t-1}^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_t}\right)^{1-\epsilon^w},$
35)	Monetary policy rule	$\frac{R^n_t}{R^n_{ss}} = \left(\frac{R^n_{t-1}}{R^n_{ss}}\right)^{\rho_i}\left[\left(\frac{\pi_t}{\bar{\pi}_t}\right)^{\kappa^\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\kappa^y}\right]^{1-\rho_i}\zeta^r_t,$
36)	Aggregate resources constraint	$Y_t = \frac{Y_t^{INT}}{\xi_t^p} = C_t + I_t + g_t^S Y,$
37)	Technology frontier evolution (NEs)	$z_t = g_t^z\, z_{t-1},$
38)	Shock to NEs' technology	$\ln(g_t^z) = (1-\rho^z)\ln(g^z) + \rho^z\ln(g_{t-1}^z) + \varepsilon_t^z,$
39)	Shock to INCs' technology	$\ln(\Psi_t) = \rho^\Psi\ln(\Psi_{t-1}) + \varepsilon_t^\Psi,$
40)	Shock to inflation target	$\ln(\overline{\pi}_t) = \ln(1-\rho^\pi)\,\overline{\pi}_{ss} + \rho^\pi\ln(\overline{\pi}_{t-1}) + \varepsilon_t^\pi,$
41)	Shock to monetary policy	$\ln(\zeta_t^r) = \rho^r\ln(\zeta_{t-1}^r) + \varepsilon_t^r,$
42)	Shock to labot supply	$\ln(\zeta_t^l) = \rho^l\ln(\zeta_{t-1}^l) + \varepsilon_t^l,$
43)	Shock to MEI	$\ln(\mu_t) = \rho^\mu\ln(\mu_{t-1}) + \varepsilon_t^\mu,$
44)	Shock to public expenditure	$\ln(g_t^S) = \rho^{g^s}\ln(g_{t-1}^S) + \varepsilon_t^{g^s}.$

G.2 The de-trended model

The model economy follows a Balanced Growth Path (BGP). Output Y_t, consumption C_t, capital K_t, investment I_t and wage w_t grow at the endogenous rate g^t. Further, the technology frontier z_t and the technology thresholds $\hat{A}_t^j$ grow at the exogenous rate g_z^t. The remaining variables are stationary. In order to compute the deterministic steady state and the de-trended model, we have to identify the relation that binds the different growth rates.

Households

We can start our computation from the households' first-order conditions. Since we know that C grows at the same rate as Y, we can show that the Lagrangian multiplier follows this path:

$$\widetilde{\lambda}_t = \frac{\lambda_t}{g_t} = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)^{-\sigma} \exp\left(\frac{\sigma-1}{1+\varphi}L_t^{1+\varphi}\right)$$

$$\widetilde{MRS}_t = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)(\psi L_t^{\varphi})$$

From the households' Euler conditions, we can find the de-trended Euler equations on capital:

$$\frac{\lambda_t}{E_t\{\lambda_{t+1}\}} = \beta\left[E_t\frac{\{r_{t+1}^k\}}{Q_t} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]$$

$$\frac{\widetilde{\lambda}_t g_{t+1}}{\beta\, E_t\left\{\widetilde{\lambda}_{t+1}\right\}} = \left(\frac{Q_{t+1}r_{t+1}^k}{Q_t} + \frac{Q_{t+1}}{Q_t}(1-\ \delta)\right)$$

INT-firms

Once we have defined the costs of production we can compute the productivity thresholds:

$$\hat{A}_t^j = \left[\frac{w_t\phi^j - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(p_t^z)^\gamma}{p_t}$$

This implies:

$$\hat{A}_t^j = \left[\frac{g_t w_t\phi^j - E_t\left\{g_t\frac{\widetilde{\Lambda}_{t+1}}{g_{t+1}}H_{t+1}\widetilde{V}_{t+1}^{av,s}g_{t+1}\right]}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma_s}\cdot\left[\frac{\left[\frac{\widetilde{r}_t^k}{\alpha}\right]^{\alpha}\left[\frac{g_t\widetilde{w}_t}{(1-\alpha)}\right]^{(1-\alpha)}}{p_t^{\frac{1}{\gamma}}}\right]^{\gamma}$$

$$\hat{A}_t^j = g_t^{1-\alpha\gamma}\left[\frac{w_t\phi^j - E_t\left[\widetilde{\Lambda}_{t+1}H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\widetilde{p}_t^z)^\gamma}{p_t}$$

$$\widetilde{\hat{A}_t^{NE,INC}} = \left[\frac{\tilde{w}_t \phi^j - E_t \left\{ \widetilde{\Lambda}_{t+1} H_{t+1} \widetilde{V}_{t+1}^{av} \right\}}{\gamma^{\frac{\gamma}{1-\gamma}} \left(1-\gamma\right)} \right]^{1-\gamma} \frac{\left(\tilde{p}_t^z\right)^{\gamma}}{p_t}$$

Since the number of firms is assumed to be stationary ,it will follow that $g_t^{1-\alpha\gamma} = g_t^z$

$$\eta_t = \left(\frac{z}{\widetilde{\hat{A}_t^{NE}}} \right)^{\xi_S} + \eta_{t-1} \left(\frac{\widetilde{\hat{A}_{t-1}^{I}} g^z \left(1 - \delta^{INC}\right) \Psi_t}{g_t^z \widetilde{\hat{A}_t^{I}}} \right)^{\xi_S}$$

The remaining de-trending steps are straightforward.

Set of de-trended equations

Tables from (G4) to (G6) summarize the system of de-trended equations:

Table G4: List of de-trended equations/1

	Descriptions	Equations
1)	Marginal utility of consumption	$\widetilde{\lambda}_t = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)^{-\sigma} \exp\left(\frac{\sigma-1}{1+\varphi} L_t^{1+\varphi}\right),$
2)	Marginal rate of substitution	$\widetilde{MRS}_t = \left(\widetilde{C}_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)\left(\psi L_t^{\varphi}\right),$
3)	Euler equation from capital	$\frac{\widetilde{\lambda}_t g_{t+1}}{\beta E_t\left\{\widetilde{\lambda}_{t+1}\right\}} = \left(\frac{Q_{t+1} r_{t+1}^k}{Q_t} + \frac{Q_{t+1}}{Q_t}\left(1-\ \delta\right)\right),$
4)	Euler equation	$i_t = \frac{\pi_{t+1}}{\beta}\frac{g_{t+1}\widetilde{\lambda}_t}{\widetilde{\lambda}_{t+1}},$
5)	FOC variable capital utilization	$r_t^k = \gamma_1 + \gamma_2\left(U_t - 1\right),$
6)	Euler equation for investments	$1 = Q_t \mu_t^i \left[1 - \left(S'\left(\frac{\widetilde{I}_t g_t}{\widetilde{I}_{t-1}}\right)\frac{\widetilde{I}_t g_t}{\widetilde{I}_{t-1}} + S\left(\frac{\widetilde{I}_t g_t}{\widetilde{I}_{t-1}}\right)\right)\right] + \beta E_t\left\{\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t g_{t+1}} Q_{t+1}\mu_{t+1}^i S'\left(\frac{\widetilde{I}_{t+1}g_{t+1}}{\widetilde{I}_t}\right)\left(\frac{\widetilde{I}_{t+1}g_{t+1}}{\widetilde{I}_t}\right)^2\right],$
7)	Capital law of motion	$\widetilde{K}_{t+1} = \mu_t^i\left(1 - S\left(\frac{\widetilde{I}_t g_t^K}{\widetilde{I}_{t-1}}\right)\right)\widetilde{I}_t + \frac{(1-\delta)\widetilde{K}_t}{g_t^K},$
8)	Production bundle cost	$\widetilde{p}_t^z = \left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{\widetilde{w}_t}{(1-\alpha)}\right]^{(1-\alpha)},$
9)	Incumbents' productivity threshold	$\widetilde{\hat{A}}_t^{INC} = \left[\frac{\tilde{w}_t\phi^{INC} - E_t\left\{\widetilde{\Lambda}_{t+1}H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\tilde{p}_t^z)^{\gamma}}{p_t},$
10)	New entrants' productivity threshold	$\widetilde{\hat{A}}_t^{NE} = \left[\frac{\tilde{w}_t\phi^{NE} - E_t\left\{\widetilde{\Lambda}_{t+1}H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\tilde{p}_t^z)^{\gamma}}{p_t},$
11)	Discounted value of future dividends	$\widetilde{V}_{t+1}^{av} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\widetilde{\hat{A}}_{t+1}^{INC}\ \gamma}{\left(\widetilde{p}_{t+1}^z\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - \widetilde{w}_{t+1}\phi^{INC} + E_{t+1}\left[\widetilde{\Lambda}_{t+2}H_{t+2}\widetilde{V}_{t+2}^{av}\right],$
12)	Survival probability	$H_t = \left(\frac{\widetilde{\hat{A}}_{t-1}^{INC}\ g^z\left(1-\delta^{INC}\right)\Psi_t}{\widetilde{\hat{A}}_t^{INC}\quad g_t^z}\right)^{\xi},$

Table G5: List of de-trended equations/2

	Descriptions	Equations
13)	New entrants	$NE_t = \left(\frac{z}{\widetilde{\hat{A}}_t^{NE}} \right)^{\xi},$
14)	Incumbents	$INC_t = \eta_{t-1} H_t,$
15)	Exit	$EX_t = \eta_{t-1} \left(1 - H_t\right),$
16)	Active firms	$\eta_t = NE_t + INC_t,$
17)	Share of entry	$entry_t = \frac{NE_t}{\eta_t},$
18)	Share of exit	$exit_t = \frac{EX_t}{\eta_t},$
19)	INT-output	$\widetilde{Y}_t^{INT} = \frac{\xi \left\{ \left[\widetilde{w}_t \phi^{NE} - E_t \left\{ \Lambda_{t+1} H_{t+1} \widetilde{V}_{t+1}^{av} \right\} \right] NE_t + \left[\widetilde{w}_t \phi^{INC} - E_t \left\{ \widetilde{\Lambda}_{t+1} H_{t+1} \widetilde{V}_{t+1}^{av} \right\} \right] INC_t \right\}}{p_t [\xi(1-\gamma)-1]},$
20)	Capital demand	$\widetilde{K}_t = \widetilde{K}_t^{INT} = \frac{\alpha\gamma}{r_t^k} p_t \widetilde{Y}_t^{INT},$
21)	Labor demand	$L_t = L_t^{INT} = \frac{(1-\alpha)\gamma}{\widetilde{w}_t} p_t \widetilde{Y}_t^{INT} + NE_t \phi^{NE} + INC_t \phi^{INC},$
22)	Average productivity	$\widetilde{TFP}_t = \frac{\xi}{\xi-1} \left(NE_t \widetilde{\hat{A}}_t^{NE} + INC_t \widetilde{\hat{A}}_t^{I} \right),$
23)	Average firms' efficiency	$\widetilde{\hat{A}}_t^{av} = \frac{\widetilde{TFP}_t}{\eta_t}$
24)	Efficiency dispersion	$\widetilde{\Sigma^A}_t = \frac{\xi}{(\xi-2)\eta_t} \left[NE_t \left(\widetilde{\hat{A}}_t^{NE} \right)^2 + INC_t \left(\widetilde{\hat{A}}_t^{INC} \right)^2 \right]$
25)	Solow Residual	$\widetilde{SR}_t = \frac{\widetilde{Y}_t}{\left[\left(\widetilde{K}_t \right)^{\alpha} (L_t)^{1-\alpha} \right]^{\gamma}},$
26)	Set of Calvo price eq. (1)	$\widetilde{a}_{1,t} = \widetilde{Y}_t \left(\Pi_t^{C*} \right) + \beta \, \Gamma_p \frac{\Pi_t^{C*}}{\Pi_{t+1}^{C*}} \left(\frac{\pi_t^{\mu_p} \bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}} \right)^{1-\epsilon_t^p} \frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t} \widetilde{a}_{1,t+1},$
27)	Set of Calvo price eq. (2)	$\widetilde{a}_{2,t} = \tilde{P}_t \, \widetilde{Y}_t + \frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t} \; \beta \, \Gamma_p \left(\frac{\pi_t^{\mu_p} \bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_{t+1}} \right)^{\left(-\epsilon_t^p\right)} \widetilde{a}_{2,t+1},$
28)	Set of Calvo price eq. (3)	$\widetilde{a}_{1,t} = \frac{\epsilon_t^p \, \widetilde{a}_{2,t}}{\epsilon_t^p - 1},$
29)	Set of Calvo price eq. (4)	$1 = (1 - \Gamma_p) \left(\Pi_t^{C*} \right)^{1-\epsilon_t^p} + \Gamma_p \left(\frac{\pi_{t-1}^{\mu_p} \bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t} \right)^{1-\epsilon_t^p},$
30)	Set of Calvo price eq. (5)	$\xi_t^p = (1 - \gamma) \left(\Pi_t^{C*} \right)^{\left(-\epsilon_t^p\right)} + \gamma \left(\frac{\pi_{t-1}^{\mu_p} \bar{\pi}_{ss}^{(1-\mu_p)}}{\pi_t} \right)^{\left(-\epsilon_t^p\right)} \xi_{t-1}^p,$

Table G6: List of de-trended equations/3

	Descriptions	Equations
31)	Set of Calvo wages eq. (1)	$\widetilde{a}^w_{1,t} = \widetilde{\lambda}_t \widetilde{w}_t^{\epsilon^w} L_t + \beta\Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{\epsilon^w-1} \widetilde{a}^w_{1,t+1},$
32)	Set of Calvo wages eq. (2)	$\widetilde{a}^w_{2,t} = \varphi \widetilde{w}_t^{(1+\theta)v_w} L_t^{1+\theta} + \beta\ \Gamma_w \left(\frac{\pi_t^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{\pi_{t+1}}\right)^{(1+\theta)\epsilon^w} \widetilde{a}^w_{2,t+1},$
33)	Set of Calvo wages eq. (3)	$(\widetilde{w}_t^{\#})^{1+\epsilon^w\theta} = \frac{\epsilon^w}{\epsilon^w-1}\frac{\widetilde{a}^w_{2,t}}{\widetilde{a}^w_{1,t}},$
34)	Set of Calvo wages eq. (4)	$\widetilde{w}_t^{1-\epsilon^w} = (1-\Gamma_w)\left(\widehat{w}_t^{\#}\right)^{1-\ \epsilon^w} + \Gamma_w\left(\frac{\widetilde{w}_{t-1}\pi_{t-1}^{\mu_w}\bar{\pi}_{ss}^{(1-\mu_w)}}{g_t\pi_t}\right)^{1-\epsilon^w},$
35)	Monetary policy rule	$\frac{R_t^n}{R_{ss}^n} = \left(\frac{R^n_{t-1}}{R^n_{ss}}\right)^{\rho_i}\left[\left(\frac{\pi_t}{\overline{\pi}_t}\right)^{\kappa^\pi}\left(\frac{\widetilde{Y}_t}{g_t\widetilde{Y}_{t-1}}\right)^{\kappa^y}\right]^{1-\rho_i}\zeta_t^r,$
36)	Aggregate resources constraint	$\widetilde{Y}_t = \frac{\widetilde{Y}_t^{INT}}{\xi_t^p} = \widetilde{C}_t + \widetilde{I}_t + g_t^S Y,$
37)	Growth rates	$g_t = (g_t^z)^{\frac{1}{1-\alpha\gamma}},$
38)	Shock to NEs' technology	$\ln(g_t^z) = (1-\rho^z)\ln(g^z) + \rho^z\ln(g_{t-1}^z) + \varepsilon_t^z,$
39)	Shock to INCs' technology	$\ln(\Psi_t) = \rho^\Psi\ln(\Psi_{t-1}) + \varepsilon_t^\Psi,$
40)	Shock to inflation target	$\ln(\overline{\pi}_t) = \ln(1-\rho^\pi)\,\overline{\pi}_{ss} + \rho^\pi\ln(\overline{\pi}_{t-1}) + \varepsilon_t^\pi,$
41)	Shock to monetary policy	$\ln(\zeta_t^r) = \rho^r\ln(\zeta_{t-1}^r) + \varepsilon_t^r,$
42)	Shock to labor supply	$\ln(\zeta_t^l) = \rho^l\ln(\zeta_{t-1}^l) + \varepsilon_t^l,$
43)	Shock to MEI	$\ln(\mu_t) = \rho^\mu\ln(\mu_{t-1}) + \varepsilon_t^\mu,$
44)	Shock to public expenditure	$\ln(g_t^S) = \rho^{g^s}\ln(g_{t-1}^S) + \varepsilon_t^{g^s}.$

G.3 Key derivations

Equation (2.25) - Firms' continuation value

We start from the (2.12) and (2.13) to get

$$\begin{aligned}
V_{t+1}^{av} &= E_t\left\{V\left(A_{t+1}^{j}\right)\right\} = \\
&= \int_{\widehat{A}_{t+1}^{INC}}^{+\infty} V_{t+1}\left(A_{t+1}^{INC,j}\right) f_t\left(A_{t+1}^{INC,j}\right) d\left(A_{t+1}^{INC,j}\right) = H_{t+1}V_{t+1}^{av} \\
&= \int_{\widehat{A}_{t+1}^{INC}}^{+\infty} (1-\gamma)\left[A_{t+1}^{f,j}\frac{p_{t+1}\gamma^{\gamma}}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} f_t\left(A_{t+1}^{INC,j}\right) d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \int_{\widehat{A}_{t+1}^{INC}}^{+\infty}\left[A_{t+1}^{f,j}\right]^{\frac{1}{1-\gamma}} f_t\left(A_{t+1}^{INC,j}\right) d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \int_{\widehat{A}_{t+1}^{INC}}^{+\infty}\left[A_{t+1}^{f,j}\right]^{\frac{1}{1-\gamma}-\xi-1} d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \int_{\widehat{A}_{t+1}^{INC}}^{+\infty}\left[A_{t+1}^{f,j}\right]^{\frac{1}{1-\gamma}-\xi-1} d\left(A_{t+1}^{INC,j}\right) - w_{t+1}\phi_t^{INC} + E_t\left\{\Lambda_{t+2}V_{t+2}\left(A_{t+2}^{f,j}\right)\right\} = \\
&= \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\frac{(1-\gamma)}{\gamma}\left[\frac{p_{t+1}\hat{A}_{t+1}^{INC}\gamma}{\left(p_{t+1}^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_{t+1}\phi_{t+1}^{INC} + E_{t+1}\left\{\Lambda_{t+2}H_{t+2}V_{t+2}^{av}\right\}.
\end{aligned}$$

Equation (2.26) - Productivity thresholds

Also in this case, we get the condition from (2.12) and (2.13)

$$\begin{aligned}
V_t\left(\hat{A}_t^{j}\right) &= (1-\gamma)\left[\hat{A}_t^{j}\frac{p_t\gamma^{\gamma}}{\left(p_t^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} - w_t\phi^{j} + E_t\left\{\Lambda_{t+1}V_{t+1}^{av}\right] = 0 \\
& (1-\gamma)\left[\hat{A}_t^{j}\frac{p_t\gamma^{\gamma}}{\left(p_t^{z}\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} = w_t\phi^{j} - E_t\left\{\Lambda_{t+1}V_{t+1}^{av}\right] \\
\hat{A}_t^{j} &= \left[\frac{w_t\phi^{j} - E_t\left\{\Lambda_{t+1}H_{t+1}V_{t+1}^{av}\right]}{(1-\gamma)\,p_t}\right]^{1-\gamma}\left[\frac{p_t^{z}}{p_t\gamma}\right]^{\gamma}
\end{aligned}$$

Equation (2.32) - Aggregate INT-output

We start from the idiosyncratic production function.

$$y_t^{f,j} = A_t^{f,j}\left[(k_t^{f,j})^{\alpha}(l_t^{f,j})^{(1-\alpha)}\right]^{\gamma}$$

Aggregating for NEs idiosyncratic productivity, we get

$$
\begin{aligned}
Y_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{f,NE}\left[\left(k_t^{f,NE}\right)^{\alpha}\left(l_t^{f,NE}\right)^{1-\alpha}\right]^{\gamma} dF\left(A_t^{f,NE}\right) \\
&= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{f,NE}\left[\frac{p_t}{p_t^z}A_t^{f,NE}\gamma\right]^{\frac{\gamma}{1-\gamma}} dF\left(A_t^{f,NE}\right) \\
&= \left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{NE}}^{+\infty}\left(A_t^{f,NE}\right)^{\frac{1}{1-\gamma}} dF\left(A_t^{f,NE}\right) \\
&= \xi z_t^{\xi}\left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{NE}}^{+\infty}\left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}-\xi-1} d\left(A_t^{f,NE}\right) \\
&= \frac{\xi\left(1-\gamma\right)}{\xi(1-\gamma)-1}NE_t\left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma p_t}{p_t^z}\right)^{\frac{\gamma}{1-\gamma}},
\end{aligned}
$$

and, aggregating for INCs idiosyncratic productivity, we get

$$
\begin{aligned}
Y_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{f,INC}\left[\left(k_t^{f,INC}\right)^{\alpha}\left(l_t^{f,INC}\right)^{1-\alpha}\right]^{\gamma} dF\left(A_t^{f,INC}\right) \\
&= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{f,INC}\left[\frac{p_t}{p_t^z}A_t^{f,INC}\gamma\right]^{\frac{\gamma}{1-\gamma}} dF\left(A_t^{f,INC}\right) \\
&= \left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{INC}}^{+\infty}\left(A_t^{f,INC}\right)^{\frac{1}{1-\gamma}} dF\left(A_t^{f,INC}\right) \\
&= \xi\left[\hat{A}_{t-1}^{INC}g_z\left(1-\delta\right)\Psi_t\right]^{\xi}\left[\frac{p_t}{p_t^z}\gamma\right]^{\frac{\gamma}{1-\gamma}}\int_{\hat{A}_t^{INC}}^{+\infty}\left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\gamma}-\xi-1} d\left(A_t^{f,INC}\right) \\
&= \frac{\xi\left(1-\gamma\right)}{\xi(1-\gamma)-1}INC_t\left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma p_t}{p_t^z}\right)^{\frac{\gamma}{1-\gamma}}.
\end{aligned}
$$

Elasticity of productivity thresholds

In order to measure the sensitivity of the two productivity thresholds to their different components, we compute the first-order approximation of equation (2.26):

$$
\widetilde{\hat{A}}_t^j = \left[\frac{\widetilde{w}_t\phi^j - E_t\left\{\widetilde{\Lambda}_{t+1}H_{t+1}\widetilde{V}_{t+1}^{av}\right\}}{\gamma^{\frac{\gamma}{1-\gamma}}\left(1-\gamma\right)}\right]^{1-\gamma}\frac{(\widetilde{p}_t^z)^{\gamma}}{p_t}.
$$

In the de-trended steady state,

$$
\widetilde{\hat{A}}^j = \left[\frac{\widetilde{w}\phi^j - H\widetilde{V}^{av}}{\gamma^{\frac{\gamma}{1-\gamma}}(1-\gamma)}\right]^{1-\gamma}\frac{(\widetilde{p}^z)^{\gamma}}{p}.
$$

Log-linearizing,[26]

$$\hat{a}_t^j = \frac{1-\gamma}{w\phi^j - HV^{av}} \left[\phi^j \hat{w}_t - HV^{av}(\hat{\Lambda}_{t+1} + \hat{H}_{t+1} + \hat{V}_{t+1}^{av})\right] - (\hat{p}_t - \gamma \hat{p}_t^z).$$

From our calibration, $\phi^{NE} < \phi^{INC}$ implies that the sensitivity of the thresholds to the approximated wedge between participation costs and expected future profits is larger for NEs. On the other hand, sensitivity to the price/cost margin is the same for each type of firm.

H Comparison with standard NK model

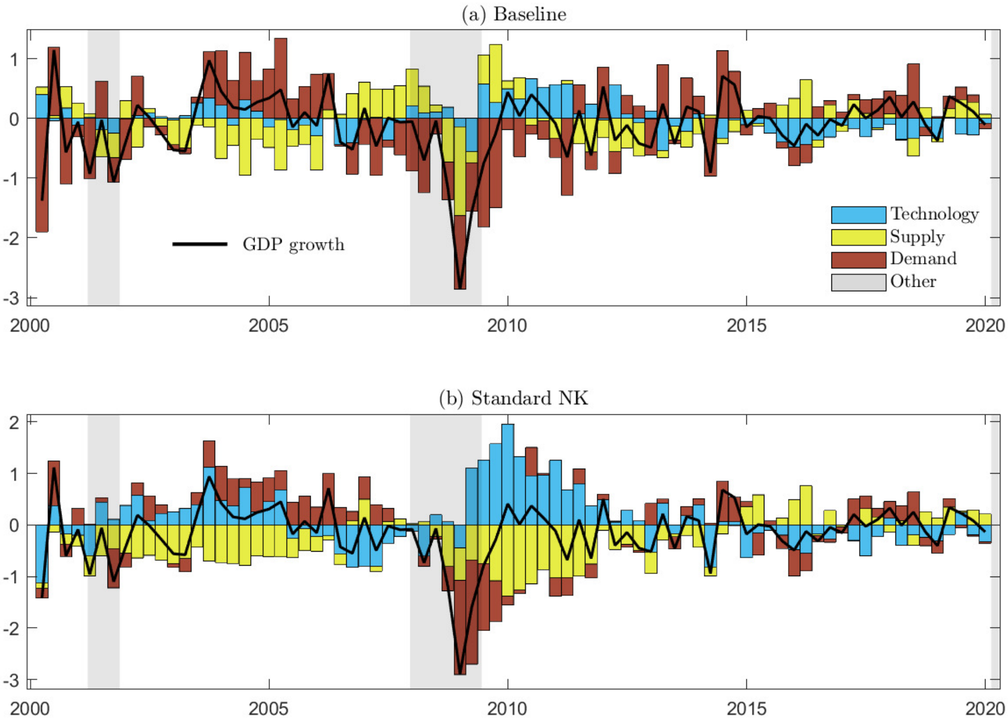
The main purpose here is to benchmark our interpretation of the U.S. business cycle against the established narrative based on the standard NK model. First, we briefly discuss posterior estimates[27] and impulse response functions. Interestingly enough, our model generates higher internal persistence: consumption habits (h), price and wage stickiness (Γ_p and Γ_w), investment adjustment and capital utilization costs are somewhat larger, but fall in the ballpark of existing estimates in the DSGE literature. On the contrary, the exogenous persistence of the standard NK model, identified in the shocks' autocorrelation coefficients, is generally more pronounced than in our model. With respect to the common shocks, the IRFs of the two models are very similar (reported in Figure J3).

We next turn to the historical decomposition of GDP growth obtained with the two models (Figure H1). To sharpen the analysis we focus on the post-2000 period. After 2013, the two models convey similar messages, but important differences are easy to spot in the previous years. According to the NK estimates, markup shocks are persistently contractionary, whereas technology shocks pull in the opposite direction with an almost symmetrical pattern. Thus the NK model conveys a story where pre-2013 growth is determined by a combination of technology improvements and persistently adverse markup shocks. These contemporaneous and opposite effects are particularly large in the occurrence and in the immediate aftermath of the GFC. The contribution of technology shocks to the post-2008:IV recovery appears implausibly large and in sharp contrast with results obtained in contributions such as Fernald (2014) and Vinci and Licandro (2021). By contrast, our baseline model does not generate equally persistent patterns and technology shocks play a lesser role. Demand shocks are relatively more important. Their positive contribution to growth in the 2003-2006 period is consistent with the popular narrative about the importance of the credit boom in the run-up to the GFC.

[26] Where $\hat{x}_t$ is the de-trended log-deviation of the generic variable x_t. For the sake of a clear notation the log-deviation of $\hat{A}_t^j$ is labeled by $\hat{a}_t^j$.

[27] See Table J1.

Figure H1: Historical shock decomposition comparison: GDP growth, 2000:I-2019:IV



Note: The solid line is observed GDP growth in log-deviations from its estimated steady state. The colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks for panel (a) and (b), and risk premium shocks for panel (b); "Supply" includes price markup shocks for panel (a) and (b), labor supply shocks for panel (a), and wage markup shocks for panel (b); "Other" includes contribution from initial values). Panel (a): Baseline model estimation. Panel (b): Standard NK model estimation.

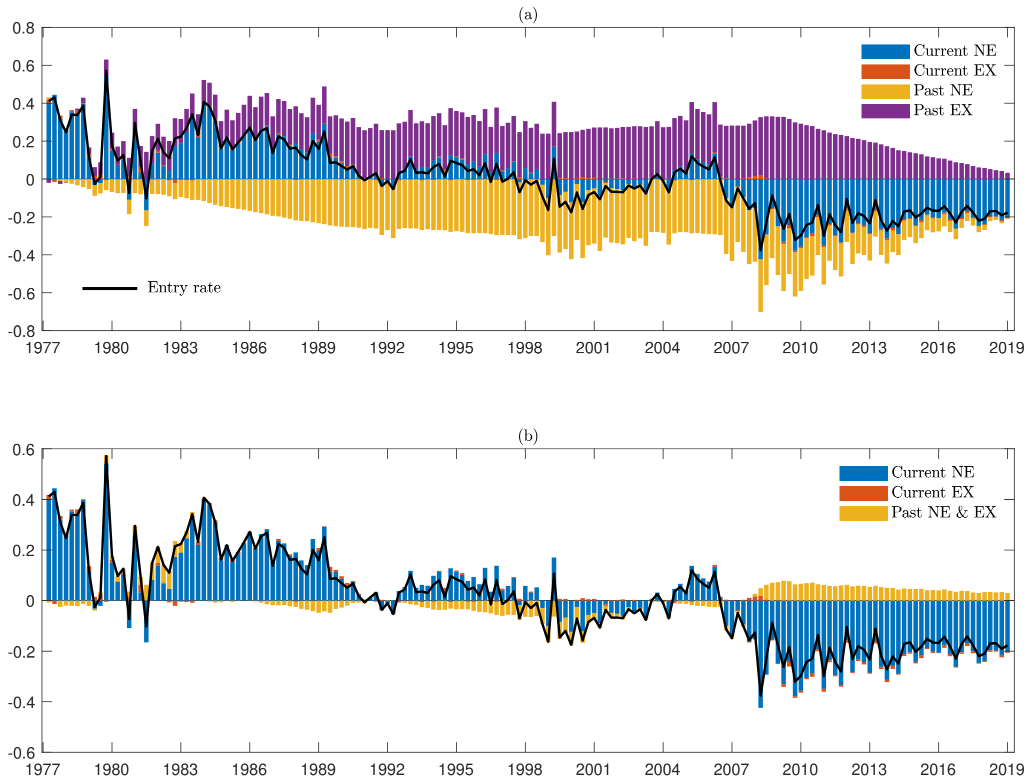
I Implications of exit mismatch for the estimation of entry

Up to first-order approximation, condition (2.3) can be decomposed as follows:

$$\widehat{entry} = H\widehat{NE}_t - (1-H)\,\widehat{EX}_t - (1-H)\sum_{j=1}^{T_0}\left(\widehat{NE}_{t-j} - \widehat{EX}_{t-j}\right), \tag{2.58}$$

where $\hat{x}_t$ denotes log-deviations from the steady state, H is the survival probability of firms in the deterministic steady state, and T_0 denotes the initial sample period. Figure I1 suggests that the current estimate of exit has virtually no effect on our interpretation of the observed entry rate. The accumulated past balance between exit and entry has a limited effect and matters only after the GFC. In this period it is driven by the accumulation of exit flows, but its importance in determining $\widehat{entry}$ remains limited.

Figure I1: Entry rate decomposition, 1978:I-2019:IV



Note: In both panels, the solid line depicts log-deviations from the steady state of the quarterly entry-rate smoothed estimate at the posterior mean. The colored bars display the model-implied contribution to those deviations coming from (i) current new entrants, (ii) current firm exits, (iii) past accumulated new entrants, and (iv) past accumulated firm exits. Panel (b) groups contributions from (iii) and (iv).

J Additional tables and figures

J.1 Tables

Table J1: Posterior estimates comparison

	Prior			Posterior mean				
	Dist.	Mean	Stdev.	Baseline	Shadow rate	Short sample	Unobserved entry	Standard NK
σ	norm	1.500	0.3750	1.173	1.168	1.233	1.116	1.118
φ	norm	2.000	0.5000	2.354	2.297	2.323	2.039	2.131
h	beta	0.700	0.1000	0.840	0.847	0.842	0.799	0.813
$100(\beta^{-1}-1)$	gamm	0.250	0.2000	0.169	0.149	0.198	0.181	0.389
$\bar{\pi}_{ss}$	gamm	0.750	0.4000	0.716	0.707	0.720	0.715	0.705
$100(g^z-1)$	norm	0.400	0.1000	0.350	0.351	0.430	0.311	0.379
κ_π	norm	1.500	0.2500	1.736	1.762	1.638	1.651	1.787
κ_y	norm	0.200 (0.125)	0.0500	0.239	0.240	0.222	0.246	0.006
κ_{Δ_y}	norm	0.125	0.0500	-	-	-	-	0.025
ρ_i	beta	0.750	0.1000	0.764	0.770	0.750	0.751	0.784
Γ_p	beta	0.650	0.1000	0.834	0.833	0.804	0.813	0.711
μ_p	beta	0.500	0.1500	0.247	0.271	0.371	0.296	0.206
Γ_w	beta	0.650	0.1000	0.815	0.821	0.876	0.805	0.788
μ_w	beta	0.500	0.1500	0.315	0.330	0.336	0.774	0.564
γ_I	norm	4.000	1.5000	9.415	9.694	8.574	7.515	7.542
σ_a	beta	0.500	0.1500	0.884	0.887	0.777	0.863	0.766
ϕ_p	norm	1.250	0.1250	-	-	-	-	1.663
Δ_L	norm	1.000 (0.000)	2.0000	-	-	1.106	-	-2.735
ρ^μ	beta	0.500	0.2000	0.556	0.548	0.460	0.795	0.815
ρ^r	beta	0.500	0.2000	0.305	0.325	0.288	0.239	0.273
ρ^p	beta	0.500	0.2000	0.984	0.982	0.976	0.964	0.973
η^p	beta	0.500	0.2000	0.363	0.359	0.401	0.482	0.845
ρ^l	beta	0.500	0.2000	0.172	0.173	0.134	0.225	-
ρ^w	beta	0.500	0.2000	-	-	-	-	0.933
η^w	beta	0.500	0.2000	-	-	-	-	0.874
ρ^Ψ	beta	0.500	0.2000	0.247	0.240	0.255	0.173	-
ρ^a	beta	0.500	0.2000	-	-	-	-	0.930
ρ^{g^s}	beta	0.500	0.2000	0.961	0.961	0.975	0.966	0.957
ρ^{rp}	beta	0.500	0.2000	-	-	-	-	0.147

Note: Terms in round brackets refer to the prior specifications used in the estimation of the benchmark NK model, when different from ours.

The interpretation of κ_y differs between the NK benchmark and our model. In the former, κ_y determines the Taylor rule response to output gap deviations from the steady state, where the output gap is defined as the difference between the actual and the flexible-price output level. κ_{Δ_y} stands for the monetary policy weight on output gap growth. Conversely, monetary policy in our model targets output growth through κ_y.

ϕ_p is the estimated share of fixed costs and ρ^{rp} is the persistence of risk premium shocks, both absent in our model, while ρ^w and ρ^a are the autocorrelation coefficients of the wage markup and stationary technology processes, respectively. These two shocks can be thought of as counterparts of our labor supply and incumbent shocks.

Δ_L enters the observation equation of hours worked as a "correction" term when the steady state of L does not equal the observed sample mean.

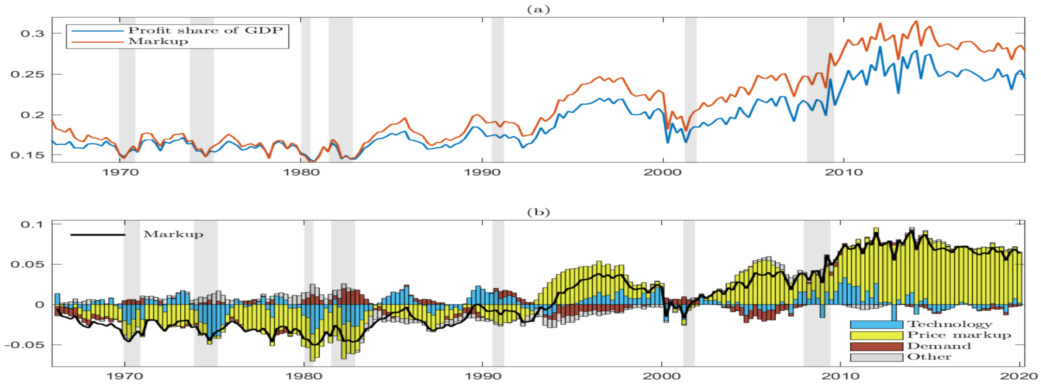
Table J2: Productivity measures variance decomposition

	Shocks			
	Incumbents	Entry	Supply	Demand
TFP growth	8.2%	2.4%	22.0%	67.4%
Average efficiency growth	91.9%	0.4%	1.5%	6.3%
Efficiency dispersion growth	93.1%	0.4%	1.3%	5.3%

Note: Unconditional variance decomposition at the posterior mean. Non-technology shocks are grouped into two categories: supply and demand. "Supply" includes price markup and labor supply shocks; "Demand" includes monetary policy, inflation target, MEI, and government spending shocks.

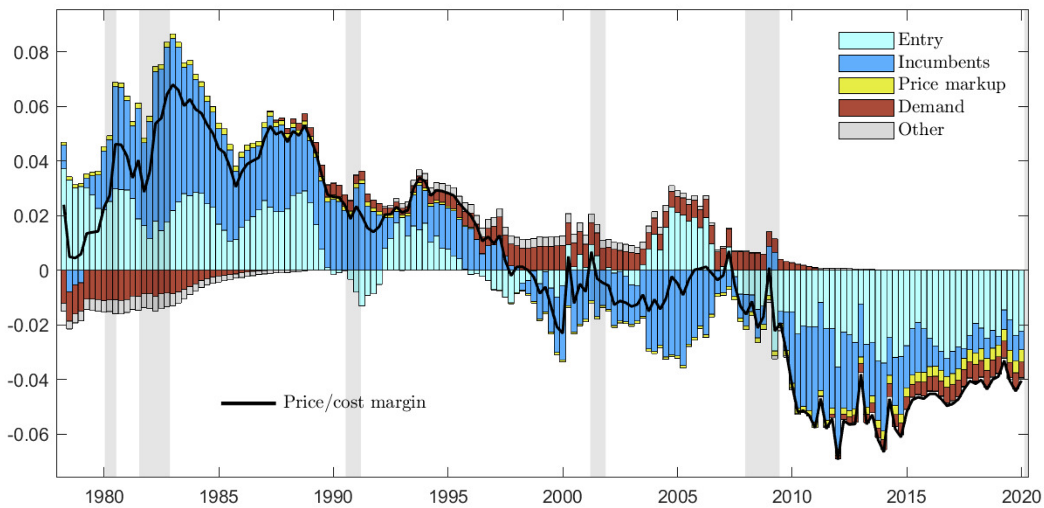
J.2 Figures

Figure J1: Model predictions for the profit share of GDP and markups, 1966:I-2019:IV



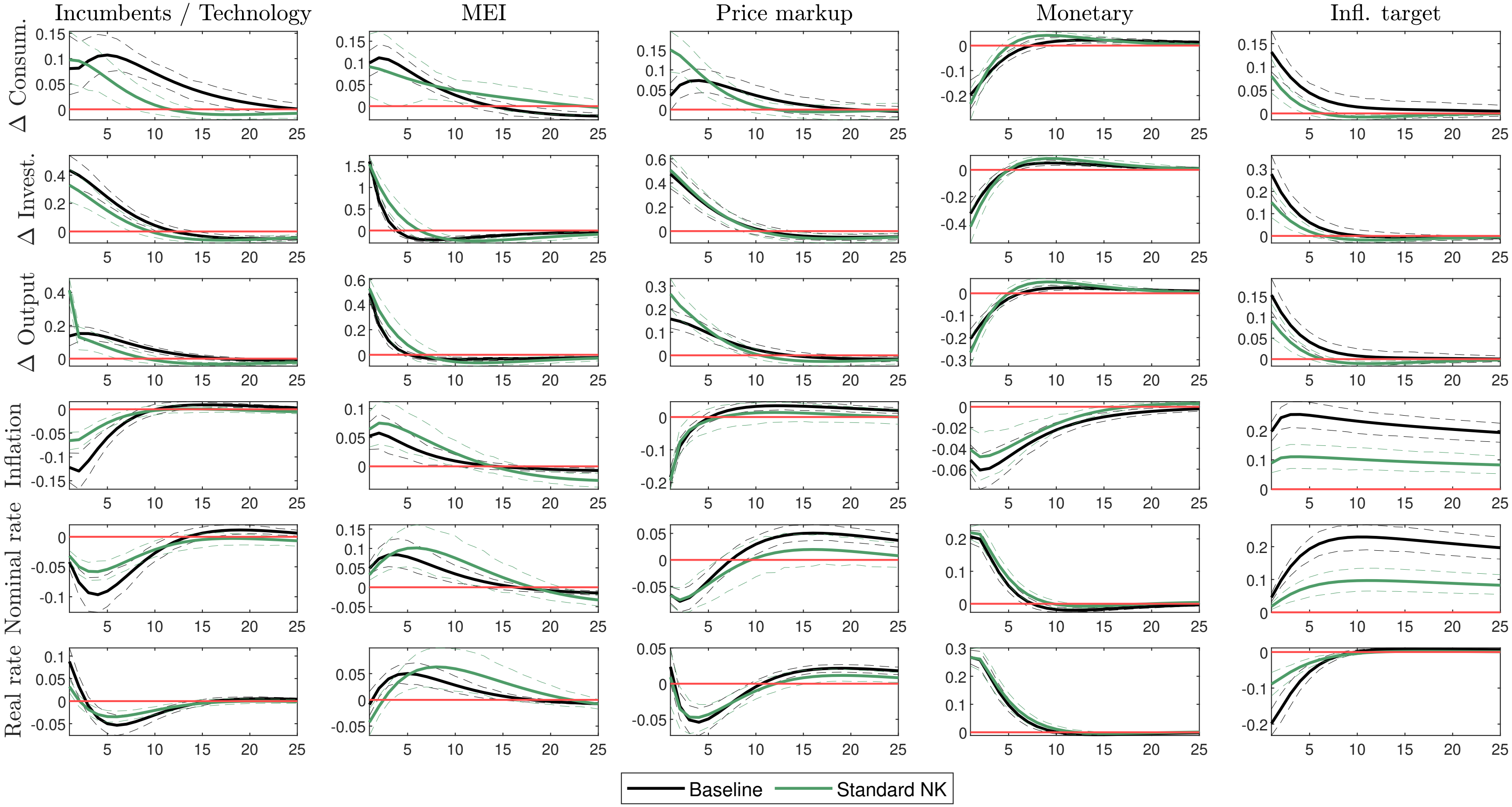
Note: Panel (a): quarterly smoothed estimates, at the posterior mean, of profits of INT-firms and retailers as a share of GDP (blue), and of retailers' price markup over marginal costs (orange). Panel (b): the solid line is markup in log-deviations from its steady state (quarterly estimate at the posterior mean); the colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values).

Figure J2: Historical shock decomposition: price/cost margin of INT-firms, 1978:I-2019:IV



Note: The solid line is the price/cost margin in log-deviations from its steady state (quarterly estimate at the posterior mean); the colored bars are the contributions of the grouped shocks ("Demand" includes monetary policy, inflation target, MEI, and government spending shocks; "Other" includes labor supply shocks and contribution from initial values).

Figure J3: IRFs comparison (baseline vs standard NK model)



Note: Quarterly estimated mean impulse responses (solid lines) with 90% HPD intervals (dashed lines) to one-standard-deviation shocks.