SOME INVERSE PROBLEMS IN GROUND WATER MODELLING

Giovanni F Crosta
Department of Environmental Sciences
Universita' degli Studi
Milano

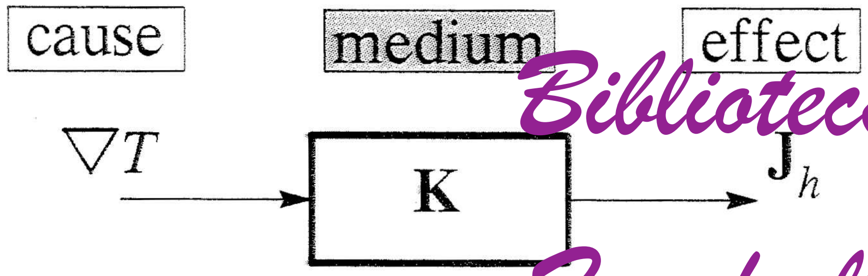
MOTIVATION

(Past) to model the drinking water supply of Milan
e.g., water flow and contaminant transport in the aquifer.
(Recent) *CNR–ENEL* Research Project
Advanced Modelling for Environmental Impact Assessment

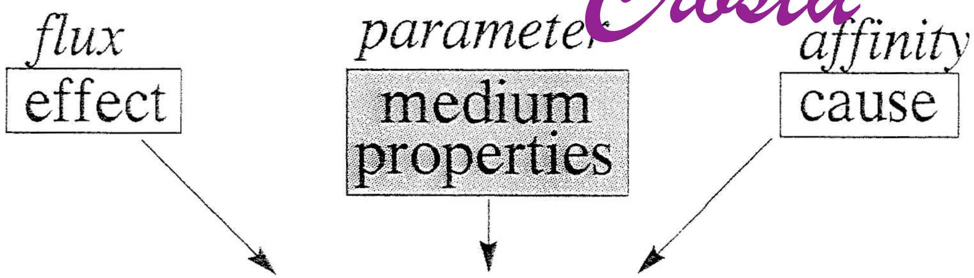
TALK PLAN

1. A model of ground water flow.
2. The inverse problem: the identification of a coefficient
   i.e., position dependent hydraulic conductivity.
3. Some recent results:
   uniqueness, stability, properties of algorithms.

MODELLING PARADIGM: THE INGREDIENTS



From linear irreversible thermodynamics, relate cause to effect



(Fourier)	$\mathbf{J}_h = -\mathbf{K} \nabla T$
(Ohm)	$\mathbf{J}_e = -\sigma \nabla V$
(Fick)	$\mathbf{J}_m = -\mathbf{D} \nabla c$
(Darcy)	$\mathbf{J}_w = -\mathbf{A} \nabla u$

MODELLING PARADIGM: THE DIRECT PROBLEM

In the (two – dimensional, bounded) *domain* $D \subset IR^2$, *recast the equation into divergence form:*

parameters
medium properties
KNOWN

potential
effect
UNKNOWN

source term
cause
KNOWN

$$-\nabla \cdot (\mathbf{A} \nabla u) = S\, u_t + f$$

IC, BC

additional data
cause
KNOWN

Types of media :

I) $\mathbf{A} = \mathbf{A}(\mathbf{x})$, $\mathbf{x} \in D$;   II) $\mathbf{A} = \mathbf{A}(\mathbf{x}, t)$, $t \in (0, T]$;

III) $\mathbf{A} = \mathbf{A}(u)$ (state – dependent coefficient).

Properties of the solution u

existence, uniqueness :

if the boundary of D, f, S and $\mathbf{A}$ are smooth => $\exists\, !\, u$

stability :

u continuously depends on all data.

The direct problem: $\{D \times (0, T]; f; \mathbf{A}; S;$ IC, BCs$\}$ –> u is well–posed.

MODELLING: PARADIGM AND PARADOX

SCOPE

Description: interpret the natural system' s behaviour under the currently applied controls e.g., well discharge rates.
Prediction: determine the effects produced by different controls, without carrying out an actual experiment.

In either case, the only unknown in the problem shall be the potential, u.

D, IC, BCs, f : available. Assume S known, constant.

PARADOX

QUESTION: HOW DOES ONE DETERMINE **A** ?

REM. : u can be measured (piezometric head)
PROPOSED ANSWER: apply {IC, BCs ; f}, measure u and try to determine **A** .

This is an inverse problem of coefficient identification
ASIDE: structural *vs.* parameter identification.

MODELLING PARADIGM:
THE INVERSE PROBLEM

Interchange the roles of ∇u *and* $\mathbf{A}$:

parameter
effect
UNKNOWN

potential
datum
KNOWN

source term
cause
KNOWN

$$-\nabla\cdot(\mathbf{A}\nabla u) = u_t + f$$

IC = ?

BC = ?

additional data
cause
????

QUESTION: DOES THIS MAKE ANY SENSE AT ALL ?

ANSWER: GENERALLY NO !

MODELLING PARADOX:
THE INVERSE PROBLEM

$$-\nabla\cdot(\mathbf{A}\nabla u) = u_t + f$$

 + IC ? BC ?

QUESTION: WHAT GOES WRONG ?

ANSWER :

I) *non existence*

given arbitrary $\{D\times(0,\ T],\ f,\ u\}$, **A** need not exist (non positivity, non smoothness, ...) regardless of data accuracy.

II) *non uniqueness*

if an admissible **A** exists, it need not be unique .

III) *instability* (non–continuous dependence on data)

minor perturbations in u (due e.g., to measurement errors) yield substantially different **A** 's .

REM.: *intrinsic* instability *vs.* *numerical* instability .

The problem $\{\, D \times (0,\ T]\ ;\ f\ ;\ u\,\}$ –> **A** is ill–posed