TIME IN PRODUCTION ECONOMICS: A BRIEF HISTORICAL EXCURSUS

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In this paper, we critically review how alternative theories treat the time profile of production processes. We discuss how production and time are modelled in the Classical economics, in the von Neumann-Sraffa representation of production, and in mainstream production theories. Next, we focus on two time-specific analyses of production, developed by Nicholas Georgescu-Roegen and Gordon Winston. Georgescu-Roegen's flow-fund model deals with the relationship between production organization, scale and efficiency. Winston's analysis of production can be considered as a complement of the flow-fund model, because it combines the time-specific representation of the production process in the fund-flow model with the cost implications of time-specific and duration-specific prices of the productive services.

1. INTRODUCTION

The present paper aims at briefly and critically reviewing the ways in which different models in economics implicitly or explicitly conceptualize the time-related aspects of production.[1] After analysing the role of time in production in the main theoretical framework in economics, from the Classical school to Neoclassical general equilibrium theories, we consider two time-specific representations of production: Georgescu-Roegen's (1970, 1971) fund-flow model and Winston's (1982) analysis of production.

The paper is structured as follows. In Section 2, we analyse the role played by time and the analytical description of production as a process unfolding in time in production models that are not 'time-specific' and have been developed outside what is usually considered 'mainstream

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[1] For the different although partly related issue of the role of time in consumption in economics see, for instance, STEEDMAN 2001, who puts forward a time-specific consumption theory, building on the insights in Gossen's original work and GEORGESCU-ROEGEN'S 1983 introductory essay on Gossen. An explicit consideration of time in a Neoclassical framework is found in FRISCH 1964.

economics'. In particular, in Section 2.1, we discuss the relationship between production and time in classical economics; in Section 2.2, we consider the role of time in production in the so-called von Neumann-Sraffa approach; in Section 2.3, we deal with the representation of production processes in Austrian and neo-Austrian economics. Section 3 addresses all the main analytical representations of production in mainstream microeconomics. Section 4 deals with two time-specific analyses of production: Section 4.1 reviews the main features and implications of Georgescu-Roegen's fund-flow model, whereas Section 4.2 discusses the main insights coming from Winston's time-specific analysis. Finally, Section 5 concludes by summing up the main results.

2. Production and time in Classical, Sraffian and Austrian economics

This section addresses the relationship between production and time in Classical and Austrian economics. In extending the original agricultural production model to manufacturing, Smith ([1776] 1976) and Ricardo ([1817] 1951) implicitly abandon the natural period of production in agriculture (the calendar year) and consider a conventional period. Von Neumann's (1945) and Sraffa's (1960) theories consider the economic system at a particular moment in time. In their theories, fixed capital is reduced to circulating capital by considering what remains of a fixed capital item at the end of the period as a product jointly produced with the final good. Pasinetti (1981, 1993) develops a theory of a vertically integrated production process, which is useful for dynamic analysis in which technical coefficients change. Finally, this section considers the Austrian representation of the production process in which the inputs (i.e. labour and land) are dated, non-produced, factors of production.

2.1 Production and time in the Classic economists

Building on the seminal contributions by Petty (1986 [1662]), Cantillon (1931 [1755]) and Quesnay (1972 [1759]), whose economic analysis is focused on the creation and distribution of the social surplus in the agricultural sector, the Classical economists, in their conceptualization of the production process, have mimicked the typical pattern of production processes in agriculture.

In the 'corn' production, the whole process takes place in a single self-contained 'natural period', the agriculture year: «a year is assumed in political economy as the period which includes a revolving cycle of production» (Mill 1826, 185). All the inputs (circulating capital, like seed-corn and the means of subsistence of workers) are assumed to be advances made at the beginning of the production period and thus enter the process from the start. The output (the corn) is produced only at the end of the period and the surplus is distributed.

This is the same time profile of the production process that Smith (1976 [1776]) and Ricardo (1951 [1817]) had in mind.[2] As noted by Hicks (1985), by extending this model of production from agriculture to manufacturing, Smith and Ricardo lose «the natural period of agriculture, which in the Original Model had made the single period self-contained», and the accounting period becomes conventional: «it can have nothing to do with the periods of production of the various goods. So the stock that is brought in at the beginning of the period can no longer be reckoned to consist entirely of finished goods. It must largely consist of goods in process» (Hicks 1985, 32).[3]

2.2 TIME IN THE VON NEUMANN-SRAFFA REPRESENTATION OF PRODUCTION

The Classical view of the production process can be found also in the von Neumann-Sraffa representation of production (Lager 2000; Kurz and Salvadori 1997).[4]

In order to analyse the steady-state equilibrium of an economic system operating at constant returns to scale and expanding at a constant growth rate, von Neumann (1945) assumes that all production processes have the *same* unit length, for «processes of longer duration» are «broken down into single processes of unit duration introducing if necessary intermediate products as additional goods» (1945, 2). As in Torrens (1821), in von Neumann (1945) and Sraffa (1960) fixed capital is reduced to circulating capital by considering what remains of a fixed capital item at the end of the period as a product jointly produced with the main commodity.[5] All the advances are made at the beginning of the production period, including the means of subsistence of workers.[6]

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[2] On the role played by the corn model in Ricardo's theory of value and distribution see, among others, KURZ and SALVADORI 1997 and DE VIVO 1996.

[3] A different interpretation of Ricardo's conceptualization of manufacturing processes has been put forward by MORISHIMA 1989, who claims that Ricardo assumes an instantaneous production process in manufacturing. This assumption is inferred by Morishima from the fact that Ricardo states that manufactured goods «may be increased almost without limit» and, according to MORISHIMA 1996, this implies that «Ricardo assumes that production of necessaries other than food is instantaneous», because, «where production is instantaneous, producers can always meet the demand, however large it may be, the price being constant» (1996, 93). In fact, Morishima's interpretation of Ricardo seems hardly tenable (see KURZ and SALVADORI 1992; 1998). Moreover, an instantaneous process does not necessarily imply an infinite potential output in a finite length of time (see Section 4.1.4).

[4] MARX's [1884] 1978, 581-99 schemes of reproduction played an important role for the development of the concept of circular flow and were the precursors of the input-output models. On this see KURZ and SALVADORI 2000b. For a thorough discussion on the relation between VON NEUMANN 1945 and SRAFFA 1960 see KURZ and SALVADORI 1997; 2001.

[5] On the reduction of fixed capital to circulating capital in the von Neumann-Sraffa framework, treating it as a joint product, see, among others, KURZ and SALVADORI 1997, Ch. 7, 9 and the references therein.

[6] This temporal structure is explicitly discussed in the dynamic analysis within the von Neumann-Sraffa framework (e.g. KURZ and SALVADORI 1997; 2000a). On the contrary, the time-profile of the production process is completely overlooked in the standard Leontief's input-output framework: «the rounds [of production in the input-output model] do not take place in calendar time, with the second round following the first. ... Artificial computational time is involved» DORFMAN *et al.* 1958, 253-254, and «the traditional [input-output] multiplier does not stipulate the time taken to realize effects, assuming instead that they usually occur almost immediately» MULES 1983, 197. Dynamic input-output models

In this framework, production processes are generally conceived as series of dated quantities of inputs and outputs. Time is divided in discrete intervals of unit duration (a period) and a generic production process k lasting T_k periods can be represented as:

$$(\boldsymbol{x}^k{}_1, \boldsymbol{x}^k{}_2, \ldots, \boldsymbol{x}^k{}_{Tk}; \boldsymbol{y}^k{}_1, \boldsymbol{y}^k{}_2, \ldots, \boldsymbol{y}^k{}_{Tk}) \qquad (1)$$

where $\boldsymbol{x}^k{}_t = (x^k{}_{1t}, x^k{}_{2t}, \ldots, x^k{}_{It})$ is the vector of inputs, i.e. advances made available at the beginning of period t in process k, and $\boldsymbol{y}^k{}_t = (y^k{}_{1t}, y^k{}_{2t}, \ldots, y^k{}_{Mt})$ is the vector of outputs released by the same process at the end of period t, and $\boldsymbol{x}^k{}_t$ and $\boldsymbol{y}^k{}_t$ can include semi-finished goods.

The same process k can be equivalently represented as a series of linked «point-input point-output processes» $(\boldsymbol{x}^k{}_t; \boldsymbol{y}^k{}_t)$, i.e. production processes lasting one period (e.g. Lager 2000):

$$((\boldsymbol{x}^k{}_1; \boldsymbol{y}^k{}_1), (\boldsymbol{x}^k{}_2; \boldsymbol{y}^k{}_2), \ldots, (\boldsymbol{x}^k{}_{Tk}; \boldsymbol{y}^k{}_{Tk})) \qquad (2)$$

where $y^k{}_{i,t} = x^k{}_{i,t+1}$ if $y^k{}_{i,t}$ is a semi-finished good.

The period t is the smallest registered interval of time: all the goods employed in the process during the unit period are assumed to be required and made available at the beginning of the period and all the outputs are assumed to be released at the end of it; the length of the unit period is chosen so as to be equal across all the processes simultaneously activated in the production system (e.g. Kurz and Salvadori 1997).

As noted by Leontief (cited by ten Raa 1986a), the discretization always gives rise to an aggregation issue with a related bias. The bias derives from interpreting possibly continuous input (output) functions of time as step functions, where inputs (outputs) that are different in terms of actual time of availability (delivery) are lumped together and treated as equal.[7]

A Classical perspective of production is also found in Pasinetti's model of a vertically integrated production process. A production process is defined as *vertically integrated* when it includes all the operations carried out along the production *filière* (or cluster), through the different intermediate stages leading up the finished output (Pasinetti 1981, 33-4). Pasinetti remarks that «inter-industry analysis, *à la* Leontief (1951) or *à la* Sraffa (1960) », is an analytical tool that considers the economic system at a particular moment in time. This tool is not suitable for investigating movements through time and changes in technology because «the coefficients of the inter-industry systems of equations change as soon as there is any change in technology, thereby causing the

that explicitly consider the temporal structure of production are in JOHANSEN 1978; ÅBERG and PERSSON 1981; MULES 1983; TEN RAA 1986a, b; 2006, Ch. 13; AULIN-AHMAVAARA 1990; ROMANOFF and LEVINE 1986; 1990.

system of equations to break down». If the aim of the analysis is to study the effects of technical change, we have «to switch from inter-industry analysis … to vertically integrated analysis» that allows investigations in time since «the coefficient of the systems that represent the vertically integrated sectors continue to hold, independent of technical change» (Pasinetti 1993, 13-14).

2.3 PRODUCTION AND TIME IN AUSTRIAN ECONOMICS

A representation of the production process as a series of dated quantities of inputs and outputs in discrete time has been also employed in the so-called Austrian and neo-Austrian approach (e.g. Böhm-Bawerk [1899] 1921; Wicksell 1934; von Hayek 1941; Hicks 1970; 1973; 1985).

The main difference between the Classical perspective and the von Neumann-Sraffa perspective, on the one hand, and the (neo-)Austrian representation of the production process, on the other, is that, in the latter the inputs $\boldsymbol{x}$ in Eq. (1) are only dated primary, non-produced, factors of production (i.e. labour and land).

This fully vertically integrated representation of production, where production processes are conceived as one-way avenues from original inputs to consumption goods, is the representation upon which the Austrian theory of capital, founded by Böhm-Bawerk (1921 [1889]) and further elaborated by Wicksell (1934) and Hicks (1970; 1973), is based.[8]

3. MAINSTREAM PRODUCTION ECONOMICS AND TIME

The analytical representation of the production process based on functions and vectors relating input and output mostly overlooks the time-related aspects of production, in spite of the fact that, like every natural process, production takes time and unfolds in historical time. In modern microeconomics textbooks, the most common models tend to disregard a surprising amount of information on the timing of events in production processes. This information is lost in economics «not because it is selectively disregarded after being judged irrelevant to the understanding of an economic issue; temporal information is lost automatically, filtered out by the way economics treat time» (Winston 1982, 3).

[7] TEN RAA 1986b, 812. See also TEN RAA 1997, 218 ff.; TEN RAA and SHESTALOVA 2011, 71 ff.

[8] It is worth stressing that, as shown by HAGEMANN and KURZ 1976, it is not possible to reduce production processes to finite series of dated quantities of non-produced factors of production (i.e. labour) when there are *basic commodities*, i.e. commodities entering directly or indirectly in the production of all the other goods. The conditions for representing production processes as infinite series of non-produced inputs showing some regularities have been analysed, among others, by SCHEFOLD 1971; 1976. For a thorough discussion see KURZ and SALVADORI 1997, Ch. 6.

In what follows, we shall briefly review the different models commonly used in mainstream production economics and then consider the time-related aspects of production as conceptualized in general equilibrium models and models of economic growth.

3.1. PRODUCTION IN MAINSTREAM ECONOMICS

In mainstream economics, production processes in single-output firms is usually represented by means of *production functions,* or *production frontiers* (Coelli *et al.* 2005): real functions relating each vector of input $\boldsymbol{x}$ to a level of output y:

$$y = f(\boldsymbol{x}). \tag{3}$$

where each x_i and y are cardinal measures of 'homogeneous' goods or services, and y is the maximum 'technologically feasible' output that corresponds to the input combination $\boldsymbol{x}$.[9] Inputs and outputs are assumed to be perfectly divisible, and production functions to be twice-continuously differentiable.

The concept of production function is generalized to the case of joint production using *transformation functions:* implicit functions relating vectors of inputs $\boldsymbol{x}$ and outputs $\boldsymbol{y}$, given the 'technological' constraints (e.g. Mas-Colell *et al.* 1995, Ch. 5; Varian 1992, Ch. 4; Coelli *et al.* 2005, Ch. 2):[10]

$$T(\boldsymbol{x}, \boldsymbol{y}) = 0. \tag{4}$$

A multi-input multi-output production process is also represented as a *production vector* (*input-output vector*, *netput vector*, or *production plan*): a vector $\boldsymbol{o} = (o_1, o_2, ..., o_L)$ that describes the net outputs of the L commodities from the production process, with the convention that positive (negative) numbers denote outputs (inputs) (e.g. Mas-Colell *et al.*, 1995, Ch. 5). When the set of inputs is distinct from that of outputs, assuming the elements are ordered in such a way that the first are inputs and the other outputs, the production vector is re-expressed as $(\boldsymbol{x}, \boldsymbol{y})$, where all the elements are non-negative, $\boldsymbol{x}$ is the vector of inputs and $\boldsymbol{y}$ the vector of outputs.

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[9] As usually stated, production functions incorporate the assumption of *technical efficiency* at every possible scale (given the scale of production, each input combination is used in the most efficient way in a 'technical' sense), whereas what is left to be dealt with by economics is the issue of *allocative efficiency*, i.e. the choice of the scale of production and the input mix (and the output mix in case of joint production), given the price schedules of the inputs and the outputs (e.g. COELLI *et al.* 2005, Ch. 1).

[10] Production functions expressed in implicit form, i.e. $f(x) - y = 0$, are a special case of transformation functions for single-output firms.

Within this theoretical framework, the *production technology* is conceived as the set $\boldsymbol{O}$ of all 'technologically feasible' production vectors $\boldsymbol{o}$.[11] The production set $\boldsymbol{O}$ is usually assumed to be convex: any convex linear combination of two feasible production vectors is also a feasible production vector;[12] and this implies: i) non-increasing returns to scale, if inaction is possible ($\mathbf{0}$ is a feasible production plan); ii) convex input sets, hence convex *isoquants* (e.g. Mas-Colell *et al.* 1995, Ch. 5).[13]

When $\boldsymbol{O}$ is a convex cone (it is convex and exhibits constant returns to scale), a ray in the L-dimensional space is thought as an activity that can be run at any scale of operation and the finite number of activities can be run simultaneously with no interference to obtain an infinite number of possible production plans. This gives rise to the so-called linear activity model that underpins the application of linear programming to production theory (e.g. Dorfman *et al.* 1958).[14]

For single-production processes, an alternative representation of the production process ($\boldsymbol{x}$, y_j), producing the quantity y of commodity j, is obtained by dividing the input vector $\boldsymbol{x}$ by the output y_j. This way, the production process is equivalently represented by the input vector $\boldsymbol{a}_j$ (= $\boldsymbol{x}/y_j$). The elements of $\boldsymbol{a}_j$ are known as *input-output coefficients* (*direct input coefficients,* or *technical coefficients*). Under the assumption of constant returns to scale, each a_{ij} is conceived as a generic 'input requirement' of commodity i per unit of output of commodity j (e.g. ten Raa 2006).[15]

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[11] When inputs and outputs are distinct, the production technology $\boldsymbol{O}$ is equivalently defined by: i) the output correspondence ($\mathbf{x}$), associating each input vector $\boldsymbol{x}$ with the set of output vectors $\boldsymbol{y}$ obtainable from it; ii) the input correspondence $L(\boldsymbol{y})$, associating each output vector $\boldsymbol{y}$ with the set of input vectors that can deliver it. In this framework, alternative representations of multi-output multi-input production processes use *distance functions*: a) the *output distance function,* defined on the output set ($\boldsymbol{x}$), is the technically feasible maximal proportional expansion of the output vector $\boldsymbol{y}$ given the input vector $\boldsymbol{x}$; b) the *input distance function*, defined on the input set $L(\boldsymbol{y})$, is the technically feasible maximal proportional contraction of the input vector $\boldsymbol{x}$ given the output vector $\boldsymbol{y}$. Distance functions are equal to unity for production plans belonging to the production frontier (e.g. Färe and Primont 1995; Coelli *et al.* 2005).

[12] Production sets are assumed to be non-empty, closed and satisfying the no free lunch assumption ($\boldsymbol{O}$ does not contain $\boldsymbol{o} \geq \mathbf{0}$ different from $\mathbf{0}$). The production technology $\boldsymbol{O}$ is said to exhibit *non-increasing* (*non-decreasing*) *returns to scale* if any feasible production vector $\boldsymbol{o}$ can be scaled down (up) obtaining another feasible production plan $\alpha\boldsymbol{o}$, with $0 \leq \alpha \leq 1$ ($\alpha \geq 1$); and to exhibit *constant returns to scale* if it exhibits both non-increasing and non-decreasing returns to scale.

[13] For single-output processes, $\boldsymbol{O}$ is convex if the production function is concave. The production technology exhibits constant returns to scale if the production function is linearly homogeneous: $f(\lambda\, \boldsymbol{x}) = \lambda\, y$.

[14] Linear programming is also used in Data Envelopment Analysis (DEA), where firms' production efficiency in terms of output (or input) distance functions is estimated under the assumption of a piece-wise linear convex frontier (see, for instance, Coelli *et al.* 2005 Ch. 6). The (single-production) linear activity model with one primary (non-produced) factor (e.g. labour) frames also the so-called (*non*) *substitution theorem*, which states that, when the production set is a convex cone, the same set of linear combinations of activities (one for each final good) is efficient regardless of the pattern of final demand. In this case, given the profit rate, relative prices are invariant to final demand changes (e.g. Salvadori 2008; ten Raa 2006).

[15] In case of joint production, the input vector $\boldsymbol{x}$ can be normalized using some linear combination of the elements of the output vector $\boldsymbol{y}$, although in this case the interpretation is less straightforward (see, for instance, ten Raa 2006, Ch. 7; Miller and Blair 2009, Ch. 5).

In all the above representations of the production process, inputs and outputs are thought of as *flows*, all measured over the same time period or *instantaneous flow rates* in continuous-time models. In continuous-time models production is assumed to be instantaneous.

For the inputs that are employed, but not 'consumed' in the production process (e.g. labour, land, fixed capital), the flow is usually implicitly conceived as a *constant flow of services* stemming from the employment of the factor in the production process, hence measurable in time units like any other constant flow. For instance, labour time in days per year, 'capital services' in days per year (e.g. Varian 1992, 1-2).

Under the implicit assumption of proportionality between the quantity of the factor employed and the constant rate of flow, these factor inputs are also measured as *stocks* (e.g. number of employees, number of machines, hectares of land).

3.2. PRODUCTION AND TIME IN MAINSTREAM ECONOMICS

In the above representation of production activities there is usually no explicit reference to time. However, mainstream models of production which deal directly with intertemporal issues are inter-temporal in nature, such as the Arrow-Debreu general equilibrium model, that is, the modern version of the general equilibrium theory. Moreover, the time dimension is present in all the models of economic growth that analyse production-related intertemporal issues assuming sequential trading.

3.2.1 ARROW-DEBREU GENERAL EQUILIBRIUM MODEL

In Arrow and Debreu (1954), the homogeneity of goods is evaluated in terms of: intrinsic quality; location; circumstances of delivery; and, what is important for us, delivery time.

Since two goods to be delivered at different dates are treated as different commodities, each production plan $\boldsymbol{o}$ can be also conceived as a description of the temporal pattern of a technologically feasible production process in discrete time. Setting aside the issues related to the location and the circumstances of delivery, with T periods and L commodities each element $o_{l,t}$ of a $1\times((1+T)\ L)$ production vector $\boldsymbol{o} = (o_{1,0},..., o_{L,0}, \ldots, o_{1,T},..., o_{L,T})$ describes the net output of commodity l to be delivered (and therefore used up for net inputs or released for net outputs) in period t. The implicit description of a production process is a set of dated quantities of inputs and outputs in discrete time.

This notwithstanding, in the Arrow-Debreu general equilibrium model, the time dimension is 'flattened' and time disappears from the picture, for in the model, under the assumption of agents' perfect foresight and complete markets – in particular, the existence of complete forward markets –, all commodities are traded and production plans set at the initial date, once and for all.

3.2.2 Neoclassical models of economic growth

The models that deal explicitly with production-related intertemporal issues assuming sequential trading are almost all the models of economic growth (Acemoglu 2009). In these models, production is represented by time-indexed production functions:

$$y(t) = f_t\,(\boldsymbol{x}(t)). \tag{5}$$

where the output y at time t is a function (possibly not constant in time) of the input vector $\boldsymbol{x}$ at t.
In fact, the interpretation of Eq. (5) is different in continuous-time models and discrete-time models. In the former, production is assumed to be instantaneous, the max instantaneous flow rate of output y is assumed to be a function of the instantaneous flow rates of inputs and the stocks of factors employed in the process. On the contrary, in discrete-time models the period of production is assumed to be uniform (e.g. the agricultural year) and Eq. (5) can be interpreted in terms of max total outflows associated with total inflows and factor employment within the unit period $[t, t+1)$. Thus, the period of production is assumed to be of uniform length: input enters, or must be available, at the beginning of the revolving cycle of production, factors are continually employed all along the production period and outputs are obtained and/or delivered at the end of the production period.

4. Time-specific analyses of production

Two economic analyses of production specifically based on a conceptualization of production as a process unfolding in time have been put forward by Georgescu-Roegen (1970; 1971) and Winston (1974a, b, c; 1977; 1982).[16]

Georgescu-Roegen's (1970, 1971) fund-flow model explicitly addresses the issues related to the organization of production processes in time and the need for coordination in time between production elements.[17]

In his analysis of production, Winston combines Georgescu-Roegen's time-specific representation of the production process with the cost implications of time-specific prices and of the

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[16] This section draws upon Vittucci Marzetti and Morroni 2017.

[17] Georgescu-Roegen presented the model at the Conference of the International Economic Association (Rome, 1965). Since then, it has appeared in some subsequent works (Georgescu-Roegen 1970; 1971; 1976 [1969]; 1990). On the

novel concept of duration-specific prices of capital services (Winston 1974a, b, c; 1977; 1982; Winston and McCoy 1974). The cost implications of time-specific prices of labour services were previously analysed by Marris (1964), while Winston's concept of duration-specific prices builds upon the analysis of the owner cost of capital by Jorgenson and Griliches (1967).

4.1 Georgescu-Roegen's fund-flow model

4.1.1 Flows, funds and functional of production

Georgescu-Roegen divides production elements into *flows* and *funds*. Flows are used in only one process as input, or they can come out from a single process as output. An inflow (outflow) always corresponds to a certain quantity of material, substance, or energy, which enters into (exits from) the process in a given instant in time.[18] On the contrary, a fund is never physically incorporated in the product; it enters and leaves the production process, providing its services in several processes over time (e.g., workers, land and capital equipment in the production of tables), and cannot be accumulated or de-cumulated in an instant (Georgescu-Roegen 1976 [1969], 72, 83-86).

Georgescu-Roegen stresses that no confusion can arise between flows and fund services, as fund services are expressed in terms of substance over time, whereas flow rates in terms of substance per time.[19]

He defines the *elementary process*, as «the process by which every unit of the product … is produced» (Georgescu-Roegen 1971, 5). An elementary process starts at time 0, when the process begins with the input of raw materials, and it ends at the instant T, with the release of a unit of the final product. For each individual element of the production process, whether fund or flow, we have a function of time defined over the closed interval $t \in [0, T]$.

In the fund-flow approach, the production process is represented using a functional:

$$O(t) = \Lambda[G_1(t), G_2(t), ..., G_I(t), F_1(t), F_2(t), ..., F_H(t), U_1(t), U_2(t), ..., U_K(t)], \tag{6}$$

where $O(t)$ is a function describing at any time t the cumulative quantity of the output (e.g. the total quantity of tables produced in the process of table production); $G_i(t)$ are time functions indicating, at any t, the cumulative quantity of the i-th outflow (e.g. waste, secondary products, emissions); $F_h(t)$

fund-flow approach see also, among the others, Morroni 1992; 1999; 2014; Scazzieri 1993; 2014; Piacentini 1995; Mir-Artigues and González-Calvet 2007; Vittucci Marzetti 2013.

[18] A flow may result from either the decumulation of a stock or from the transformation made by the production process, although Georgescu-Roegen (1970; 1976 [1969]) stresses that, whereas all stocks accumulate or de-cumulate in a flow, not all flows imply an increase or reduction in a stock (e.g. electricity).

[19] The production elements are defined on the basis of their role in the process and the time profile of their utilization: the same commodity may well be a flow in one process and a fund in another. For instance, a computer is a flow in its process of production, but a fund in the process in which it provides its services.

are time functions indicating the cumulative quantity of the h-th inflow; finally, $U_k(t)$ are functions indicating, at any time t, the degree of use of the k-th fund.

By convention, a positive sign is given to the functions of outflows, $O(t)$ and $G_i(t)$, while a negative sign to the functions of inflows $F_h(t)$ and funds $U_k(t)$ (Georgescu-Roegen 1971, 236). Hence, $O(t)$ and $G_i(\mathrm{t})$ are non-decreasing functions of t, while $F_h(\mathrm{t})$ are non-increasing functions of t. In particular, from the definition of elementary process, it follows that $O(t) = 0$ for $t \in [0, T)$ and $O(T) = 1$.

Each $U_k(t)$ can be remapped on the closed interval [-1, 0], so it can vary from 0 (presence with no use of the k-th fund) and -1 (full utilization of the k-th fund). The function $U_k(t)$ therefore shows the k-th fund idle times when the value is zero (Morroni 1992, 50-60).

Figure 1 provides an illustration of a possible shape of these functions for a generic elementary process with the following elements: i) output flows: product $G_1(t)$, waste $G_2(t)$; ii) input flows: from nature $F_1(t)$, raw material $F_2(t)$, energy $F_3(t)$; iii) funds: worker $U_1(t)$, loom $U_2(t)$; plant area $U_3(t)$.

By using the functions $U_k(t)$, the various time profiles of funds can be compared. For instance, in Figure 1, the time profiles of the three funds are different because of the unequal distribution of the fund times of presence and of utilization. The worker is present only when the process is in operation. By contrast, the loom is present all along the elementary process, even if it is inactive during the pauses when the process is suspended (in Figure 1, the presence time is indicated with a dotted line, whereas the utilization time with a continuous line).

INSERT FIGURE 1 AROUND HERE

4.1.2 TEMPORAL ARRANGEMENTS OF ELEMENTARY PROCESSES AND PRODUCTION SYSTEMS

The conceptualization of the production process in the fund-flow model makes it clear that there is no possibility of substitution between funds and flows. Moreover, since it considers the time profile of fund utilization, it allows the consideration of the issues related to the organization of production processes in time. For instance, the fund-flow model allows the analysis of the differences between the craft production and the factory system (Morroni 1992, 60-67).

In the *craft* (or *artisan*) *production*, the elementary processes are organized *in series*: performed one after another. In the example depicted in Figure 2, function $W_1(t)$ indicates, at any time t, the degree of use of the worker during the elementary process. The functions $U_k(t)$ (k = 1, 2, 3, 4) indicate the degree of use of four funds (e.g. four different tools). In this example, the worker carries out an elementary process performing four different *activities*. Activities consist of different operations, which require the performance of one or more *elementary tasks*, where an elementary task is an operation which, by definition, is not further divisible (for instance, loading or unloading an intermediate product or cutting a piece of fabric). In Figure 2, the first activity requires the execution of three elementary tasks, the second and the third require the execution of two elementary tasks and the fourth the execution of four elementary tasks. This arrangement in series is typical of artisan production. The worker uses four funds: one for each activity. These funds are tools available in the plant, but three of them (out of four) remain constantly idle throughout the process, because the worker performs one activity at a time (Morroni 2014, 8-9).

INSERT FIGURE 2 AROUND HERE

This organization has three basic characteristics: i) high flexibility, i.e. the capacity to adjust the mix of products over time; ii) long idle times for tools; iii) long training times and on-the-job experience for workers, as they perform all the elementary tasks and hence must possess extensive productive knowledge (Morroni 2014, 8).

If the demand for the product increases up to a point where it is convenient to employ four workers (W_1, W_2, W_3, W_4), in craft production each one of them carries out *in parallel* an entire elementary process performing all the four activities according to the arrangement in series. Each worker employs four different tools and performs sequentially all the tasks required to complete each elementary process. The number of workers, of tools and the volume of production all quadruple. Both idle times of tools and the required productive knowledge remain entirely unaffected.

To reduce the idle times and the required productive knowledge, one needs to move toward an arrangement *in line*, the arrangement adopted in the *factory system*: several elementary processes start together one after the other with some predetermined lag (*cycle time*), corresponding to an equal fraction of the duration of the elementary process.

With the arrangement in line, the idleness of funds can be eliminated, or at least reduced. In particular, in order to completely remove all the inefficiencies related to the idleness of funds in an elementary process with duration T, the cycle time δ ($\leq T$) must be equal to the greatest common measure of the durations of the intervals of time of effective utilization of each fund in the elementary process. In this case, the *minimum efficient size* of the process, i.e. the number of elementary processes simultaneously activated, is T/δ. The process stabilizes after $(T - \delta)$ units of time, or $(T/\delta - 1)$ cycles; and the *minimum efficient scale*, i.e. the average amount of output per unit of time in the stabilized production, becomes $1/\delta$ (Tani 1986; Vittucci Marzetti 2013).

In an arrangement in line, it is possible to assign the different activities to different workers. As singled out by Tani (1986), an arrangement in line with *full specialization* of all the funds (i.e., an arrangement where each fund performs a different activity) is always feasible under the following conditions: if the durations of the periods of activity and idleness are all commensurable quantities, i.e. their ratio is a rational number; ii) the size of the process can be increased without bounds. However, an arrangement in line with full specialization of all funds is not the only possible division of labour compatible with the full utilization of the funds (Morroni 1992, 63; Vittucci Marzetti 2013).

Figure 3 illustrates a possible arrangement in line that completely removes idleness and leads to full specialization (in this case δ is equal to 1). In the new arrangement, each worker performs only one activity in a linear sequence. This entails a reduction in the number of elementary tasks performed by each worker with a subsequent decrease in the range of abilities and skills required. Furthermore, moving from the craft production to the factory system has made it possible to save on fixed capital: the set of funds previously needed by one worker to perform one elementary process is now enough to carry out four different elementary processes, because idle times for funds have been eliminated. Of course, the higher the cost of fixed capital, the greater the need to reduce idle times (Morroni 2014, 8-10).

INSERT FIGURE 3 AROUND HERE

4.1.3 Time profiles, production scale and flexibility

The analysis of the line production by means of the fund-flow model makes it apparent that the existence of an efficient scale of production and of possible efficiency reversals over certain ranges of increases in production levels derives not only from the presence of indivisibilities of production elements, or from some scale-dependent nature of the elementary process, but also from the particular distribution of fund utilization times in relation to the production process duration. The pattern of utilization time of single funds depends on the particular technique adopted.[20]

The ways a production unit can eliminate periods of idleness and minimize the under-utilization of the productive capacity of the different fund elements may affect its dimensions. The size of the production unit reflects the need to minimize both idle times and under-utilization of funds, by balancing the productive capacities in the different phases. In the presence of specialized and indivisible funds that characterize industrial processes, a high volume of production is necessary to make compatible their different productive capacities and to overcome bottlenecks. Following Babbage's law of multiples, once a scale is established that eliminates idle time for equipment, any expansion has to occur in discrete jumps of multiples of the scale achieved (Scazzieri 2014).[21]

In changing market conditions, production flexibility plays a pivotal role in affecting competitiveness. The analysis of the degree of flexibility requires a model that explicitly accounts for the time dimension of production processes. In particular, the fund-flow approach shows how the degree of flexibility is linked to short set-up times of machines and large warehouses, and that cutting down set-up times and warehouses' costs is thus a key element for enhancing flexibility (Morroni 1992, 180-186). The greatest flexibility is obtained when the same level of economies of scale can be enjoyed for producing single-unit lots as they can be in producing a large lot of numerous homogeneous products.

4.1.4 Production functions as representations of continuous stabilized line production processes

According to Georgescu-Roegen (1970), the implicit idea of the production process behind a standard production function is that of a *continuous stabilized line production*: the limiting case of a stabilized line production, where all funds are used continuously and all instantaneous flow rates are well defined and constant. In this case, inflows and outflows are proportional to the length of the time period and fund services per time unit are proportional to stocks. In particular, we have:

$$O(t) = o \cdot t \qquad G_i(t) = g_i \cdot t \qquad F_h(t) = f_h \cdot t \qquad S_k(t) = K_k \cdot t \tag{7}$$

[20] Because of the rigidities in the time profile of the activation of the funds involved in the elementary process, the line process has only a limited range of efficient activation scales, even when all its elements are perfectly divisible. Thus, element divisibility is a necessary though not sufficient condition for process divisibility.

[21] Let us note in passing that, if the elementary production process is decomposable in different intermediate stages, the need to coordinate the different productive capacities of the various fund elements does not necessarily involve an increase in the size of the firm (e.g. Williamson 1975). Different intermediate processes may be performed by a single production unit or by several production units, which in turn may belong to the same firm or to different firms.

where o, g_i and f_h are the (constant) instantaneous flow rates, $x_i = \lim_{\delta\to 0} (X_i(t+\delta) - X_i(t))/\delta$; $S_k(t)$ is the cumulative amount of the services of the k-th fund, and K_k is the stock of the k-th fund, assumed to be continuously and fully employed in the process.

In this case, production is instantaneous and the production process is time-invariant. As such, for a constant time interval, inflows and outflows can be alternatively measured in terms of flows or instantaneous flow rates, whereas funds can be expressed in terms of stocks or fund services measured in time units.

However, it is worth pointing out that, since the equivalence holds for a constant time interval, a correct representation of the production process should always indicate the time period. This is for two reasons. Firstly, as argued by Georgescu-Roegen (1970), the tacit presumption that the forms $y = f(\boldsymbol{x})$, where y and $\boldsymbol{x}$ denote instantaneous (out- and in-) flow rates, and $Y = g(\boldsymbol{X})$, where Y and $\boldsymbol{X}$ are outflows and inflows over a certain interval of time t, are equivalent descriptions of the same process, has the paradoxical implication of always constant returns to scale. Indeed:

$$t\,f(\boldsymbol{x}) = t\,y = Y = g(\boldsymbol{X}) = g(t\,\boldsymbol{x}). \tag{8}$$

where, setting $t = 1$, it follows that $f(.) \equiv g(.)$ and such function is linearly homogeneous. Of course, «this does not mean that the factory process operates with constant returns to scale», but that, «if we double the time during which a factory works, then the quantity of every flow element and the service of every fund will also double» (Georgescu-Roegen 1970, 7).

Secondly, although under the assumption of constant returns to scale, the production function is linearly homogeneous, and therefore an alternative description of the same production process is:

$$Y = f(\boldsymbol{X}) = Y\,f(\boldsymbol{a}) \tag{9}$$

where $\boldsymbol{a}$ ($= \boldsymbol{x}/Y$) is the vector of input-output coefficients, when the funds elements are expressed in terms of stocks, the coefficients for the funds, being a stock-flow ratio, depends on the time interval actually chosen: the longer the interval, the lower the coefficient.[22]

[22] This is the reason why, as discussed by TEN RAA 1986b and AULIN-AHMAVAARA 1990, the maximum balanced growth of LEONTIEF's 1970 dynamic input-output model for a given time period is inversely related to the length of the basic interval of time of the model (the limiting value of the balanced growth being that of the continuous version of the model). On dynamic input-output models see, among the others, DUCHIN and SZYLD 1985; LEONTIEF and DUCHIN 1986; and MILLER and BLAIR 2009 Ch.13.

4.1.5 A FUND-FLOW REPRESENTATION OF THE 'CLASSICAL' PRODUCTION PROCESS

Although, according to Georgescu-Roegen (1970), the continuous stabilized line production is the implicit process behind the production function, and this is indeed the only interpretation strictly consistent with the use of a production function in continuous-time models, we would like to point out that the use is also consistent with a more 'Classical' interpretation: given a constant 'natural period' of production of unit length entailing a complete revolving cycle of production (the agriculture year), as we discussed in Section 0, the production function in discrete-time can be interpreted in terms of maximum total outflows associated with total inflows and factor employment within that period: input enters at the beginning of the period, factors are continually employed all along the period and all the outputs are obtained at the end.

It is worth pointing out a possible correspondent fund-flow representation of this 'Classical' production process, i.e.:

$$\begin{array}{lll} F_h(t) = \varphi_h & U_k(t) = \varsigma_k & \text{for } 0 \leq t \leq 1,\, h \in \{1, 2, ..., H\},\, k \in \{1, 2, ..., K\} \\ O(t) = 0 & G_i(t) = 0 & \text{for } 0 \leq t < 1,\, i \in \{1, 2, ..., I\} \\ O(1) = \upsilon & G_i(1) = \gamma_i & i \in \{1, 2, ..., I\} \end{array} \tag{10}$$

where φ_h ($h \in \{1, 2, ..., H\}$), ς_k ($k \in \{1, 2, ..., K\}$), γ_i ($i \in \{1, 2, ..., I\}$) and υ are given constants.

This representation makes it apparent that the naïve time profile of the process excludes *ipso facto* from the analysis all the issues related to the temporal coordination of production elements. In fact, in the production process described by Eq. (10), the only inefficiencies that may arise are related to the underutilization of non-perfectly divisible funds.[23] In this case, indivisibility determines an inefficiency that stems from partial utilization of the fund productive capacity, and there is no inefficiency associated with fund idleness.[24]

The implicit assumption of such representation is that the most efficient organization of the time profile of inputs within the unit production period has already been chosen. Therefore, it is not suitable to analyse the issues of the temporal organization of production within that period.

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[23] Indivisibility of production elements is in fact the only source of increasing returns to scale usually considered by standard microeconomic theories (on this see also MORRONI 1992).

[24] The inclusion of working capital among the fund elements has been criticized by Landesmann and Scazzieri 1996 for the passive role of the semi-processed goods in the process. However, as noted by MIR-ARTIGUES and GONZÁLEZ-CALVET 2007, its exclusion can generate issues of internal consistency in the analytical representation of line production processes. For an interesting attempt to formalize the concept of working capital in an input-output framework, which usually overlooks this element, see TEN RAA *et al.* 1989.

4.2 WINSTON'S TIME-SPECIFIC ANALYSIS OF PRODUCTION

4.2.1 ELEMENTARY UNIT TIME AND INFORMATION LOSS

Winston (1982) singles out three different meanings of time used in economics: i) *analytical time*, describing «the temporal framework of any economic analysis»; ii) *perspective time*, related to the temporal perspective of economic agents (past, present and future); iii) *commodity time*, i.e. time as a scarce commodity to be allocated between competing uses, as in Becker (1976). As far as analytical time is considered, Winston (1982) puts forward the distinction between: i) *calendar unit time* (*T*) of conventional analysis; and ii) *elementary unit time* (*t*), i.e. «the largest time unit that can be used to decompose T and still reveal all the relevant economic relationships» (1982, 24). This distinction between calendar unit of time and elementary unit of time is based upon Hood's (1948) dichotomy between exogenous and endogenous time measures in economics. The elementary unit time is a «functional endogenous interval determined by the particular economic process under study» (Winston 1982, 24) and it is hardly the same for two different economic processes.

The elementary unit time determines the level of 'temporal abstraction' in the analysis of the production process: the larger it is, the larger the abstraction and the information loss, but also the larger the analytical tractability and the smaller the amount of data required to analytically describe the process. At one extreme, too finely grained units generate an «indigestible mass of temporal information about economic events» (1982, 19). This could be the case of Georgescu-Roegen's representation of the production process, where time is continuous and the level of temporal abstraction in the description of the elementary process is practically zero, but the model is very demanding in terms of analytical tractability and data requirement.[25] At the other extreme, we have the «more subtle and more serious problem» that «too-long unit time», not appropriate to the underlying production process, «obliterates the temporality of events» (Winston 1982, 19). This is the case, for instance, of the unwitting use of the agricultural year, «the natural period of agriculture», as the elementary time unit of manufacturing processes.[26]

What is worth stressing is that the analytical representation of a production process via a production function or an input-output vector with respect to a given elementary time unit entails dismissing all the time-related issues of the process within that unit. The implicit assumption is that either the process is a continuous stabilized line production, thus the production process is time-

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[25] For attempts to reduce the analytical complexity while maintaining the basic insights of the fund-flow model see MORRONI 1992; PIACENTINI 1995; 1997.

[26] A related but different mistake is the analytical representation of agricultural processes by means of instantaneous production functions, thus implicitly assuming a continuous stabilized line process typical of manufacturing processes.

invariant (Section 4.1.4), or that, within the elementary time unit, the process has got the naïve time profile discussed in Section 4.1.5.

4.2.2 Time-specific analysis of production and prices: time-invariant, time-specific and duration-specific prices

As we discussed in Section 4.1.2, Georgescu-Roegen's fund-flow model shows how an arrangement in line of elementary processes can be employed and is actually employed in the factory system to achieve full utilization of the fund elements characterized by a particular pattern of utilization time linked to the characteristics of the elementary process that imply some idle times, because of the specific technique adopted. In this framework, full utilization is always the most efficient use of a fund element,[27] and this necessarily follows from the assumption of 'economic invariableness' of funds: by definition, fund elements enter and exit the process without any economic impingement providing productive services. In this respect the model has been harshly criticized by Lager (2000, 2006), Kurz and Salvadori (2003), who have rightly pointed out how such an assumption excludes *ipso facto* from the analysis important issues related to durable means of production, like optimal patterns of utilization over time and economic lifetimes (Vittucci Marzetti 2013).[28]

Interestingly, Georgescu-Roegen does not analyse the role of prices in actual production processes. Winston's (1982) time-specific analysis of production fills this lacuna of the fund-flow model and can be considered as complementary to it.

To start with, let us note that Winston (1982) does not adopt Georgescu-Roegen's distinction between funds and flows. As he puts it, «while retaining the spirit of the analysis, it is not necessary to worry about the distinction between drawing down goods inventories (his 'stocks') and the utilization in production of the services of capital equipment of a labour crew (his 'funds'): *their relevant differences are reflected in time-specific analysis in their price behaviour*» (1982, 48, our emphasis). Therefore, according to Winston, the distinction between funds and flows is not necessary as it naturally emerges from the different schemes normally adopted to price the different production elements, although, as a matter of fact, Winston's (1982) definition of 'stocks of capital

[27] As Georgescu-Roegen put it, «one of the most important aspects of the economics of production is how to minimize these periods of fund idleness, whether we are thinking of man, capital equipment, or land. … The economics of production reduces to two commandments: first, produce by the factory system and, second, let the factory operate around the clock» (1970, 6-8).

[28] For instance, the issue of so-called 'premature truncation', i.e. the possibility of a durable means of production becoming economically obsolete before the end of its technically feasible lifetime (e.g. Kurz and Salvadori 1997; 2003).

and workers' (42), i.e. production factors that provide services in the production process without depreciation caused by utilization, is almost overlapping with that of funds.

Winston distinguishes two kinds of production flows (both defined with respect to a chosen elementary unit time): i) *service flows*, «including capital and labour services and inputs like electric power»; ii) *goods flows*, «including nuts and bolts and cloth and sugar» (1982, 47).[29]

Then, he goes on to analyse the pricing of the different production factors. He identifies three different typologies: i) *time-invariant prices*, prices of goods flows that do not vary over the unit time T; ii) *time-specific prices*, prices of goods and service flows (e.g. electricity and labour services) that vary over the unit time; and, finally, iii) *duration-specific prices*, i.e. prices of the service flows delivered by stocks when the market price is related to the stock (the price is paid for the ownership of the stock or its actual availability over the unit time T), and not to the actual flow of productive services actually delivered.

The notion of duration-specific prices is crucial in Winston's time-specific analysis of production involving fixed capital items. In this respect, Winston (1982, 54-6) distinguishes three different 'prices of capital': i) the purchase price of a unit of capital stock, P_m (e.g. the purchase price of a truck); ii) the price of owning a unit of capital stock for a period of unit time T, P_k: this is what is usually referred to as the 'rental price of capital', or 'owner cost of capital' (e.g. the rental price of a truck for a week); iii) the price of the capital service flows used in production, p_k. This last price is a duration-specific price, as it is inversely related to the rate of effective utilization of the capital stock in production (e.g. the price per kilometre of effective distance covered by the truck in a week, if we assume that the services delivered by the truck are proportional to the distance covered by it and the latter is in turn proportional to the time of effective utilization of the truck).[30]

Assuming that the rate of depreciation of the capital item over the period T, δ_T, is related only to factors that do not depend on the actual use of the item (i.e. technological obsolescence), the price of the service flows actually delivered by the capital unit is equal to:[31]

[29] WINSTON (1982) further distinguishes between *storable flows*, i.e. flows that can be accumulated (*accumulated flows*) or depleted, and *perishable flows*, i.e. flows that cannot be accumulated. While storable flows are 'temporally footloose' within the calendar unit time, perishable flows are time-specific (52).

[30] This entails assuming that the truck load is always the same, no matter the actual time of its utilization, and it always travels at the same speed.

[31] If one assumes that the rate of depreciation of the capital item depends in turn on its level of actual utilization in the process, δ_T becomes itself a function of T_p. One important drawback of this framework is that the rate of depreciation of a capital item is in fact not exogenous, like it is usually considered in the models that assume 'depreciation by evaporation', or 'depreciation by radioactive decay', but depends on the overall structure of the production system and the income distribution (for a thorough discussion within the von Neumann-Sraffa framework see KURZ and SALVADORI 1997, Ch. 9).

$$p_k = P_k/T_p = P_m(r_T + \delta_T)/T_p \tag{11}$$

where T_p ($\leq T$) is the period of actual use of the capital item during T and r_T is the market interest rate, equal to the opportunity cost of tying up resources in Neoclassical economics.

The important point raised by Winston (1982) is that the existence of rental markets for stocks does not do away with the distinction between stocks and stock-delivered service flows; each time what is actually rented in the market is (or is also) the availability of the productive resource over the period, independent from any actual use of the resource in the period. So, for instance, in our truck example, if the price paid to rent a truck for a week is independent from any actual use of the truck in the week or, in any case, the total rental price includes some fixed component independent from any actual use of the truck, then the pricing scheme implies that the leaseholder pays for the periods of idleness of the truck.

It is worth noting that if only the productive services actually delivered by the stocks were rented (e.g. in the truck example, we pay for the truck only for the kilometres actually covered in the week), production costs would not be dependent on stock idle times and there would be no need to temporally coordinate production processes moving from production in series to line production systems to reduce idle times of stocks (funds in Georgescu-Roegen's jargon), because this pricing scheme would transfer idleness costs from the lessee to the lessor. On the contrary, with duration-specific prices, each time there is no element with time-specific prices involved in production, cost minimization always entails minimizing the idle times for the production elements involving such duration-specific prices. The minimization of this idleness always involves some form of temporal arrangement of elementary production processes, when there are rigidities in the time profile of their utilization in the single elementary process, although these issues tend to be completely overlooked in the mainstream representation of the production process (Section 3) and in the von Neumann-Sraffa framework (Section 2.2).

5. Concluding remarks

Although production takes time and unfolds in time, the time-specific aspects of production are not usually plainly discussed in the majority of economic analyses of production. Production models in economics normally tend to disregard almost all the information about the timing of the events in the production process.

In the present paper, we have critically reviewed the alternative analytical descriptions of the production process developed in the different streams of economic theory (from Classical economics to mainstream microeconomics) from the narrow perspective of how they treat and model production in time, and what time-specific aspects of production they do (not) consider. In the final section, we have analysed two time-specific analyses of production, i.e. analysis of production based on a conceptualization of the production process as a process unfolding in micro-time: Georgescu-Roegen's (1970; 1971) fund-flow model and Winston's (1982) optimal utilization model.

Georgescu-Roegen (1970; 1971) identifies the different possible arrangements of processes in time and formally shows how and why line production «deserves to be placed side by side with money as the two most fateful economic innovations for mankind» (Georgescu-Roegen 1970, 8), and initiates the studies on the relation between line production systems, production scale, factor specialization and productive knowledge, giving formal substance to «Adam Smith's famous theorem that the division of labour depends upon the extent of the market» (Young 1928, 529; cf. Scazzieri 1993; Morroni 2014).

Winston's (1982) time-specific analysis of production naturally complements Georgescu-Roegen's model, because it combines the time-specific representation of the production process in the fund-flow model with the cost implications of time- and duration-specific prices of the productive services. Winston's analysis shows when, how and why pricing schemes make time-specific aspects of the production process relevant, making a connection between the actual organization of production in time, the time-specific aspects of pricing and the cost-minimizing techniques, thus originating a time-specific duality theory of production.

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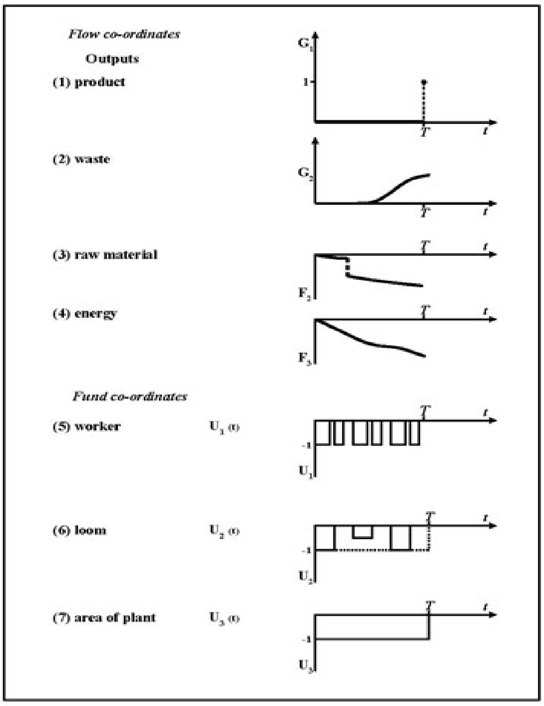
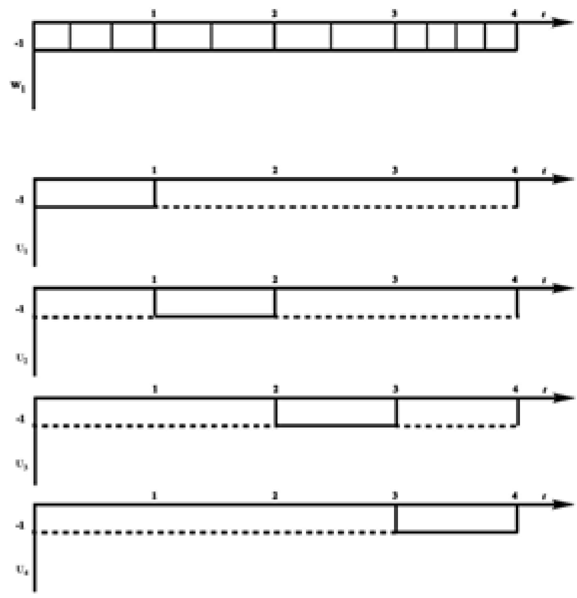
Figure 1. Flow and fund coordinates of a generic elementary process



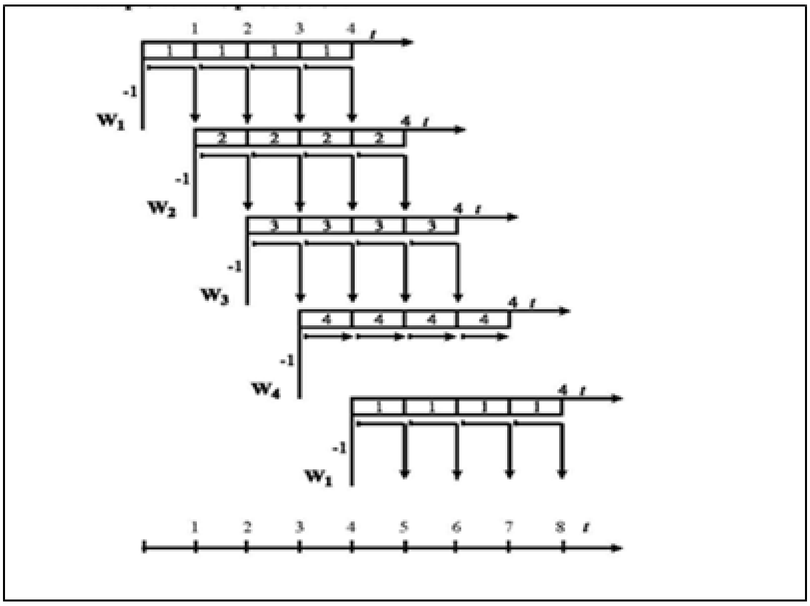
Source: based on Georgescu-Roegen 1970, 89.

Figure 2. Craft production: one worker, four activities and four funds



Source: Morroni 2104, 9.

Figure 2. Production in line: four workers, four activities and four funds



Source: Morroni 2014, 10.