PROBLEM-BASED LEARNING AND MATHEMATICS: RETHINKING LABORATORY AT THE TIMES OF CRISIS

Marina Cazzola
Università degli Studi di Milano-Bicocca
Milano Italy
marina.cazzola@unimib.it

Abstract

As in other papers presented at previous IATED conferences (see [1], and [2]), we start from the assumption that the use of active learner-centered methodologies at school, especially with the use of pedagogical laboratories, is essential for an effective teaching of mathematics. In order to stimulate the use of active methodologies in future teachers, the pre-service teacher training program at the University of Milano-Bicocca includes compulsory laboratories, that, at least for the math discipline, are implemented according to Problem-based learning (PBL) principles.
The design of such laboratories is conceived so to allow for group work and to promote intensive peer interaction between students. The lab class consists of at most 30 students, who are split into smaller groups. Each group is assigned the problem and has to tackle it, trying to provide a solution, being given the necessary amount of time. The tutor walks around the classroom among the desks, observing the students, listening to their arguing and answering their questions when necessary.
With the outbreak of the COVID-19 emergency, the laboratories scheduled for the April and May 2020 had to be redesigned in order to be remotely held.
An analysis of the laboratories realized in the last 10 years has been carried out in order to highlight the essential features that contributed to the effectiveness of such laboratories. Then a deep study was made in order to identify which of the available ICT tools could allow to reproduce these characteristics online.
The following features were considered essential:

- Group work and cooperation: sharing ideas is the foundation of the learning process in a PBL environment. An online working environment that allowed for a easy interaction had to be selected. Such an environment had to be used both for synchronous and for asynchronous interactions, and for collaborative writing of documents.
- The role of the problem: in a PBL environment the problem is the source of the learning process. An analysis has been carried out in order to understand which of the problems used in the previous experiences would better exploit the use of ICT. Some of these had to be adapted, e.g. introducing some meta questions, in order to make better use of the new online environment.

We will describe in detail the design and implementation of a laboratory on symmetry held remotely in April and May 2020, highlighting the ICT tools chosen in order to better enhance the laboratory's PBL setting.
Keywords: problem-based learning, teacher training, mathematics, covid-19, symmetry

1. PBL AND TEACHER TRAINING IN MATHEMATICS

As we already pointed out in [1] and [2], research in mathematics education is universally aware of the key role of active methodologies for an effective teaching and learning of mathematics. The key aspect in a well-designed learning path is the opportunity for students to experiment with their mathematical knowledge. The central role in such a path should be taken by laboratories, in which pupils are actively engaged in problem solving tasks. In spite of this well-known fact, we can observe a trend of teachers that, once employed in school, tend to rely on self-perpetuating "traditional" methods. The introduction of active learning activities in the pre-service training of prospective teachers is thus a string to pull in order to try to reverse such trend: "a teacher who acquired whatever he knows in mathematics purely receptively can hardly promote the active learning of his students" [3, vol. 2, p. 113].

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Such an idea of learning through *problem solving* activities, if corresponding to the model that can be rooted as back as the early works of the mathematician Polya, can be situated in the problem-based learning (PBL) methodology (e.g. see [4] for a definition of PBL, see also [5] for a discussion on these aspects of Polya's works). Typically, a PBL activity is organized according to these steps (e.g. see [1], [2], [6] or [7]):

- pupils are given a problem;
- they discuss the problem and/or work on the problem in small groups, collecting information useful to solve the problem;
- all the pupils gather to compare findings and/or discuss conclusions; new problems could arise from this discussion, in this case
- pupils go back to work on the new problems, and the cycle starts again.

1.1 The teacher training program at Milano-Bicocca

The experience we want to describe is part of the training program for prospective primary school teacher at the University of Milano-Bicocca. Since 2011, prospective teachers have to attend a five year long combined Bachelor and Master degree. In the Milano-Bicocca implementation, this degree program includes three modules in mathematics (e.g. see [1], and [2]):

- *Foundations of mathematics* (year 1, 8 credits): lectures focus mainly on purely mathematical contents (numbers, arithmetic and probability), while exercise classes are devoted to the explanation and resolution of standard math exercises;
- *Basic mathematics for teaching* (year 2, 9 credits): lectures focus on mathematical contents (measure and geometry), and, as above, exercise classes are devoted to standard exercises; moreover, students are required to attend a compulsory laboratory in which active methodologies are shown in action (students experience *learning in a lab*);
- *Mathematics teaching* (year 4, 5 credits): lectures focus mainly on methodologies (active learning and PBL) and teaching examples (analysis of problems); moreover, students are required to attend a compulsory laboratory in which a further analysis of teaching methodologies is carried on (prospective teachers experience *teaching through a lab*).

Apart from the mathematical courses, the program includes numerous pedagogical modules which cover instructional methods and recommend the idea of active learning. The total amount of credits of the 5-year program, including school internship activities and the preparation of a final dissertation, is 300 credits.
All the laboratories linked to the math classes are implemented according to PBL principles: it is in the lab that students are really expected to experience and understand PBL. In this paper we will describe one of the laboratories linked to the module *Basic mathematics for teaching*, focusing on the particular setup required because of the COVID-19 emergency.

2. THE COVID-19 EMERGENCY

With the spread of the COVID-19 emergency all the activities linked to the module *Basic mathematics for teaching* had to be held remotely. We refer to [8] for a description of the changes the implementation of lectures and exercise classes had to undergo during the 2020 spring semester. The switch from face-to-face to remote learning was particularly delicate for the labs, some of which required a deep rethinking.
Before the emergency, students (about 400) were split into smaller groups (usually of about 30 pupils) and the actual contents dealt in the laboratory attended by each group could vary. That is, tutors had as much freedom as possible in the design of their laboratory, provided they followed a PBL model. Each lab included group work activities, during which, physical presence in a standard classroom allowed for the tutor, and to walk among groups, being thus able to better monitor students' work, listening to their arguing and answering their questions. Also, many of the laboratories promoted the use of manipulatives. Because of these characteristics, the start of the laboratories, planned for March, was postponed as much as possible (hoping in a loosening of the security measures), and only after the lessons ended we got the news that the emergency would not stop, and all of the laboratories activities would have had to be held remotely. Therefore, all the kind of physical interactions had to be dropped.
Many of the tutors were scared at the idea of having to rethink their laboratories in a very short time. The issues raised by the tutors were not only about the extra workload due to the need to rethink the contents

(this was particularly true, for example, for those laboratories that focused on manipulatives), but also the need to acquire a level of ICT knowledge sufficient enough for conducting laboratory activities.
The two lecturers and 11 tutors were involved in the process of redesigning laboratories, that loosely followed a path of 4 steps.

- First, a discussion about the characteristics of the laboratories realized in the previous 10 years was conducted. Many (virtual) meetings were held in order exchange experiences, and to analyse which of the existing labs could be proposed without much changes in a remote learning model. This working team got the conclusion that an essential feature of the laboratories was the group work phase and the exchange between students.
- Secondly, the PBL structure of the laboratories had to be maintained. As some of the labs needed full revision, the lecturers suggested a few themes related to the course syllabus that nicely fitted in a PBL project and these themes were discussed with the tutors during the meetings.
- Thirdly, each tutor analyzed his/her own laboratory and either devised a plan for remotely holding it or planned a completely new one based on one of the suggested themes.
- Finally, the ICT tools available were analyzed. The main issue was the need of a conference system for synchronous video calls. Also, the possibility to collaborate editing of documents was required.

We will give detail of the various aspects of this process in the following section with a full example. As much as the choice of the ICT tools is concerned, our university provided Moodle, and both the Google and the Microsoft suites (e.g. see [8]). These tools were used in all the steps of the revision process described above, so tutors had the chance to familiarize with all of them. One of the main issues was the need to split an active video conference into smaller groups, to allow for the actual group work. Our university did not provide Zoom, that is well known for having this function built in, but the effect can be achieved both with Google Meet and with Microsoft Teams, that allow for concomitant multiple calls. The tutors were given the freedom to choose the setup: most of the tutors opted for Google Classroom, mainly because they had previous experiences with it. Actually, one of the tutors did use Zoom with his own personal account.

3. AN EXAMPLE OF A LABORATORY

We will describe in detail the design and implementation of a laboratory on Symmetry held remotely in April and May 2020.

3.1 Symmetry

Symmetry nicely fit in the *Basic mathematics for teaching* course syllabus, as "geometry of transformation" is one of the main topics covered by the lectures. We could count on an established expertise in teaching and popularizing symmetry due to the realizations of the Matematita research center (http://www.matematita.it/). In particular we need to mention the exhibition "Symmetry, playing with mirrors" (http://www.matematita.it/realizzazioni/mostre.php?NL=en) on display at the University of Milan (e.g. see [9, p. 37]). The activity around the Milan copy of this exhibition, spanning over more than two decades, allowed us to acknowledge that everybody is familiar with symmetry (examples of symmetries are visible in many aspects of our everyday life), but that symmetry allows as well for experiencing deep mathematical concepts. You can start from the simple observation of nice pictures, and end up talking not only about Euclidean geometry, but also about abstract algebra, focusing on groups and their classification (e.g. see [10]). Easy to obtain manipulatives, such as simple mirrors, provide very powerful concrete model for the abstract transformations known as "reflections", that generate all of the isometries in the plane, and are thus a key element for the abstract theory of symmetry (and in fact the word "mirror" plays a key role in the very name of the exhibition). Experience shows that you can design activities that are engaging both for small kids and for a more expert audience at the same time.

3.2 Symmetry and ICT

The center Matematita has among its objects of study the use of new technologies for the communication of mathematics, and experiences in this field can be dated as early as the CD of the virtual part of the exhibition "matemilano" (e.g. see [11] and [12]). In such a CD videos showing the generation of the finite symmetry groups in the plane ("rosettes") are provided (http://www.matapp.unimib.it/~marina/

CDMM/Matemilano/simmetria.html). You can also find a Java kaleidoscope for generating wallpaper patterns (application that unfortunately is no longer working as Java disabled some of its functionalities for security reasons). From a purely ICT point of view, in order to study the symmetry of a certain image all you need is an image manipulation program. You only need to superimpose the image you want to study with a copy of it transformed by some isometry. Many programs that are publicly available have nice user interfaces to achieve this, and can even execute more complex tasks, such as building an image with a given symmetry. We mention GeCla (http://www.atractor.pt/mat/GeCla/GeCla-_en.html) by Atractor ([9]), as it is especially designed to link the image processing to the theory. GeoGebra can be used to build symmetry patterns too. With all these tools available, the theme of symmetry proved to be suitable for a transposition from a face to face presentation to a virtual one, without losing its strengths.

3.3 Symmetry and PBL

Before the emergency, many laboratories about symmetry were implemented, and a series of ready to use turnkey laboratory kits for teachers were designed (http://www.matematita.it/realizzazioni/materiale_didattico.php?NL=en). Nice problems about symmetry can be formulated and exploited in a PBL environment:

- given an image, list all of its symmetries, that is all of the isometries that map the image onto itself;
- given a set of images, sort them according to their types of symmetry;
- (reverse problem) given a symmetry pattern, draw an image according to that scheme.

Before the emergency, we used to explicitly refer to mirrors and provide them as tools to be used in tackling the problems. As pointed out in the previous section, many ICT tools are available that can provide a “mirror effect”. Thus, it is easy to reformulate the questions above so that they can be the base on which the PBL activity is developed, without actual manipulatives, but in a remote virtual environment.

3.4 PBL and ICT

During the revision process the group work phase was identified as a key element of the laboratory, it was therefore necessary to identify and select the ICT tools that best allowed for cooperative work. For example, remote simultaneous multi user editing of the same document was required to improve combined efforts between the students. More specifically, when groups were given the task of producing a document to be shown to the other groups, the possible utilities provided by our University would have been:

- a wiki in Moodle;
- a Powerpoint presentation shared in OneDrive;
- a Google Presentation shared in Google Drive.

For this lab we opted for the Google Suite, that provides a full office-like set of collaborative tools (Google Docs, Google Sheets, Google Slides, Google Drawings, . . .). A Google Classroom was set for this lab and a Google Meeting session was active for the whole time during each session. We set up the Meeting with Calendar in order to reserve it to the given list of participants. Moreover, the extra task to create working environments for the group work was resolved with the opening of concomitant multiple Meets, activated as well using Calendar in order to reserve each of them to the corresponding group members.

3.5 Implementation of the laboratory

The laboratory was organized in 3 sessions of 4 hours each. More specifically, here is a schema of the activities:

- Session 1: isometries and rosettes
  - short introduction and explanation of the aims of the laboratory (15 mins)
  - short introduction to the Google Classroom Suite (15 mins)
  - brainstorm on the word “isometry” (20 mins)
  - brainstorm on the word “symmetry” (20 mins)
  - creation of the groups, Classroom and Drive setup (20 mins)
  - group work on *Problem 1* (1 hour and 30 mins)
  - discussion in large group and comments by the tutor (1 hour)

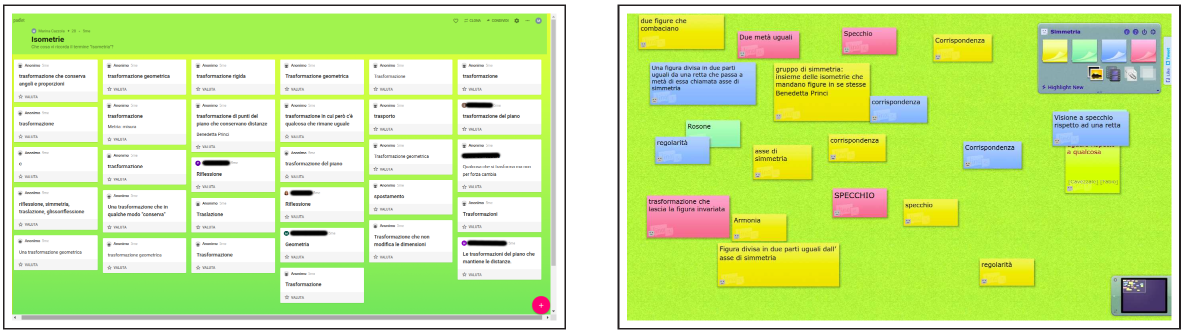

Fig. 1: Brainstorms

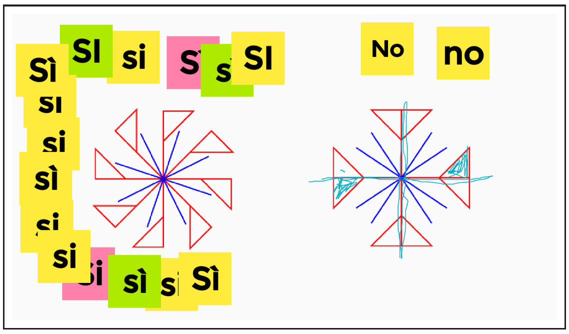

Fig. 2: Image display and collaborative analysis through Jamboard

At the end of the session, the tutor assigns as homework the revision of some GeoGebra applets used during the lectures to create rosettes (https://www.geogebra.org/m/usq7nkez).

- Session 2: friezes
  - group work on *Problem 2* (2 hours)
  - discussion in large group (2 hours)

  At the end of the session, the tutor assigns as homework an exploration of the software GeCla (http://www.atractor.pt/mat/GeCla/GeCla-_en.html)
- Session 3: friezes (revision)
  - presentation by the tutor (1 hour)
  - group work on *Problem 2* (2 hours)
  - discussion in large group (50 mins)
  - conclusions by the tutor and final discussion (10 mins)

After the end of the laboratory, students had 10 days to hand in a final report with a personal review of the experience.

3.5.1 The problems used

The fundamental element for the effectiveness of a PBL laboratory is the choice of a suitable problem and therefore we will give some detail on the ones used for this laboratory.
During the first session, after the preliminary brainstorms on the ideas of "isometry" and "symmetry" (see Fig. 1), the groups were given a set of 30 images and *Problem 1* was in fact the task to classify them according to symmetry, highlighting similarities and differences among them. Although students had full access to the Google Suite and an example of use of Jamboard (e.g. see Fig. 2) to display images was actually given, the groups did not use any visual tool to explain their findings in the final discussion.
The underlying theme of the laboratory was the classification of frieze groups, that is a nontrivial task. Such task was therefore split into two sub tasks, to be object of each of the following sessions. On the second session groups were asked to "describe the characteristics of a figure that admits a translation in its symmetry group". Moreover, as the final discussion of the first session was totally lacking in the visual register and therefore did not turn out to be very effective, this time an explicit request was made to produce a presentation to be shown in the final discussion. In other words, *Problem 2* had both a

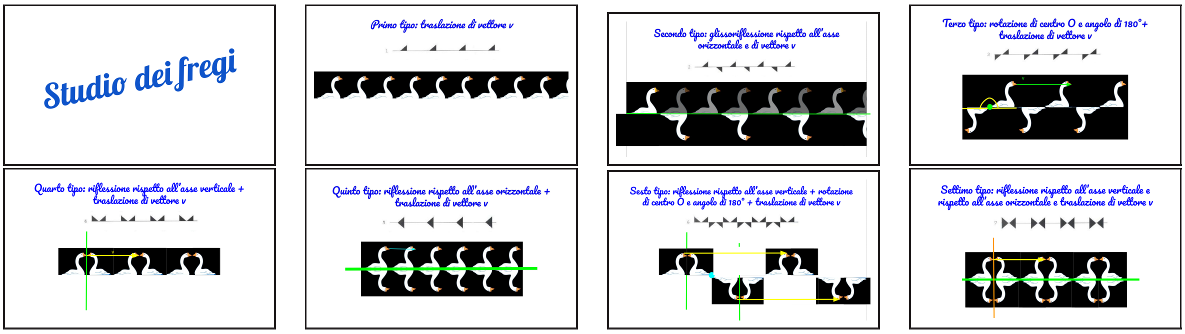

Fig. 3: Seven and only seven

mathematical core (friezes), and a “meta” aspect, involving the competence of communication of mathematics. The final discussion verted on these presentations and the word “frieze” emerged, although not made explicit in the assignment. Also, the “seven and only seven” result about the classification of frieze groups emerged. All the presentations contained mathematical mistakes, many of which were spotted by the students themselves during the discussion.
Finally, on the third session the groups were asked to refine their presentations focusing on an explicit description of the classification of the seven frieze groups. These presentation (e.g. see Fig. 3) animated the final discussion.

3.5.2 Some remarks

We believe it is useful to end this section with a few comments about a few points that have emerged during the laboratory.

- The timetable of this lab was not well planned, due to conflicts within the students' timetable. The second session took place three weeks after the first session, with a very large interval. Moreover, there was no coordination between lab and lectures timetable, so some of the students had knowledge about isometries and rosettes, and some had not (and this clearly emerged during the brainstorms). This is the reason why the first session is disconnected from the others and is in fact only meant to verify and somehow level the students' knowledge.
- The group work on *Problem 1* in the first session seems to have had a positive effect on the students, who showed a better knowledge of the subject in the following session. An explanation for this knowledge improvement was certainly the time lapse (three weeks) and the progress of the other activities related to the course (lessons and exercises). But we believe that the stimulus given by the personal engagement in the search for a solution of *Problem 1* also played a role, as stated by the students themselves in their final report.
- The first session was also peculiar because a lot of time had to be spent in organizing the virtual environment. Students had very little previous experience with Google Meet and almost no experience at all with Classroom.
- The statement of *Problem 2* used during the second session is vague and does not explicitly ask for a specific procedure: “describe the characteristics of a figure that admits a translation in its symmetry group”. This is exactly what we mean by ill-structured problem in Savery's sense: “the problem simulations used in problem-based learning must be ill-structured and allow for free inquiry” [4, p. 13]. Even with this unclear definition of the task, the idea of frieze and of the existence of only seven different type of frieze groups emerged after the group work phase.
- The presentations by the groups given in the second session contained mathematical errors. Some of these were spotted and corrected by the students in the final discussion, during which the different groups gathered to share and compare their findings. But many were not. After many thoughts, we understood that this was an opportunity to talk about the role of errors in teaching, so the tutor opened the last session with a detailed revision of the presentations. Major errors were identified by the tutor, and groups were given the additional task of fixing them. Having been given more time, the task of identifying the errors should have probably been left to the students (e.g. via a further whole class discussion). Nevertheless, the tutor managed to create a suitable class climate so that students lived this marking of errors, not as a criticism, but as a real chance for learning. This is evident in many comments the students wrote in their final report.

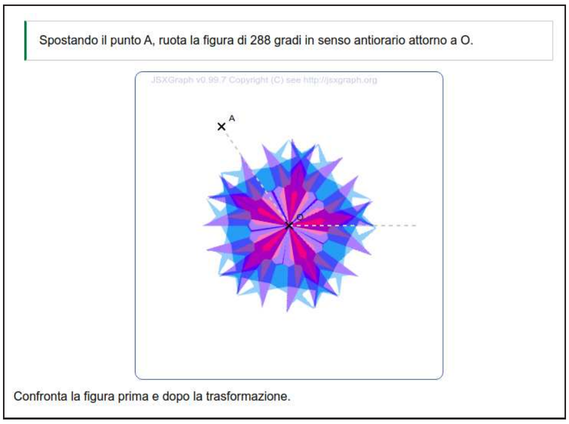

Fig. 4: The “Rotations” WIMS exercises

- Many of the presentations created in the second session drew heavily from internet searches (although the presentations did not contain any clear reference to the sources) and some of the mistakes were actually derived from the sources used (many web pages on symmetry do contains errors!). The tutor stressed that Internet can be deceitful and Internet browsing requires an additional careful revision process.
- As long as students' ICT competencies are concerned, in the first laboratory session we expected students to be able to plan the communication for the final discussion and choose the proper ICT tool for presentation autonomously. They did not, and an explicit requirement to PowerPoint and Google Slides was required. Also, students did not really take advantage from the sharing facilities of Google Drive, and some groups kept exchanging files via email.
- The effectiveness of an activity such as the one proposed in *Problem 1* is strongly dependent on the choice of images to be presented to the students. Matematita has dedicated a large share of its activity to iconographic reference and to the creation of a publicly accessible images archive [13]. With respect to the choice of images, the needs that emerged during this laboratory inspired the creation of the exercises on “Rotations” in the WIMS module “OEF Rosettes” [14] (e.g. see Fig. 4; see also [15] for an introduction to WIMS). Those interested can access the set of images used for this laboratory by viewing those exercises.
- As long as the choice of a virtual environment is concerned, the main fault of Google Classroom was the lack of group management. Natively you only have a group for the teachers and a group for the whole class of students. For example, a Google Classroom is linked to a folder in Google Drive, with permission set either for the teachers or for all the class participants. The only way to form groups is to assign a homework to a selection of students, manually inserting the list each time. This is connected to Google Drive and in this way you can assign a shared single document to the group, but not a folder. You can share a folder with a group (and this was convenient during the first session for giving each group a set of image files) only going directly to Google Drive and manually inserting the list of group members. This is the reason why the phase of creation of the group and setup in the first session lasted 20 minutes. Similarly, Google Meet is not group aware. Group Meeting can be made working the way we did, but, again, inserting manually the group members each time.
- It could be worth to test the availability of the features described above in Microsoft Teams, which apparently seems to have a better native user management. Superficial user management can be a security risk. For instance, access restrictions to a Google Meet only rely on the 10 characters random code that identify the session. If this code is made public, an intruder can try to access the meeting and even if not given access, this attempt can interfere with the lecture, as the teacher has to make some action.
- As already mentioned, another possibility for the video conferences is Zoom, that we could not test and that is known to have the possibility to split the conference in smaller groups (although last summer concerns about the security of user management in Zoom arose). An additional useful feature of Zoom is the grid view, that can be obtained in Google Meet via a plugin, but that in Microsoft Teams was limited to 9 participants (rumors says this limit will increase to 49).
- We ended up using the Google Suite, but it turned out far from being the “perfect” environment. At some points, we integrated it with programs available on the Internet, such as Padlet (`http://`

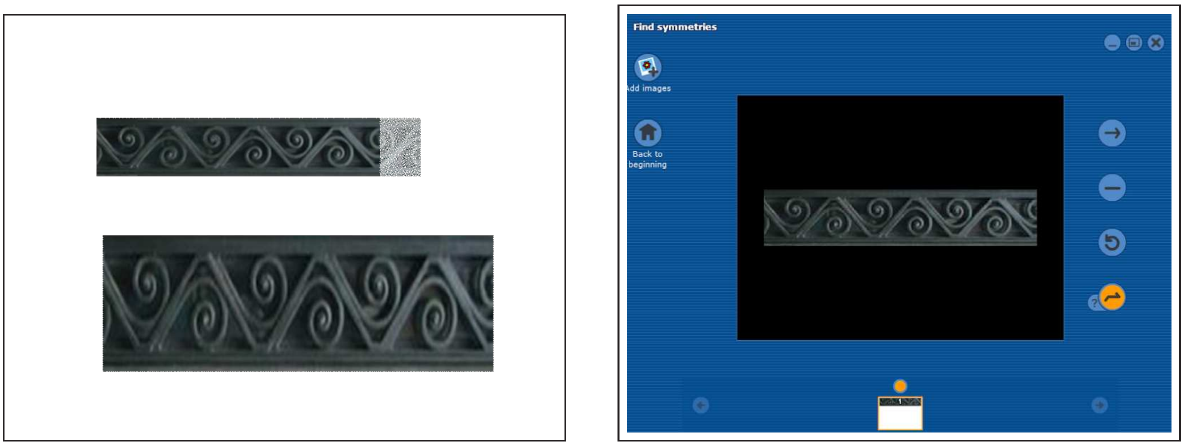

Fig. 5: Google Drawings vs GeCla

www.pladlet.com) and Linoit (http://www.linoit.com) for the brainstorms (e.g. see Fig. 1), and we could have explored many more. This aspect of continuous research and integration of ICT tools (and not only ICT tools) is in our opinion a necessary skill for prospective teachers, and this point of view has been often carried out during the laboratory: no “turnkey” ready to use solution exists and teachers have to make their own choices in order to set up their working environment.

- The remark above applies also to mathematical content related software. We choose not to present GeCla right from the beginning, but first to show how to use Google Drawings to act on images (see Fig 5 on the left). The idea was to postpone the presentation of a “turnkey” ready to use solution, but to build the knowledge of the actions that needed to be carried out and therefore understand at least schematically the basic functions of the program. That is try to give the idea that a teacher should not accept the “black box” solution, but should instead try to have a glimpse of the content of the box.
- Some groups did use both GeoGebra and GeCla to build the images to be used in their presentations.

4. CONCLUSIONS

This paper should be report of the solutions we could quickly provide to build a teaching environment during the emergency. In our opinion, this is only a starting point and many some aspects should be reviewed and further investigated. Nevertheless, we believe that a few facts can be taken for granted. Firstly, of all, this experience shows that the PBL model for laboratory can be transposed from face-to-face learning to remote learning, without losing its effectiveness. Secondly, the theme of symmetry proves once again to be extremely flexible, suitable for building engaging experiences for students, and for building knowledge beyond its scope. Research in didactics of mathematics should identify more and more themes with these characteristics. Moreover, the remote learning environment required both in the teachers and in the students a higher level of ICT competencies then a standard learning environment. It is therefore a good environment for acquiring and refining ICT skills. Among these skills, we consider crucial the ability to search for information on the Internet and verify that what selected actually make sense. As long as ICT tools are concerned, we did not identify a unique environment suitable for all the required tasks, but it was necessary to use additional programs. Furthermore, ICT are evolving very fast and only lifelong training can make users able to use the most suitable tool.

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