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ESSAYS ON MONETARY POLICY, STOCK MARKET AND HETEROGENEOUS EXPECTATIONS

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*A mia mamma Marcella
il cui amore non scorderò mai*

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Abstract

This dissertation investigates the relationship among heterogeneous expectations, stock prices and monetary policy. In particular, we attempt to answer the question on whether or not central banks should respond to stock prices other than to inflation and output gap.

The first chapter presents a perpetual youth model à la Blanchard (1985) and Yaari (1965) following Nisticò (2012). This type of model generates a financial wealth channel through which stock prices fluctuations affect the dynamics of the aggregate consumption, and thus the equilibrium solution. We model expectations as in Brock and Hommes (1997) and De Grauwe (2011). Agents are boundedly rational, they adopt simple rules to make forecasts and evaluate their past performances using a fitness measure. The model generates endogenous waves of optimism and pessimism due to the correlation among beliefs. Moreover, the presence of this heterogeneity removes the classic trade-off between output gap and inflation typical of Rational Expectations models. We also show that, contrary to the Bernanke and Gertler's (1999) prescription, central banks should respond to stock prices fluctuations. However, to be beneficial, this “leaning against the wind” strategy in the stock market has to be moderate.

In the second chapter, we adopt the same baseline model of the first part. We build on Nisticò (2012) and allow for the inclusion of diverse beliefs following the Rational Beliefs theory by Kurz (1997). With respect to the previous work, beliefs are modeled at a micro-level and enter in the equilibrium solution. Although agents do not observe the true dynamics of the economy, they are still rational in the sense that their beliefs are compatible with the observable empirical distribution of past data. In this framework, stock prices fluctuations affect real economy through two different channels: the financial wealth channel and the expectational channel. We simulate the model under both Rational Expectations and Rational Beliefs. Contrary to Bernanke and Gertler's (1999) prescription, we find that a mild “leaning against the wind” strategy in the stock market is beneficial for both output gap and inflation stabilization. Moreover, all results under Rational Beliefs exhibit a higher volatility and the magnitude of responses to shock is amplified by beliefs dynamics. Widespread optimism boosts inflation as well as output gap and can generate a bubble in stock prices. However, the effect on the real economy of such exuberance might be reduced by a more “aggressive” policy.

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Chapter 1

Monetary Policy and Heterogeneous Expectations: Should the Central Bank Target Asset Prices?

1.1 Introduction

The last financial crisis renewed the attention on the role central banks should have in guaranteeing financial stability. Many economists (e.g. Levine (2012), Allen and Carletti (2010), Taylor (2009)) believe that one of the causes of the recent financial meltdown is the failure of the US Federal Reserve under Alan Greenspan to react to bubbles in the stock and housing markets.

In the economic literature, the debate is still open. The issue has been analyzed from both theoretical and empirical perspective but it remains an unsolved question.

On one side of the debate there is the work of Bernanke and Gertler (1999). They claim that in order to achieve both price and financial stability, an aggressive inflation targeting policy is enough to stabilize the economy and there is no need for asset price targeting. To include a stock price target not only is irrelevant if we use an aggressive monetary policy but it may be destabilizing when an accommodative monetary regime is chosen. They claim that since it is not easy to detect bubbles, targeting stock prices may lead the Central Bank to resist to positive fundamental fluctuations which should be instead accommodated. They use a Dynamic New Keynesian model with financial frictions (Bernanke et al. (1999)). The connection between asset prices and the real economy works mainly through the balance sheet channel (credit channel) where the transmission of the economic shock is exacerbated by the financial acceleration mechanism.

On the other side of the debate, there is the work of Cecchetti et al. (2000, 2002, 2003).

We would like to thank Maurizio Motolese, Domenico Massaro, Mordecai Kurz, Giulia Piccillo and PhD seminar participants at Catholic University of Milan for insightful comments and suggestions.

They maintain that if central banks react to asset price misalignments, monetary policy would be improved. This result is obtained by conducting a series of simulations on the BG model. They use a wide range of policy responses which allows them to obtain a policy prescription different from that of Bernanke et al. (1999). However, reacting to asset price misalignments is not the same as targeting them. They agree with Bernanke and Gertler in saying that the Central Bank should “not react mechanically and in the same way to all changes in stock prices” but they strongly refuse the justification according to which measuring bubbles is too difficult so monetary policy should ignore them. Assenza et al. (2011) try to tackle the problem from another perspective. Instead of augmenting the IS curve as in Bernanke and Gertler (1999), they produce a New Keynesian augmented Philips curve. Their objective is creating a framework such that stock prices and inflation move in opposite directions so to isolate and favor the role of asset price targeting. Despite their efforts, they find that targeting asset prices has a destabilizing effect especially on inflation volatility. Carlstrom and Fuerst (2007) obtain a similar result. They use a standard NK-DSGE model and focus their attention on the effect that a Taylor rule with asset price target has on the determinacy of the model. They find that the equilibrium indeterminacy increases by including the asset price target in the monetary policy rule. In both models, the stock price equation is redundant for the equilibrium allocation, unless monetary policy explicitly targets them ¹.

A way to solve this redundancy is offered by Nisticò (2012). He overcomes the problem by using a perpetual youth model of the type of Blanchard (1985) and Yaari (1965). This structure allows him to have an augmented IS curve (different from that of Bernanke and Gertler). In this framework, stock prices fluctuations impact the real economy via wealth effects on consumption making the stock prices equation non-redundant for the equilibrium system. His results show that the role of stock prices in the Taylor rule may depend on the kind of shock that triggers central banks to react: if the underlying shock is a supply shock, there is no need for a stock price targeting, if, on the other hand, it is a demand shock, the target is required. Moreover, a rule based on stock-prices growth rate instead of one based on stock prices gap has a better stabilizing effect even with respect to the standard Taylor rule.

We build on Nisticò (2012) when we derive our model. However, we depart from the classical rational expectations hypothesis and introduce heterogeneous expectations à la Brock and Hommes (1997).

All previous models share a common characteristic, they all assume expectations to be rational. However, if we drop the homogeneous rational expectations hypothesis, the monetary policy prescription might change. For example, Kurz et al. (2016) shows how, when we assume Rational Beliefs (see Kurz (1994)), the smooth trade-off between inflation and the output gap volatility no longer holds. The diverse beliefs structure interacts with the policy parameter space,

¹Of course, this does not mean that asset prices do not have any impact on the economy. For example, in Assenza et al. (2011), their effect works indirectly through the cost channel.

affecting monetary policy decisions. Hence, when we depart from the assumption of the single representative agent holding rational expectations, some considerations are needed. The inclusion of bounded rational agents adds an additional source of uncertainty in the model. This uncertainty influences the economy response to monetary policy requiring a new stabilization strategy, in particular when we include the asset price targeting.

In the past thirty years, many attempts have been made to try to reconcile the economic theory with the financial market empirical evidences. The Rational Expectations doctrine fails to reproduce results consistent with the data. Hence, alternative solutions have been investigated. Kurz and Motolese (2001) , build an OLG financial market model where agents hold Rational Beliefs. As we are intended to do, they introduce a new source of uncertainty (Endogenous Uncertainty) which arises from the diversity of beliefs. In this way, they can explain some regularities of the financial market, so far left unsolved ². On the role of beliefs heterogeneity as important factor for predictions of asset returns and volatility, there is the empirical work of Anderson et al. (2005). They empirically implement a dynamic general equilibrium model with heterogeneous beliefs and show the importance of the dispersion in earnings forecasts as pricing factor. The relationship between heterogeneous expectations and financial markets is also studied by Brock and Hommes (1997, 1998). They use an *Adaptive Believe System* where agents may choose among a finite set of beliefs and select the predictor by using a ‘fitness’ measure. Their agents are boundedly rational, that is they have only partial information about the distribution of the economy. However, they are still rational in the sense that they choose the predictor which returns the best performance. Results show that heterogeneous expectations may lead to market instability and may generate complicated dynamics for asset prices and returns.

We follow Brock and Hommes (1997) and in particular De Grauwe (2011) for the introduction of heterogeneous expectations in Nisticò’s model. To describe agents’ expectations, he uses a DSGE model with two simple heuristics. Agents are boundedly rational which means that they do not know the true distribution of the economic variables they forecast but they are rational in the sense that they are willing to learn from their mistakes and change their positions accordingly. To this aim, he introduces a selection mechanism which allows individuals to switch between rules according to their past performances. An interesting result is related to that of Kurz et al. (2016): the smooth monotonic trade-off between inflation and output gap volatility no longer holds. Therefore, a flexible inflation targeting policy better stabilizes economic fluctuations than one which responds only to inflation. In the work “Lectures on Behavioral Macroeconomics”, De Grauwe extends the previous analysis by introducing stocks in the economy. The non-monotonic relation between output gap and inflation volatility still holds. In addition, he shows that central banks can improve their stabilization policy by reacting to stock prices fluctuations.

As already mentioned, our paper is similar to those of De Grauwe (2008, 2011, 2012) as

²We refer to the financial puzzles as the equity premium puzzle, the risk-free rate puzzle and the excess volatility puzzle.

concerns the expectations framework. However, we depart from his work for the type of baseline model we use. In his papers, De Grauwe (2008, 2011, 2012) adopts a very simple and ad hoc model where he simply adds ex-post stock prices to both the IS equation and the Philips curve. To avoid this ex-post procedure, we use a perpetual youth model which allows us to derive a non redundant asset prices equation and IS curve augmented with stock prices. Moreover, it allows stock prices to depend not only on forecasts of inflation and output gap but also on their expected future value.

Attempting to answer the question on whether or not central banks should target asset prices, this paper contributes to the literature on the interaction between heterogeneous expectations and monetary policy. Belonging to this literature are the papers by Branch and McGough (2009), Massaro (2013) and by Branch and Evans (2011). Branch and McGough (2009) adopt a New Keynesian model in which they incorporate heterogeneous expectations. In order to do that they impose a set of assumptions on the expectations operator which allows them to obtain a reduced form model similar to the standard IS and AS relations. As noted by Massaro (2013), their agents “choose optimal plans that satisfy the associated Euler equations instead of looking at the intertemporal budget constraint.” He claims that it is exactly this behavioral choice which compel them to have these restrictive assumptions on beliefs. In Massaro’s (2013) model, agents’ decision rules as well as aggregate equations depend on long horizon forecast. This type of setting does not require any beliefs assumptions. Our model is similar to Massaro’s (2013); we do not have the IS curve from the Euler equation only but we also take into account the aggregate budget constraint. However, with respect to his work, our agents make forecasts only one period ahead. Because of the structure of the perpetual youth model, we do not need all of the Branch and McGough’s (2009) assumptions. For sure we cannot apply the assumption which imposes all agents to have common expectations on expected differences in limiting wealth. The only assumption we need to make the aggregation possible is that the distribution of beliefs is independent of the cohorts. Although quite restrictive, the assumptions introduced by Branch and McGough (2009) aim at obtaining a consistent microfounded model which includes also expectations. We do not seek the same consistency, our purpose is to provide a microfoundation for the relationship between stock market and real economy leaving for future research that of beliefs. Both Branch and McGough (2009) and Massaro (2013) focus their analysis on the equilibrium determinacy issue. In Branch and McGough’s (2009) paper, results show that heterogeneity may or may not be destabilizing for the economy depending on how the adaptive predictor weighs past data. Similarly, Massaro (2013) finds that in a world with heterogeneous beliefs, the Taylor principle no longer guarantees a unique equilibrium.

Branch and Evans (2011) use a different approach. They build a New Keynesian model with heterogeneous expectations adopting the Misspecification Equilibrium framework from Branch and Evans (2006). Agents are assumed to learn as econometricians do and then select their forecasting predictor. They show that the introduction of heterogeneous expectations produces

regime-switching volatility and either multiple misspecification equilibria or equilibria exhibiting Intrinsic Heterogeneity.

Our work is related to a recent paper by Milani (2017). He modifies Nisticò (2012) model allowing for labor rigidity and assuming agents to hold subjective, near rational, expectations. He shows that stock prices effect on economic fluctuations works mainly through the expectations channel and not through the wealth channel.³

This paper contributes to two strands of literature: on one side, it relates to the debate on the relationship between monetary policy and stock prices, on the other side, it is linked to the literature on monetary policy and heterogeneous expectations. The model we used is a perpetual youth model à la Blanchard (1985) and Yaari (1965) with heterogeneous expectations à la Brock and Hommes (1997). Our main results summarize as follows. As in De Grauwe (2011), the expectations mechanism generates endogenous cycles characterized by waves of optimism and pessimism. With the introduction of diverse beliefs, the output gap-inflation trade-off typical of Rational Expectations models disappears. Finally, contrary to the Bernanke and Gertler's (1999) prescription, we find that a mild "leaning against the wind" policy with respect to asset prices improves both output gap and inflation stabilization.

The remainder of the paper proceeds as follows. Section 2 presents the baseline model. In Section 3, we analyze the types of expectations used in the model. Section 4 is devoted to the results. Section 5 summarizes and concludes.

1.2 The Model

The model is a perpetual youth model of the type of Nisticò's (2010), Airaudo et al. (2015) and Castelnovo and Nisticò (2010). The interesting feature of these models is that the stock prices equation is no longer redundant for the equilibrium allocation which means that asset prices enter the IS equation. Our contribution to this strand of literature is the introduction of heterogeneous expectations à la Brock and Hommes (1997)⁴.

1.2.1 Demand Side

The model is a perpetual youth model à la Blanchard (1985) and Yaari (1965). The economy is characterized by an infinite number of cohorts each of which may or may not actively participate in the financial market. The probability of exiting the market is constant and equal to γ ⁵. Only the agents who are in the market for more than one period have access to the financial resources.

³The posterior mean estimation of the wealth channel is quite low. This result contrasts with that of Castelnovo and Nisticò (2010). However, this is not surprising, since the two papers model agents' expectations differently.

⁴See also De Grauwe (2012).

⁵There is no population growth within the cohort and each cohort has a constant size equal to γ . Moreover, the total population is normalized to 1.

The difference between newcomers and old agents (old traders) lies on their wealth.

We focus our attention on the old-representative consumer of the j -period cohort ⁶. The maximization problem is the following:

$$E_t \sum_{k=0}^{\infty} \beta^k (1 - \gamma)^k [\ln C_{j,t+k} + \delta \ln (1 - N_{j,t+k})] \quad (1.1)$$

where $\beta \in [0, 1], \gamma \in [0, 1], \delta > 0$. The utility is log-separable utility between consumption $C_{j,t+k}$ and leisure $(1 - N_{j,t+k})$. Each household maximizes his expected lifetime utility which is discounted by the rate of impatience β and the uncertainty of staying in financial market, $1 - \gamma$. Since we are dealing with old traders, the budget constraint includes also the financial assets:

$$P_t C_{j,t} + E_t \mathcal{F}_{t,t+1} B_{j,t+1}^* + P_t Q_t Z_{j,t+1} \leq W_t N_{j,t} + \Omega_{j,t}^* \quad (1.2)$$

where $B_{j,t+1}^* = P_t B_{j,t}$ and $\Omega_{j,t}^*$ is the nominal financial wealth carried over from the previous period and it is defined as follows:

$$\Omega_{j,t}^* \equiv \frac{1}{1 - \gamma} [B_{j,t}^* + P_t (Q_t + D_t) Z_{j,t}] \quad (1.3)$$

where $\gamma \in [0, 1]$. The financial wealth is composed by two types of assets: the state-contingent bonds and the equity shares issued by monopolistic firms. We assume the equity shares $Z_{j,t}$ to be a stock price market index, as for example S&P 500, whose real price is Q_t . As concerns bonds, they are discounted by a factor $\mathcal{F}_{t,t+1}$ and return one unit of currency in period $t + 1$. The following no-arbitrage condition holds:

$$(1 + r_t) E_t \mathcal{F}_{t,t+1} = 1 \quad (1.4)$$

Notice that the nominal financial wealth is multiplied by a factor of $\frac{1}{1 - \gamma}$. This is due to the fact that in each period, a fraction of traders exits the financial market and their wealth is redistributed among the remaining agents ⁷.

Solving the maximization problem, the budget constraint holds with the equality and the F.O.Cs. with respect to bond, equity and labor are respectively:

$$E_t \mathcal{F}_{t,t+1} = \beta E_t \frac{P_t C_{j,t}}{P_{t+1} C_{j,t+1}} \quad (1.5)$$

$$P_t Q_t = E_t \mathcal{F}_{t,t+1} P_{t+1} (Q_{t+1} + D_{t+1}) \quad (1.6)$$

⁶Notice that index j identifies the j -cohort representative agent, who is associated to the period he enters the economy. Hence, j represents the representative agent of a certain cohort which has entered the market in period $j < t$ (old traders). For the newcomers we have that $j = t$.

⁷Since the number of cohorts sum up to 1, the financial wealth of the cohort that goes out of the market is divided by the remaining market participants, i.e. $1 - \gamma$. Newcomers are of course excluded from this redistribution.

$$\delta C_{j,t} = \frac{W_t}{P_t} (1 - N_{j,t}) \quad (1.7)$$

The optimal plan satisfies also the transversality condition:

$$\lim_{k \rightarrow +\infty} E_t \left\{ \mathcal{F}_{t,t+k} (1 - \gamma)^k \Omega_{j,t+k}^* \right\} = 0 \quad (1.8)$$

The key element of the model is the difference between newcomers' and old traders' wealth. This difference creates a wedge between the stochastic discount factor, $\mathcal{F}_{t,t+1}$ and the average marginal rate of substitution in consumption, i.e. eq.(1.5) holds within cohort but not in aggregate. The old traders have a consumption equal to⁸:

$$C_{j,t} = \frac{1}{(1 + \delta) \Sigma_t} [\Omega_{j,t} + h_t] \quad (1.9)$$

where $\Omega \equiv \Omega^*/P$ and $\Sigma_t = \sum_{k=0}^{\infty} \beta^k (1 - \gamma)^k$. h_t^* represents the human wealth and is equal to $h_t^* = E_t \sum_{k=0}^{\infty} (1 - \gamma)^k \mathcal{F}_{t,t+k} W_{t+k}$ ⁹. We removed the subscript j since h is independent of age, i.e. $h_{j,y}^* = h_t^*$ ¹⁰. Consumption of the newcomers derives only from their salary, so we have :

$$C_{j,t} = \frac{1}{(1 + \delta) \Sigma_t} h_t \quad (1.10)$$

1.2.1.1 Aggregate Economy

In our model we do not have the government sector, hence the aggregate resource constraints are:

$$Y_t = C_t \quad (1.11)$$

$$P_t Y_t = W_t N_t + P_t D_t \quad (1.12)$$

As concerns the financial market, the bond market clearing condition is $B_t = 0$ while the equity market clears at $Z_t = 1$.

Aggregation across cohorts is computed as the weighted average $X_t = \sum_{j=-\infty}^t \gamma (1 - \gamma)^{t-j} X_{j,t}$. The weight is the size of the cohort, γ , and it is multiplied by the time each cohort has been trading in the market. Cohorts which are in the market for a long time have weights lower than new ones.

From the maximization problem, the main aggregate equations are:

$$P_t C_t + \mathcal{F}_{t,t+1} B_{t+1}^* + P_t Q_t Z_{t+1} = W_t N_t + \Omega_t^* \quad (1.13)$$

⁸For details on the derivation look at the Appendix A.

⁹ h_t represents the human wealth in real terms.

¹⁰This is because human wealth depends on wage which is the same in every period for all the population.

$$\delta C_t = \frac{W_t}{P_t} (1 - N_t) \quad (1.14)$$

$$C_t = \frac{1}{(1 + \delta) \Sigma_t} (\Omega_t + h_t) \quad (1.15)$$

Recall that it is not possible to aggregate over Euler equations because of the different level of financial wealth. Hence, the aggregate consumption equation is obtained by combining together eq.(1.13) and eq.(1.15)¹¹.

$$(\Sigma_t - 1) C_t = (1 - \gamma) \Sigma_{t+1} E_t \mathcal{F}_{t,t+1} \Pi_{t+1} C_{t+1} + \gamma \frac{1}{(1 + \delta)} E_t \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} \quad (1.16)$$

From the definition of financial wealth, market clearing condition and the stock market price equation, we can rewrite the previous equation as¹²:

$$(\Sigma_t - 1) C_t = (1 - \gamma) \Sigma_{t+1} E_t \mathcal{F}_{t,t+1} \Pi_{t+1} C_{t+1} + \gamma \frac{1}{(1 + \delta)} Q_t \quad (1.17)$$

1.2.1.2 Linearization

As we have already mentioned, the peculiarity of this model is the presence, in each period, of two different groups of agents: the old traders and the newcomers. They are identical expect for their access to the financial market. Those cohorts for which $j < t$ consume a higher amount of resources since they benefit from an extra component of wealth. This diverse level of consumption causes the discount factor of newcomers to be different from the discount factor of old traders. Hence, on aggregate, the classical equality between subjective discount factor, β , and the inverse of the gross interest rate, no longer holds. In this economy, the equality modifies as follows¹³:

$$\tilde{\beta} \equiv \frac{\beta}{(1 + \psi)} = \frac{1}{(1 + r)} \quad (1.18)$$

where $\psi = \gamma \frac{1 - \beta(1 - \gamma)}{(1 + \delta)(1 - \gamma)} \frac{\Omega}{C}$.

From the demand-side of our economy the linearized equations are the following:

$$q_t = E_t \pi_{t+1} - r_t + \tilde{\beta} E_t q_{t+1} + (1 - \tilde{\beta}) E_t d_{t+1} + \eta_t \quad (1.19)$$

$$c_t = w_t - p_t - \varphi n_t \quad (1.20)$$

¹¹Appendix B provides a detailed description of the computations.

¹²We use $E_t \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} = Q_t$ to modify eq. (1.16).

¹³We obtained the equality from the steady state of eq. (1.16)

where $\varphi = \frac{N}{1-N}$.

$$d_t = \frac{Y}{D} y_t - \frac{WN}{PD} (w_t - p_t + n_t) \quad (1.21)$$

$$c_t = \frac{1}{(1+\psi)} [E_t \pi_{t+1} - i_t + E_t c_{t+1}] + \frac{\psi}{(1+\psi)} q_t \quad (1.22)$$

We add to eq.(1.19) a shock, η_t . This shock can be considered capturing all the fluctuations in the asset prices which are not related to fundamentals, as for instance, exogenous bubbles and fads ¹⁴.

1.2.2 Supply Side

The supply side of the economy is composed by two sectors: the retail sector and the wholesale sector. As in the classical New Keynesian DSGE model, the retail sector sells final goods in a perfect competitive market, while the wholesale sector produces intermediate goods by mean of labor in a monopolistic market.

1.2.2.1 Retail Sector

In the retail sector, firms produce final goods Y_t through a CRT technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{(\varepsilon-1)}{\varepsilon}} \right]^{\frac{\varepsilon}{(\varepsilon-1)}} \quad (1.23)$$

where $\varepsilon > 1$ is the intertemporal elasticity of substitution which reflects the degree of competition in the market of inputs. In equilibrium the input demand function and the aggregate price index are respectively:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t \quad (1.24)$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \right]^{\frac{1}{(1-\varepsilon)}} \quad (1.25)$$

1.2.2.2 Wholesale Sector

In the wholesale sector, firms produce differentiate goods through the following production function:

$$Y_t(i) = A_t N_t(i) \quad (1.26)$$

where $A_t = Ae^{a_t}$ is the total factor productivity with $a_t = \rho_a a_{t-1} + \epsilon_a$, $\epsilon_a \sim WN$.

Prices are set according to the Calvo (1983) assumption. Only a fraction of firms $(1 - \theta)$ are

¹⁴Milani (2017) finds that when expectations are not rational, asset price shocks affect significantly output through the expectational channel. Between the 1960s and 1970s, stock price shocks accounted on average for almost 40% for the output volatility. The percentage decreases to 20% between the 1990s and 2000s.

allowed to change their price in each period. The dynamic problem faced by an optimizing firm is:

$$\max_{P_t^*(i)} \sum_{k=0}^{\infty} \theta^k E_t \mathcal{F}_{t,t+k} \left\{ P_t^*(i) \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} - TC_{t+k}(Y_{t+k}(i)) \right\} \quad (1.27)$$

where $TC_{t+k}(Y_{t+k}(i)) = \frac{W_t}{P_t} N_t = \frac{W_t}{P_t} \frac{Y_t}{A_t}$ is the total cost. From the F.O.C., setting $(1 + \mu) = \frac{\varepsilon}{\varepsilon - 1}$, and log-linearize, we have :

$$p_t^* = (1 - \theta \tilde{\beta}) \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k (mc_{t+k} + p_{t+k}) \quad (1.28)$$

Because we have adopted the Calvo (1983) assumption, the linearized aggregate price index will be:

$$\pi_t = (1 - \theta) (p_t^* - p_{t-1}) \quad (1.29)$$

From the total cost definition we derive the marginal cost which is equal to $MC_t = \frac{W_t}{P_t} \frac{1}{A_t}$. Linearized the marginal cost and the aggregate resource constraint $Y_t = C_t$, we get:

$$mc_t = (1 + \varphi) (y_t - a_t) \quad (1.30)$$

If we depart from the Calvo (1983) assumption, we can compute the potential level of output which is the output we obtain when prices are flexible, i.e. $y_t^n = a_t$. Combining together eqs.(1.28)-(1.30) and the definition of the natural output we obtain the equation for the Philips curve ¹⁵

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + k x_t \quad (1.31)$$

where $k = \lambda(1 + \varphi)$.

1.2.3 Complete Model

The purpose of this paper is to answer the question on whether or not the Central Bank should respond to asset prices fluctuations. In this respect, we adopt a perpetual youth model à la Blanchard (1985) and Yaari (1965) so to have in the complete model a non-redundant stock price equation. This is made possible by the turnover in the financial market guaranteed by a $\gamma \neq 0$. If $\gamma = 0$, the model is a standard New Keynesian model characterized by a complete dichotomy between real economy and financial market. For this reason, in order to have stock prices in the final system, De Grauwe (2008, 2012) simply adjusts the model ex-post by adding stock prices to both the IS and the Philips curve. We wanted to depart from this choice and provide a micro-model useful to justify the link between real economy and financial market.

¹⁵Details on the derivations are in the Appendix C.

In order to derive the IS curve, we first substitute the aggregate resource constraint $y_t = c_t$ into the aggregate consumption equation. Then, we write the natural version of the equation and subtract it from the output equation we have previously obtained. Hence, the IS curve has the following form:

$$x_t = \frac{1}{(1+\psi)} E_t x_{t+1} - \frac{1}{(1+\psi)} [i_t - E_t \pi_{t+1} - r_t^n] + \frac{\psi}{(1+\psi)} s_t \quad (1.32)$$

We built the stock price equation in a similar way. First of all, we substitute the dividend equation into the asset price equation¹⁶. Then, we proceed as in the previous case. In this way we obtain:

$$s_t = \tilde{\beta} E_t s_{t+1} - [i_t - E_t \pi_{t+1} - r_t^n] + \xi E_t x_{t+1} + \eta_t \quad (1.33)$$

where $s_t = q_t - q_t^n$ is the stock price gap. The stock price gap captures the effect that structural distortions have on the financial conditions of the economy.

The main equations of the model are:

$$x_t = \frac{1}{(1+\psi)} E_t x_{t+1} - \frac{1}{(1+\psi)} [i_t - E_t \pi_{t+1} - r_t^n] + \frac{\psi}{(1+\psi)} s_t \quad (1.34)$$

$$s_t = \tilde{\beta} E_t s_{t+1} - [i_t - E_t \pi_{t+1} - r_t^n] + \xi E_t x_{t+1} + \eta_t \quad (1.35)$$

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + k x_t + \nu_t \quad (1.36)$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \phi_s s_t + u_t \quad (1.37)$$

where $\xi = (1 - \tilde{\beta}) \frac{\mu - (1+\varphi)}{\mu}$ and all shocks are assumed to follow an AR(1) process.

Notice that $r_t^n = E_t \Delta a_{t+1} + \frac{\psi}{1 + \psi - \tilde{\beta} \rho_\eta} \eta_t$ and $q_t^n = \frac{1}{1 + \psi - \tilde{\beta} \rho_\eta} \eta_t + a_t$.

1.3 The Structure of Heterogeneous Expectations

In this model, we use heterogeneous expectations à la Brock and Hommes (1997) similar to De Grauwe (2011). Agents are boundedly rational, that is they do not observe the true distributions of the economy. In order to make forecasts, they use simple rules which lead to biased predictions. To rule out the possibility that every prediction is possible, we make agents choose between two heuristics. Since agents still rational, they cannot simply randomly forecast the future. To this aim, we use an adaptive learning mechanism which allows individuals to choose between heuristics according to a ‘fitness’ criterion. Individuals evaluate their past predictions and choose whether to keep using a certain heuristic or switching to the other one.

We assume that the distribution of expectations is independent of γ . The newcomers have

¹⁶The stock price equation becomes : $q_t = E_t \pi_{t+1} - i_t + \tilde{\beta} E_t q_{t+1} + \xi E_t x_{t+1} + (1 - \tilde{\beta}) E_t y_{t+1}^n$.

the same expectations as those who exit the market.

1.3.1 Technology Shock Expectations

Unlike De Grauwe (2011), we include forecasting rules also for the technology shock. We assume that when agents make forecasts, they can be either optimistic or pessimistic. That is, they systematically bias their expectations upwards or downwards with respect to the rational ones. The two heuristics are:

$$E_t^{opt} a_{t+1} = \rho_a a_{t-1} + m_t \quad (1.38)$$

$$E_t^{pes} a_{t+1} = \rho_a a_{t-1} - m_t \quad (1.39)$$

where $m_t = d_{mt}/2$. The term d_{mt} represents the divergence of beliefs among agents about the technology shock and it is defined as follows:

$$d_{mt} = \zeta_{m1} + \zeta_{m2} \sigma(a_t) \quad (1.40)$$

where $\zeta_{m1} \geq 0$, $\zeta_{m2} \geq 0$ and $\sigma(a_t)$ is the unconditional standard deviation of the technology shock¹⁷. The greater is the volatility of technology shock the higher is the uncertainty, and accordingly the divergence among agents' beliefs.

The market forecast is the weighted average of the two expectation rules where the weights are endogenously determined as follows:

$$E_t a_{t+1} = \alpha_{opta,t} E_t^{opt} a_{t+1} + \alpha_{pesa,t} E_t^{pes} a_{t+1} \quad (1.41)$$

where $\alpha_{opta,t} + \alpha_{pesa,t} = 1$. Although individual expectations are biased, the market forecast will turn out to be unbiased on average.

As we already mentioned, agents do not choose randomly their forecasting rules but they base their decisions on the evaluation of past performances. In order to model this behavior we apply the discrete choice theory (see Anderson et al. (1992); Brock and Hommes (1997); Manski and McFadden (1981)). The evaluation is determined computing the mean squared forecasting error of each heuristic:

$$U_{opta,t} = - \sum_{k=1}^{\infty} \omega_k \left[a_{t-k} - E_{t,t-k}^{opt} a_{t-k} \right]^2 \quad (1.42)$$

$$U_{pesa,t} = - \sum_{k=1}^{\infty} \omega_k \left[a_{t-k} - E_{t,t-k}^{pes} a_{t-k} \right]^2 \quad (1.43)$$

where ω_k are geometrically declining weights; recent performances have a higher weight than past performances. Applying the discrete choice theory we obtain the fraction of optimistic and

¹⁷We assume the standard deviation to be computed on the previous 50 periods.

pessimistic agents ¹⁸:

$$\alpha_{opta,t} = \frac{e^{\chi U_{opta,t}}}{e^{\chi U_{opta,t}} + e^{\chi U_{pesa,t}}} \quad (1.44)$$

$$\alpha_{pesx,t} = \frac{e^{\chi U_{pesx,t}}}{e^{\chi U_{optx,t}} + e^{\chi U_{pesx,t}}} \quad (1.45)$$

where χ is the intensity of choice. If $\chi = 0$, it means that agents choose among forecasting rules randomly, that is past performances no longer matter. On the other hand, if $\chi = \infty$, the probability of choosing a certain heuristic is deterministic. The parameter χ can also be considered as a measure of the willingness to learn from the past. The higher is χ , the more agents are apt to learn from past performances. We assume χ to be common to all expectations.

In summary, the expectations mechanism works as follows. Each individual makes a forecast on the technology shock according to a certain rule. After observing the realizations of the shock, he evaluates his choice and decides whether to switch or not to the other heuristic. In this way, the fraction of optimistic and pessimistic is endogenously determined and changes over time.

1.3.2 Output Gap Expectations

The forecasting rules for the output gap are similar to those for the technology shock. Agents can be either pessimistic or optimistic, hence they systematically bias their forecast upwards or downwards.

$$E_t^{opt} x_{t+1} = g_t \quad (1.46)$$

$$E_t^{pes} x_{t+1} = -g_t \quad (1.47)$$

where $g_t = \frac{d_{gt}}{2}$. d_{gt} is the divergence of beliefs among agents about the output gap and is defined as follows:

$$d_{gt} = \zeta_{g1} + \zeta_{g2}\sigma(x_t) \quad (1.48)$$

The rest follows as in the previous case.

1.3.3 Asset Prices Expectations

Also in this case, agents are divided between optimistic and pessimistic. The expectations are:

$$E_t^{opt} q_{t+1} = f_t \quad (1.49)$$

$$E_t^{pes} q_{t+1} = -f_t \quad (1.50)$$

¹⁸Eq.(1.44) and eq.(1.45) are the limiting probabilities of choosing a certain predictor in a stochastic discrete choice model for predictors selection.

where $f_t = \frac{d_{ft}}{2}$. d_{ft} is the divergence of beliefs among agents about the stock prices and is defined as follows:

$$d_{ft} = \zeta_{f1} + \zeta_{f2}\sigma(q_t) \quad (1.51)$$

The market forecast and the fraction of pessimistic and optimistic agents are obtained as as described above.

1.3.4 Inflation Expectations

Inflation forecasting rules are modeled in a slightly different way with respect the other cases. We follow De Grawe (2011)¹⁹ and assume agents to be either targeters or extrapolators. The targeters' forecast is based on the Central Bank's target which we assume to be equal to 0. The extrapolators use a heuristic based on past information. The two expectation rules are:

$$E_t^{targ} \pi_{t+1} = \pi^* = 0 \quad (1.52)$$

$$E_t^{ext} \pi_{t+1} = \pi_{t-1} \quad (1.53)$$

The market forecast is again a weighted average of the two expectations where the weights are obtained using the discrete choice theory.

1.4 Monetary Policy and Heterogeneous Expectations

1.4.1 Monetary Policy and Stock Prices

Should central banks target asset prices? According to Bernanke and Gertler (1999) the answer is no. In this section, we present the simulation results of the model. First, we give a summary of the parameter choice; we are not aiming to a full calibration of the economy, since we have no data for expectations but we try to stay as close as possible to real data.

We use a quarterly parametrization as in the classical convention. The subjective discount factor β is set to be equal to 0.99 which is consistent with a steady state quarterly interest rate of 0.1. For the intertemporal elasticity of substitution ε , we set it at a value of 6 which implies a mark-up of 0.2%. The key parameter γ is fixed at 0.13. We take this value from Castelnovo and Nisticò (2010).²⁰ We choose δ , the leisure parameter, to be equal to 1 so to have the total hours of work in steady state equal to almost half of the time endowment. The probability that firms are able to change their price in the current period is set to 2/3 which implies an average

¹⁹See also Brazier et al. (2008).

²⁰They estimate a NK-DSGE model of the type of Nisticò (2010) by using Bayesian techniques.

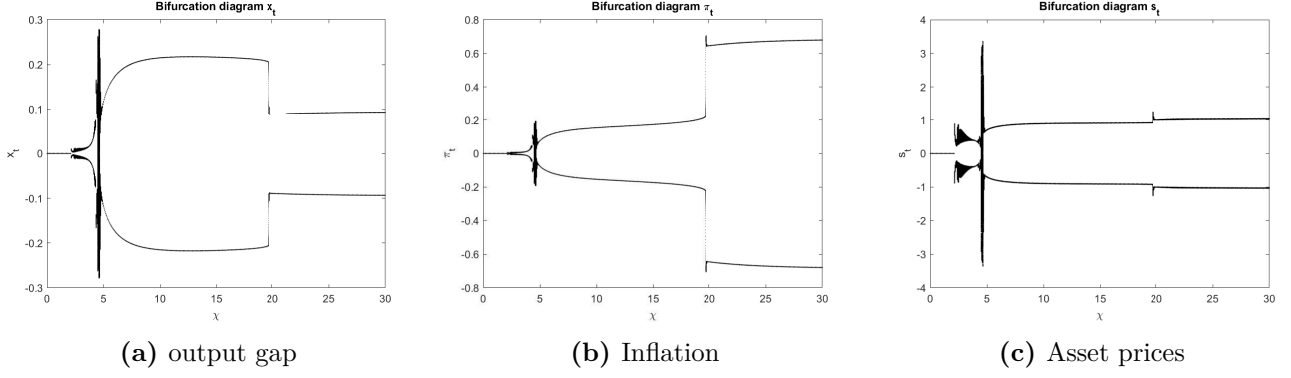


Figure 1.1: Bifurcation diagrams.

duration of three quarters, as reported by the empirical evidences ²¹.

As we have mentioned before, χ is the intensity of choice parameter and it tells us how much past performances matter for the expectations decisions. We use bifurcation theory to determine the value of χ in order to avoid situations of multiple equilibria and chaos. Figure 1.1 shows the bifurcation diagrams for output gap, inflation and asset prices. Following these results, we set χ equal to 1²² which guarantees a unique stable equilibrium.

As concerns the forecasting performances, we define the parameter ω which can be interpreted as the agents' memory as follows: $\omega = (1 - \rho)\rho^{k-1}$ with $0 \leq \rho \leq 1$. When $\rho = 0$, agents have no memory, while for $\rho = 1$ they have an infinite memory. De Grauwe (2011) conducts a sensitivity analysis for this memory parameter. He finds that for $\rho = 0$, there is a high correlation between animal spirits and output gap which slowly declines towards zero for $\rho = 1$. We conduct the same analysis on our model. The results turn out to be consistent with those of De Grauwe's, hence we choose an average value of 0.5.

As concerns shocks, we calibrated them as follows. The cost push shock is the same as Smets and Wouters (2007), $\rho_\nu = 0.9$ and $\sigma_\nu = 0.0045$. To set the monetary policy shock, we refer to the Federal Reserve policy choices, i.e. $\rho_u = 0.8$ and $\sigma_u = 0.002$ (see also Rudebusch, 2002). As for the technology shock we depart from the classical RBC model calibration introduced by King and Rebelo (1999) and follow Kurz et al. (2016) where $\rho_a = 0.9$ and $\sigma_a = 0.0045$. The lower standard deviation corresponds to the exclusion of the diverse beliefs volatility from the Solow's residuals. Finally, for the stock prices shock we set $\rho_\eta = 0.9$ and $\sigma_\eta = 0.01$.

In 1993, Taylor describes the monetary policy through an interest rate rule of the following type : $i : t = 0.04 + 1.5(\pi_t - 0.2) + 0.5(y_t - \bar{y}_t)$. We follow Taylor and adopt the same policy parameters for output gap and inflation. As concerns the stock prices coefficient, we set it equal to 0.025.

Figure 1.2 shows the time pattern of output gap, asset prices and the fraction of agents who use the optimistic rule. As in De Grauwe (2011, 2012), the model generates endogenous cycles due

²¹See estimates in Galí et al. (2001) and Sbordone (2002).

²²Although low, $\chi = 1$ is sufficient to guarantee a high correlation between animal spirits and business cycle.

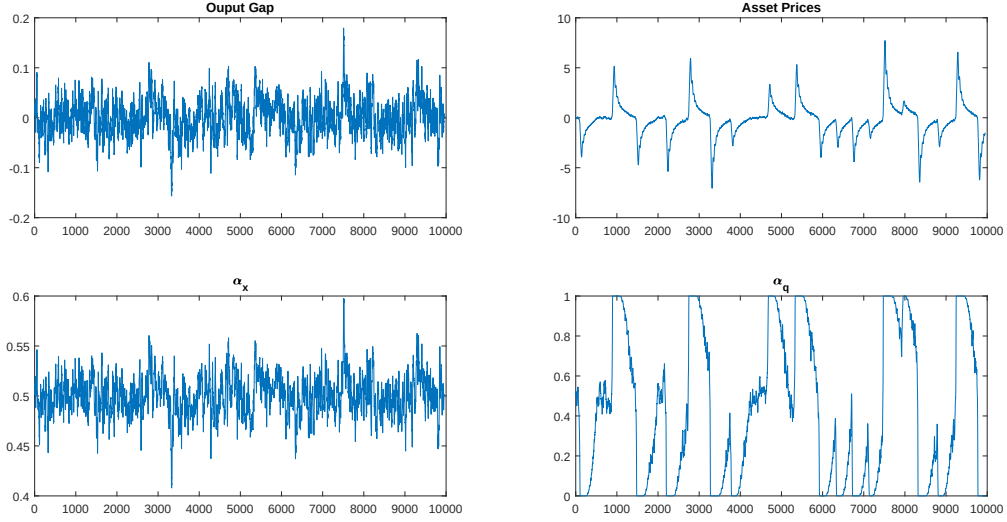


Figure 1.2: Output gap, asset prices and animal spirits with asset prices targeting.

to the partially self-fulfilling structure of the economy. In particular, these endogenous waves are created by the switching mechanism between heuristics. A positive shock makes the optimistic rule delivers a higher payoff so that more agents choose this rule, switching from the pessimistic one. The increase in the fraction of agents who uses an optimistic heuristic stimulates aggregate demand which will strengthen the position of the optimistic individuals. This is particularly evident for asset prices where animal spirits oscillations are very large.

As already mentioned, this work attempts to answer the question on whether or not central

Taylor		
Policy	Output gap	Inflation
$\phi_s = 0.025$	0.036	0.024
$\phi_s = 0$	0.054	0.035

Table 1.1: Output gap and asset prices volatility under the Taylor rule.

banks should target asset prices. To this aim we build a series of tables to study the volatility of inflation and output gap under different monetary policy rules.

Table 1.1 shows the results obtain under the classical Taylor rule. We observe that a monetary policy with an asset prices target better stabilizes business cycle and inflation than one without it. To check the robustness of this result and to better compare our findings with those of Bernanke and Gertler's, we consider different combinations of coefficients and simulate the model again (Tables 1.2 and 1.3). Among these combinations, we include also those used by Bernanke and Gertler (1999) and by Cecchetti et al. (2000).

First thing to notice is that the lowest level of volatility is reached when asset prices are

targeted, and in particular when their coefficient is equal to 0.04. This result strengthens the previous one and contradicts Benanke and Gertler's prescription. However, if we take into account

Volatility of output gap						
ϕ_x	$\phi_\pi \backslash \phi_s$	0	0.1	$\phi_s = 0.025$	$\phi_s = 0.04$	$\phi_s = 0.075$
0	1.1	0.376	0.518	0.375	0.239	0.339
	1.25	0.142	0.444	0.081	0.060	0.162
	1.75	0.070	0.140	0.048	0.044	0.072
	2	0.065	0.106	0.044	0.040	0.060
0.5	1.1	0.074	0.112	0.030	0.027	0.046
	1.25	0.059	0.094	0.038	0.035	0.060
	1.75	0.050	0.072	0.033	0.030	0.046
	2	0.042	0.057	0.031	0.029	0.045
1	1.1	0.043	0.069	0.030	0.027	0.046
	1.25	0.047	0.058	0.029	0.025	0.043
	1.75	0.036	0.051	0.026	0.024	0.036
	2	0.035	0.047	0.024	0.023	0.034

Table 1.2: Output gap volatility under different policy rules.

only either their combinations (green) or those of Cecchetti (blue) we get the opposite outcome. In both cases the best policy appears to be an aggressive one with a flexible inflation target which confirms Bernanke and Gertler's (1999) conclusions. It must be noted that their monetary rules present a value for the asset prices coefficient which is quite high. Setting ϕ_s equal to 0.1 means that if the stock market goes up 10 basis point, the Central Bank responds with a 1% increase of the interest rate. This is a quite strong assumption in a quarterly model.

In their paper, Bernanke and Gerlter (1999) claim that a strict aggressive policy on inflation is sufficient to stabilize the macro-economy. If we look at tables 1.2 and 1.3, we observe that, an aggressive policy with no asset prices target is always outperformed by a policy which includes stock prices and is less aggressive on inflation. Hence, a strict inflation targeting policy cannot be used as a substitute for a more accommodative policy with asset prices target. The same holds true for the output gap.

Figure 1.3 captures the relationship between stock prices and output gap and inflation volatility. It reveals a U-shaped curve where the minimum level of volatility occurs at $\phi_{p^s} = 0.04$. If central

²³We try different seeds and the results are robust for these changes.

²⁴Results are sensitive to the choice of the expectations' parameters, in particular to the choice of those concerning asset prices. A too large discrepancy among stock prices beliefs leads the economy to an explosive path. However, if this discrepancy is not too big, targeting asset prices is beneficial under different values of the expectations' parameters. Since there are no evidences for the parametrization of expectations, we choose values of ζ so to have reasonable range of volatility.

Volatility of Inflation						
ϕ_x	$\phi_\pi \backslash \phi_s$	0	0.1	$\phi_s = 0.025$	$\phi_s = 0.04$	$\phi_s = 0.075$
0	1.1	2.078	3.162	2.291	1.023	1.910
	1.25	0.095	1.650	0.052	0.037	0.111
	1.75	0.043	0.093	0.028	0.025	0.045
	2	0.039	0.067	0.025	0.022	0.037
0.5	1.1	0.048	0.074	0.029	0.027	0.045
	1.25	0.039	0.062	0.026	0.024	0.039
	1.75	0.032	0.046	0.021	0.020	0.031
	2	0.026	0.037	0.019	0.018	0.030
1	1.1	0.031	0.048	0.023	0.022	0.032
	1.25	0.031	0.039	0.022	0.020	0.031
	1.75	0.025	0.034	0.018	0.018	0.025
	2	0.023	0.031	0.017	0.016	0.023

Table 1.3: Inflation volatility under different policy rules.

banks react to stock prices, they will improve stabilization of both business cycle and inflation.²³. However, to be beneficial, the “leaning against the wind” strategy in the stock market has to be moderate.²⁴

Notice that when we set $\gamma = 0$ ²⁵, we no longer have a U-shape form. In this case all agents have access to the financial market and the stock price equation becomes again redundant for the equilibrium solution. The overall volatility is an increasing function of the asset prices coefficient.

Also when we depart from the heterogeneous expectations hypothesis and impose rational expectations, results are different. Due to the presence of the trade-off between output gap and inflation volatility²⁶, answering the question on whether or not central banks should target asset prices is not straightforward. Unlike the behavioral model, there is not a unique value of ϕ_s for which both inflation and output gap volatility are at their minimum level²⁷.

1.4.2 Trade-off between Output Gap and Inflation Volatility

In this section, we investigate the presence of the trade-off between output gap and inflation volatility taking into account both heterogeneous and rational expectations.

Figure 1.4²⁸ shows the pattern of output gap and inflation volatility for different values of

²⁵Figure D7 in Appendix D.

²⁶We will talk about the trade-off in the next section.

²⁷Figure D8 in the Appendix

²⁸The value of the asset prices and of the inflation coefficients are respectively 0.025 and 1.5. All the other figures share the same values.

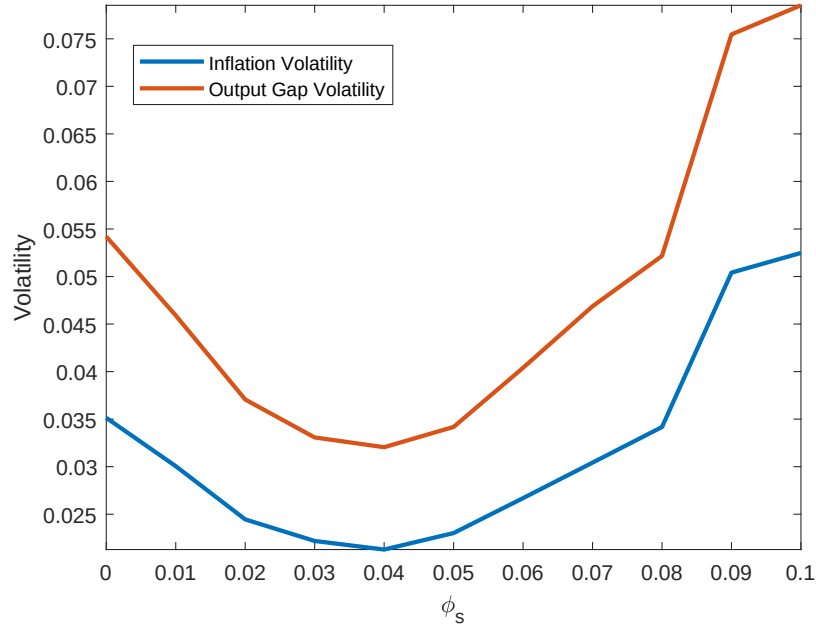


Figure 1.3: Inflation and output gap volatility for different level of ϕ_s using Taylor coefficients.

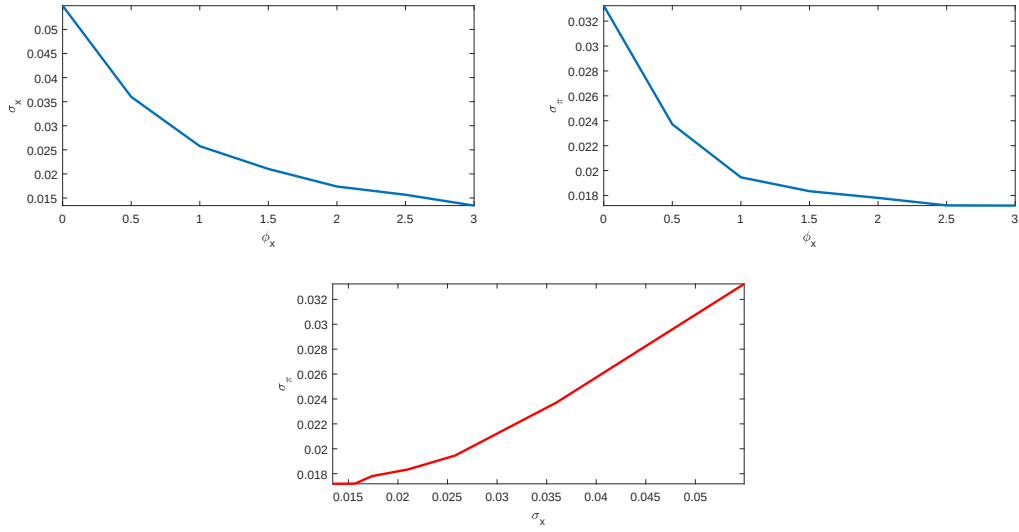


Figure 1.4: Output gap and inflation volatility in the Behavioral model.

the output coefficient in the behavioral model.²⁹ One observes that as the output gap coefficient increases from 0 to 3, both volatilities decrease. When we include heterogeneous expectations in the model the classical output gap-inflation trade-off disappears. On the other hand, under rational expectations we obtain the standard result. The increase in the output gap parameter generates an opposite response in the two volatilities (trade-off). Stabilization of the output gap

²⁹In the appendix, it is possible to find the same graphs (D1-D2) for values of the inflation coefficient that goes from 1.1 to 3. We start from 1.1 because lower values not always guarantee determinacy.

comes at the price of higher inflation volatility: there is no Divine Coincidence when rational expectations are assumed. Hence, as suggested by Galí (2008) and Woodford (2003), the optimal policy for a central bank which is concerned only with price stability is not react to output gap fluctuations at all. When we introduce heterogeneous expectations the policy implications change: even if the Central Bank only cares about price stability, targeting output gap is beneficial for the achievement of the Central Bank's goal.

As illustrated by Hommes et al. (2015) the reason for these different policy implications has to be ascribed to the nature of expectations. In the rational expectation model, beliefs are set to be equal to either their target or their steady state level. In the behavioral model, expectations depend on past and current values of variables. It is precisely through this dependence that targeting the output gap becomes beneficial.

To illustrate this, they provide the following example. Let us assume that a combination of shock increases the output gap over its steady state level but leaves inflation at the target. No further shocks are assumed for the next periods. The rational expectations model suggests that the Central Bank targets only inflation. Since expectations are not affected by the past, targeting output gap will be only destabilizing. On the other hand, in the behavioral model, expectations do depend on the past. Hence, an increase in the output gap makes agents to revise their expectations upwards producing an upward pressure on prices. In this case, targeting output gap is beneficial to the stabilization of the economy since it offsets the upwards pressure on inflation.

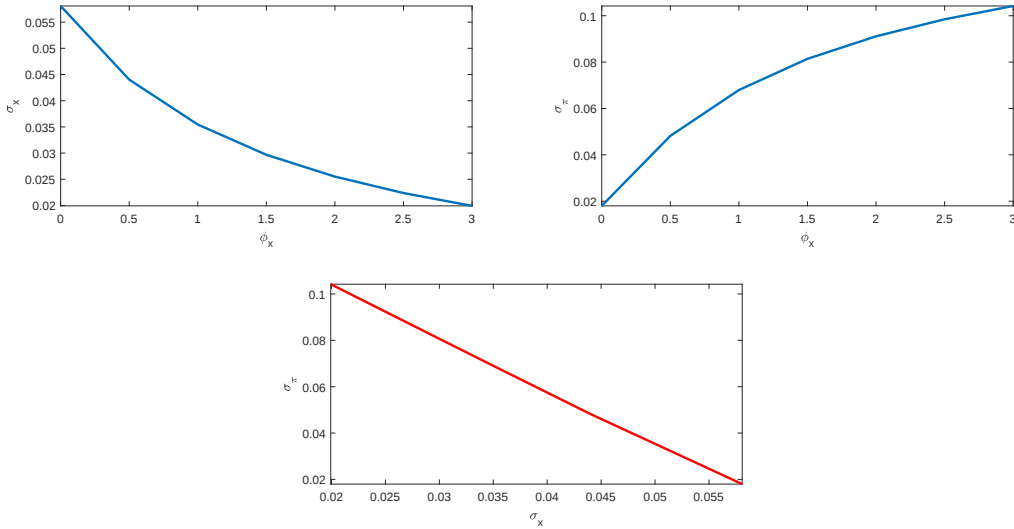


Figure 1.5: Output gap and inflation volatility in the Rational Expectations model.

As already mentioned, in our behavioral model, the trade-off between output gap and inflation volatility completely disappears. This result is slightly different from that of other heterogeneous expectations works as De Grauwe (2011), Kurz et al. (2016) and Hommes et al. (2015). In these models, the relationship between inflation and output gap volatility presents a U-shaped form. A

reason for this difference with respect to De Grauwe (2011, 2012) and Hommes et al.(2015) is to ascribe to the way we calibrate the shocks. Our shocks have much smaller standard deviations. If we use standard deviations of 0.05 for all shocks, we obtain their same result.

The above trade-off graphs are built taking into account a monetary policy in which stock prices are targeted. We rebuild the same graphs ³⁰ ruling out the target. In the behavioral model, the relations between output gap and inflation volatility does not change but we observe a diverse magnitude. The overall volatility is higher under the no targeting policy. This result is in line with the analysis in the previous section. In the rational expectation model, the two cases return

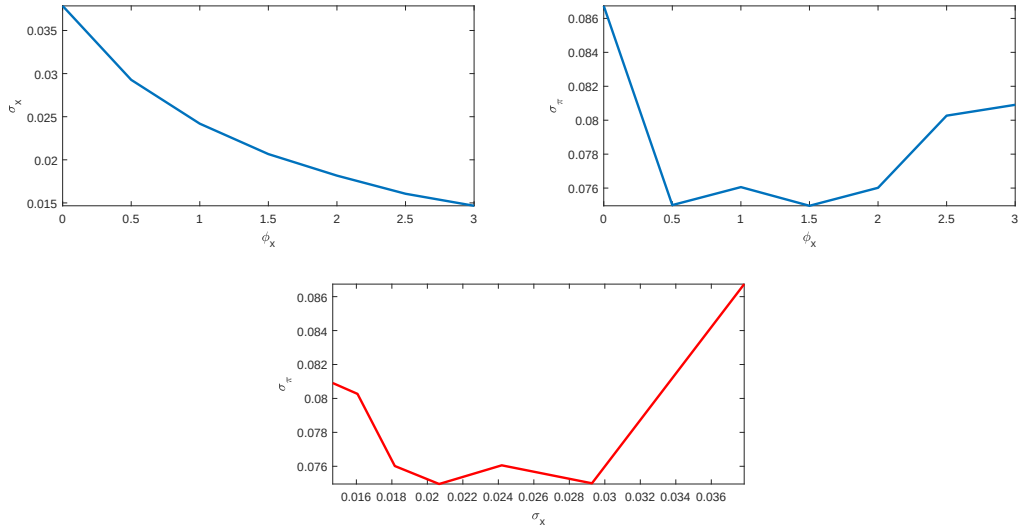


Figure 1.6: Output gap and inflation volatility: zero credibility.

the same result.

The previous results on the output gap-inflation trade-off are obtained for a value of the inflation coefficient equal to 1.5. We change this value to check if we get the same outcomes³¹. In the behavioral model, the curves have the same shape but are shifted down as ϕ_π increases. As regards the rational expectations model, as ϕ_π rises, the curves of the inflation volatility decrease while the curves of the output gap volatility increase, as expected by the trade-off. The trade-off does not change for different values of ϕ_π .

Results are also robust for different values of the intensity of choice parameter.

De Grauwe (2011, 2012) claims that the inflation target credibility is an important source of macroeconomic stability. So far, we have conducted the analysis in an environment in which agents maintain their skepticism about the credibility of the inflation-targeting regime. Following De Grauwe (2011, 2012), we conduct the analysis by taking into account two extreme cases: 100% credibility and no credibility at all. When all agents are extrapolators (figure 1.6) the relationship

³⁰Figures D3-D4 in the appendix.

³¹Figures D5-D6 in the appendix.

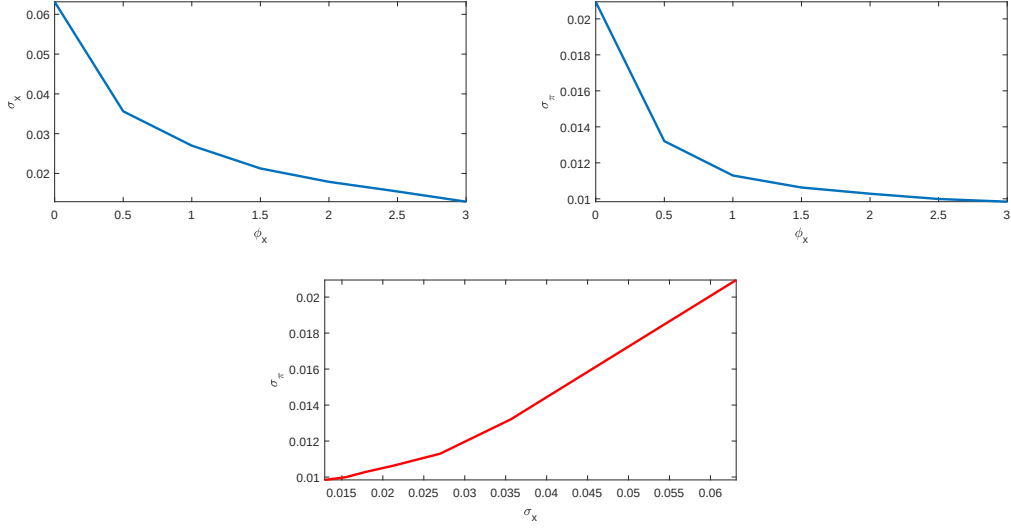


Figure 1.7: Output gap and inflation volatility: perfect credibility.

between output gap and inflation volatility is no longer linear. For low level of the output gap coefficient, both volatilities decrease. When the policy becomes too aggressive on output gap, the volatility of inflation starts increasing and we obtain the classical trade-off. A mild target of output gap increases the inflation credibility reducing the waves of optimism and pessimism thereby stabilizing inflation and output gap. A too aggressive policy has the opposite effect. In the case of 100% credibility, the relationship between output gap and inflation volatility is again monotonically decreasing. Moreover, if we look at the magnitude, with homogeneous expectations equal to the target, the volatility of both output gap and inflation is lower than in all the other cases. Inflation credibility is an important ingredient of economic stability.

Figure 1.8 shows the relationship between output gap and inflation volatility for different values of the asset prices coefficient. In the behavioral model, the curves bear out what we found in the previous section: the point of lower volatility is reached for $\phi_s = 0.04$. In the rational expectation model, “leaning against the wind ” strategies for asset prices have no visible impact.

1.5 Conclusions

In the standard New Keynesian model, a Taylor rule with a stock price target destabilizes the economy and increases the scope for equilibrium indeterminacy. This result is shown by Carlstrom and Fuerst (2007) and Assenza et al. (2011). Responding to stock prices produces, in Carlstrom and Fuerst (2007), a reduction of the equilibrium determinacy space while in Assenza et al. (2011), it has destabilizing effects especially on inflation volatility. The problem with these models is that they do not have any linkage between stock prices and real economy and thus, there is no reason why central banks should respond to asset prices fluctuations.

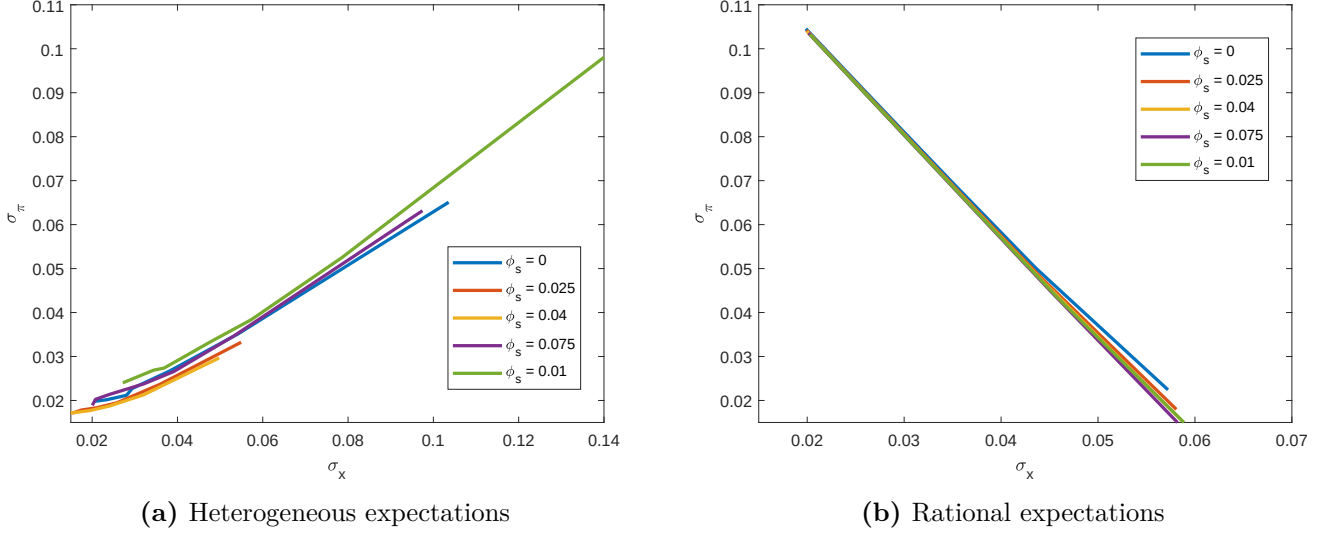


Figure 1.8: Inflation-output gap volatility trade-off for different values of ϕ_s .

The aim of this paper is to investigate the relationship between monetary policy and financial market. We have attempted to answer the question on whether or not central banks should respond to stock prices other than to inflation and output. Many studies and empirical evidences (Kahneman and Tversky (1973); Frankel and Froot (1990); Kurz and Motolese (2011)) report that not only agents' forecasting behavior does not coincide with that proposed by the Rational Expectations theory but also that there is a heterogeneity among individual beliefs. In light of the empirical evidences, we conducted our analysis in a heterogeneous expectations' framework.

In order to overcome the problem of stock prices redundancy of New Keynesian models we adopted a perpetual youth model à la Blanchard (1985) and Yaari (1965) following Nisticò (2012). The presence of two groups of agents with different wealth creates a wedge between the current and expected level of consumption which depends not only on the real interest rate but also on the market value of the financial resources. This structure generates a financial wealth channel through which stock prices fluctuations affect the dynamics of the aggregate consumption, and thus the equilibrium solution. Expectations are built as in Brock and Hommes (1997).

Our main results are summarized as follows. The model generates endogenous cycles due to the partially self-fulfilling structure of the economy. These endogenous waves are created by the switching mechanism between heuristics as in De Grauwe (2011,2012). The introduction of heterogeneity modifies the output gap-inflation trade-off typical of the Rational Expectations models. With diverse beliefs this trade-off disappears: including the output gap in the rule has a positive impact not only on business cycle but also on inflation volatility. Moreover, we find that the choice of inflation targeting regime plays an important role in the stabilization effect of monetary policy. When the inflation targeting is credible, output gap and inflation fluctuation are greatly reduced. As concerns the conduct of monetary policy, our results contradict the "benign neglect" approach of Bernanke and Gertler. The lowest level of the overall volatility is reached

for a positive value of the asset prices coefficient. This proves that an inflation targeting policy benefits from a response to stock prices. However, to be beneficial, this “leaning against the wind” strategy in the stock market must be moderate.

References

- Airaud, M., Nisticò, S., and Zanna, L.-f. (2015). Learning , Monetary Policy and Asset Prices. *Journal of Money, Credit and Banking*, 47(7):1273–1307.
- Allen, F. and Carletti, E. (2010). An Overview of the Crisis: Causes, Consequences, and Solutions. *International Review of Finance*, 10:1–27.
- Anderson, E. W., Ghysels, E., and Juergens, J. L. (2005). Do Heterogeneous Beliefs Matter for Asset Pricing ? *Review of Financial Studies*, 18(3):875–924.
- Anderson, S., De Palma, A., and Thisse, J.-F. (1992). *Discrete Choice Theory of Product Differentiation*. MIT Press, Cambridge.
- Assenza, T., Berardi, M., and Delli Gatti, D. (2011). Was Bernanke Right ? Targeting Asset Prices may not be a Good Idea after all. *Working Paper*.
- Bernanke, B. S. and Gertler, M. (1999). Monetary Policy and Asset Price Volatility. *Economic Review (Fed of Kansas City)*, 84(Fourth Quarter):17–51.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, M. and Woodford, J., editors, *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Amsterdam: North Holland.
- Blanchard, O. J. (1985). Debt, Deficits, and Finite Horizons. *Journal of Political Economy*, 93(2):223–247.
- Branch, W. A. and Evans, G. W. (2006). Intrinsic heterogeneity in expectation formation. *Economic Theory*, 127:264–295.
- Branch, W. A. and Evans, G. W. (2011). Monetary policy and heterogeneous expectations. *Economic Theory*, 47(2-3):365–393.
- Branch, W. A. and McGough, B. (2009). A New Keynesian Model with Heterogeneous Expectations. *Journal of Economic Dynamics & Control*, 33:1036–1051.
- Brazier, A., Harrison, R., King, M., and Yates, T. (2008). The Danger of Inflating Expecta-

tions of Macroeconomic Stability: Heuristic Switching in an Overlapping-Generations Monetary Model. *International Journal of Central Banking*, 4(2):219–254.

Brock, W. a. and Hommes, C. H. (1997). A Rational Route to Randomness. *Econometrica*, 65(5):1059–1095.

Brock, W. A. and Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics & Control*, 22:1235–1274.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12:383–398.

Carlstrom, C. T. and Fuerst, T. S. (2007). Asset prices, nominal rigidities, and monetary policy. *Review of Economic Dynamics*, 10(2):256–275.

Castelnuovo, E. and Nisticò, S. (2010). Stock market conditions and monetary policy in a DSGE model for the U.S. *Journal of Economic Dynamics & Control*, 34:1700–1731.

Cecchetti, S. G. (2003). What the FOMC Says and Does When the Stock Market Booms. *Mimeo*.

Cecchetti, S. G., Genberg, H., Lipsky, J., and Wadhwani, S. (2000). Asset Prices and Central Bank Policy. Technical report, CEPR and ICMB, Geneva.

Cecchetti, S. G., Genberg, H., and Wadhwani, S. (2002). Asset Prices in a Flexible Inflation Targeting Framework. In Hunter, W.C., Kaufman, G.G., Pomerleano, M., editor, *Asset Price Bubbles: Implications for Monetary, Regulatory, and International Policies*. MIT Press, Cambridge, MA.

De Grauwe, P. (2008). Stock Prices and Monetary Policy. *Ceps*, (304).

De Grauwe, P. (2011). Animal spirits and monetary policy. *Economic Theory*, 47:423–457.

De Grauwe, P. (2012). *Lectures on behavioral macroeconomics*. Princeton University Press.

Frankel, J. and Froot, K. (1990). Chartists, Fundamentalists and the Demand for Dollars. In Courakis, A. and Taylor, M., editors, *Private Behaviour and Government Policies in Interdependent Economies*, pages 73–126. Oxford University Press.

Galí, J. (2008). *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press., Princeton.

Galí, J., Gertler, M., and Lopez-Salido, J. D. (2001). European inflation dynamics. *European Economic Review*, 45(7):1121–1150.

- Hommes, C. H., Massaro, D., and Weber, M. (2015). Monetary Policy Under Behavioral Expectations: Theory and Experiment.
- Kahneman, D. and Tversky, A. (1973). On the Psychology of Prediction. *Psychological Review*, 80(4).
- King, R. G. and Rebelo, S. T. (1999). Resuscitating Real Business Cycles. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics*, pages 927–1007. North Holland,.
- Kurz, M. (1994). On rational belief equilibria. *Economic Theory*, 4(6):859–876.
- Kurz, M. and Motolese, M. (2001). Endogenous uncertainty and market volatility. *Economic Theory*, 544(17):497–544.
- Kurz, M. and Motolese, M. (2011). Diverse Beliefs and Time Variability of Risk Premia. *Economic Theory*, 47:293–335.
- Kurz, M., Motolese, M., Piccillo, G., and Wu, H. (2016). Monetary Policy with Diverse Private Expectations. *CESifo Working Paper Series*.
- Levine, R. (2012). The Governance of Financial Regulation: Reform Lessons from the Recent Crisis. *International Review of Finance*, 12(1):39–56.
- Manski, C. F. and McFadden, D. L. (1981). *Structural Analysis of Discrete Data and Econometric Applications*. Cambridge: The MIT Press.
- Massaro, D. (2013). Heterogeneous expectations in monetary DSGE models. *Journal of Economic Dynamics and Control*, 37:680–692.
- Milani, F. (2017). Learning about the interdependence between the macroeconomy and the stock market. *International Review of Economics and Finance*, 49:223–242.
- Nisticò, S. (2012). Monetary policy and stock-price dynamics in a DSGE framework. *Journal of Macroeconomics*, 34:126–146.
- Rudebusch, G. (2002). Term structure evidence on interest rate smoothing and monetary policy inertia. *Journal of Monetary Economics*, 49:1161–1187.
- Sbordone, A. M. (2002). Prices and unit labor costs: A new test of price stickiness. *Journal of Monetary Economics*, 49:265–292.
- Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: a bayesian DSGE approach. *American Economic Review*, 97(3):586– 606.
- Taylor, J. B. (2009). The Financial Crisis and the Policy Responses. *NBER working paper*.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.

Yaari, M. (1965). Uncertain life, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32.

Appendix A Individual Consumption

We can rewrite the budget constraint using eq.(1.6) and eq.(1.7) as:

$$(1 + \delta) P_t C_{j,t} + (1 - \gamma) E_t \mathcal{F}_{t,t+1} \Omega_{j,t+1}^* = W_t + \Omega_{j,t}^* \quad (\text{A-1})$$

Solving forward eq.(A-2) and using eq.(1.5) we get :

$$P_t C_{j,t} = \frac{1}{(1 + \delta) \Sigma_t} [\Omega_{j,t}^* - h_t^*] \quad (\text{A-2})$$

Rewriting eq.(A-2) in real terms we obtain eq. (1.10)

Appendix B Aggregate Consumption

To obtain the aggregate consumption equation, first write eq.(A-2) in aggregate terms:

$$(1 + \delta) C_t + E_t \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} = \frac{W_t}{P_t} + \Omega_t \quad (\text{B-1})$$

where $\Omega_t \equiv [B_t + (Q_t + D_t) Z_t]$. Then, substitute eq.(B-1) into eq.(1.15):

$$(\Sigma_t - 1) C_t = \frac{1}{(1 + \delta)} \left[E_t \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} + h_t - \frac{W_t}{P_t} \right] \quad (\text{B-2})$$

Lead eq.(1.15) forward one period and multiply it by $\Sigma_{t+1} \mathcal{F}_{t,t+1} (1 - \gamma)$ and take the conditional expectations.

$$\frac{1}{(1 + \delta)} [h_t^* - W_t] = (1 - \gamma) \Sigma_{t+1} E_t \mathcal{F}_{t,t+1} P_{t+1} C_{t+1} - (1 - \gamma) \frac{1}{(1 + \delta)} E_t \mathcal{F}_{t,t+1} \Omega_{t+1}^* \quad (\text{B-3})$$

Write eq.(B-2) in nominal form and plug eq.(B-3) into it:

$$(\Sigma_t - 1) P_t C_t = (1 - \gamma) \Sigma_{t+1} E_t \mathcal{F}_{t,t+1} P_{t+1} C_{t+1} + \gamma \frac{1}{(1 + \delta)} E_t \mathcal{F}_{t,t+1} \Omega_{t+1}^* \quad (\text{B-4})$$

Rewriting eq.(B-4) in real terms, we have eq.(1.17).

Appendix C Philips Curve

From eq.(1.30), using the definition of potential output, we get:

$$mc_t = (1 + \varphi) x_t \quad (\text{C-1})$$

where $x_t = y_t - y_t^n$ is the output gap. The optimal price rule can be rewritten in recursive form as follows:

$$p_t^* = (1 - \theta \tilde{\beta}) (mc_t + p_t) + \theta \tilde{\beta} p_{t+1}^* \quad (\text{C-2})$$

From the eq.(C-2) and eq.(1.29) we get:

$$\pi_t = \lambda mc_t + \tilde{\beta} E_t \pi_{t+1} \quad (\text{C-3})$$

where $\lambda = \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta}$. Substituting eq.(C-1) into eq.(C-3), we obtain the Philips curve.

Appendix D Figures

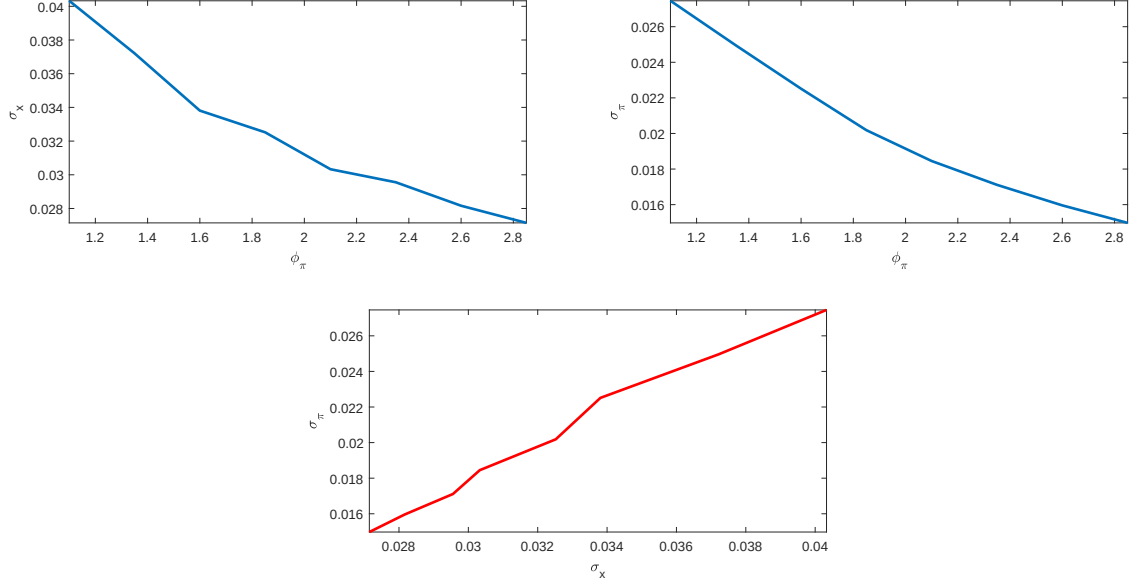


Figure D1: Output gap and inflation volatility in the Behavioral model.

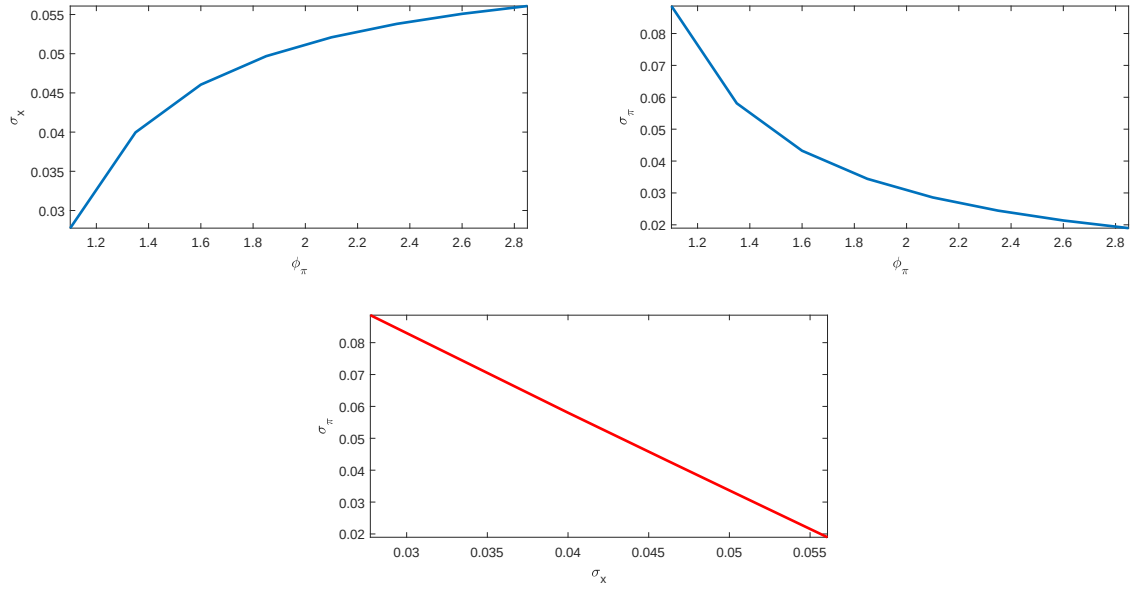


Figure D2: Output gap and inflation volatility in the Rational Expectations model.

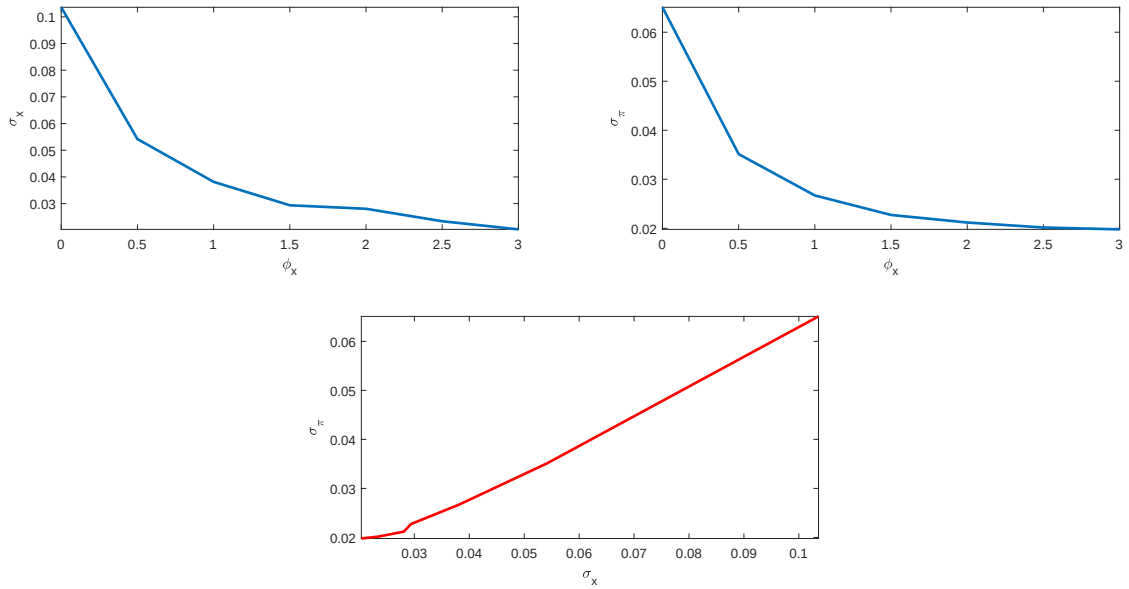


Figure D3: Output gap and inflation volatility in the Behavioral model without asset prices targeting.

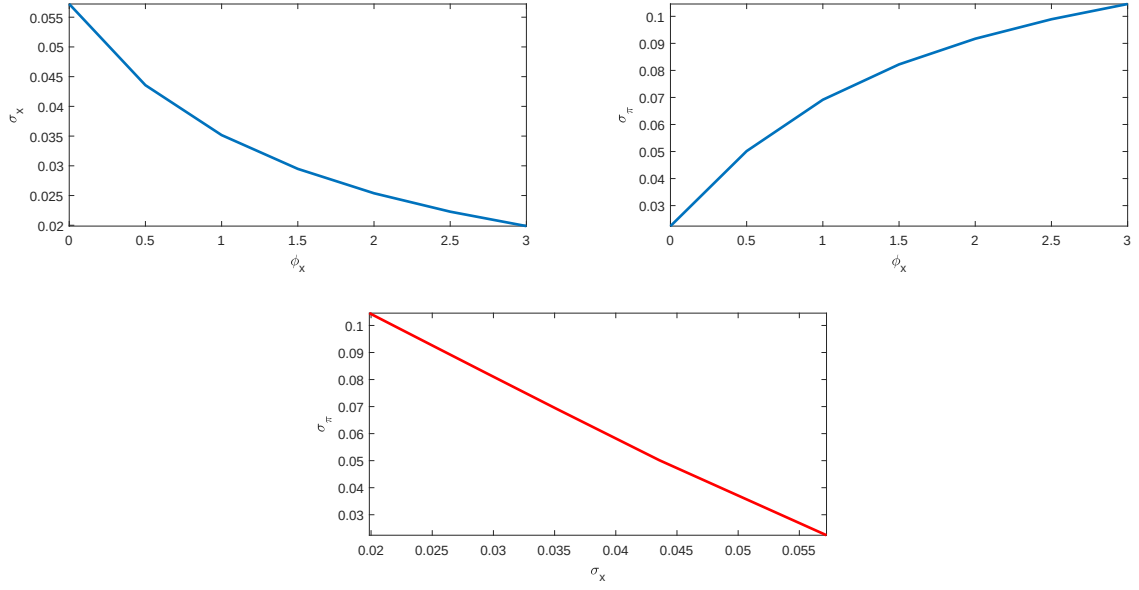


Figure D4: Output gap and inflation volatility in the Rational Expectations model without asset prices targeting.

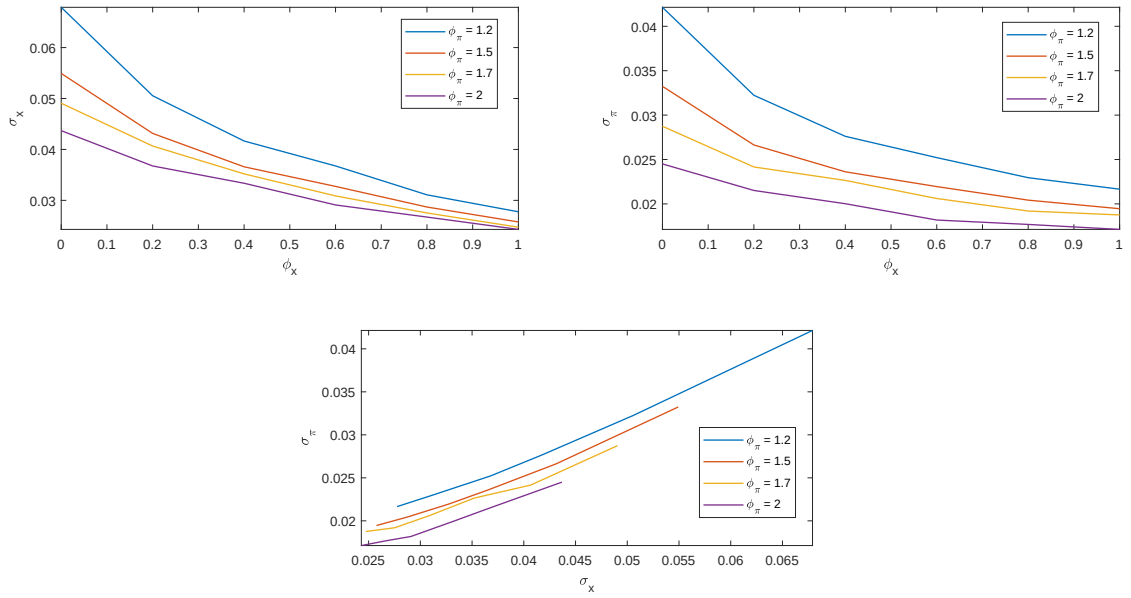


Figure D5: Output gap and inflation volatility in the Behavioral model.

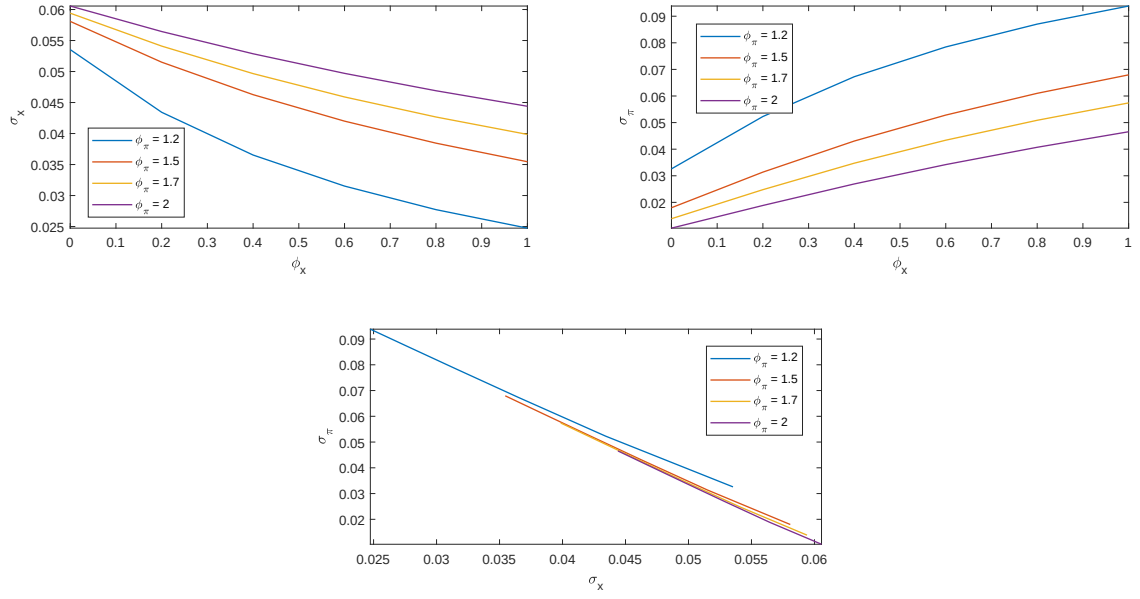


Figure D6: Output gap and inflation volatility in the Rational Expectations model.

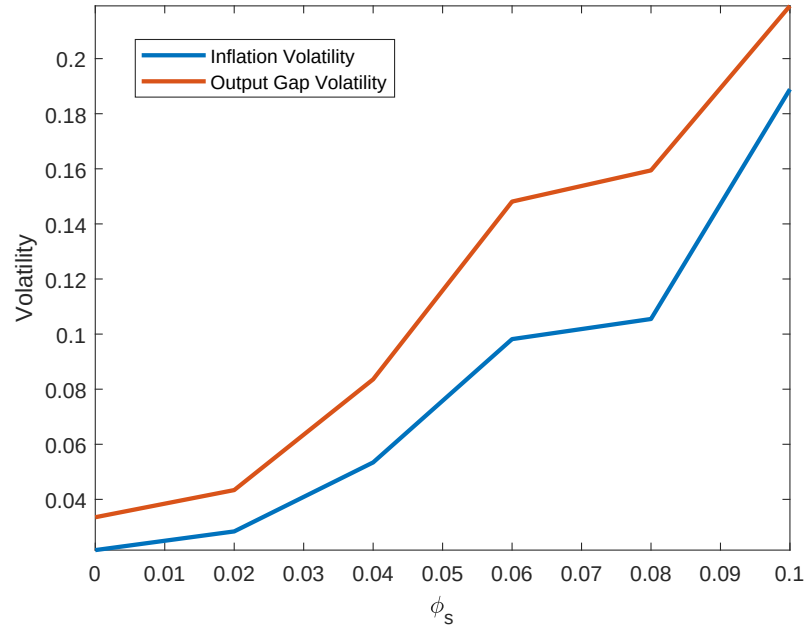


Figure D7: RB : Output gap and inflation volatility for different values of ϕ_s and $\gamma = 0$.

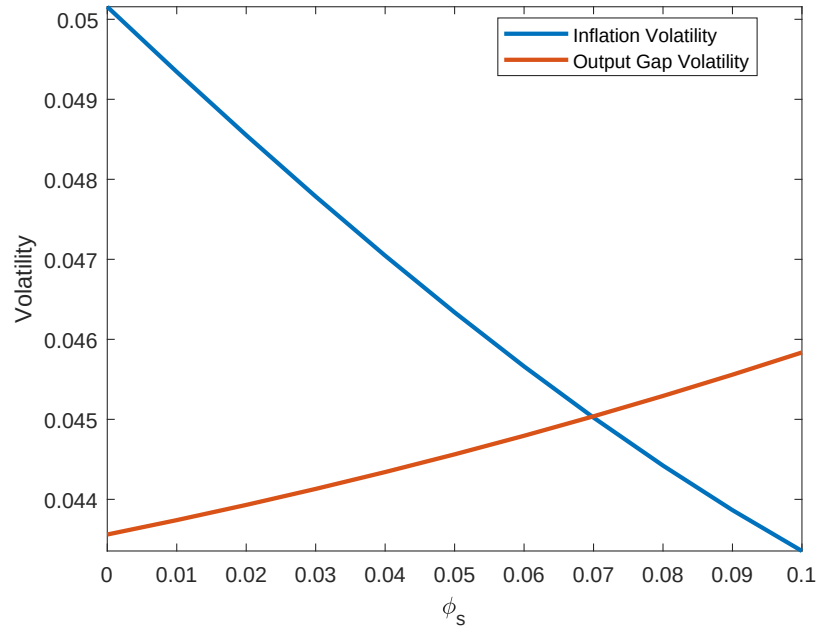


Figure D8: RE : Output gap and inflation volatility for different values of ϕ_s

Chapter 2

Rational Beliefs, Stock Market and Monetary Policy *

2.1 Introduction

The burst of the last financial crisis and its worldwide effects renewed the attention on the relationship between monetary policy and financial markets. In particular, it reopens the debate on whether central banks should take asset prices into account when they formulate monetary policy.

Before the crisis, the consensus view agrees with the stance taken by Bernanke and Gertler (1999): monetary policy should not respond to stock prices fluctuations unless they signal changes in expected inflation. They show that policy rules which include stock prices are always outperformed by rules which prescribe a simple strict inflation targeting. A year later, Cecchetti et al. (2000) conduct a series of simulations on Bernanke and Gertler's (1999) model which deliver a different result. They take into account a wider range of policies and find that if central banks react to asset prices misalignments ¹, monetary policy would be improved. The answer of Bernanke and Gertler does not take too long to arrive. In Bernanke and Gertler (2001), they simulate their model again but enlarge the number of combinations of policy parameters under study. The conclusion remains the same. They ascribe the difference in the two papers' results to the way in which is modeled the ability of central banks to differentiate between underlying sources of movements in endogenous variables. They do not allow for this differentiation. Cecchetti et al. (2002) recognizes

*This chapter is a joint work with Prof. Maurizio Motolese, Department of Economics and Finance, Catholic University of Milan.

We would like to thank Carsten Krabbe Nielsen, Mordecai Kurz, Giulia Piccillo and PhD seminar participants at Catholic University of Milan for insightful comments and suggestions.

¹Notice that Cecchetti et al. (2000, 2002) suggest that central banks should react to asset prices misalignments and not respond to them mechanically and indiscriminately. They are aware of the pitfalls involving this job, but they claim that they are not different from those that arise estimating theoretical constructs such potential GDP or the equilibrium real interest rate.

the cause of disagreement. However, they strongly defend their assumption claiming it should be central bank's job to try to distinguish among the underlying disturbances affecting the economy.

Assenza et al. (2011) try to tackle the problem from another perspective. Instead of augmenting the IS curve as in Bernanke et al. (1999), they obtain a NK augmented Philips curve. Their objective is creating a framework such that stock prices and inflation move in opposite directions so to isolate and favor the role of asset prices targeting. In this setup, a stock market boom shows up as a supply shock instead of a demand shock. Despite their efforts, they find that targeting asset prices has a destabilizing effect especially on inflation volatility. Including asset prices in the rule makes monetary policy "too accommodating" and this may be welfare-reducing.

On the same side of Bernanke and Gertler (2001) are also Bullard and Schaling (2002) and Carlstrom and Fuerst (2007). They enter the debate analyzing the issue of equilibrium determinacy. Their papers show that the introduction of an asset price target in the rule enlarges the indeterminacy space. These results are not surprising and depend on the structure of their model. Both works adopt a classical New-Keynesian model with an infinitely-lived representative agent. This type of model does not allow any structural relation between financial markets and economic activity making stock prices fluctuations irrelevant for consumption decisions and thus for the equilibrium solution. In this framework, there is no rationale for central banks to move the interest rate in response to stock price changes.

Our paper contributes to this unsettled debate. In order to avoid the stock prices redundancy issue created by the standard New Keynesian framework, we borrow from Nisticò (2012). He adopts a perpetual youth model à la Blanchard (1985) and Yaari (1965). The economy is characterized by Ricardian agents who have access to financial markets and non Ricardian agents who simply consume everything they earn. The financial wealth of the two groups is then different and this difference plays a crucial role in creating a structural linkage between financial markets and real activity. In particular, it creates a wedge between the current and the expected aggregate consumption which depends not only on the real interest rate, as in the classical New Keynesian model, but also on the market value of financial wealth. In this framework, stock prices fluctuations affect the dynamics of the aggregate consumption, and thus the equilibrium solution. Nisticò (2012) analyzes the interplay between monetary policy and asset prices targeting along three dimensions: equilibrium determinacy, implied dynamic response of the economy to selected shocks, implications for macroeconomic stability. He focuses his attention on two types of monetary rules which differ only in the definition of the stock prices target. In the first rule the target is the stock price gap defined as deviations of the stock-price index from its flexible-price level, in the second rule, it is the growth rate of stock prices. The two rules perform very differently: while with the stock-price gap, targeting asset prices might have a destabilizing effects and decrease the determinacy region, with the growth-rate, targeting asset prices not only not entail any risk of indeterminacy but also may improve the stability of inflation and interest rate.

Over the past decades, the New Keynesian model has acquired large relevance in the mon-

etary policy analysis. However, most of these models have been developed within the rational expectations framework in which agents possess the same expectations about future events. This hypothesis has been proven wrong in two directions. On the one side, many empirical evidences and laboratory experiments (e.g. Kahneman and Tversky, 1973) show that human behavior can be described by simple heuristics which may produce significant biases. On the other side, there are many researches that documented heterogeneity in individual expectations. Frankel and Froot (1990) and Allen and Taylor (1992) show that predictions on exchange rate movements are made by financial professionals who use different forecasting models. More recently, Carroll (2003), Branch (2004) and Pfajfar and Santoro (2010) provide evidences of heterogeneity in the process of inflation expectation formation using survey data while Kurz and Motolese (2011) show that fluctuations in market belief which depend on the correlation among individual diverse beliefs have a significant impact on the time variability of risk premia.

Alternative approaches to the Rational Expectations (in short, RE) hypothesis are offered by those heterogeneous expectations theories that assume that agents possess a bounded rationality. Examples of bounded rational models are those of Branch and Evans (2011), De Grauwe (2011) and Branch and McGough (2009). They assume that agents not only lack of a full knowledge of the truth but also they have cognitive limitations and make their forecasts using simple heuristics.

When we assume heterogeneous expectations, we are de facto introducing a new source of uncertainty². Fluctuations in the economy are no longer explained only by persistence of classical exogenous shocks (e.g. technology shock, cost-push shock, etc.) but also by persistence in market beliefs. In the context of monetary policy, this extra uncertainty has an impact on central bank's reaction to shocks and may alter their stabilization strategy.

Branch and McGough (2009) study the relationship between heterogeneous expectations and monetary policy, focusing on the analysis of equilibrium determinacy. They introduce some general assumptions on the expectation operator which allow the reduced model to be the same as the usual New Keynesian RE model with instead of conditional expectations, a convex combination of the heterogeneous expectations operators. In their model, they have two types of beliefs: rational and adaptive ($E_t(x_t) = \theta x_{t-1}$). They find that the effect of heterogeneity on the equilibrium determinacy depends on the distribution and nature of expectations³.

In Branch and McGough (2009), the fraction of agents adopting a certain heuristic is fixed. Branch and Evans (2011) and De Grauwe (2011, 2012) relax this assumption and let the proportion of agents using a certain heuristic vary over time. They assume that agents are willing to learn from their mistakes but they adopt two different approaches. Branch and Evans (2011)

²Fundamental for the propagation of this endogenous volatility is the correlation among beliefs. Without correlation there would be no beliefs aggregation and the average market belief would be constant and irrelevant to dynamics. See Kurz (2011).

³They analyze two cases. First they assume agents to hold adaptive expectations in the sense that they dampen recent observations ($\theta < 1$), then to be extrapolators in the sense that they place greater weight on past data ($\theta > 1$). In the first case, the introduction of adaptive agents in the economy has a stabilizing effect; in the second case, it has a destabilizing effect and reduces the determinacy region with respect to the standard RE case.

follows the statistical learning literature (Sargent (1993); Evans and Honkapohja (2001)); agents are assumed to learn as econometricians do and then select their forecasting predictor. They show that the introduction of heterogeneous expectations, through the choice of underparameterized forecasting models, produces regime-switching volatility and either multiple misspecification equilibria or equilibria exhibiting Intrinsic Heterogeneity ⁴. The extra endogenous volatility may be reduced with the adoption of an aggressive inflation targeting policy. De Grauwe (2011, 2012) uses a different approach called “trials and error” learning or “adaptive learning”. In his model, agents choose between two forecasting rules. The selection mechanism occurs through an adaptive learning process of the type of Brock and Hommes (1997). This mechanism creates waves of optimism and pessimism in the economy, introducing an extra source of uncertainty in the model. In this framework, the best stabilization policy prescribes a flexible inflation target.

In the book ‘Lectures on behavioral macroeconomics’, De Grauwe augments his previous model and introduces stock prices. He finds that a mild leaning against the wind with respect to asset prices reduces the scope for waves of optimism and pessimism and thus improves the macroeconomics stabilization. He also shows that the introduction of heterogeneous expectations in the model modifies the relationship between output and inflation volatility. The smooth trade-off between inflation volatility and the output volatility no longer holds ⁵.

In this paper, we depart from the Rational Expectations approach but we do not adopt a theory of Bounded Rationality either. In these latter models forecasting rules are about endogenous variables and are employed directly at a macro level. Our approach is different and follows the Rational Beliefs theory by Kurz (1994, 1997), Kurz and Motolese (2001) and Nielsen (1996). This theory assumes that although agents do not know the true dynamics of the economy, they are still rational in the sense that their predictions are consistent with the empirical distribution they observe. Moreover, beliefs are set at the micro level and contribute to the equilibrium solution.

Our work is related to the recent papers by Airaudo et al. (2015) and Milani (2017). Both use as baseline model Nisticò (2012) and discard the Rational Expectations assumption. Airaudo et al. (2015) assume that agents make forecasts based on simple learning adaptive rules. They use as evaluation criteria the equilibrium determinacy from Blanchard and Kahn (1980) and the learnability from Evans and Honkapohja (2001)). The main result shows that reacting to stock prices enlarges the policy space for which the equilibrium is both determinate and learnable, as long as the policy is not too aggressive. Milani (2017) adopts a structural New Keynesian model where agents hold subjective, near-rational expectations and learn about economic relationships over time (e.g., Evans and Honkapohja, 2001). He estimates the model using Bayesian methods on monthly U.S. data. The posterior estimation reveals that the effect of stock prices on the business

⁴A similar result is found by Massaro (2013). He shows that the under heterogeneous expectations the Taylor principle no longer guarantees a unique equilibrium.

⁵The same result is obtained by Kurz et al. (2018).

cycle works mainly through the expectational channel instead of through the wealth channel⁶.

Focusing on a context of diverse beliefs, in this paper we attempt to answer the question of whether or not central banks should respond to stock prices other than to inflation and output gap. We present a New Keynesian Model (in short, NKM) with heterogeneous expectations based on RB theory. We solve the model both under RE and under RB. Our main results summarize as follows. Contrary to Bernanke and Gertler’s (1999) prescription, we find that a mild “leaning against the wind” strategy in stock market is beneficial for both output gap and inflation stabilization. This different result is to ascribe to the presence of a financial wealth channel and an expectational channel with the latter being an aggregate feature of the diverse RB model. Once we silence the wealth channel under RE we have a complete dichotomy between the stock market and the real economy. Under RE, both output gap and inflation volatility become monotonically increasing in the asset pricing parameter ϕ_{ps} of the monetary policy rule, hence we are back to the Bernanke and Gertler’s (1999) prescription. This is not the case under RB where the expectational channel being operative guarantees a linkage between the stock market and the real economy which makes the asset prices targeting policy desirable. All results under RB exhibit a higher volatility and the magnitude of responses to shock is amplified by beliefs dynamics. Widespread optimism boosts inflation as well as output gap and can generate a bubble in stock prices. The effect on the real economy of such exuberance might be reduced by a more “aggressive” policy.

The remainder of the paper proceeds as follows. Section 2.2 describes the New Keynesian Model (in short, NKM) first at the micro level with aggregation across cohorts as in Nisticò (2012). After aggregating across cohorts we aggregate across the diverse belief types. The structures of the diverse beliefs and of the exogenous shocks are described in Section 2.3. Section 2.4 presents the final aggregated macro model and shows computations of the endogenous effect of market belief on the dynamics of output, inflation and stock prices. In Section 2.5, we illustrate our results first under RE and then under diverse Rational Beliefs (in short RB). Section 2.6 concludes.

2.2 A New Keynesian Model with Stock Market and Belief-driven Fluctuations

2.2.1 The Monopolistic Firms

We follow the standard setup of a NKM with a continuum of agents and products. Agents are consumer-producers. Each household j owns and manages a single firm⁷.

Firm i produces an intermediate good i sold at price $P_{i,t}$. The firms are monopolistic competi-

⁶The posterior mean estimate of γ is 0.0084. This value is quite low, especially with respect to Castelnuovo and Nisticò (2010). Airaudo et al. (2015) show that this difference is related to the different way expectations are modeled.

⁷Note that as households we index agents by superscripts $j \in [0, 1]$ but as firms with distinct products we index them by subscripts $i \in [0, 1]$.

tors who select optimal prices of their intermediate goods given demand, wage rate and production technology which uses only labor and is defined by

$$Y_{i,t} = A_t N_{i,t} \quad (2.1)$$

where $A_t > 0$ is a random variable with $\mathbb{E}^m(A_t) = 1$. We are interested in the case of diverse beliefs. Therefore, we allow agents to form expectations under different model probabilities. The probability measure m in (2.1) is the stationary *empirical* probability deduced from past data which is common knowledge across agents. As explained later in Section 2.3, agent j 's belief is a model of how his subjective conditional probability is different from m .

Final consumption of household j derives from transforming intermediate outputs produced by all firms and it is given by

$$C_t^j = \left[\int_0^1 (C_{i,t}^j)^{\frac{(\varepsilon-1)}{\varepsilon}} di \right]^{\frac{\varepsilon}{(\varepsilon-1)}}, \quad \varepsilon > 1 \quad (2.2)$$

where ε is the intertemporal elasticity of substitution. Each household maximizes consumption C_t^j for any given expenditure level $F_t^j \equiv \int_0^1 P_{i,t} C_{i,t}^j di$. That yields:

$$C_{i,t}^j = \left[\frac{P_{i,t}}{P_t} \right]^{-\varepsilon} C_t^j \quad (2.3)$$

where P_t is the price of final consumption, which is also *The Price Level*, defined in equilibrium by

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{(1-\varepsilon)}} \quad (2.4)$$

2.2.2 Households

Households have preferences over consumption and leisure. They demand consumption goods, supply labor and can trade in two financial assets: one-period contingent bonds and equity shares. The monopolistic firms issue equity shares which are bundled in a single asset: a stock market index with a total supply normalized to 1. Households trade equity shares of the stock market index.

The structure of population (with size normalized to 1) is designed to evolve similarly to the perpetual youth model by Blanchard (1985) and Yaari (1965). Time is discrete and at each date each agent faces a probability $\gamma \in [0, 1]$ of exiting the market and being replaced by a newcomer before the beginning of the next period. There is no population growth and the cohort size is equal to γ . Newcomers, therefore, differ from old agents (old traders) in their wealth. Only agents who are in the market for longer than one period have access to financial markets. We also allow for agents to hold diverse beliefs as described later in Section 2.3. We assume the distribution of

beliefs to be independent of cohorts. Therefore, the h -period-old consumer-trader holding belief type $j \in [0, 1]$ maximizes the following⁸:

$$\mathbb{E}_t^j \sum_{k=0}^{\infty} \beta^k (1 - \gamma)^k \left[\ln C_{h,t+k}^j + \delta \ln (1 - L_{h,t+k}^j) - \frac{\tau_s}{2} (S_{h,t+k}^j - 1)^2 - \frac{\tau_b}{2} \left(\frac{B_{h,t+k}^{*j}}{P_{t+k}} \right)^2 \right] \quad (2.5)$$

under a sequence of budget constraints of the form:

$$C_{h,t}^j + \frac{B_{h,t}^{*j}}{P_t} + P_t^s S_{h,t}^j \leq W_t L_{h,t}^j + \frac{\Omega_{h,t}^{*j}}{P_t} \quad (2.6)$$

where $\beta \in [0, 1]$, $\delta > 0$ is the leisure weight, $L_{h,t}^j$ is labor supplied, W_t is the real wage rate, $B_{h,t}^{*j} = B_{h,t}^j P_t$ is the nominal one-period bond holdings, $S_{h,t}^j$ is stock holdings, P_t^s is the stock price and $\Omega_{h,t}^{*j}$ is the nominal financial wealth carried over from the previous period and defined by:

$$\Omega_{h,t}^{*j} \equiv \frac{1}{1 - \gamma} \left[B_{h,t-1}^{*j} (1 + r_{t-1}) + P_t (P_t^s + D_t) S_{h,t-1}^j \right] \quad (2.7)$$

where r_t is the nominal interest rate⁹. Functions $\frac{\tau_s}{2} (S_{h,t+k}^j - 1)^2$ and $\frac{\tau_b}{2} \left(\frac{B_{h,t+k}^{*j}}{P_{t+k}} \right)^2$ in (2.5) are small penalties on excessive stock short-selling and excessive borrowing, respectively. They work as substitutes for transversality conditions. Kurz et al. (2013) show a similar penalty function can be used to avoid a Ponzi equilibrium with unbounded borrowing¹⁰. The stock market, markets for contingent bonds and for labor are competitive and wages are flexible. The optimality conditions of the s -period-old consumer-trader holding belief type j are:

$$\frac{1}{(1 + r_t)} = \beta \mathbb{E}_t^j \frac{P_t C_{h,t}^j}{P_{t+1} C_{h,t+1}^j} - \tau_b \frac{B_{h,t}^{*j}}{P_t} \frac{C_{h,t}^j}{(1 + r_t)} \quad (2.8)$$

$$P_t^s = \beta \mathbb{E}_t^j \left[\frac{C_{h,t}^j}{C_{h,t+1}^j} (P_{t+1}^s + D_{t+1}) \right] - \tau_s (S_{h,t}^j - 1) C_{h,t}^j \quad (2.9)$$

$$\delta C_{h,t}^j = W_t (1 - L_{h,t}^j) \quad (2.10)$$

As in Nisticò (2012), we can write nominal individual consumption as a linear function of total nominal financial and human wealth and obtain:

$$P_t C_{h,t}^j = \frac{1}{\Sigma(1 + \delta)} \left[(\Omega_{h,t}^{*j} + T_{h,t}^{*j}) + \nu_t^{*j} \right] \quad (2.11)$$

⁸In our notation the superscript $*$ denotes nominal variables.

⁹The model under study is that of a cashless economy in which the central bank enforces a nominal interest rate by a policy rule specified later.

¹⁰In the case of two financial assets the penalties also guarantee bond holdings and stock holdings to be determined at steady state.

where $T_{h,t}^{*j}$ is a negligible wealth adjustment due to the penalty functions on excessive stock short-selling and excessive borrowing (see Appendix A for how to derive (2.11) and for further details).

2.2.2.1 Aggregation across Cohorts

We have assumed the distribution of beliefs to be independent of cohorts. Therefore, for each belief type j we can aggregate per-capita levels across cohorts for each variable $\Xi_h^j = C_h^j, L_h^j, B_h^j, S_h^j, \Omega_h^j, T_h^j$ as weighted average

$$\Xi_t^j \equiv \sum_{h=-\infty}^t \gamma(1-\gamma)^{t-h} \Xi_{h,t}^j$$

Therefore, the budget constraint for each belief type j is given by

$$C_t^j + B_t^j + P_t^s S_t^j = W_t L_t^j + \frac{1}{\Pi_t} B_{t-1}^j (1 + r_{t-1}) + (P_t^s + D_t) S_{t-1}^j \quad (2.12)$$

also,

$$\frac{1}{(1+r_t)} = \mathbb{E}_t^j M_{t+1}^j - \tau_b B_t^j \frac{C_t^j}{(1+r_t)} \quad (2.13)$$

$$P_t^s = \mathbb{E}_t^j [M_{t+1}^j \Pi_{t+1} (P_{t+1}^s + D_{t+1})] - \tau_s (S_t^j - 1) C_t^j \quad (2.14)$$

$$\delta C_t^j = W_t (1 - L_t^j) \quad (2.15)$$

where $M_{t+1}^j = \beta \frac{P_t C_t^j}{P_{t+1} C_{t+1}^j}$ and $\Pi_{t+1} = \frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}$. We can also write the j -type nominal financial wealth

$$\Omega_t^{*j} \equiv [B_{t-1}^{*j} (1 + r_{t-1}) + P_t (P_t^s + D_t) S_{t-1}^j] \quad (2.16)$$

which in real terms is

$$\Omega_t^j \equiv \left[\frac{B_{t-1}^j}{\Pi_t} (1 + r_{t-1}) + (P_t^s + D_t) S_{t-1}^j \right] \quad (2.17)$$

and finally aggregation across cohorts of (2.11) results in

$$P_t C_t^j = \frac{1}{\Sigma(1+\delta)} [(\Omega_t^{*j} + T_t^{*j}) + \nu_t^{*j}] \quad (2.18)$$

or in real terms

$$C_t^j = \frac{1}{\Sigma(1+\delta)} [(\Omega_t^j + T_t^j) + \nu_t^j] \quad (2.19)$$

which leads to (see Appendix B for details):

$$(\Sigma - 1)C_t^j = \Sigma(1 - \gamma) \mathbb{E}_t^j (M_{t+1}^j C_{t+1}^j \Pi_{t+1}) + \frac{\gamma}{(1 + \delta)} \mathbb{E}_t^j (M_{t+1}^j \Omega_{t+1}^j \Pi_{t+1}) \quad (2.20)$$

2.2.3 Optimal Pricing of Firm i

The variable real cost function of firm i is $W_t \left(\frac{Y_{i,t}}{A_t} \right)$, therefore the real marginal cost is

$$\zeta_t = \frac{W_t}{A_t} \quad (2.21)$$

Optimal pricing by firms is subject to some price rigidity which we introduce via the Calvo (1983) mechanism. The market demand function of commodity i is the following:

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t} \right]^{-\varepsilon} Y_t \quad (2.22)$$

which is obtained by aggregating (2.3) over households j . The production function used to produce the intermediate commodities is (2.1) where output is subject to random labor productivity shocks $A_t = e^{\hat{a}_t}$. We assume $\hat{a}_t$ to be Markov and will specify its stochastic dynamics later in Section 2.3.

According to Calvo (1983) mechanism, a randomly selected set of firms $\Theta_t \in [0, 1]$ of measure $(1 - \theta)$ are allowed to change their price at t while Θ_t^C is the set of firms that cannot.

ASSUMPTION 1. *In a Calvo (1983) pricing process, the set of firms in Θ_t is selected independently across agents hence the distribution of agents in terms of output or beliefs is the same whether one looks at those who adjust prices or those who do not adjust prices¹¹.*

By Assumption 1 the mean price of firms in Θ_t^C is the price level at $t - 1$. Hence, the price level is composed as follows:

$$P_t^{1-\varepsilon} = \int_{\Theta_t} P_{i,t}^{*(1-\varepsilon)} di + \int_{\Theta_t^C} P_{i,t-1}^{(1-\varepsilon)} di = \int_{\Theta_t} P_{i,t}^{*(1-\varepsilon)} di + \theta P_{t-1}^{(1-\varepsilon)} \quad (2.23)$$

where $P_{i,t}^*$ is the optimal price of firm i hence,

$$1 = \int_{\Theta_t} \left(\frac{P_{i,t}^*}{P_t} \right)^{(1-\varepsilon)} di + \theta \left(\frac{P_{t-1}}{P_t} \right)^{(1-\varepsilon)} \quad (2.24)$$

Firm i profit function is then defined by

$$\Phi_{i,t} = \frac{1}{P_t} [P_{i,t} Y_{i,t} - W_t^* N_{i,t}] = \left[\left(\frac{P_{i,t}}{P_t} \right) - \frac{W_t}{A_t} \right] Y_{i,t} = \left[\left(\frac{P_{i,t}}{P_t} \right)^{1-\varepsilon} - \frac{W_t}{A_t} \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \right] Y_t \quad (2.25)$$

Firm i chooses the optimal price $P_{i,t}^*$ so as to maximize discounted future profits given its own

¹¹Assumption 1 ensures the belief distribution is orthogonal to the Calvo lottery. For more details the interested reader is referred to the discussion of Assumption 1 in Kurz et al. (2013).

belief. Formally it solves the problem

$$\max_{P_{i,t}^*} \mathbb{E}_t^i \sum_{k=0}^{\infty} \theta^k M_{t+k}^i \left[\left(\frac{P_{i,t}^*}{P_{t+k}} - \zeta_{t+k} \right) \left(\frac{P_{i,t}^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right] \quad (2.26)$$

where $M_{t+k}^i = \beta^k \frac{P_t C_t^i}{P_{t+k} C_{t+k}^i}$ and $\zeta_{t+k} = \frac{W_{t+k}}{A_{t+k}}$ is the marginal cost. The first order conditions associated with (2.26) take the form

$$\mathbb{E}_t^i \sum_{k=0}^{\infty} \theta^k M_{t+k}^i \left[(1 - \varepsilon) \left(\frac{P_{i,t}^*}{P_{t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{P_{t+k}} + \varepsilon \zeta_{t+k} \left(\frac{P_{i,t}^*}{P_{t+k}} \right)^{-(1+\varepsilon)} \frac{Y_{t+k}}{P_{t+k}} \right] = 0 \quad (2.27)$$

or equivalently

$$\mathbb{E}_t^i \sum_{k=0}^{\infty} \theta^k M_{t+k}^i Y_{t+k} \left(\frac{P_t}{P_{t+k}} \right)^{-\varepsilon} \left[(1 - \varepsilon) \left(\frac{P_{i,t}^*}{P_t} \right) \frac{P_t}{P_{t+k}} + \varepsilon \zeta_{t+k} \right] \frac{1}{P_{i,t}^*} \left(\frac{P_{i,t}^*}{P_t} \right)^{-\varepsilon} = 0 \quad (2.28)$$

and after canceling the end terms in (2.28) solve for $Q_{i,t} = \frac{P_{i,t}^*}{P_t}$

$$Q_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t^i \sum_{k=0}^{\infty} \theta^k M_{t+k}^i Y_{t+k} \zeta_{t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon}}{\mathbb{E}_t^i \sum_{k=0}^{\infty} \theta^k M_{t+k}^i Y_{t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon-1}} \quad (2.29)$$

ASSUMPTION 2. *Aggregate profits equal dividends. All dividends are distributed to shareholders proportionally to their stock holdings:*

$$\Phi_t = \int_0^1 \Phi_{i,t} di = D_t = Y_t - W_t N_t \quad (2.30)$$

where $N_t = \int_0^1 N_{i,t} di$ is aggregate labor employed.

2.2.4 Steady State and the log-linearized model¹²

2.2.4.1 The Steady State

The economy converges in the long run to a riskless (non-stochastic) steady state under full price flexibility (i.e. $\bar{Q} = 1$) and zero inflation $\bar{\Pi} = \bar{P}/\bar{P} = 1$. Steady state values for the two financial

¹²In what follows keep in mind that for any variable V , $\bar{V}$ denotes its steady state value, and $\hat{v}_t = (V_t - \bar{V})/\bar{V}$ its percent deviation from steady state, unless differently specified. Also, see Appendix C for more details.

assets are guaranteed by the small penalties introduced in (2.5) and are $\bar{B} = 0$ and $\bar{S} = 1$. At steady state also $\bar{Y} = \bar{C}$, $\bar{A} = 1$ and $\bar{L} = \bar{N}$. From (2.29) we get the steady state value of marginal cost

$$\bar{\zeta} = \frac{\varepsilon - 1}{\varepsilon} = \frac{\bar{W}}{\bar{A}} \quad (2.31)$$

and the steady state gross markup

$$(1 + \mu) \equiv \frac{\varepsilon}{\varepsilon - 1} = \frac{1}{\bar{\zeta}} \quad (2.32)$$

Steady state conditions of marginal cost (2.31) and of gross markup (2.32) imply the following steady state value for the real wage rate

$$\bar{W} = \frac{\bar{A}}{1 + \mu} \quad (2.33)$$

From equation (2.13) and (2.20) we have the following steady state condition:

$$\frac{1}{1 + \bar{r}} = \bar{M} = \frac{\beta}{1 + \psi} = \tilde{\beta} \quad (2.34)$$

where $\psi = \frac{\gamma}{(1 + \delta)} \frac{1 - \beta(1 - \gamma)}{(1 - \gamma)} \frac{\bar{\Omega}}{\bar{C}}$. Steady state aggregate dividends (or profits) in (2.30) are

$$\bar{D} = \bar{Y} \left(\frac{\mu}{1 + \mu} \right) \quad (2.35)$$

Total output is

$$\bar{Y} = \bar{A} \bar{N} \quad (2.36)$$

and evaluating at steady state equations (2.14), (2.15) and (2.16) we get the following conditions:

$$\bar{Y} = \frac{\bar{A}}{\delta(1 + \mu) + 1} \quad (2.37)$$

$$\bar{D} = \frac{\mu}{(1 + \mu)} \bar{Y} = \frac{\bar{A}\mu}{(1 + \mu)[\delta(1 + \mu) + 1]} \quad (2.38)$$

$$\bar{P}^s = \frac{\tilde{\beta}}{(1 - \tilde{\beta})} \bar{D} = \frac{\bar{A}\tilde{\beta}\mu}{(1 - \tilde{\beta})(1 + \mu)[\delta(1 + \mu) + 1]} \quad (2.39)$$

$$\bar{\Omega} = \frac{1}{(1 - \tilde{\beta})} \bar{D} = \frac{\bar{A}\mu}{(1 - \tilde{\beta})(1 + \mu)[\delta(1 + \mu) + 1]} \quad (2.40)$$

2.2.4.2 The Log-linearized Model

Log-linearization of the real financial wealth equation (2.17) results in

$$\hat{\omega}_t^j = \frac{1}{\tilde{\beta}} \hat{b}_{t-1}^j + \hat{s}_{t-1}^j + \tilde{\beta} \hat{p}_t^s + (1 - \tilde{\beta}) \hat{d}_t \quad (2.41)$$

where $\hat{b}_t^j = \frac{B_t^j}{\Omega}$. After leading (2.41) one period forward and taking conditional expectations we have

$$\mathbb{E}_t^j \hat{\omega}_{t+1}^j = \frac{1}{\tilde{\beta}} \hat{b}_t^j + \hat{s}_t^j + \mathbb{E}_t^j \tilde{\beta} \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \mathbb{E}_t^j \hat{d}_{t+1} \quad (2.42)$$

From agent j 's optimality condition (2.14) we get

$$\hat{p}_t^s = \mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{\Omega} \bar{Y} \hat{b}_t^j - \tau_s \frac{(1 - \tilde{\beta})(1 + \mu)}{\tilde{\beta}} \hat{s}_t^j + \tilde{\beta} \mathbb{E}_t^j \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \mathbb{E}_t^j \hat{d}_{t+1} \quad (2.43)$$

where $\hat{r}_t = \frac{r_t - \bar{r}}{1 + \bar{r}}$. Equation (2.43) defines the stock-price dynamics, which we assume are also affected by some additive exogenous disturbances captured by the random variable η_t , hence we rewrite (2.43)

$$\hat{p}_t^s = \mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{\Omega} \bar{Y} \hat{b}_t^j - \tau_s \frac{(1 - \tilde{\beta})(1 + \mu)}{\tilde{\beta}} \hat{s}_t^j + \tilde{\beta} \mathbb{E}_t^j \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \mathbb{E}_t^j \hat{d}_{t+1} + \eta_t \quad (2.44)$$

From (2.15)

$$\hat{c}_t^j = \hat{w}_t - \varphi \hat{l}_t^j \quad (2.45)$$

where $\varphi = \frac{\bar{L}}{(1 - \bar{L})} = \frac{1}{(1 + \mu) \delta}$.

From (2.20)

$$\hat{c}_t^j = \mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \frac{1}{(1 + \psi)} \mathbb{E}_t^j \hat{c}_{t+1}^j + \tau_b \bar{\Omega} \bar{Y} \hat{b}_t^j + \frac{\psi}{(1 + \psi)} \mathbb{E}_t^j \hat{\omega}_{t+1}^j \quad (2.46)$$

which, after inserting (2.42) on the right hand side, becomes

$$\hat{c}_t^j = \mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \frac{1}{(1 + \psi)} \mathbb{E}_t^j \hat{c}_{t+1}^j + \left(\frac{\psi}{(1 + \psi)} \frac{1}{\tilde{\beta}} + \tau_b \bar{\Omega} \bar{Y} \right) \hat{b}_t^j + \frac{\psi}{(1 + \psi)} \left(\hat{s}_t^j + \tilde{\beta} \mathbb{E}_t^j \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \mathbb{E}_t^j \hat{d}_{t+1} \right) \quad (2.47)$$

Log-linearization of the budget constraint (2.12) results in

$$\hat{b}_t^j = \frac{1}{\tilde{\beta}} \hat{b}_{t-1}^j - \tilde{\beta} \hat{s}_t^j + \hat{s}_{t-1}^j + (1 - \tilde{\beta}) \frac{1 + \varphi}{\mu \varphi} \hat{w}_t - (1 - \tilde{\beta}) \frac{1 + \varphi}{\mu} \left(1 + \frac{1}{(1 + \mu) \varphi} \right) \hat{c}_t^j + (1 - \tilde{\beta}) \hat{d}_t \quad (2.48)$$

Consider the optimal pricing equation (2.29) for the case of agent j managing firm j . Its log-linearization leads to a relation between optimal price at t and expected optimal price at $t + 1$ in

the case j can adjust price at $t + 1$:

$$\hat{q}_{j,t} = (1 - \tilde{\beta}\theta) \hat{\zeta}_t + \tilde{\beta}\theta \mathbb{E}_t^j (\hat{q}_{j,t+1} + \hat{\pi}_{t+1}) \quad (2.49)$$

2.3 The Exogenous Shocks and the Endogenous Structure of Beliefs

The economy is subject to two exogenous shocks: the labor productivity shock $\hat{a}_t$ and the stock market shock η_t . We assume the true process of productivity and stock market shocks is not known and agents do not have full knowledge of the economy's stochastic dynamics. Here we assume the true process is Markov and has the following transition

$$\begin{aligned} \hat{a}_{t+1} &= \lambda_a \hat{a}_t + \lambda_a^s \varsigma_t + \tilde{\rho}_{t+1}^a, \\ \eta_{t+1} &= \lambda_\eta \eta_t + \lambda_\eta^s \varsigma_t + \tilde{\rho}_{t+1}^\eta, \end{aligned} \quad \begin{pmatrix} \tilde{\rho}_{t+1}^a \\ \tilde{\rho}_{t+1}^\eta \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \tilde{\sigma}_a^2 & 0 \\ 0 & \tilde{\sigma}_\eta^2 \end{bmatrix} \right) \quad (2.50)$$

with time varying parameters, expressed by zero mean ς_t , common to all shocks. The true process is characterized by regime shifts and it is non-stationary. Agents cannot learn (2.50). However, the true process (2.50) is stable and agents can learn its *empirical distribution*. We follow the RB approach (see Kurz (1994, 1997, 2009)). In this context, agent's lack of full knowledge about any observable implies they have to form a belief about its future dynamics. Computation of the empirical distribution leads to the stationary probability m we have already encountered in (2.1). The stationary distribution m is common knowledge across agents. All agents use past data to compute the joint unique stationary empirical probability m of $(\hat{a}_t, \eta_t)$. We assume m has the following Markov transition

$$\begin{aligned} \hat{a}_{t+1} &= \lambda_a \hat{a}_t + \rho_{t+1}^a, \\ \eta_{t+1} &= \lambda_\eta \eta_t + \rho_{t+1}^\eta, \end{aligned} \quad \begin{pmatrix} \rho_{t+1}^a \\ \rho_{t+1}^\eta \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \right), \text{ i.i.d} \quad (2.51)$$

Broadly speaking, the stationary measure (2.51) is the long run average of (2.50). It is important to stress the role of (2.50) and (2.51). Agents do not know (2.50) and cannot learn it since the economy is subject to structural changes and may have time-varying parameters $(\lambda_a, \lambda_a^s, \lambda_\eta, \lambda_\eta^s)$. What can be learned is the economy's long run performance guided by m in (2.51) and simulations are run according to it. Agents are entitled to believe (2.51) is not the truth and form diverse conditional beliefs. Not all beliefs are allowed according to the RB theory. The RB rationality principle puts some restrictions on the set of admissible beliefs: a belief is considered rational if it induces economic dynamics with an empirical distribution equal to m in (2.51). The set of admissible diverse Rational Beliefs is therefore larger than that under Rational Expectations which we assume it requires a common belief that m is the truth.

2.3.1 Agents' Individual Beliefs

Agents' beliefs are subjective conditional expectations of ς_t , described by state variables g_t^j . We follow Kurz et al. (2018) and assume, for simplicity, a single belief variable although multiple states may be used. The notation $(\hat{a}_{t+1}^j, \eta_{t+1}^j)$ expresses j 's *perception* of $t+1$ shocks before observing them. Therefore, agent j perceives $(\hat{a}_{t+1}^j, \eta_{t+1}^j)$ at date t to be distributed as

$$\begin{aligned} \hat{a}_{t+1}^j &= \lambda_a \hat{a}_t + \lambda_a^g g_t^j + \rho_{t+1}^{aj} \\ \eta_{t+1}^j &= \lambda_\eta \eta_t + \lambda_\eta^g g_t^j + \rho_{t+1}^{\eta j} \end{aligned} \quad \begin{pmatrix} \rho_{t+1}^{aj} \\ \rho_{t+1}^{\eta j} \end{pmatrix} \sim N \left(\mathbf{0}, \begin{bmatrix} \hat{\sigma}_a^2 & 0 \\ 0 & \hat{\sigma}_\eta^2 \end{bmatrix} = \Sigma_1^j \right) \quad (2.52)$$

(2.52) shows g_t^j specifies the *difference* between j 's date t forecasts and the forecasts under m . The RB principle requires (2.52) to reproduce (2.51) over time, which restricts the variance-covariance matrix and the dynamics of g_t^j . We define g_t^j by

$$g_{t+1}^j = \lambda_Z g_t^j + \lambda_Z^a [\hat{a}_{t+1} - \lambda_a \hat{a}_t] + \lambda_Z^\eta [\eta_{t+1} - \lambda_\eta \eta_t] + \rho_{t+1}^{gj}, \quad \rho_t^{gj} \sim N(0, \sigma_g^2), \quad \rho_t^{gj} \text{ correlated across } j \quad (2.53)$$

This transition is central to the development and is taken as an agent's primitive characteristic. How general is (2.53)? If we first ignore the learning feedback the equation is reduced to

$$g_{t+1}^j = \lambda_Z g_t^j + \rho_{t+1}^{gj}, \quad \rho_t^{gj} \sim N(0, \sigma_g^2), \quad \rho_t^{gj} \text{ correlated across } j \quad (2.54)$$

Combining (2.54) and (2.52) one shows that RB restrictions (see Appendix D) on Σ_1^j make the time average of (2.52) reproduce (2.51). What about the empirical relevance of (2.53) and (2.54)? In Kurz and Motolesse (2011) the data reveal the Markov property with high persistence $\lambda_Z \geq 0.7$ and in Kurz (2008) a Bayesian learning model is used to prove (2.54) as a conclusion. Hence, given the Markov property of (2.51) condition (2.54) is as general as one can have, except for allowable burst of short term deviations from (2.54) that average to 0 over time and cannot be explained by any systematic theory. To address j 's short term surprises of forecasting $(\hat{a}_{t+1}^j, \eta_{t+1}^j)$, we allow in (2.53) learning feedback terms $\lambda_Z^a [\hat{a}_{t+1} - \lambda_a \hat{a}_t] + \lambda_Z^\eta [\eta_{t+1} - \lambda_\eta \eta_t]$ that permit placing more weight on recent data. Agents know that any real time learning violates the RB principle since it puts into forecasts information which is not in the market. Since learning feedback average zero over time, (2.53) actually agree, on average, with (2.51) (see Appendix D for further details and discussion).

We assume each agent is anonymous therefore his belief has no effect on the market. Furthermore, although each g_t^j is not publicly observed, the distribution of all g_t^j is observed hence mean market belief $Z_t = \int_0^1 g_t^j dj$ is observed. Like all observed variables, they have an empirical distribution and induce diverse beliefs about their future. The RB approach shows that *agents forming beliefs about mean market belief* do expand their state spaces but do not trigger an infinite regress since Z_t is common knowledge. Before deriving the empirical distribution of mean market belief Z_t , we need to introduce the following assumption to specify the stochastic structure of the

random term ρ_{t+1}^{gj} that was unspecified in the transition function of g_{t+1}^j in (2.53).

ASSUMPTION 3. $\rho_{t+1}^{gj} = \Upsilon_{t+1}(1 + \epsilon_{t+1}^{gj}) \Rightarrow \rho_{t+1}^Z = \Upsilon_{t+1}$ where the common component Υ_{t+1} is a sequence of i.i.d. random variables with mean 0 and variance σ_Z^2 . In addition, ϵ_{t+1}^{gj} are i.i.d. with mean 0 and variance σ_ϵ^2 and are uncorrelated across agents, independent of Υ_{t+1} . Both are uncorrelated with (g_t^j, Z_t) .

The empirical distribution of mean market belief is deduced from (2.53) with stationary transition

$$Z_{t+1} = \lambda_Z Z_t + \lambda_Z^a [\hat{a}_{t+1} - \lambda_a \hat{a}_t] + \lambda_Z^\eta [\eta_{t+1} - \lambda_\eta \eta_t] + \rho_{t+1}^Z, \quad \rho_{t+1}^Z = \int_0^1 \rho_{t+1}^{gj} dj \quad (2.55)$$

By Assumption 3 note $\rho_{t+1}^Z \neq 0$ since, due to correlation across agents, the law of large numbers does not hold. Since correlation is not determined by individual rationality it is a *belief externality*. Agents' uncertainty about future market belief Z_{t+1} is central to our approach; hence, the stationary transition in (2.51) are supplemented, for completeness, with stationary transition (2.55).

Agents may not believe (2.55) is the true transition of Z_t and form their own belief about its future according to

$$Z_{t+1}^j = \lambda_Z Z_t + \lambda_Z^a [\hat{a}_{t+1} - \lambda_a \hat{a}_t] + \lambda_Z^\eta [\eta_{t+1} - \lambda_\eta \eta_t] + \lambda_Z^g g_t^j + \rho_{t+1}^{Zj} \quad (2.56)$$

The full perception model $(\hat{a}_{t+1}^j, \eta_{t+1}^j, Z_{t+1}^j, g_{t+1}^j)$ is then described by transition functions

$$\begin{aligned} \hat{a}_{t+1}^j &= \lambda_a \hat{a}_t + \lambda_a^g g_t^j + \rho_{t+1}^{aj} \\ \eta_{t+1}^j &= \lambda_\eta \eta_t + \lambda_\eta^g g_t^j + \rho_{t+1}^{\eta j} \\ Z_{t+1}^j &= \lambda_Z Z_t + \lambda_Z^a [\hat{a}_{t+1} - \lambda_a \hat{a}_t] + \lambda_Z^\eta [\eta_{t+1} - \lambda_\eta \eta_t] + \lambda_Z^g g_t^j + \rho_{t+1}^{Zj} \\ g_{t+1}^j &= \lambda_Z g_t^j + \lambda_Z^a [\hat{a}_{t+1} - \lambda_a \hat{a}_t] + \lambda_Z^\eta [\eta_{t+1} - \lambda_\eta \eta_t] + \rho_{t+1}^{gj} \end{aligned} \quad \begin{pmatrix} \rho_{t+1}^{aj} \\ \rho_{t+1}^{\eta j} \\ \rho_{t+1}^{Zj} \\ \rho_{t+1}^{gj} \end{pmatrix} \sim N(\mathbf{0}, \Sigma^j) \quad (2.57)$$

where $\Sigma^j = \left[\begin{array}{c|cc} \Sigma_1^j & \mathbf{0} & \\ \hline \mathbf{0} & \hat{\sigma}_Z^2 & \hat{\sigma}_{Zg} \\ & \hat{\sigma}_{Zg} & \sigma_g^2 \end{array} \right]$. Since we have assumed only one unobserved state variable ς_t ,

the sign of belief parameters in (2.57) matter for them to have comparable meaning. Let agent j 's state of *optimism* be associated with $g_t^j > 0$. In the labor productivity shock $\hat{a}_t$ it is clear being optimistic means $\lambda_a^g > 0$ hence we set $\lambda_a^g = 1$ (which is a normalization). Also in the case of the stock market shock we can naturally set $\lambda_\eta^g > 0$, that is we interpret optimisms being expecting higher capitalization of the market (i.e. higher prices). About the learning feed-back parameters, consistency with the parameter choice of λ_a^g and λ_η^g requires that we set $\lambda_Z^a > 0$ and $\lambda_Z^\eta > 0$.

What about individual expectations of future mean market belief or the mean belief of others Z_t ? We have already acknowledged agents may not believe (2.55) to be the true transition of Z_t

and form their own belief about it. Agent j 's belief about date $t + 1$ mean beliefs of others, (i.e. Z_{t+1}), is modeled by the parameter λ_Z^g which measures how $\mathbb{E}_t^j [Z_{t+1}]$ in (2.57) changes with g_t^j . Now suppose that $g_t^j > 0$: agent j is optimistic about the current state of the economy and its effect on future state variables. Given observed Z_t , what does agent j expect to believe tomorrow relative to the belief of others? We define $Z_t - g_t^j$ to be the *conditional optimism of j relative to the mean market belief*¹³. We know from (2.57) that relative optimism has a persistence rate $\lambda_Z < 1$ so that, apart from other factors, it tends to decline. We also know from (2.57) that

$$\mathbb{E}_t^j [Z_{t+1} - g_{t+1}^j] = \lambda_Z [Z_t - g_t^j] + \lambda_Z^g g_t^j$$

and this shows that the sign of λ_Z^g alters the speed at which conditional relative optimism declines. This implies two different and opposing belief patterns of the agent:

The reserved view: $\lambda_Z^g > 0$. Agent j when optimistic expects others to be relatively *more* optimistic than him at $t + 1$ while when pessimistic expects others to be relatively *less* optimistic (more pessimistic) than him. In short, he has a **reserved view**, expecting others to react more sharply to shocks than he does.

The active view: $\lambda_Z^g < 0$. It implies that shifting belief of an agent leads him to expect others to shift less than himself: when optimistic he expects the market to be *less* relatively optimistic and when pessimistic he expects the market to be *more* relatively optimistic. He has an **active view**, expecting others to react less sharply to shocks than he does.

The diverse RB theory we are using is flexible enough to permit different parameter values for belief parameters. Therefore, the theory could be relevant for different social contexts in which patterns of belief may vary. Since all what follows uses standard NKM parameters relevant to US data, we follow Kent and Piccillo (2017) who use Bayesian estimation, aggregate macro data and survey data for the US economy to estimate the parameters of a NKM with Rational Beliefs and conclude the parameter λ_Z^g must be positive. The same conclusion is reached in Kurz et al. (2018) who claim the belief parameter λ_Z^g can be identified using forecast data on GDP growth rate collected by the *Survey of Professional Forecasters*. Hence, we set $\lambda_Z^g > 0$.

2.4 Equilibrium and the Aggregate Log-linear Model

From the aggregate dividends in (2.30) we have

$$\hat{d}_t = \frac{\mu - (1 + \varphi)}{\mu} \hat{y}_t + \frac{(1 + \varphi)}{\mu} \hat{a}_t \quad (2.58)$$

¹³Note the term *conditional* since all comparisons are made given known information at t .

Observe that in equilibrium the following conditions must hold:

$$\int_0^1 \hat{c}_t^j dj = \hat{c}_t = \hat{y}_t = \int_0^1 \hat{y}_{j,t} dj \quad (2.59)$$

$$\int_0^1 \hat{l}_t^j dj = \hat{l}_t = \hat{n}_t = \int_0^1 \hat{n}_{j,t} dj \quad (2.60)$$

$$\int_0^1 \hat{s}_t^j dj = 0 \quad (2.61)$$

$$\int_0^1 \hat{b}_t^j dj = 0 \quad (2.62)$$

We also need to specify the equilibrium relationship between aggregated optimal price and inflation. Considering $Q_{j,t} = \frac{P_{j,t}^*}{P_t}$ we log-linearize (2.24)

$$\int_{\Theta_t} \hat{q}_{j,t} dj = \theta \hat{\pi}_t \quad (2.63)$$

By Assumption 1 and when every firm selects its optimal price, the mean over the population is related to (2.63) through the relation:

$$\int_{\Theta_t} \hat{q}_{j,t} dj = (1 - \theta) \int_0^1 \hat{q}_{j,t} dj \Rightarrow \int_0^1 \hat{q}_{j,t} dj = \hat{q}_t = \frac{\theta}{(1 - \theta)} \hat{\pi}_t \quad (2.64)$$

Aggregating (2.45) we get

$$\hat{y}_t = \hat{w}_t - \varphi \hat{n}_t \quad (2.65)$$

and considering the aggregate log-linear production function $\hat{y}_t = \hat{a}_t + \hat{n}_t$, (2.65) becomes

$$\hat{w}_t = (1 + \varphi) \hat{y}_t - \varphi \hat{a}_t \quad (2.66)$$

After inserting (2.66) and (2.58) in (2.48), this can be rewritten as

$$\hat{b}_t^j = \frac{1}{\tilde{\beta}} \hat{b}_{t-1}^j - \tilde{\beta} \hat{s}_t^j + \hat{s}_{t-1}^j + (1 - \tilde{\beta}) \left[\frac{1 + \varphi (1 + \mu)}{\mu \varphi} \right] (\hat{y}_t - \hat{c}_t^j) \quad (2.67)$$

After log-linearizing the real marginal cost (2.21) we get

$$\hat{\zeta}_t = \hat{w}_t - \hat{a}_t \quad (2.68)$$

Combining (2.68) and (2.66) we get

$$\hat{\zeta}_t = (1 + \varphi) (\hat{y}_t - \hat{a}_t) \quad (2.69)$$

In the case of fully flexible prices (2.29) implies a constant real marginal cost at its steady state value, i.e. $\zeta = \frac{\varepsilon - 1}{\varepsilon}$ and $\hat{\zeta} = 0$, therefore, from (2.69), we can deduce that the **natural rate**

of output is $\hat{y}_t^n = \hat{a}_t$ and the output gap is $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$. Hence, equation (2.49) the log-linear optimal pricing condition of agent j becomes

$$\hat{q}_{j,t} = (1 - \tilde{\beta}\theta) (1 + \varphi) \hat{x}_t + \tilde{\beta}\theta \mathbb{E}_t^j (\hat{q}_{j,t+1} + \hat{\pi}_{t+1}) \quad (2.70)$$

2.4.1 The Aggregate Macro Model

Before deriving the aggregate macro model we need to highlight the difference between the state space for individual j 's decision functions and the one relevant for aggregates. The set of state variables for agent j is

$$I_t^j = (\hat{a}_t, \eta_t, g_t^j, Z_t, \hat{s}_{t-1}^j, \hat{b}_{t-1}^j)$$

Due to the equilibrium conditions listed above, the assumption of individual beliefs g_t^j aggregating to Z_t and the assumption of anonymity, state space I_t^j implies the following vector of state variables for the macro variables

$$I_t = (\hat{a}_t, \eta_t, Z_t, Z_t, 0, 0)$$

We also need to explain how to aggregate individual expectations. We follow Kurz et al. (2013). While aggregation is straightforward for any individual variable $\hat{v}_t^j$ as reported in (2.59)-(2.62), aggregation of the individual forecasts $\mathbb{E}_t^j (\hat{v}_{t+1}^j)$ requires some care. We introduce the following

DEFINITION 1. *For any random variable v ,*

$$\bar{\mathbb{E}}_t(v) = \int_0^1 \mathbb{E}_t^j(v) dj$$

defines the average forecast.

Keep in mind that the operator $\bar{\mathbb{E}}_t$ is not a conditional expectations, it does not obey the law of iterated expectations but it simply is an average forecast (see Kurz (2008), Kurz and Motolese (2011) and Kurz et al. (2013) for more details).

Aggregation of (2.47) can be written as

$$\hat{y}_t = \int_0^1 \mathbb{E}_t^j \hat{\pi}_{t+1} dj - \hat{r}_t + \frac{1}{(1 + \psi)} \int_0^1 \mathbb{E}_t^j \hat{c}_{t+1}^j dj + \frac{\tilde{\beta}\psi}{1 + \psi} \int_0^1 \mathbb{E}_t^j \hat{p}_{t+1}^s dj + \frac{(1 - \tilde{\beta})\psi}{1 + \psi} \int_0^1 \mathbb{E}_t^j \hat{d}_{t+1} dj \quad (2.71)$$

Definition 1 can be easily applied to the terms on the right hand side of (2.71) which contain expectations of macro economic aggregates such as $\hat{\pi}_{t+1}$, $\hat{p}_{t+1}^s$ and $\hat{d}_{t+1}$. Instead, aggregation of individuals' forecasts of the deviation of their future consumption from steady state can be

computed but it is not a natural macro economic aggregate. We therefore rewrite (2.71) as follows

$$\hat{y}_t = \bar{\mathbb{E}}_t \hat{\pi}_{t+1} - \hat{r}_t + \frac{\left[\int_0^1 \bar{\mathbb{E}}_t^j \hat{c}_{t+1} dj + \int_0^1 \bar{\mathbb{E}}_t^j (\hat{c}_{t+1}^j - \hat{c}_{t+1}) dj \right]}{(1 + \psi)} + \frac{\tilde{\beta}\psi}{1 + \psi} \bar{\mathbb{E}}_t \hat{p}_{t+1}^s + \frac{(1 - \tilde{\beta})\psi}{1 + \psi} \bar{\mathbb{E}}_t \hat{d}_{t+1} \quad (2.72)$$

Let $\Psi_t(\hat{c}) = \int_0^1 \bar{\mathbb{E}}_t^j (\hat{c}_{t+1}^j - \hat{c}_{t+1}) dj$ and rewrite (2.72) as

$$\hat{y}_t = \bar{\mathbb{E}}_t \hat{\pi}_{t+1} - \hat{r}_t + \frac{\bar{\mathbb{E}}_t \hat{y}_{t+1}}{(1 + \psi)} + \frac{\Psi_t(\hat{c})}{(1 + \psi)} + \frac{\tilde{\beta}\psi}{1 + \psi} \bar{\mathbb{E}}_t \hat{p}_{t+1}^s + \frac{(1 - \tilde{\beta})\psi}{1 + \psi} \bar{\mathbb{E}}_t \hat{d}_{t+1} \quad (2.73)$$

Aggregation of (2.44) results in the aggregate dynamics of the real stock price

$$\hat{p}_t^s = \bar{\mathbb{E}}_t \hat{\pi}_{t+1} - \hat{r}_t + \tilde{\beta} \bar{\mathbb{E}}_t \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \bar{\mathbb{E}}_t \hat{d}_{t+1} + \eta_t \quad (2.74)$$

from (2.74) we deduce

$$\tilde{\beta} \bar{\mathbb{E}}_t \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \bar{\mathbb{E}}_t \hat{d}_{t+1} = \hat{p}_t^s - \bar{\mathbb{E}}_t \hat{\pi}_{t+1} + \hat{r}_t - \eta_t \quad (2.75)$$

inserting (2.75) into (2.73) we finally get the IS curve in which output dynamics are also related to real stock price fluctuations and to beliefs dynamics embedded in the mean forecast operator $\bar{\mathbb{E}}_t$ and the term $\Psi_t(\hat{c})$:

$$\hat{y}_t = \frac{\bar{\mathbb{E}}_t \hat{y}_{t+1}}{(1 + \psi)} + \frac{\bar{\mathbb{E}}_t \hat{\pi}_{t+1} - \hat{r}_t}{(1 + \psi)} + \frac{\Psi_t(\hat{c})}{(1 + \psi)} + \frac{\psi}{1 + \psi} (\hat{p}_t^s - \eta_t) \quad (2.76)$$

Now consider (2.58), we can rewrite the equation for the aggregate dynamics of the real stock price (2.74) as follows

$$\hat{p}_t^s = \tilde{\beta} \bar{\mathbb{E}}_t \hat{p}_{t+1}^s + \bar{\mathbb{E}}_t \hat{\pi}_{t+1} - \hat{r}_t + \xi \bar{\mathbb{E}}_t \hat{x}_{t+1} + (1 - \tilde{\beta}) \bar{\mathbb{E}}_t \hat{a}_{t+1} + \eta_t \quad (2.77)$$

where $\xi = (1 - \tilde{\beta}) \frac{\mu - (1 + \varphi)}{\mu}$.

Similarly we proceed in aggregating (2.70) to derive the Phillips curve. First write the aggregate equation of (2.49) as follows

$$\hat{q}_t = (1 - \tilde{\beta}\theta) (1 + \varphi) \hat{x}_t + \tilde{\beta}\theta \bar{\mathbb{E}}_t \hat{\pi}_{t+1} + \tilde{\beta}\theta \int_0^1 (\hat{q}_{j,t+1} - \hat{q}_{t+1}) dj + \tilde{\beta}\theta \hat{q}_{t+1} \quad (2.78)$$

Let $\Psi_t(\hat{q}) = \int_0^1 \bar{\mathbb{E}}_t^j (\hat{q}_{j,t+1} - \hat{q}_{t+1}) dj$ and rewrite (2.78) as

$$\hat{q}_t = (1 - \tilde{\beta}\theta) (1 + \varphi) \hat{x}_t + \tilde{\beta}\theta \bar{\mathbb{E}}_t \hat{\pi}_{t+1} + \tilde{\beta}\theta \Psi_t(\hat{q}) + \tilde{\beta}\theta \hat{q}_{t+1} \quad (2.79)$$

recall from equilibrium condition (2.64) $\hat{q}_t = \frac{\theta}{(1-\theta)}\hat{\pi}_t$, and finally we obtain the following aggregate Phillips Curve

$$\hat{\pi}_t = \kappa\hat{x}_t + \tilde{\beta}\bar{\mathbb{E}}_t\hat{\pi}_{t+1} + \tilde{\beta}(1-\theta)\Psi_t(\hat{q}) \quad (2.80)$$

where $\kappa = \frac{(1-\tilde{\beta}\theta)(1-\theta)}{\theta}(1+\varphi)$. In summary, the Aggregate Macro Model, to which we need to add the Monetary Policy rule, is:

$$\left. \begin{array}{ll} \text{IS Curve} & \hat{y}_t = \frac{\bar{\mathbb{E}}_t\hat{y}_{t+1}}{1+\psi} + \frac{\bar{\mathbb{E}}_t\hat{\pi}_{t+1} - \hat{r}_t}{1+\psi} + \frac{\Psi_t(\hat{c})}{1+\psi} + \frac{\psi}{1+\psi}(\hat{p}_t^s - \eta_t) \\ \text{Stock Price} & \hat{p}_t^s = \tilde{\beta}\bar{\mathbb{E}}_t\hat{p}_{t+1}^s + \bar{\mathbb{E}}_t\hat{\pi}_{t+1} - \hat{r}_t + \xi\bar{\mathbb{E}}_t\hat{x}_{t+1} + (1-\tilde{\beta})\bar{\mathbb{E}}_t\hat{a}_{t+1} + \eta_t \\ \text{Phillips Curve} & \hat{\pi}_t = \kappa\hat{x}_t + \tilde{\beta}\bar{\mathbb{E}}_t\hat{\pi}_{t+1} + \tilde{\beta}(1-\theta)\Psi_t(\hat{q}) \end{array} \right\} \quad (2.81)$$

2.4.1.1 The Monetary Policy Rule

We complete the model by introducing the linearized monetary policy rule. In the following sections we study the macroeconomic performance of the model under a policy rule which may include a response to asset market prices. We consider the rule proposed by Taylor (1993) with an explicit target of stock price fluctuations.

$$\hat{r}_t = \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t + \phi_{p^s}\hat{p}_t^s \quad (2.82)$$

Its obvious that in (2.82) setting $\phi_{p^s} > 0$ denotes the case in which the monetary authority is actively targeting asset prices fluctuations. $\phi_{p^s} = 0$ is the case of no asset prices targeting.

We can now rewrite the Macro Model in terms of output gap $\hat{x}_t = \hat{y}_t - \hat{a}_t$:

$$\left. \begin{array}{ll} \text{IS Curve} & \hat{x}_t = \frac{\bar{\mathbb{E}}_t\hat{x}_{t+1}}{1+\psi} + \frac{\bar{\mathbb{E}}_t\hat{a}_{t+1}}{1+\psi} + \frac{\bar{\mathbb{E}}_t\hat{\pi}_{t+1} - \hat{r}_t}{1+\psi} + \frac{\Psi_t(\hat{c})}{1+\psi} + \frac{\psi}{1+\psi}(\hat{p}_t^s - \eta_t) - \hat{a}_t \\ \text{Stock Price} & \hat{p}_t^s = \tilde{\beta}\bar{\mathbb{E}}_t\hat{p}_{t+1}^s + \bar{\mathbb{E}}_t\hat{\pi}_{t+1} - \hat{r}_t + \xi\bar{\mathbb{E}}_t\hat{x}_{t+1} + (1-\tilde{\beta})\bar{\mathbb{E}}_t\hat{a}_{t+1} + \eta_t \\ \text{Phillips Curve} & \hat{\pi}_t = \kappa\hat{x}_t + \tilde{\beta}\bar{\mathbb{E}}_t\hat{\pi}_{t+1} + \tilde{\beta}(1-\theta)\Psi_t(\hat{q}) \\ \text{Monetary Rule} & \hat{r}_t = \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t + \phi_{p^s}\hat{p}_t^s \end{array} \right\} \quad (2.83)$$

2.4.2 The final Aggregate Macro Model and the Effects of Diverse Beliefs

The macro model in (2.83) requires a solution for the terms $\Psi_t(\hat{c})$ and $\Psi_t(\hat{q})$ and computation of the mean forecasts $\bar{\mathbb{E}}_t\hat{\pi}_{t+1} = \int_0^1 \mathbb{E}_t^j\hat{\pi}_{t+1}dj$, $\bar{\mathbb{E}}_t\hat{x}_{t+1} = \int_0^1 \mathbb{E}_t^j\hat{x}_{t+1}dj$ and $\bar{\mathbb{E}}_t\hat{p}_{t+1}^s = \int_0^1 \mathbb{E}_t^j\hat{p}_{t+1}^s dj$. We follow Kurz et al. (2013) and introduce the following Theorem

THEOREM 1. *In the equilibrium of the log-linearized economy with monetary rule (2.82)*

- (i) *there exist parameters $(\lambda_c^\Psi, \lambda_q^\Psi)$ such that $\Psi_t(\hat{c}) = \lambda_c^\Psi Z_t$ and $\Psi_t(\hat{q}) = \lambda_q^\Psi Z_t$*

(ii) there exist parameters $(\Gamma^x, \Gamma^\pi, \Gamma^{p^s})$ such that

$$\int_0^1 \mathbb{E}_t^j \hat{x}_{t+1} dj - \mathbb{E}_t^m \hat{x}_{t+1} = \Gamma^x Z_t \quad \int_0^1 \mathbb{E}_t^j \hat{\pi}_{t+1} dj - \mathbb{E}_t^m \hat{\pi}_{t+1} = \Gamma^\pi Z_t \quad \int_0^1 \mathbb{E}_t^j \hat{p}_{t+1}^s dj - \mathbb{E}_t^m \hat{p}_{t+1}^s = \Gamma^{p^s} Z_t$$

Proof. see Appendix E. □

Theorem 1 shows there exist belief parameters which impact the dynamics of the economy and are embedded in the equilibrium conditions of the log-linear model with diverse beliefs. These parameters create a link between the macroeconomic model and the log-linearized micro equilibrium. To define the macroeconomic model we have to solve the log-linearized micro equilibrium and deduce from it the equilibrium parameters needed for the macro model. Any change in monetary policy calls for a solution of the micro equilibrium before tracing its effects on the macro economy.

Using Theorem 1 we can rewrite the Aggregate Macro Model as follows

$$\left. \begin{aligned} \text{IS Curve} \quad & \hat{x}_t = \frac{\mathbb{E}_t^m \hat{x}_{t+1}}{1 + \psi} + \frac{\mathbb{E}_t^m \hat{\pi}_{t+1} - \hat{r}_t}{1 + \psi} + \frac{\Lambda_x}{1 + \psi} Z_t + \frac{\psi}{1 + \psi} (\hat{p}_t^s - \eta_t) - \vartheta_x \hat{a}_t \\ \text{Stock Price} \quad & \hat{p}_t^s = \tilde{\beta} \mathbb{E}_t^m \hat{p}_{t+1}^s + \mathbb{E}_t^m \hat{\pi}_{t+1} - \hat{r}_t + \xi \mathbb{E}_t^m \hat{x}_{t+1} + \Lambda_{p^s} Z_t + \vartheta_{p^s} \hat{a}_t + \eta_t \\ \text{Phillips Curve} \quad & \hat{\pi}_t = \kappa \hat{x}_t + \tilde{\beta} \mathbb{E}_t^m \hat{\pi}_{t+1} + \Lambda_\pi Z_t \\ \text{Monetary Rule} \quad & \hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t + \phi_{p^s} \hat{p}_t^s \end{aligned} \right\} \quad (2.84)$$

where $\vartheta_x = \frac{1 + \psi - \lambda_a}{1 + \psi}$, $\vartheta_{p^s} = (1 - \tilde{\beta}) \lambda_a$, $\Lambda_x = \Gamma^x + \Gamma^\pi + \lambda_c^\Psi + \lambda_a^g$, $\Lambda_{p^s} = \tilde{\beta} \Gamma^{p^s} + \Gamma^\pi + \xi \Gamma^x + (1 - \tilde{\beta}) \lambda_a^g$ and $\Lambda_\pi = \tilde{\beta} \Gamma^\pi + \lambda_q^\Psi$ (see Appendix E for details).

Note that (2.84) is written under the empirical probability measure m which satisfies the law of iterated expectations and the Blanchard and Kahn (1980) conditions are applicable to ensure determinacy.

A special case of the macroeconomic system in (2.84) is that of $\gamma = 0$, when no agent exits the market. When $\gamma = 0$ also $\psi = 0$ and the direct effect of stock price fluctuations on the dynamics of the IS curve disappears. Under such conditions there exists a full dichotomy between the stock market and the real economy, therefore any targeting of asset prices by the Central Bank is totally ineffective. However, the dichotomy disappears once we introduce diverse beliefs. The macroeconomic system in (2.84) shows that the mean market belief Z_t has an amplification effect on the dynamics of the economy. Z_t also contains market predictions of unobservable regime changes (i.e. changes in ς_t in (2.50)) and might be a valuable signal for monetary policy decision. Correlated beliefs in the market create an endogenous correlation between stock price fluctuations and real variables. Therefore, even when $\gamma = 0$ with diverse rational beliefs, targeting of asset prices by the monetary authority might become desirable as we discuss later in Section 2.5.2.1.

2.5 Monetary policy and asset prices targeting

The relationship between monetary policy and financial markets and the repercussions of stock market fluctuations on the real economy certainly depend on the effectiveness of the wealth and the expectational channels. An increase in financial wealth can increase consumption via a wealth channel (see Mian and Sufi, 2011): consumers increase consumption spending as a direct result of an increase in the value of their assets. On the other hand, changes in stock prices may also affect consumers' confidence and their expectations of future business conditions: asset prices affect the economy via the expectational channel (see Milani, 2017). In our setup both channels are active and are represented by the terms $\frac{\Lambda_x}{1+\psi}Z_t$ and $\frac{\psi}{1+\psi}(\hat{p}_t^s - \eta_t)$ in the IS curve in (2.84). The term $\frac{\Lambda_x}{1+\psi}Z_t$ is a pure expectational factor while changes in $\frac{\psi}{1+\psi}(\hat{p}_t^s - \eta_t)$ might be caused by fluctuations in stock prices due to changes in technology, shocks in the stock market and/or market perceptions of future business conditions. It follows that the need for asset prices targeting by the monetary authority depends on how effective the two terms are. Under RE, as long as $\gamma > 0$, only the second term is active while with diverse RB the pure expectational channel of term one might dominate.

In what follows, we first give a summary of the parameter choice and then present the simulation results of the model. Trying to answer the question whether Central Banks should target asset prices we start by reporting the RE results first and compare them with those in Bernanke and Gertler (2001). We then continue by focusing on the aggregate effects of diverse RB and on how dynamics of optimistic (pessimistic) market belief Z_t interact with the conduct of monetary policy.

We use a quarterly parameterization as in almost all NK DSGE models. We do not aim at a precise calibration of the model by replicating the observed moments in the data. We aim at providing some qualitative equilibrium features of the interaction between diverse private expectations, stock market dynamics and monetary policy. The New Keynesian parameters of the model are standard (e.g. Galí, 2008; Woodford, 1997). The subjective discount factor β is set to be equal to 0.99 which corresponds to a steady state quarterly interest rate of about 1%. For the intertemporal elasticity of substitution ε , the value is 6 which implies a mark-up of 20%. The leisure weight parameter δ is set equal to 1. The proportion θ of firms which are not allowed to change their prices for the current quarter is set equal to 2/3 which correspond to an average duration of tree quarters, as reported by empirical evidence¹⁴. The penalties are set as follows: $\tau_b = 10e^{-4}$ and $\tau_s = 10e^{-3}$.

The exogenous shocks are parameterized as follows. In the case of the technology shock, we depart from the standard Real Business Cycle (in short, RBC) model calibration introduced by King and Rebelo (1999) for which $\sigma_a = 0.0072$ (as measure by Solow residual). Many authors

¹⁴See estimates in Galí and D. (2001) and Sbordone (2002).

(Basu, 1996; Eichenbaum, 2010; King and Rebelo, 1999; Summers, 1986) make a persuasive case in favor of the fact that the residual does not capture exclusively the effect of the technology. On this ground, we follow Kurz et al. (2018) and set $\sigma_a = 0.0045$ where the lower standard deviation corresponds to the exclusion of the diverse beliefs volatility from the Solow's residuals. The persistence of the technology shock λ_a has the standard value of 0.9. In the case of the stock price shock the persistence is set to $\lambda_\eta = 0.9$ and its standard deviation $\sigma_\eta = 0.01$ (in line with those in Castelnuevo and Nisticò, 2010).

The value set for the probability of exiting the financial markets takes into account Castelnuevo and Nisticò's (2010) posterior estimations. They found the turnover rate to range between 0.0798 and 0.1827. We set $\gamma = 0.15$. Table 2.1 summarizes the choice of parameters common to RE and RB cases.

Description		Value
<i>Structural parameters</i>		
τ_b	Bonds penalty function	$10e^{-4}$
τ_s	Stocks penalty function	$10e^{-3}$
θ	Price rigidity	$2/3$
ε	Elasticity of substitution	6
δ	Leisure weight	1
β	Discount factor	0.99
γ	Probability of exiting financial markets	0.15
<i>Exogenous shock processes</i>		
λ_a	Persistence in a_t	0.9
λ_η	Persistence in η_t	0.9
σ_a	Std. dev. of ρ_t^a	0.0045
σ_η	Std. dev. of ρ_t^η	0.01

Table 2.1: Common parameters choice

2.5.1 The RE solution and the Bernanke and Gertler (2001) prescription

According to Bernanke and Gertler (1999, 2001) the monetary authority should not respond to stock prices fluctuations unless they signal changes in expected inflation. We find a quite different picture under RE when comparing dynamics of implied volatilities of the output gap and of inflation with those in Bernanke and Gertler (2001). We report in Table 2.2 the unconditional standard deviations, in percentage points, of the output gap and of inflation under different policy rule specifications, going from the accommodative rules in the first six rows to the “aggressive” inflation targeting rules in the last nine rows. The three numbers in the first three left columns of Table 2.2 represent the monetary policy rule parameter choice. In columns four and six we

report the standard deviation of the output gap, σ_x , and the standard deviation of inflation, σ_π , respectively. In columns five and seven we compute the gain in increased stability of output gap and inflation, respectively¹⁵. The gain is computed as the ratio between the standard deviation under the no asset price targeting (i.e. $\phi_{ps} = 0$) and that under the limited targeting with (i.e. $\phi_{ps} = 0.043$). In all cases reported, both output gap volatility and inflation volatility are U-shaped functions of ϕ_{ps} and around $\phi_{ps} = 0.043$ they reach their minimum as illustrated in Figure 2.1. All standard deviations are computed for the economy in which both technology shocks and stock market shocks are present.

	Policy Rule			σ_x	$\text{gain}(x)$ $\frac{\sigma_x \phi_{ps}=0}{\sigma_x \phi_{ps}=0.043}$	σ_π	$\text{gain}(\pi)$ $\frac{\sigma_\pi \phi_{ps}=0}{\sigma_\pi \phi_{ps}=0.043}$
	ϕ_x	ϕ_π	ϕ_{ps}				
Accommodative	0.00	1.01	0.000	1.2009		2.7563	
	0.00	1.01	0.043	0.3730	3.2193	0.8562	3.2193
	0.00	1.01	0.100	2.1190		4.8637	
	0.50	1.01	0.000	0.9411		2.1600	
	0.50	1.01	0.043	0.2840	3.3131	0.6520	3.3131
	0.50	1.01	0.100	1.5341		3.5211	
Aggressive	0.00	2.00	0.000	0.1995		0.4580	
	0.00	2.00	0.043	0.0557	3.5809	0.1279	3.5809
	0.00	2.00	0.100	0.2672		0.6132	
	0.50	2.00	0.000	0.1908		0.4379	
	0.50	2.00	0.043	0.0532	3.5840	0.1222	3.5840
	0.50	2.00	0.100	0.2549		0.5851	
	1.00	3.00	0.000	0.1032		0.2368	
	1.00	3.00	0.043	0.0285	3.6156	0.0655	3.6157
	1.00	3.00	0.100	0.1350		0.3099	

Table 2.2: Macro Volatilities under RE with accommodative and aggressive inflation targeting policy rule.

Comparing Table 2.2 with Bernanke and Gertler (2001, Table 3) we conclude that under the wealth channel of our model (a wealth effect on consumption spending) a moderate targeting would be desirable, a little “leaning against the wind” would be beneficial even with “aggressive” inflation targeting rules. Standard deviations of output gap as well as of inflation under no asset price targeting are larger than those with asset price targeting by a factor of more than 3 as reported by the gains in Table 2.2. Therefore, volatilities across the different scenarios would be reduced by about 71% when adopting a moderate asset price targeting. Furthermore, results in Table 2.2 suggest that rules based on “aggressive” inflation targeting and a moderate response to

¹⁵Any value larger than 1 corresponds to an improvement

stock market fluctuations perform best across the different scenarios. Figures 2.2 and 2.3 report the responses of output gap, inflation, nominal interest rate and stock price to a one standard deviation positive stock market shock. Figures 2.2 and 2.3 show that with either accommodative or “aggressive” rules the magnitude of the responses to a positive stock market shock is smaller when a moderate targeting of stock market fluctuations is added.

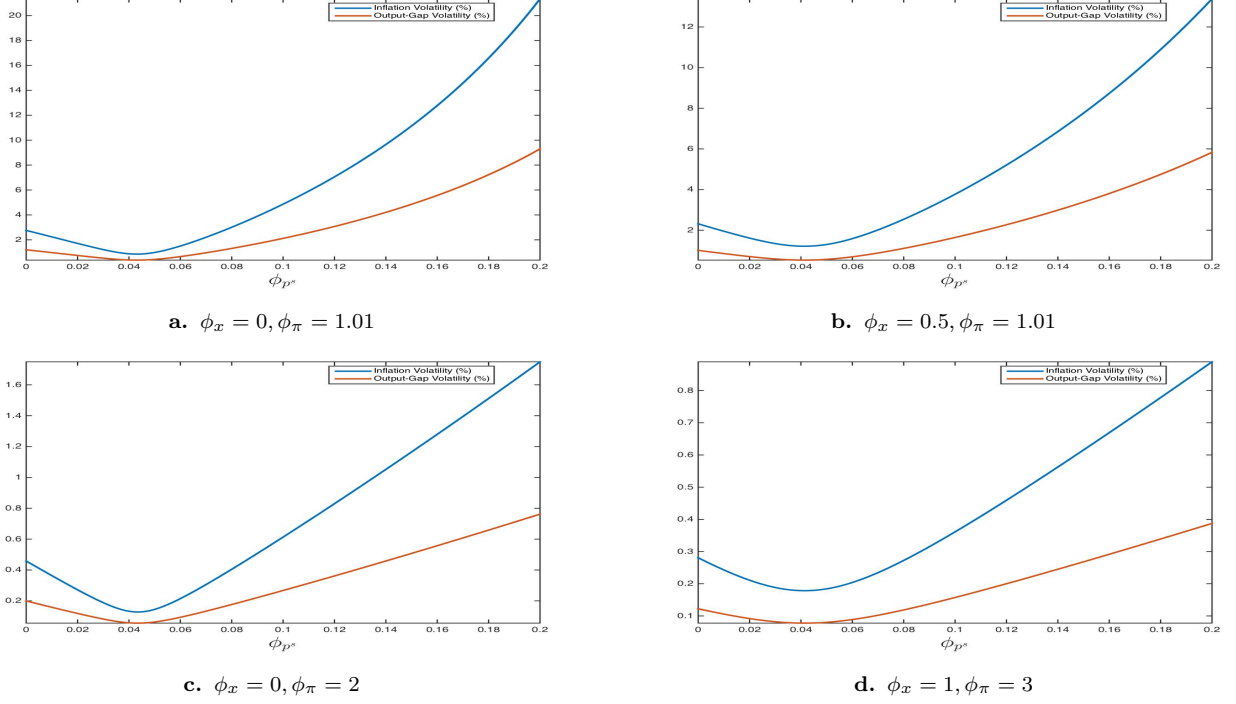


Figure 2.1: RE inflation and output gap volatilities with “accommodative” (panels a and b) and “aggressive” inflation targeting rules (panels c and d).

We explore a larger number of possible parameterizations of the policy rule and let $0 \leq \phi_x \leq 2.0$ and $1.1 \leq \phi_\pi \leq 2.0$. We then report in Tables 2.3 and 2.4 volatilities of the output gap and inflation under the several policy configurations without any asset prices targeting ($\phi_{p^s} = 0$) and with asset prices targeting ($\phi_{p^s} = 0.043$). The gray cells in Tables 2.3 and 2.4 show there exist a point-wise dominance of the rules which embed asset prices targeting¹⁶.

2.5.1.1 The RE case of $\gamma = 0$

So far we have assumed $\gamma = 0.15$ to make the financial wealth channel meaningful and the stock prices equation non redundant for the macroeconomic system. If we solve for the RE equilibrium assuming $\gamma = 0$, the model returns to be a classical NKM where there is a complete dichotomy between the stock market and the real economy. As illustrated in Figure 2.4, for the standard case

¹⁶Notice there exist no trade-off between output gap and inflation volatility in our setup. However, adding a cost-push shock to the model would bring about the trade-off, as shown by Blanchard and Galí (2007). Our results still stand after adding a cost-push shock and, as in Kurz et al. (2018), under diverse RB the trade-off ceases to be monotonic.

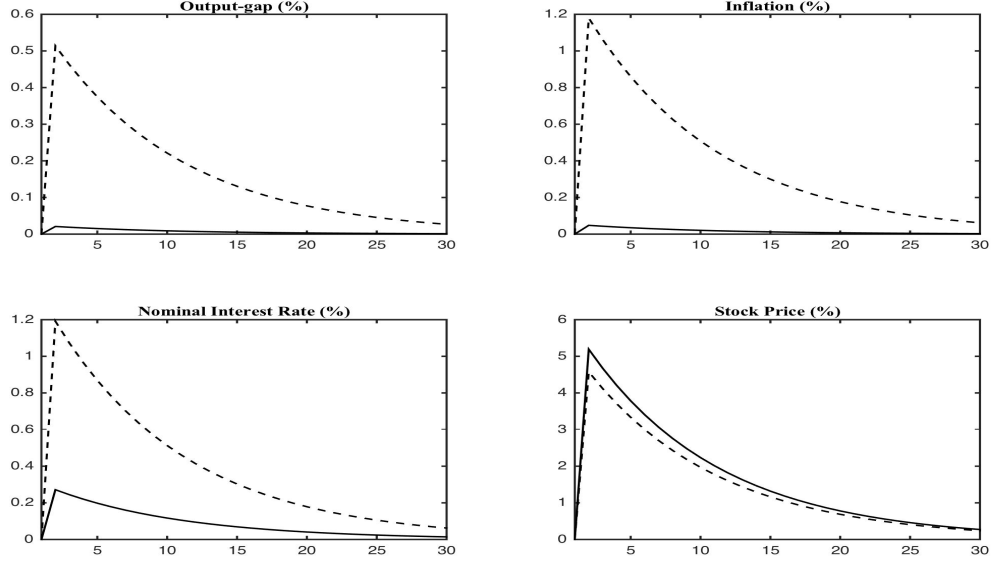


Figure 2.2: The responses to a one standard deviation positive innovation to the stock price shock η . The solid lines show responses under the accommodative monetary rule $\hat{r}_t = 1.01\hat{\pi}_t + 0.043\hat{p}_t^s$ with moderate asset prices targeting; the dashed lines show responses under the accommodative monetary rule $\hat{r}_t = 1.01\hat{\pi}_t$ without any asset prices targeting.

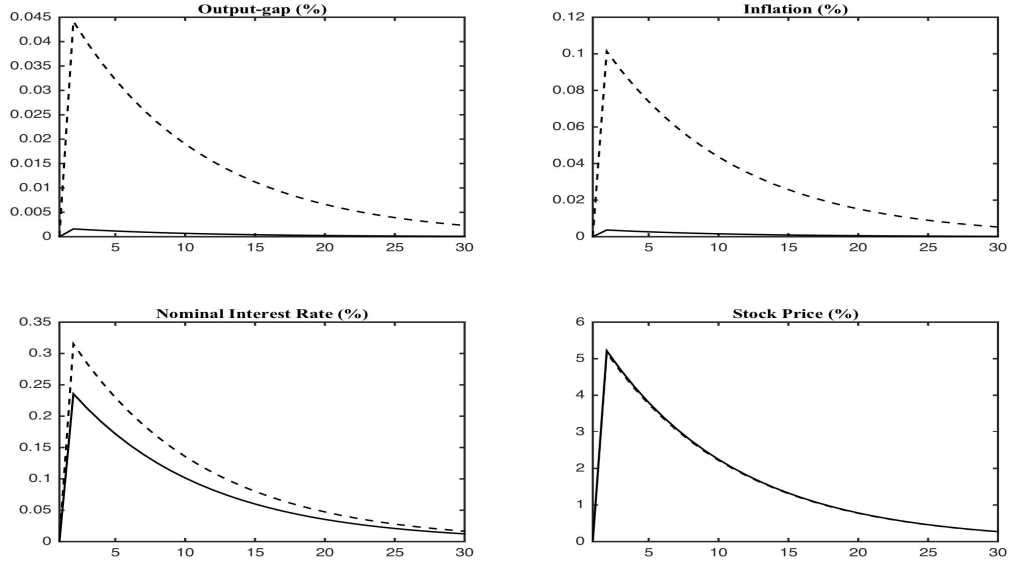


Figure 2.3: The responses to a one standard deviation positive innovation to the stock price shock η . The solid lines show responses under the “aggressive” monetary rule $\hat{r}_t = 3\hat{\pi}_t + \hat{x}_t + 0.043\hat{p}_t^s$ with moderate asset prices targeting; the dashed lines show responses under the “aggressive” monetary rule $\hat{r}_t = 3\hat{\pi}_t + \hat{x}_t$ without any asset prices targeting.

Output-Gap Volatility											
		$\phi_{p^s} = 0$					$\phi_{p^s} = 0.043$				
$\phi_\pi \backslash \phi_x$		0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
1.1		0.825	0.693	0.598	0.526	0.469	0.246	0.204	0.174	0.152	0.135
1.2		0.612	0.536	0.477	0.430	0.392	0.178	0.155	0.137	0.123	0.111
1.5		0.345	0.319	0.298	0.279	0.262	0.098	0.090	0.084	0.078	0.074
1.7		0.267	0.252	0.238	0.225	0.214	0.075	0.071	0.067	0.063	0.060
2.0		0.200	0.191	0.183	0.175	0.169	0.056	0.053	0.051	0.049	0.047

Table 2.3: Output-gap volatility (percentage standard deviation) under RE with ($\phi_{p^s} = 0.043$) and without ($\phi_{p^s} = 0$) asset prices targeting

Inflation Volatility											
		$\phi_{p^s} = 0$					$\phi_{p^s} = 0.043$				
$\phi_x \backslash \phi_\pi$		0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
1.1		1.893	1.591	1.372	1.207	1.077	0.564	0.468	0.399	0.348	0.309
1.2		1.404	1.231	1.096	0.987	0.899	0.409	0.356	0.315	0.282	0.256
1.5		0.791	0.733	0.683	0.639	0.601	0.224	0.207	0.193	0.180	0.169
1.7		0.613	0.577	0.546	0.518	0.492	0.172	0.162	0.153	0.145	0.138
2.0		0.458	0.438	0.420	0.403	0.387	0.128	0.122	0.117	0.112	0.108

Table 2.4: Inflation volatility (percentage standard deviation) under RE with ($\phi_{p^s} = 0.043$) and without ($\phi_{p^s} = 0$) asset prices targeting

with $\phi_x = 0.5$ and $\phi_\pi = 1.5$, both inflation and output gap volatilities are increasing functions of the asset prices targeting coefficient in the monetary rule: the Bernanke and Gertler (2001) prescription holds. However, as it will be discussed later in Section 2.5.2.1, under diverse RB the Bernanke and Gertler (2001) prescription ceases to hold.

2.5.2 The RB solution and the need for asset price targeting

We now turn our attention on the role of diverse beliefs on the relationship between monetary policy and stock prices fluctuations. Before presenting the results we summarize in Table 2.5 the parameter choice for the diverse beliefs processes¹⁷. We have already motivated such parameters in Section 2.3 and .

While in the RE framework of Section 2.5.1 only the financial wealth channel, represented by

¹⁷As discussed earlier in Section 2.3 the learning feedback parameters are set to positive values: $\lambda_Z^a > 0$ and $\lambda_Z^\eta > 0$. However, the results about stock prices targeting we present stand also in the absence of learning feedback, i.e. $\lambda_Z^a = 0$ and $\lambda_Z^\eta = 0$, as well as for cases in which the feedback is larger than the value set in Table 2.5.

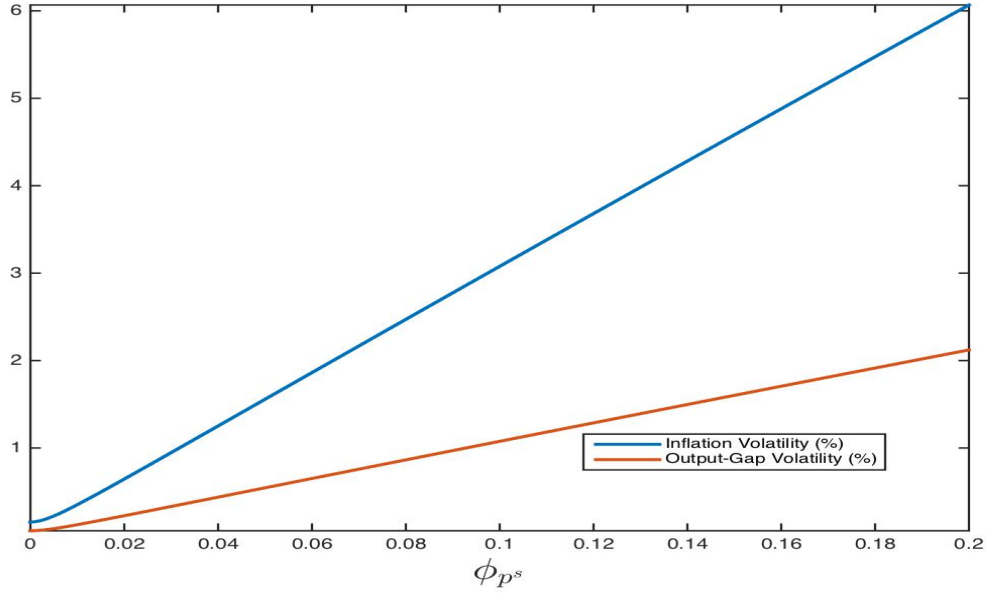


Figure 2.4: Output gap and inflation volatility with $\phi_x = 0.5$ and $\phi_\pi = 1.5$

Description		Value
<i>Beliefs parameters</i>		
λ_a^g	Agent's subjective belief parameter in $\hat{a}_t$	1
λ_η^g	Agent's subjective belief parameter in η_t	0.5
λ_Z	Persistence in Z_t	0.8
λ_Z^a	Learning feedback from $\hat{a}_t$	0.1
λ_Z^η	Learning feedback from η_t	0.1
λ_Z^g	Agent's subjective belief parameter in Z_t	0.1
σ_g	Standard deviation of ρ_t^{gj}	0.003
ρ	Correlation across j belief types	0.9

Table 2.5: The parameter choice for the belief processes

the term $\frac{\psi}{1+\psi}(\hat{p}_t^s - \eta_t)$ in the IS curve, was present, in the diverse RB framework also the pure expectational channel is active, i.e. term $\frac{\Lambda_x}{1+\psi}Z_t$ in the IS curve. Notice how changes in market beliefs Z_t affect all aggregate variables simultaneously in the macro model (2.84). Both types of channels in the diverse RB framework feed into the linkage between the stock market and the real economy. Table 2.6 presents the unconditional standard deviations of output gap and inflation under the different combinations of the policy rule analyzed by Bernanke and Gertler (2001).

From an inspection of Table 2.6 reacting to stock prices fluctuations is even more desirable than under the RE case, especially for accommodative policies. The impact of asset prices is now largely amplified by the presence of diverse beliefs and this is shown in the higher volatilities reported in Tables 2.6, 2.7 and 2.8 when compared to Tables 2.2, 2.3 and 2.4 under RE. Unlike

		Policy Rule						
		ϕ_x	ϕ_π	ϕ_{p^s}	σ_x	$\text{gain}(x)$ $\frac{\sigma_x \phi_{p^s}=0}{\sigma_x \phi_{p^s}>0}$	σ_π	$\text{gain}(\pi)$ $\frac{\sigma_\pi \phi_{p^s}=0}{\sigma_\pi \phi_{p^s}>0}$
Accommodative	0.00	1.01	0.000	65.1955			693.5308	
	0.00	1.01	0.043	1.4975	43.5369		3.5081	197.6963
	0.00	1.01	0.045	0.4958	131.5062		14.4241	48.0813
	0.00	1.01	0.100	28.2012			332.5478	
	0.50	1.01	0.000	10.8528			115.1274	
	0.50	1.01	0.044	0.9926	10.9336		0.9983	115.3260
	0.50	1.01	0.049	0.4273	25.3984		10.8775	10.5839
	0.50	1.01	0.100	8.6697			114.6246	
Aggressive	0.00	2.00	0.000	0.2307			1.6827	
	0.00	2.00	0.023	0.2019	1.1428		1.0500	1.6026
	0.00	2.00	0.059	0.2702	0.8541		0.3004	5.6017
	0.00	2.00	0.100	0.4422			1.1943	
	0.50	2.00	0.000	0.2240			1.6600	
	0.50	2.00	0.024	0.1951	1.1478		1.0014	1.6577
	0.50	2.00	0.058	0.2533	0.8841		0.2805	5.9171
	0.50	2.00	0.100	0.4172			1.1968	
	1.00	3.00	0.000	0.1168			0.8307	
	1.00	3.00	0.022	0.1024	1.1405		0.5368	1.5476
	1.00	3.00	0.059	0.1389	0.8411		0.1578	5.2661
	1.00	3.00	0.100	0.2256			0.5783	

Table 2.6: Macro Volatilities under diverse RB with accommodative and aggressive inflation targeting policy rule

the RE case, under RB the minimum volatility of inflation and output gap is reached at different values of the stock prices coefficient ϕ_{p^s} in the monetary rule (see Figure 2.5). Therefore, we report in Table 2.6 asset prices targeting solutions under the values of ϕ_{p^s} which guarantee the minimum volatility of the output gap and the minimum volatility of inflation. Moreover, under RB, such minima vary across the different policies because there exists a deep interaction between policy and market beliefs (see Kurz et al. (2018)).

Stabilization of inflation suggests that a mild “leaning against the wind” strategy in the stock market would be beneficial for both accommodative and “aggressive” inflation targeting policies. In particular, if we take into account the fact that central banks are more concerned with inflation stabilization, Table 2.6 shows the inflation volatility could be reduced by about 99% in case of accommodative policies and by about 81% for “aggressive” policy rules. However, if the bank’s objective gives higher weight to output gap stabilization, Table 2.6 shows a gain in terms of output gap volatility can also be achieved when the inflation targeting policy is too “aggressive”. Output gap volatility could be reduced by about 98% in case of accommodative policies and by about

12% for “aggressive” policy rules.

Tables 2.7 and 2.8 report the standard deviations of inflation and output gap under different policy rule specifications with $(\phi_{ps} = 0.029)^{18}$ and without $(\phi_{ps} = 0)$ asset prices targeting.

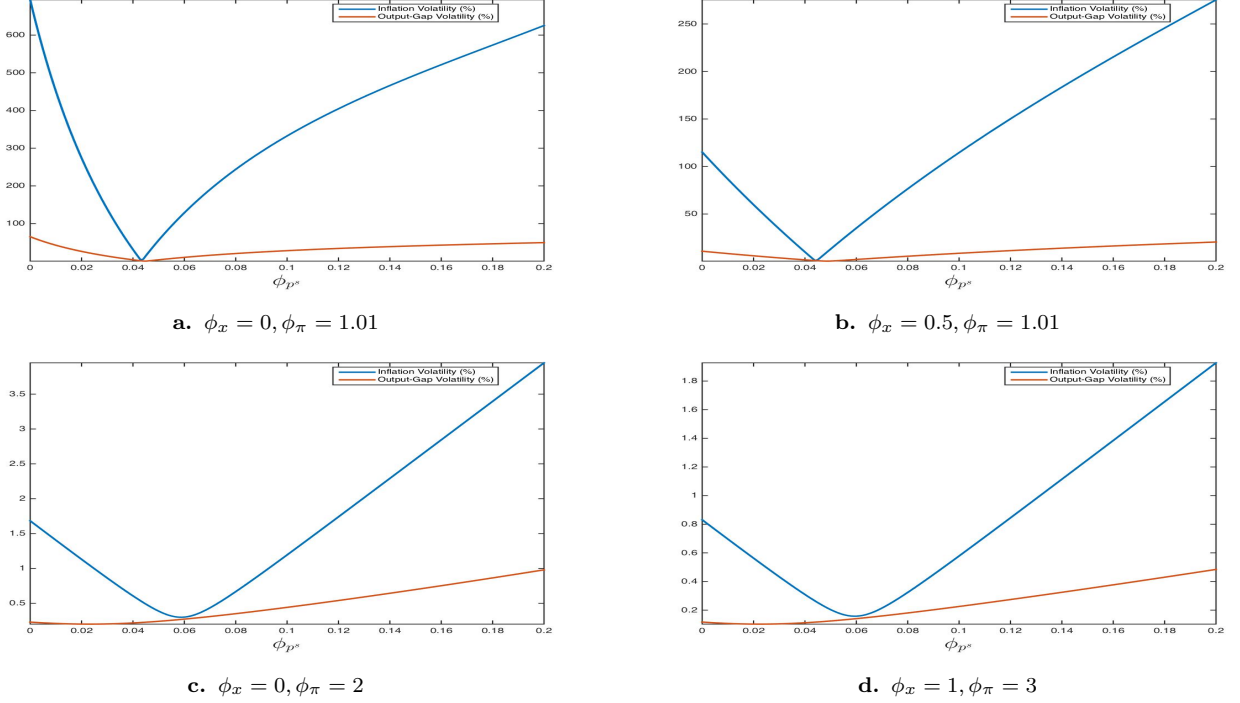


Figure 2.5: RB inflation and output gap volatilities with “accommodative” (panels a and b) and “aggressive” inflation targeting rules (panels c and d).

The gray cells, in Tables 2.7 and 2.8, show that whatever policy rule is chosen, reacting to asset prices fluctuations better stabilizes both output gap and inflation volatility. Notice that increasing “aggressiveness” of inflation targeting in the policy rules reduces the overall volatility. This may suggest that by being “aggressive” the monetary authority is implicitly targeting the expectations exuberance of market beliefs.

Finally we show in Figures 2.6 and 2.7 the responses of output gap, inflation, nominal interest rate and stock price to a one standard deviation positive stock market shock. We report the responses under diverse RB with either accommodative or “aggressive” rules. The magnitude of the responses is smaller when a moderate targeting of stock market fluctuations is added. Of course the magnitude of the responses is higher than under RE due to the endogenous volatility stemming from diverse beliefs.

¹⁸The values of ϕ_{ps} that ensure minima in either output gap volatility or inflation volatility change across policy configurations. In Tables 2.7 and 2.8 we have set $\phi_{ps} = 0.029$ which corresponds to the minimum level of output gap volatility using Taylor coefficients, i.e. $\phi_x = 0.5$ and $\phi_\pi = 1.5$.

Output-Gap Volatility											
		$\phi_{p^s} = 0$					$\phi_{p^s} = 0.029$				
$\phi_\pi \backslash \phi_x$		0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
1.1		1.800	1.584	1.415	1.279	1.168	1.189	1.021	0.896	0.799	0.721
1.2		0.953	0.880	0.818	0.767	0.722	0.740	0.664	0.603	0.553	0.512
1.5		0.429	0.409	0.392	0.376	0.362	0.369	0.346	0.327	0.310	0.294
1.7		0.319	0.307	0.296	0.286	0.277	0.279	0.265	0.253	0.242	0.233
2.0		0.231	0.224	0.218	0.212	0.207	0.204	0.197	0.190	0.183	0.178

Table 2.7: Output-gap volatility (percentage standard deviation) under RB with ($\phi_{p^s} = 0.029$) and without ($\phi_{p^s} = 0$) asset prices targeting

Inflation Volatility											
		$\phi_{p^s} = 0$					$\phi_{p^s} = 0.029$				
$\phi_x \backslash \phi_\pi$		0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
1.1		17.643	15.645	14.053	12.756	11.679	7.217	6.342	5.661	5.115	4.666
1.2		8.522	8.020	7.576	7.179	6.823	3.863	3.569	3.323	3.112	2.929
1.5		3.366	3.281	3.201	3.125	3.053	1.695	1.624	1.562	1.505	1.454
1.7		2.403	2.358	2.316	2.275	2.236	1.242	1.202	1.166	1.132	1.101
2.0		1.683	1.660	1.638	1.617	1.597	0.889	0.868	0.847	0.828	0.811

Table 2.8: Inflation volatility (percentage standard deviation) under RB with ($\phi_{p^s} = 0.029$) and without ($\phi_{p^s} = 0$) asset prices targeting

2.5.2.1 The RBE case of $\gamma = 0$

As mentioned earlier in Section 2.5.1.1, when $\gamma = 0$ the financial wealth channel disappears. In the absence of any relevant impact of asset prices on the macroeconomy, there is no reason to target them neither to stabilize business cycle nor inflation. The RE result confirmed such hypothesis. However, when we introduce diverse RB this is no longer true. Fig. 2.8, with $\phi_x = 0.5$ and $\phi_\pi = 1.5$, shows inflation volatility continues to be a U-shaped function of ϕ_{p^s} instead of being monotonously increasing as in Figure 2.4 under RE, while the output gap volatility has virtually no gain. Hence, a mild targeting of asset prices is beneficial to inflation stabilization. In particular, reacting to asset prices reduces inflation volatility by about 80%. However, if we strengthen the expectational channel by increasing the feedback parameters λ_Z^a and λ_Z^η from 0.1 to 0.15 we also get a meaningful gain in terms of output gap volatility as shown in Figure 2.9, with a gain of about 75% reduction of inflation volatility and of about 51% of output gap volatility.

The difference between the two policy prescriptions is to ascribe to the role of beliefs. We

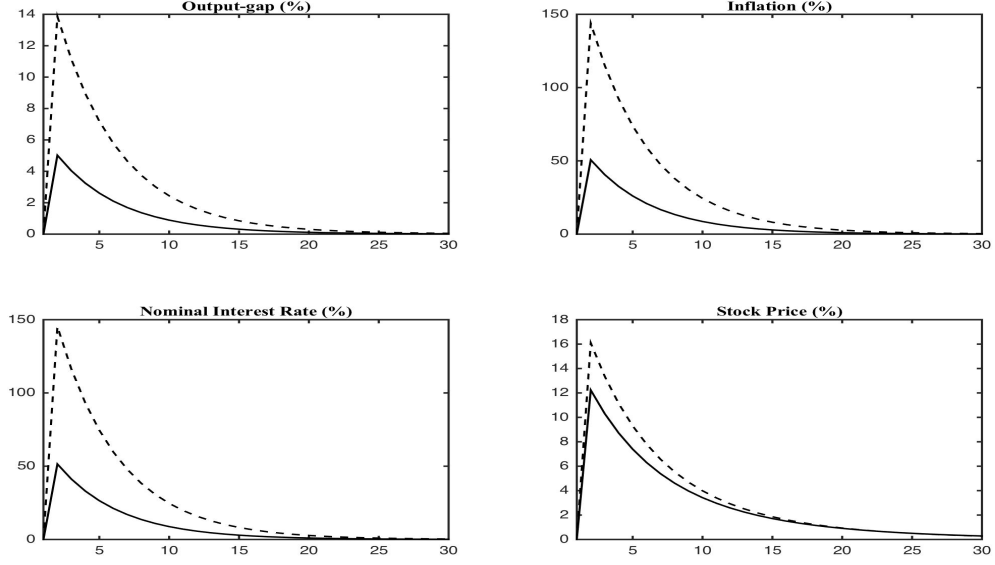


Figure 2.6: The responses under diverse RB to a one standard deviation positive innovation to the stock price shock η . The solid lines show responses under the accommodative monetary rule $\hat{r}_t = 1.01\hat{\pi}_t + 0.045\hat{p}_t^s$ with moderate asset prices targeting; the dashed lines show responses under the accommodative monetary rule $\hat{r}_t = 1.01\hat{\pi}_t$ without any asset prices targeting.

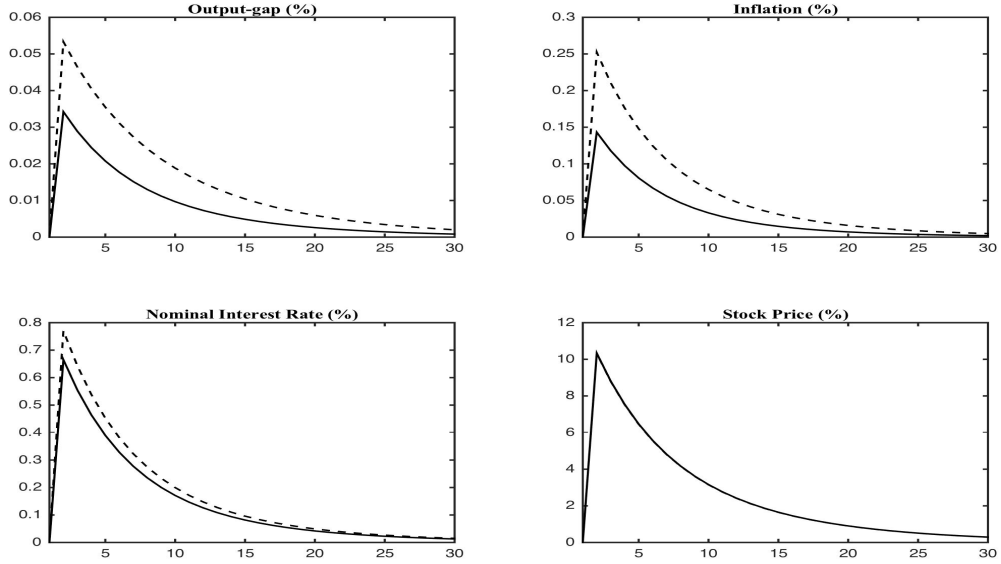


Figure 2.7: The responses under diverse RB to a one standard deviation positive innovation to the stock price shock η . The solid lines show responses under the “aggressive” monetary rule $\hat{r}_t = 3\hat{\pi}_t + \hat{x}_t + 0.022\hat{p}_t^s$ with moderate asset prices targeting; the dashed lines show responses under the “aggressive” monetary rule $\hat{r}_t = 3\hat{\pi}_t + \hat{x}_t$ without any asset prices targeting.

assume agents form expectations on a single belief index about overall business conditions and when optimistic about the stock market they also foresee improvements in technology shocks. This forecasting setup generates an endogenous co-movement between financial and real variables¹⁹. Hence, assuming diverse beliefs restores the linkage between financial market and real economy which disappeared once we set $\gamma = 0$ under RE. In this framework, reacting to stock prices becomes again beneficial to the stabilization of both business cycle and inflation.

The co-movements between financial and real variables suggest that by observing stock price changes, central bankers might be able to extract some information about the underline market expectations of future business conditions. This is in line with Bernanke (2004), he claims that “policymakers watch financial markets carefully” because “asset prices and yields are potentially valuable sources of timely information about economic and financial conditions. Because the future returns on most financial assets depend sensitively on economic conditions, asset prices (...) should embody a great deal of investors’ collective information and beliefs about the future course of the economy”. In this respect, Stock and Watson (2003) provide extensive evidences that stock prices might signal important information relevant for monetary policy about both future business conditions and future inflation.

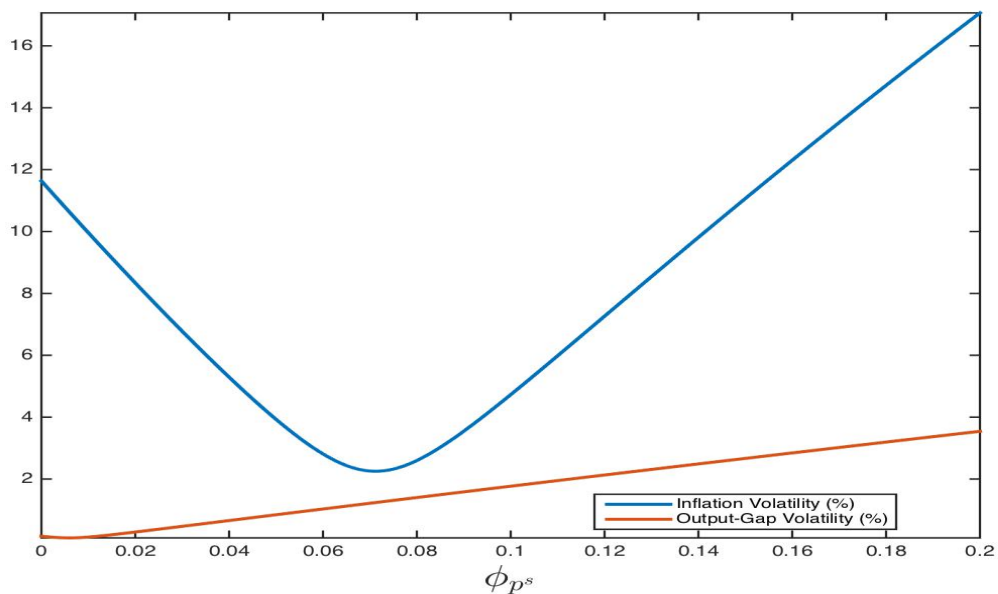


Figure 2.8: Output gap and inflation volatility with $\phi_x = 0.5$ and $\phi_\pi = 1.5$

¹⁹Evidence of the existence of pro-cyclicality of stock prices and their co-movement with real variables can be found in Adam and Merkel (2018).

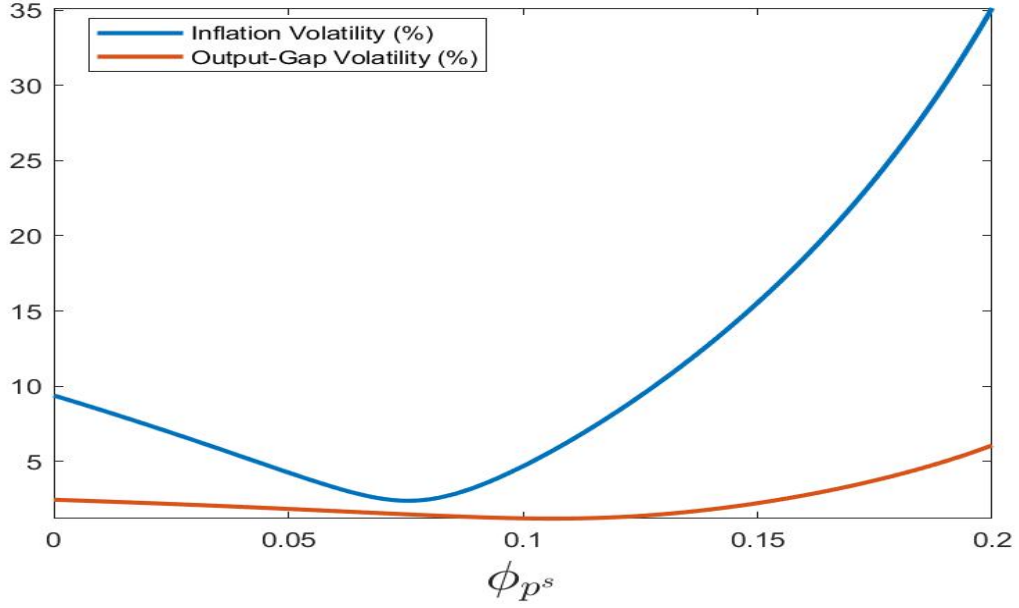


Figure 2.9: Output gap and inflation volatility with $\phi_x = 0.5$, $\phi_\pi = 1.5$ and $\lambda_Z^a = \lambda_Z^\eta = 0.15$.

2.5.2.2 The direct equilibrium effects of market state of belief Z_t

We now turn our attention to the direct equilibrium effects of the observed mean of the distribution of beliefs Z_t . We report in Figure 2.10 the responses of output gap, inflation, nominal interest rate and stock price to a one standard deviation positive mean market belief shock. As shown in Figure 2.10, widespread optimism boosts inflation as well as output gap making the economy work beyond potential. Furthermore, optimism alone can generate a bubble in asset prices. Notice that the intensity of the impact of shifts to mean market belief Z_t varies with the policy configuration and more “aggressive” rules are able to abate it.

To further clarify the aggregate effect of shifts to market belief Z_t we compare the responses of output gap, inflation, nominal interest rate and stock price to the exogenous shocks η_t and $\hat{a}_t$ under RE and diverse RB. The RE case is simulated under the policy rule $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.043\hat{p}_t^s$ which ensure the minimum volatility (as in Tables 2.3 and 2.4) and the RB case is simulated with the policy rule being $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.022\hat{p}_t^s$ (as in Tables 2.7 and 2.8). We report such responses in Figures 2.11 and 2.12. The different magnitude of responses in the output gap is striking in Figure 2.12. The difference is due to the fact that the impact of the productivity shock is partially offset via the positive revision of mean market belief Z_t caused by the increased forecast error in $\hat{a}_t$. Any positive unanticipated shock to $\hat{a}_t$ cause agents to revise upward their beliefs according to (2.53) and by aggregation this results in higher optimism in the mean market belief Z_t according to (2.55). The revision of the mean market belief Z_t may also reverse the sign of the impact of productivity shocks. If we select the more accommodating policy rule $\hat{r}_t = 1.1\hat{\pi}_t + 0.5\hat{x}_t + 0.022\hat{p}_t^s$ we observe an initial positive response of output gap to productivity shock when compared to the

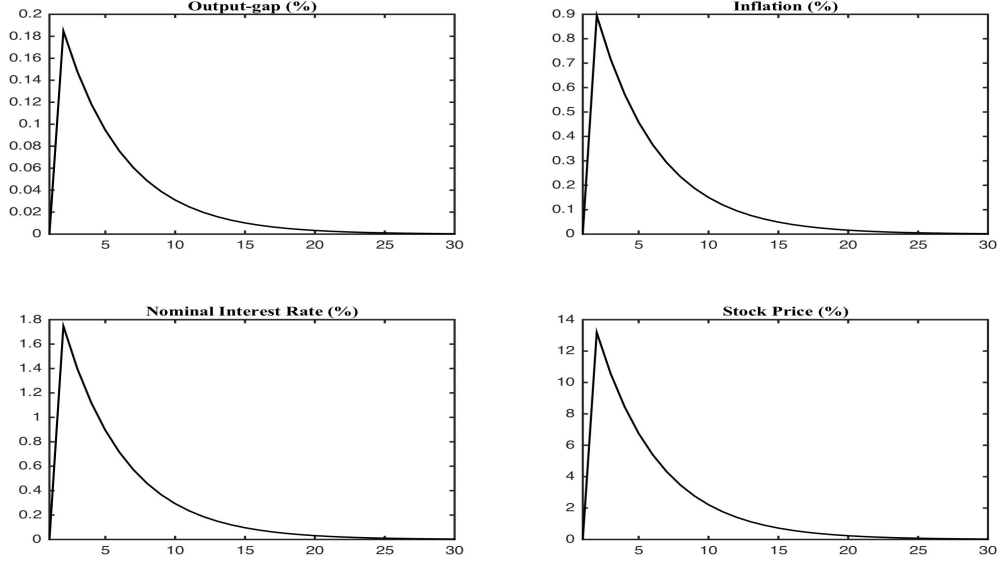


Figure 2.10: The responses under diverse RB to a one standard deviation positive innovation to the mean market belief Z_t under the monetary rule with moderate asset prices targeting: $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.022\hat{p}_t^s$.

RE response (see Figure 2.13). Moreover, as in Kurz et al. (2018), in Figure 2.12 market belief dynamics create circumstances in which output gap and inflation respond with opposite signs. This demonstrates there exist circumstances when the economy exhibits a “stagflation” response to a shock. In such circumstances policy is partly muted since the effects of higher inflation and lower output gap on the interest rate partially offset.

2.6 Conclusions

In the standard NKM, a Taylor rule with an explicit stock price targeting destabilizes the economy. This was largely believed to be the case before the financial crisis of 2008 and it was strongly advocated by Bernanke and Gertler (1999, 2001). The problem with those models is that they do not have a clear expectational channel that links stock market dynamics to the real economy.

This paper studied a NKM with heterogeneous expectations modeled according to the RB theory proposed by Kurz (1994, 1997) and investigated the relationship between monetary policy and stock market in light of the expectational channel caused by the correlated beliefs in the market. In particular, we attempted to answer the question of whether or not central banks should respond to stock prices other than only to inflation and output gap.

We solved the model both under RE and under diverse RB. In the first case, contrary to Bernanke and Gertler’s (1999) prescription, we find that a mild “leaning against the wind” strategy in stocks market is beneficial for both business cycle and inflation stabilization. Targeting asset

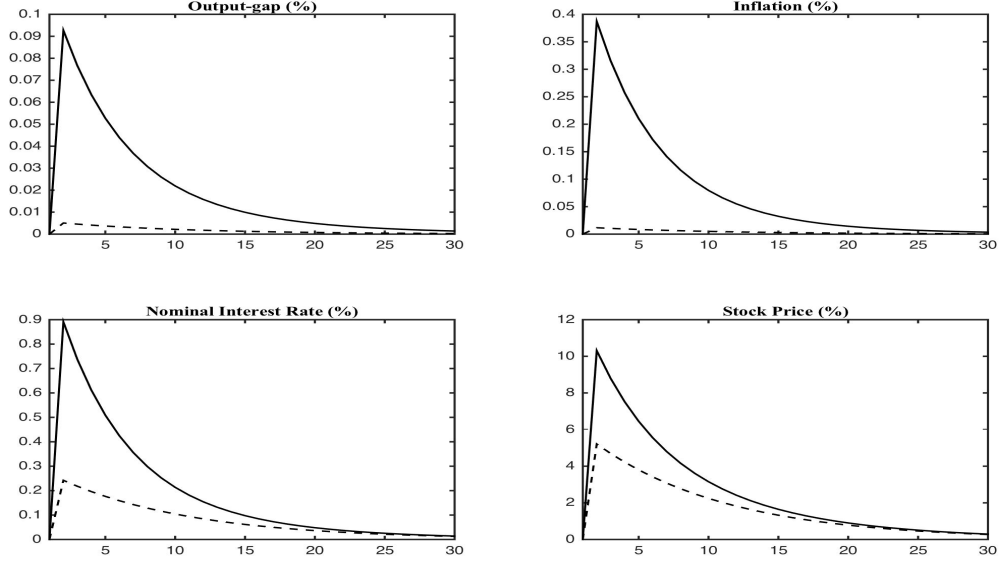


Figure 2.11: The responses to a one standard deviation positive innovation to the stock price shock η . The solid lines show responses under diverse RB with monetary rule $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.022\hat{p}_t^s$; the dashed lines show responses under RE with monetary rule $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.043\hat{p}_t^s$.

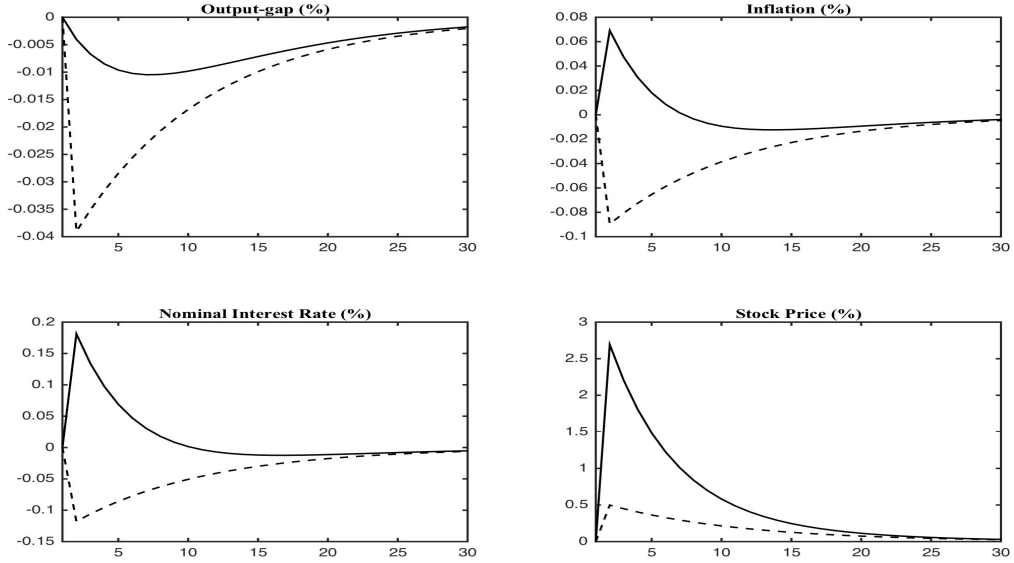


Figure 2.12: The responses to a one standard deviation positive innovation to the productivity shock $\hat{a}_t$. The solid lines show responses under diverse RB with monetary rule $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.022\hat{p}_t^s$; the dashed lines show responses under RE with monetary rule $\hat{r}_t = 1.5\hat{\pi}_t + 0.5\hat{x}_t + 0.043\hat{p}_t^s$.

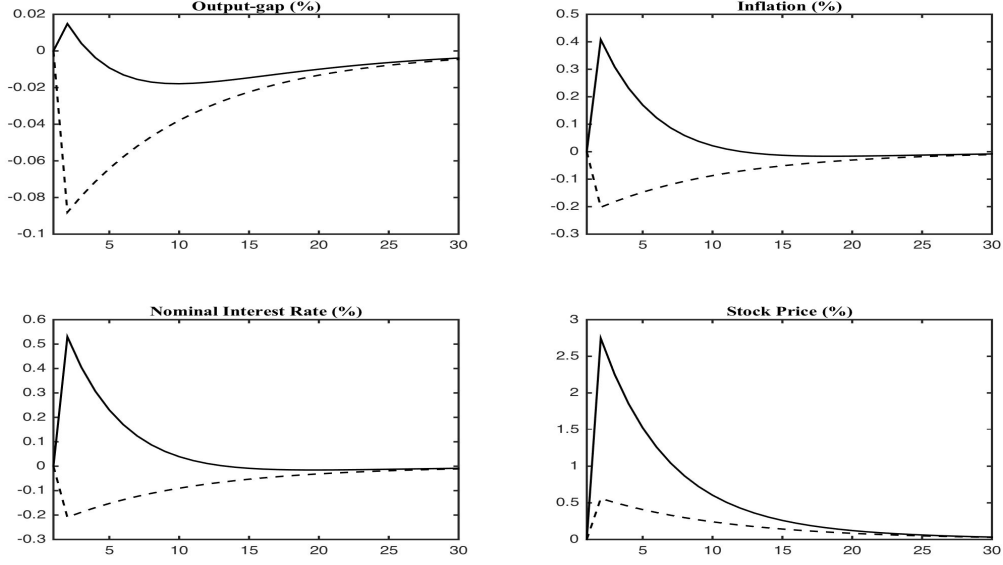


Figure 2.13: The responses to a one standard deviation positive innovation to the productivity shock $\hat{a}_t$. The solid lines show responses under diverse RB with monetary rule $\hat{r}_t = 1.1\hat{\pi}_t + 0.5\hat{x}_t + 0.022\hat{p}_t^s$; the dashed lines show responses under RE with monetary rule $\hat{r}_t = 1.1\hat{\pi}_t + 0.5\hat{x}_t + 0.043\hat{p}_t^s$.

prices produces a consistent reduction in both output gap and inflation volatility. This different result is to be ascribed entirely to the presence of a financial wealth channel that impacts on consumption spending. Bernanke and Gertler (1999) focus on a wealth channel that operates through the balance sheets of firms. However, once we shut down the financial wealth channel under RE by setting $\gamma = 0$, the connection between stocks market and real economy disappears, both output gap and inflation volatility become increasing function of ϕ_{ps} . Hence, there is no advantage for central banks to react to stock prices fluctuations.

In the second case, with the introduction of diverse RB, the effect of stock prices fluctuations on real economy depends not only on the wealth channel but also on the expectational channel. In this framework, results suggest that central banks should respond to fluctuations in asset prices. Although the minimum level of output gap and inflation volatility is not reached for the same value of ϕ_{ps} , it is still beneficial to include the target, especially if central banks are more concerned with inflation volatility. An interesting result is obtained simulating the model under RB with $\gamma = 0$. Unlike the RE case there is still a connection between the real economy and the stock market and it operates via the expectational channel. This is shown by the fact that, the inflation volatility preserves its U-shaped function with respect to ϕ_{ps} when $\gamma = 0$ and once we strengthen the expectational channel, by increasing both learning feedback parameters λ_Z^a and λ_Z^η , also the output gap recovers its U-shaped function. Hence, a moderate “leaning against the wind” strategy reduces both output gap and inflation volatility.

Moreover, results under diverse RB exhibit higher volatility and the magnitude of responses

to shocks is amplified by market belief dynamics. Widespread optimism boosts inflation as well as output gap making the economy work beyond potential. Also, higher market optimism (i.e. higher Z_t) amplifies the effect of increased productivity and, via the positive revision of mean market belief Z_t , partially offsets (or reverse) the fall in output gap observed under RE. As in Kurz et al. (2018) we also observed market belief dynamics create circumstances in which the economy exhibits a “stagflation” where output gap and inflation respond with opposite signs. In such circumstances the monetary authority is in a dilemma since the effects of higher inflation and lower output gap on the interest rate partially offset. Finally, we observed that swings in optimism alone can generate a bubble in asset prices. The effects on the real economy of such belief exuberance might be reduced by a more “aggressive” policy rule.

References

- Adam, K. and Merkel, S. (2018). Stock price cycles and business cycles. Working Paper.
- Airaudo, M., Nisticò, S., and Zanna, L. F. (2015). Learning , Monetary Policy and Asset Prices. *Journal of Money, Credit and Banking*, 47(7):1273–1307.
- Allen, H. and Taylor, M. (1992). Chartist analysis. In Milgate, M. and Newman P., E. J., editors, *The New Palgrave Dictionary of Money and Finance*, pages 73–126. MacMillan.
- Assenza, T., Berardi, M., and Delli Gatti, D. (2011). Was Bernanke Right ? Targeting Asset Prices may not be a Good Idea after all. *Working Paper*.
- Basu, S. (1996). Procyclical productivity: increasing returns or cyclical utilization? *Quarterly Journal of Economics*, 111:719–751.
- Bernanke, B. and Gertler, M. (1999). Monetary policy and asset price volatility. *Federal Reserve Bank of Kansas City Economic Review*, Fourth Quarter 84:17–51.
- Bernanke, B. S. (2004). What policymakers can learn from asset prices Remarks. Speech Before the Investment Analyst Society of Chicago.
- Bernanke, B. S. and Gertler, M. (2001). Should central banks respond to movements in asset prices? *American Economic Review*, 91:253–257.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, M. and Woodford, J., editors, *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. North-Holland, Amsterdam: North Holland.
- Blanchard, O. and Galí, J. (2007). Real wage rigidities and the new keynesian model. *Journal of Money, Credit and Banking*, 39:35–65.
- Blanchard, O. and Kahn, C. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48:1305–1311.
- Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–247.

- Branch, W. (2004). The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. *The Economic Journal*, 114(497):592–621.
- Branch, W. A. and Evans, G. W. (2011). Monetary policy and heterogeneous expectations. *Economic Theory*, 47(2-3):365–393.
- Branch, W. A. and McGough, B. (2009). A new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics & Control*, 33:1036–1051.
- Brock, W. A. and Hommes, C. H. (1997). A rational route to randomness. *Econometrica*, 65:1059–1095.
- Bullard, J. B. and Schaling, E. (2002). Why the fed should ignore the stock market. *Federal Reserve Bank of St. Louis Economic Review*, pages 35–41.
- Calvo, G. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, 12:983–998.
- Carlstrom, C. T. and Fuerst, T. S. (2007). Asset prices, nominal rigidities, and monetary policy. *Review of Economic Dynamics*, 10(2):256–275.
- Carroll, C. (2003). Macroeconomic expectations of households and professional forecasters. *Quarterly Journal of Economics*, 118:269–298.
- Castelnuovo, E. and Nisticò, S. (2010). Stock market conditions and monetary policy in a dsge model for the u.s. *Journal of Economic Dynamics & Control*, 34:1700–1731.
- Cecchetti, S. G., Genberg, H., Lipsky, J., and Wadhvani, S. (2000). Asset Prices and Central Bank Policy. Technical report, CEPR and ICMB, Geneva.
- Cecchetti, S. G., Genberg, H., and Wadhvani, S. (2002). Asset Prices in a Flexible Inflation Targeting Framework. In Hunter, W.C., Kaufman, G.G., Pomerleano, M., editor, *Asset Price Bubbles: Implications for Monetary, Regulatory, and International Policies*. MIT Press, Cambridge, MA.
- De Grauwe, P. (2011). Animal spirits and monetary policy. *Economic Theory*, 47:423–457.
- De Grauwe, P. (2012). *Lectures on behavioral macroeconomics*. Princeton University Press.
- Eichenbaum, M. (2010). Real business-cycle theory, wisdom or whimsy?. *Journal of Economic Dynamics & Control*, 15:607–626.
- Evans, G. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press.

- Frankel, J. and Froot, K. (1990). Chartists, Fundamentalists and the Demand for Dollars. In Courakis, A. and Taylor, M., editors, *Private Behaviour and Government Policies in Interdependent Economies*, pages 73–126. Oxford University Press.
- Galí, J., G. M. and D., J. L.-S. (2001). European inflation dynamics. *European Economic Review*, 45:1237–1270.
- Galí, J. (2008). *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton.
- Kahneman, D. and Tversky, A. (1973). On the Psychology of Prediction. *Psychological Review*, 80(4).
- Kent, L. and Piccillo, G. (2017). Rational beliefs in an estimated DSGE model. Working Paper.
- King, R. and Rebelo, S. (1999). Resuscitating real business cycles. In Taylor, J. and Woodford, M., editors, *Handbook of Macroeconomics, vol. 1B.*, chapter 14, pages 927–1007. North-Holland, Amsterdam.
- Kurz, M. (1994). On the structure and diversity of rational beliefs. *Economic Theory*, 4:877–900.
- Kurz, M. (1997). *Endogenous Economic Fluctuations: Studies in the Theory of Rational Belief. Studies in Economic Theory, Vol. 6*. Springer-Verlag, Berlin and New York.
- Kurz, M. (2008). Beauty contests, private information and diverse beliefs. *Journal of Mathematical Economics*, 44:762–784.
- Kurz, M. (2009). Rational diverse beliefs and economic volatility. In Hens, T. and Schenk-Hoppé, K., editors, *Handbook of Financial Markets: Dynamics and Evolution*, chapter 8, pages 439–506. North-Holland, Amsterdam.
- Kurz, M. (2011). Symposium: On the role of market belief in economic dynamics, an introduction. *Economic Theory*, 47(2):189–204.
- Kurz, M. and Motolese, M. (2001). Endogenous uncertainty and market volatility. *Economic Theory*, 17:497–544.
- Kurz, M. and Motolese, M. (2011). Diverse beliefs and time variability of risk premia. *Economic Theory*, 47:293–335.
- Kurz, M., Motolese, M., Piccillo, G., and Wu, H. (2018). Monetary policy with diverse private expectations. Working Paper.
- Kurz, M., Piccillo, G., and Wu, H. (2013). Modeling diverse expectations in an aggregated new keynesian model. *Journal of Economic Dynamics & Control*, 37:1403–1433.

- Massaro, D. (2013). Heterogeneous expectations in monetary DSGE models. *Journal of Economic Dynamics & Control*, 37:680–692.
- Mian, A. and Sufi, A. (2011). House prices, home equity-based borrowing, and the U.S. household leverage crisis. *American Economic Review*, 101:2132–2156.
- Milani, F. (2017). Learning about the interdependence between the macroeconomy and the stock market. *International Review of Economics and Finance*, 49:223–242.
- Nielsen, C. K. (1996). Rational belief structures and rational belief equilibria. *Economic Theory*, 8:399–422.
- Nisticò, S. (2012). Monetary policy and stock-price dynamics in a DSGE framework. *Journal of Macroeconomics*, 34(1):126–146.
- Pfajfar, D. and Santoro, E. (2010). Heterogeneity, learning and information stickiness in inflation expectations. *Journal of Economic Behavior and Organization*, 75(3):426–444.
- Sargent, T. (1993). *Bounded Rationality in Macroeconomics*. Oxford University Press.
- Sbordone, A. M. (2002). Prices and unit labor costs: A new test of price stickiness. *Journal of Monetary Economics*, 49(2):265–292.
- Stock, H. and Watson, M. (2003). Forecasting output and inflation: The role of asset prices. *Journal of Economic Literature*, 41(3):788–829.
- Summers, L. (1986). Some skeptical observations on real business cycle theory. *Federal Reserve Bank of Minneapolis Quarterly Review*, 10:23–27.
- Taylor, J. B. (1993). Discretion practice versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.
- Woodford, M. (1997). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press,, Princeton.
- Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32(2):137–150.

Appendix A Individual Consumption

Using (2.8) define

$$M_{h,t+1}^j = \beta \frac{P_t C_{h,t}^j}{P_{t+1} C_{h,t+1}^j} = \frac{1}{(1+r_t)} + \tau_b \frac{B_{h,t}^{*j}}{P_t} \frac{C_{h,t}^j}{(1+r_t)} \quad (\text{A-1})$$

Insert (2.9) and (2.10) into the nominal budget constraint:

$$P_t C_{h,t}^j + B_{h,t}^{*j} + P_t P_t^s S_{h,t}^j \leq W_t^n L_{h,t}^j + \Omega_{h,t}^{*j}$$

use (A-1) and obtain

$$(1+\delta)P_t C_{h,t}^j + (1-\gamma) \mathbb{E}_t^j \left[M_{h,t+1}^j \Omega_{h,t+1}^{*j} \right] - \tau_b \frac{(B_{h,t}^{*j})^2}{P_t} C_{h,t}^j - \tau_s (S_{h,t}^j - 1) S_{h,t}^j P_t C_{h,t}^j = W_t^n + \Omega_{h,t}^{*j} \quad (\text{A-2})$$

Solving (A-2) forward and using (2.8) results in

$$\Omega_{h,t}^{*j} = \mathbb{E}_t^j \sum_{k=0}^{\infty} (1-\gamma)^k M_{h,t+k}^j \left[(1+\delta)P_{t+k} C_{h,t+k}^j - \tau_b \frac{(B_{h,t+k}^{*j})^2}{P_{t+k}} C_{h,t+k}^j - \tau_s (S_{h,t+k}^j - 1) S_{h,t+k}^j P_{t+k} C_{h,t+k}^j - W_{t+k}^n \right] \quad (\text{A-3})$$

where $M_{h,t+k}^j = \beta^k \frac{P_t C_{h,t}^j}{P_{t+k} C_{h,t+k}^j}$. Let

$$\Sigma = \sum_{k=0}^{\infty} \beta^k (1-\gamma)^k = \frac{1}{1-\beta(1-\gamma)} \quad (\text{A-4})$$

$$\nu_{h,t}^{*j} = \mathbb{E}_t^j \sum_{k=0}^{\infty} (1-\gamma)^k M_{h,t+k}^j W_{t+k}^n \quad (\text{A-5})$$

$$T_{h,t}^{*j} = \mathbb{E}_t^j \sum_{k=0}^{\infty} (1-\gamma)^k M_{h,t+k}^j \left[\tau_b \frac{(B_{h,t+k}^{*j})^2}{P_{t+k}} C_{h,t+k}^j + \tau_s (S_{h,t+k}^j - 1) S_{h,t+k}^j P_{t+k} C_{h,t+k}^j \right] \quad (\text{A-6})$$

Since all agents have the same wage, human wealth is independent of the length of time being active in the financial markets, i.e. $\nu_t^{*j} = \nu_{h,t}^{*j}$. Finally, using (A-4)-(A-6), rewrite (A-3) as in the main text:

$$P_t C_{h,t}^j = \frac{1}{\Sigma(1+\delta)} \left[(\Omega_{h,t}^{*j} + T_{h,t}^{*j}) + \nu_t^{*j} \right]$$

Appendix B Deriving Equation (2.20)

Consider the aggregation across cohorts of equation (A-2) in real terms:

$$(1+\delta)C_t^j + \mathbb{E}_t^j \left[M_{t+1}^j \Omega_{t+1}^j \Pi_{t+1} \right] - \tau_b \left(\frac{B_t^{*j}}{P_t} \right)^2 C_t^j - \tau_s (S_t^j - 1) S_t^j C_t^j = W_t + \Omega_t^j \quad (\text{B-1})$$

From (2.19) we can write:

$$\Omega_t^j = \Sigma (1 + \delta) C_t^j - \nu_t^j - T_t^j \quad (\text{B-2})$$

Insert (B-2) in (B-1) and obtain

$$(\Sigma - 1) C_t^j = \frac{1}{(1 + \delta)} \left[\mathbb{E}_t^j \left(M_{t+1}^j \Omega_{t+1}^j \Pi_{t+1} \right) + \left(T_t^j - \tau_b \left(\frac{B_t^{*j}}{P_t} \right)^2 C_t^j - \tau_s (S_t^j - 1) S_t^j C_t^j + \nu_t^j - W_t \right) \right] \quad (\text{B-3})$$

From (2.18) one period forward, after taking conditional expectations and using definitions (A-5)-(A-6) we get:

$$\begin{aligned} \Sigma (1 - \gamma) \mathbb{E}_t^j \left(M_{t+1}^j C_{t+1}^j P_{t+1} \right) &= \frac{\Sigma (1 - \gamma)}{\Sigma (1 + \delta)} \mathbb{E}_t^j \left[M_{t+1}^j \left(\Omega_{t+1}^{*j} + T_{t+1}^{*j} + \nu_{t+1}^{*j} \right) \right] = \\ &= \frac{(1 - \gamma)}{(1 + \delta)} \mathbb{E}_t^j \left[M_{t+1}^j \Omega_{t+1}^{*j} \right] + \frac{1}{(1 + \delta)} \left[T_t^{*j} - \tau_b \frac{(B_t^{*j})^2}{P_t} C_t^j - \tau_s (S_t^j - 1) S_t^j P_t C_t^j + \nu_t^{*j} - W_t^* \right] \end{aligned} \quad (\text{B-4})$$

from (B-4) we can conclude that

$$\begin{aligned} \frac{1}{(1 + \delta)} \left(T_t^j - \tau_b \left(\frac{B_t^{*j}}{P_t} \right)^2 C_t^j - \tau_s (S_t^j - 1) S_t^j C_t^j + \nu_t^j - W_t \right) &= \Sigma (1 - \gamma) \mathbb{E}_t^j \left(M_{t+1}^j C_{t+1}^j \Pi_{t+1} \right) \\ &\quad - \frac{(1 - \gamma)}{(1 + \delta)} \mathbb{E}_t^j \left[M_{t+1}^j \Omega_{t+1}^j \Pi_{t+1} \right] \end{aligned} \quad (\text{B-5})$$

and after inserting the right hand side of (B-5) in (B-3) we finally get equation (2.20) in the text:

$$(\Sigma - 1) C_t^j = \Sigma (1 - \gamma) \mathbb{E}_t^j \left(M_{t+1}^j C_{t+1}^j \Pi_{t+1} \right) + \frac{\gamma}{(1 + \delta)} \mathbb{E}_t^j \left(M_{t+1}^j \Omega_{t+1}^j \Pi_{t+1} \right)$$

Appendix C Steady State and Log-linearization

Steady State: Steady state evaluation of (2.13) implies $\overline{M} (1 + \bar{r}) = 1$. Evaluating (2.20) at steady state and considering (A-4) we obtain the following

$$(\Sigma - 1) \overline{C} = (1 - \gamma) \Sigma \frac{1}{1 + \bar{r}} \overline{C} + \frac{\gamma}{1 + \delta} \frac{\overline{\Omega}}{1 + \bar{r}} \quad (\text{C-1})$$

$$1 + \bar{r} = \frac{(1 - \gamma) \Sigma}{\Sigma - 1} + \frac{\gamma}{(1 + \delta) (\Sigma - 1)} \frac{\overline{\Omega}}{\overline{C}} \quad (\text{C-2})$$

$$1 + \bar{r} = \frac{1}{\beta} + \gamma \frac{1 - \beta (1 - \gamma)}{\beta (1 - \gamma) (1 + \delta)} \frac{\overline{\Omega}}{\overline{C}} \quad (\text{C-3})$$

Let $\psi = \frac{\gamma}{(1 + \delta)} \frac{1 - \beta (1 - \gamma)}{(1 - \gamma)} \frac{\overline{\Omega}}{\overline{C}}$ and finally obtain

$$\frac{1}{1 + \bar{r}} = \frac{\beta}{1 + \psi} = \tilde{\beta} = \overline{M} \quad (\text{C-4})$$

As reported in the text steady state marginal cost is $\bar{\zeta} = \frac{\bar{W}}{\bar{A}} = \frac{1}{(1+\mu)}$. Hence $\bar{W} = \frac{\bar{A}}{1+\mu}$. Aggregate dividends at steady state are:

$$\bar{D} = \left(1 - \frac{\bar{W}}{\bar{A}}\right) \bar{Y} = \left(1 - \frac{1}{1+\mu}\right) \bar{Y} = \left(\frac{\mu}{1+\mu}\right) \bar{Y} \quad (\text{C-5})$$

Evaluation of equation (2.14) at steady state results in:

$$\bar{P}^s = \bar{M} (\bar{P}^s + \bar{D}) = \frac{\bar{M}}{1 - \bar{M}} \bar{D} = \frac{\tilde{\beta}}{1 - \tilde{\beta}} \bar{D} = \frac{\tilde{\beta}}{1 - \tilde{\beta}} \frac{\mu}{1 + \mu} \bar{Y} \quad (\text{C-6})$$

Steady state real financial wealth using equation (2.18) is

$$\bar{\Omega} = \bar{P}^s + \bar{D} = \left(\frac{\tilde{\beta}}{1 - \tilde{\beta}} + 1\right) \bar{D} = \frac{1}{1 - \tilde{\beta}} \bar{D} = \frac{1}{1 - \tilde{\beta}} \frac{\mu}{1 + \mu} \bar{Y} \quad (\text{C-7})$$

To close the computation of steady state values we need to solve for $\bar{Y}$. Using equation (2.15)

$$\begin{aligned} \delta \bar{Y} &= \bar{W} (1 - \bar{N}) \\ \bar{N} &= 1 - \frac{\delta \bar{Y}}{\bar{W}} \end{aligned} \quad (\text{C-8})$$

and knowing from the production function that $\bar{Y} = \bar{A} \bar{N}$ we finally obtain

$$\bar{Y} = \bar{A} - \frac{\bar{A}}{\bar{W}} \delta \bar{Y} = \bar{A} - \delta (1 + \mu) \bar{Y} = \frac{\bar{A}}{1 + \delta (1 + \mu)} \quad (\text{C-9})$$

Log-linearization: Log-linearization of (2.17) is obtained as follows.

$$\bar{\Omega} \hat{\omega}_t^j = (1 + \bar{r}) \hat{b}_{t-1}^j \bar{\Omega} + (\bar{P}^s + \bar{D}) \hat{s}_{t-1}^j + \bar{P}^s \hat{p}_t^s + \bar{D} \hat{d}_t \quad (\text{C-10})$$

recall from steady state calculations that $\tilde{\beta} = \frac{1}{1 + \bar{r}}$, $\frac{\bar{P}^s}{\bar{\Omega}} = \tilde{\beta}$, $\frac{\bar{D}}{\bar{\Omega}} = 1 - \tilde{\beta}$ and $\bar{\Omega} = \bar{P}^s + \bar{D}$, hence (C-10) becomes (2.41) in the text

$$\hat{\omega}_t^j = \frac{1}{\tilde{\beta}} \hat{b}_{t-1}^j + \hat{s}_{t-1}^j + \tilde{\beta} \hat{p}_t^s + (1 - \tilde{\beta}) \hat{d}_t$$

where $\hat{b}_t^j = \frac{B_t^j}{\bar{\Omega}}$. Log-linearizing agent j 's optimality conditions (2.14) we get

$$\begin{aligned} \bar{P}^s \hat{p}_t^s &= \frac{\bar{P}^s + \bar{D}}{(1 + \bar{r})} \left[\tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j - \hat{r}_t + \mathbb{E}_t^j \hat{\pi}_{t+1} \right] + \frac{1}{(1 + \bar{r})} \bar{P}^s \mathbb{E}_t^j \hat{p}_{t+1}^s + \frac{1}{(1 + \bar{r})} \bar{D} \mathbb{E}_t^j \hat{d}_{t+1} - \tau_s \bar{Y} \hat{s}_t^j \\ \hat{p}_t^s &= \frac{\bar{P}^s + \bar{D}}{\bar{P}^s} \frac{1}{(1 + \bar{r})} \left[\tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j - \hat{r}_t + \mathbb{E}_t^j \hat{\pi}_{t+1} \right] + \frac{1}{(1 + \bar{r})} \mathbb{E}_t^j \hat{p}_{t+1}^s + \frac{1}{(1 + \bar{r})} \frac{\bar{D}}{\bar{P}^s} \mathbb{E}_t^j \hat{d}_{t+1} - \tau_s \frac{\bar{Y}}{\bar{P}^s} \hat{s}_t^j \end{aligned} \quad (\text{C-11})$$

and given the steady state conditions recalled above and $\frac{\bar{Y}}{\bar{P}^s} = \frac{(1 - \tilde{\beta}) (1 + \mu)}{\tilde{\beta}}$ and $\frac{\bar{D}}{\bar{P}^s} = \frac{(1 - \tilde{\beta})}{\tilde{\beta}}$ equation (C-11) becomes (2.43) in the text:

$$\hat{p}_t^s = \mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{\Omega} \bar{Y} \hat{b}_t^j - \tau_s \frac{(1 - \tilde{\beta}) (1 + \mu)}{\tilde{\beta}} \hat{s}_t^j + \tilde{\beta} \mathbb{E}_t^j \hat{p}_{t+1}^s + (1 - \tilde{\beta}) \mathbb{E}_t^j \hat{d}_{t+1}$$

where $\hat{r}_t = \frac{r_t - \bar{r}}{1 + \bar{r}}$. Condition (2.15) is log-linearized as follows

$$\begin{aligned}\delta \bar{C} \hat{c}_t^j &= (1 - \bar{L}) \bar{W} \hat{w}_t - \bar{W} \bar{L} \hat{l}_t^j \\ \hat{c}_t^j &= \frac{(1 - \bar{L}) \bar{W}}{\delta \bar{C}} \hat{w}_t - \frac{\bar{W} \bar{L}}{\delta \bar{C}} \hat{l}_t^j\end{aligned}$$

and considering $\delta \bar{C} = \delta \bar{Y} = \bar{W} (1 - \bar{L})$,

$$\hat{c}_t^j = \hat{w}_t - \frac{\bar{L}}{1 - \bar{L}} \hat{l}_t^j$$

and setting $\varphi = \frac{\bar{L}}{1 - \bar{L}}$, we get equation (2.45) in the text

$$\hat{c}_t^j = \hat{w}_t - \varphi \hat{l}_t^j$$

To obtain the log-linear condition (2.46), consider equation (2.20) and its log-linearization is derived as follows

$$\begin{aligned}(\Sigma - 1) \bar{Y} \hat{c}_t^j &= \frac{\Sigma(1 - \gamma) \bar{Y}}{1 + \bar{r}} \left[\tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j - \hat{r}_t + \mathbb{E}_t^j \hat{\pi}_{t+1} + \mathbb{E}_t^j \hat{c}_{t+1}^j \right] + \frac{\gamma}{1 + \delta} \frac{\bar{\Omega}}{1 + \bar{r}} \left[\tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j - \hat{r}_t + \mathbb{E}_t^j \hat{\pi}_{t+1} + \mathbb{E}_t^j \hat{\omega}_{t+1}^j \right] \\ \hat{c}_t^j &= \frac{\Sigma}{\Sigma - 1} \frac{1 - \gamma}{1 + \bar{r}} \left[\mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j + \mathbb{E}_t^j \hat{c}_{t+1}^j \right] + \\ &\quad + \frac{\gamma}{1 + \delta} \frac{1}{\Sigma - 1} \frac{1}{1 + \bar{r}} \frac{\bar{\Omega}}{\bar{Y}} \left[\mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j + \mathbb{E}_t^j \hat{\omega}_{t+1}^j \right]\end{aligned}$$

note that $\frac{\Sigma}{\Sigma - 1} \frac{1 - \gamma}{1 + \bar{r}} = \frac{1}{1 + \psi}$ and $\frac{\gamma}{1 + \delta} \frac{1}{\Sigma - 1} \frac{1}{1 + \bar{r}} \frac{\bar{\Omega}}{\bar{Y}} = \frac{\psi}{1 + \psi}$, hence

$$\hat{c}_t^j = \frac{1}{1 + \psi} \left[\mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j + \mathbb{E}_t^j \hat{c}_{t+1}^j \right] + \frac{\psi}{1 + \psi} \left[\mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \tau_b \bar{Y} \bar{\Omega} \hat{b}_t^j + \mathbb{E}_t^j \hat{\omega}_{t+1}^j \right]$$

which gives equation (2.46) in the text

$$\hat{c}_t^j = \mathbb{E}_t^j \hat{\pi}_{t+1} - \hat{r}_t + \frac{1}{(1 + \psi)} \mathbb{E}_t^j \hat{c}_{t+1}^j + \tau_b \bar{\Omega} \bar{Y} \hat{b}_t^j + \frac{\psi}{(1 + \psi)} \mathbb{E}_t^j \hat{\omega}_{t+1}^j$$

Log-linearization involving the budget constraint (2.12) is

$$\begin{aligned}\bar{Y} \hat{c}_t^j + \bar{\Omega} \hat{b}_t^j + \bar{P}^s \hat{s}_t^j + \bar{P}^s \hat{p}_t^s &= \bar{L} \bar{W} \hat{l}_t^j + \bar{L} \bar{W} \hat{w}_t + (1 + \bar{r}) \bar{\Omega} \hat{b}_{t-1}^j + \bar{P}^s \hat{s}_{t-1}^j + \bar{P}^s \hat{p}_t^s + \bar{D} \hat{s}_{t-1}^j + \bar{D} \hat{d}_t \\ \frac{\bar{Y}}{\bar{\Omega}} \hat{c}_t^j + \hat{b}_t^j + \frac{\bar{P}^s}{\bar{\Omega}} \hat{s}_t^j + \frac{\bar{P}^s}{\bar{\Omega}} \hat{p}_t^s &= \frac{\bar{L} \bar{W}}{\bar{\Omega}} \hat{l}_t^j + \frac{\bar{L} \bar{W}}{\bar{\Omega}} \hat{w}_t + (1 + \bar{r}) \hat{b}_{t-1}^j + \frac{\bar{P}^s}{\bar{\Omega}} \hat{s}_{t-1}^j + \frac{\bar{P}^s}{\bar{\Omega}} \hat{p}_t^s + \frac{\bar{D}}{\bar{\Omega}} \hat{s}_{t-1}^j + \frac{\bar{D}}{\bar{\Omega}} \hat{d}_t\end{aligned}$$

recall $\frac{\bar{P}^s}{\bar{\Omega}} = \tilde{\beta}$, $\frac{\bar{D}}{\bar{\Omega}} = 1 - \tilde{\beta}$ and $\frac{\bar{Y}}{\bar{L} \bar{W}} = \frac{1}{1 + \mu}$, $(1 + \bar{r}) = \frac{1}{\beta}$ and using (2.45) we get

$$\frac{\bar{Y}}{\bar{\Omega}} \hat{c}_t^j + \hat{b}_t^j + \tilde{\beta} \hat{s}_t^j = \frac{\bar{Y}}{\bar{\Omega}} \frac{1}{1 + \mu} \left(\frac{1}{\varphi} (\hat{w}_t - \hat{c}_t^j) \right) + \frac{\bar{Y}}{\bar{\Omega}} \frac{1}{1 + \mu} \hat{w}_t + \frac{1}{\beta} \hat{b}_{t-1}^j + \hat{s}_{t-1}^j + (1 - \tilde{\beta}) \hat{d}_t \quad (\text{C-12})$$

given $\frac{\bar{Y}}{\bar{\Omega}} = (1 - \tilde{\beta}) \frac{(1 + \mu)}{\mu}$, after collecting terms (C-12) becomes equation (2.48) in the text

$$\hat{b}_t^j = \frac{1}{\tilde{\beta}} \hat{b}_{t-1}^j - \tilde{\beta} \hat{s}_t^j + \hat{s}_{t-1}^j + (1 - \tilde{\beta}) \frac{1 + \varphi}{\mu \varphi} \hat{w}_t - (1 - \tilde{\beta}) \frac{1 + \varphi}{\mu} \left(1 + \frac{1}{(1 + \mu) \varphi} \right) \hat{c}_t^j + (1 - \tilde{\beta}) \hat{d}_t$$

Log-linearization of the optimal price equation in (2.29) results from the following: first rewrite (2.29) as

$$\mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k M_{t+k}^i Y_{t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon-1} Q_{j,t} = \frac{\varepsilon}{\varepsilon - 1} \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k M_{t+k}^i Y_{t+k} \zeta_{t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon} \quad (\text{C-13})$$

then log-linearization on both sides gives

$$\begin{aligned} \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \bar{Y} \left[\tau_b \bar{Y} \bar{\Omega} \hat{b}_{t+k}^j - (1 + \bar{r}) \hat{r}_{t+k} + \hat{y}_{t+k} + (\varepsilon - 1) (\hat{p}_{t+k} - \hat{p}_t) + \hat{q}_{j,t} \right] = \\ = \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \bar{Y} \bar{\zeta} \frac{\varepsilon}{\varepsilon - 1} \left[\tau_b \bar{Y} \bar{\Omega} \hat{b}_{t+k}^j - (1 + \bar{r}) \hat{r}_{t+k} + \hat{y}_{t+k} + \hat{\zeta}_{t+k} - \varepsilon (\hat{p}_{t+k} - \hat{p}_t) \right] \end{aligned}$$

in steady state prices are fully flexible and $\bar{\zeta} \frac{\varepsilon}{\varepsilon - 1} = 1$, hence

$$\begin{aligned} \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \hat{q}_{j,t} = \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \left[\hat{\zeta}_{t+k} + (\hat{p}_{t+k} - \hat{p}_t) \right] \\ \frac{1}{1 - \theta \tilde{\beta}} (\hat{q}_{j,t} + \hat{p}_t) = \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \left[\hat{\zeta}_{t+k} + \hat{p}_{t+k} \right] \end{aligned} \quad (\text{C-14})$$

$$\hat{q}_{j,t} + \hat{p}_t = (1 - \theta \tilde{\beta}) \left(\hat{\zeta}_t + \hat{p}_t \right) + (1 - \theta \tilde{\beta}) \theta \tilde{\beta} \mathbb{E}_t^j \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \left[\hat{\zeta}_{t+k+1} + \hat{p}_{t+k+1} \right] \quad (\text{C-15})$$

and after moving (C-14) one period forward and taking conditional expectations we get an expression to substitute in the last term on the right hand side of (C-15), hence

$$\hat{q}_{j,t} + \hat{p}_t = (1 - \theta \tilde{\beta}) \left(\hat{\zeta}_t + \hat{p}_t \right) + \theta \tilde{\beta} \mathbb{E}_t^j (\hat{q}_{j,t+1} + \hat{p}_{t+1})$$

which finally results in equation (2.49) in the text

$$\hat{q}_{j,t} = (1 - \tilde{\beta} \theta) \hat{\zeta}_t + \tilde{\beta} \theta \mathbb{E}_t^j (\hat{q}_{j,t+1} + \hat{p}_{t+1})$$

Appendix D RBE Restrictions and the Role of Learning Feed-back

The RB principle (see Kurz (1994)) is a model of rational agents who deviate from m but reproduce it with sufficiently long data. An RB model of the exogenous shocks exhibits the same volatility as the empirical model

and it implies the following restrictions (for details, see Kurz et al. (2013)):

$$\begin{aligned}\text{Var}[\lambda_a^g g_t^j + \rho_{t+1}^{aj}] &= \text{Var}[\rho_{t+1}^a] \Rightarrow (\lambda_a^g)^2 \text{Var}(g) + \hat{\sigma}_a^2 = \sigma_a^2 \\ \text{Var}[\lambda_\eta^g g_t^j + \rho_{t+1}^{\eta j}] &= \text{Var}[\rho_{t+1}^\eta] \Rightarrow (\lambda_\eta^g)^2 \text{Var}(g) + \hat{\sigma}_\eta^2 = \sigma_\eta^2\end{aligned}\quad (\text{D-1})$$

By normalization $\lambda_a^g = 1$, therefore the rationality conditions (D-1) imply

$$\text{Var}(g) \leq \sigma_a^2, (\lambda_\eta^g)^2 \text{Var}(g) \leq \sigma_\eta^2, (\lambda_Z^g)^2 \text{Var}(g) \leq \sigma_Z^2, \tilde{\sigma}_a \leq \sigma_a, \tilde{\sigma}_\eta \leq \sigma_\eta, \tilde{\sigma}_Z \leq \sigma_Z \quad (\text{D-2})$$

In addition, the variance of ρ_{t+1}^Z is restricted by σ_g^2 and is specified as

$$\sigma_Z^2 \leq \sigma_g^2 \text{ with } \sigma_Z = \rho \sigma_g \text{ and } \rho = \text{Corr}(\rho_{t+1}^{gj}, \rho_{t+1}^Z) > 0 \quad (\text{D-3})$$

Keep in mind that by Assumption 3 the variance of ρ_{t+1}^{gj} is $\sigma_g^2 = \text{Var}(\Upsilon_{t+1}(1 + \epsilon_{t+1})) = \sigma_Z^2(1 + \sigma_\epsilon^2)$ while the covariance between individual and mean market belief is $\text{Cov}(\rho_{t+1}^{gj}, \rho_{t+1}^Z) = \mathbb{E}((1 + \epsilon_{t+1}^{gj})\Upsilon_{t+1}^2) = \sigma_Z^2$. It follows that $\rho = \text{Cov}(\rho_{t+1}^{gj}, \rho_{t+1}^Z)/(\sigma_g \sigma_Z) = 1/\sqrt{1 + \sigma_\epsilon^2}$ and this puts some restrictions on the choice of σ_ϵ^2 : $\sigma_\epsilon^2 = 1/\rho^2 - 1$.

The unconditional variance of g_t^j is

$$\text{Var}(g) = \frac{1}{1 - \lambda_Z^2} [(\lambda_a^g)^2 \sigma_a^2 + (\lambda_\eta^g \lambda_\eta^g)^2 \sigma_\eta^2 + \sigma_g^2] \quad (\text{D-4})$$

The parameters $(\lambda_Z^a, \lambda_Z^\eta)$ are important. They measure learning feed-back from current data with which agents deduce changes in estimated value of ς_t in (2.50) from forecast errors in (2.57). This causes revisions of the belief index g_t^j which is j 's subjective conditional expectations of ς_t . The variance of g which ignores such feed-back is therefore

$$\text{Var}^{NF}(g) = \frac{\sigma_g^2}{1 - \lambda_Z^2} \quad (\text{D-5})$$

Interest in (D-5) arises from the need to reconcile learning feed-back with the RB principle. As pointed out in Kurz et al. (2018), learning feed-back violates the RB principle and a rational agent who adjusts his belief about exogenous shocks in response to most recent data knows there is no learning within the actual data of exogenous shocks hence he must purge (D-1)-(D-5) from the effect of learning feed-back. This is the reason why, as in Kurz et al. (2018), we use (D-5) and not (D-4) as the basis for restricting individual beliefs. Sufficient conditions implied by this procedure are

$$\sigma_g \leq \sigma_a \sqrt{(1 - \lambda_Z^2)}, |\lambda_\eta^g| \leq \frac{\sigma_\eta}{\sigma_a}, |\lambda_Z^g| \leq \frac{\sigma_Z}{\sigma_a} \leq \rho \sqrt{(1 - \lambda_Z^2)} \quad (\text{D-6})$$

$$(\lambda_\varsigma^g)^2 \text{Var}^{NF}(g) + \hat{\sigma}_a^2 \geq \sigma_a^2, (\lambda_\eta^g)^2 \text{Var}^{NF}(g) + \hat{\sigma}_\eta^2 \geq \sigma_\eta^2, (\lambda_Z^g)^2 \text{Var}^{NF}(g) + \hat{\sigma}_Z^2 \geq \sigma_Z^2 \quad (\text{D-7})$$

What is the effect of the learning feed-back? Consider (2.53) where future belief of j depends upon realized future data from which he will deduce g_{t+1}^j . Such dependence of belief upon current data amplifies the effect of beliefs. In fact, suppose $g_t^j > 0$ that is agent j is optimistic today about larger ς_t and hence he perceives a larger $\hat{a}_{t+1}^j$ and a larger forecast error $(\hat{a}_{t+1}^j - \lambda_a \hat{a}_t)$ at $t + 1$. With learning feed-back he also knows that he expects to interpret this larger forecast error as a larger value of ς_{t+1} as well, and the larger is the learning feed-back parameter λ_Z^g the larger is this secondary effect of $g_t^j > 0$ on the expected value of ς_{t+1} . The mathematical result of this fact is seen by taking expectations of (2.53) using (2.52):

$$\mathbb{E}_t^j [g_{t+1}^j] = \lambda_Z g_t^j + \lambda_Z^a g_t^j + \lambda_Z^\eta \lambda_\eta^g g_t^j = (\lambda_Z + \lambda_Z^a + \lambda_Z^\eta \lambda_\eta^g) g_t^j$$

Note that, due to learning feed-back from current data the parameter $(\lambda_Z + \lambda_Z^a + \lambda_Z^\eta \lambda_\eta^g)$ of g_t^j in j 's expected g_{t+1}^j

may exceed 1. Being only within the agent's model it does not have any effect on the actual dynamic movements of either g_t^j or Z_t hence has no effect on market instability or violation of Blanchard-Kahn conditions. It is something in the mind of the agent when forecasting and merely a result of the interaction between learning feed-back and persistence of beliefs.

Appendix E Proof of Theorem 1

Proof. In order to explain the parameters $(\lambda_c^\Psi, \lambda_q^\Psi)$ and $(\Gamma^x, \Gamma^\pi, \Gamma^{p^s})$ recall the state space of agent j 's decisions is $I_t^j = (\hat{a}_t, \eta_t, g_t^j, Z_t, \hat{s}_{t-1}^j, \hat{b}_{t-1}^j)$, and given that solutions of endogenous variables in the log-linearized economy are linear in the state variables in I_t^j , individual decision functions are written as

$$\hat{c}_t^j = A_y^a \hat{a}_t + A_y^\eta \eta_t + A_y^g g_t^j + A_y^Z Z_t + A_y^s \hat{s}_{t-1}^j + A_y^b \hat{b}_{t-1}^j \equiv A_y \bullet I_t^j \quad (\text{E-1})$$

$$\hat{q}_t^j = \frac{\theta}{1-\theta} \left(A_\pi^a \hat{a}_t + A_\pi^\eta \eta_t + A_\pi^g g_t^j + A_\pi^Z Z_t + A_\pi^s \hat{s}_{t-1}^j + A_\pi^b \hat{b}_{t-1}^j \right) \equiv \frac{\theta}{1-\theta} A_\pi \bullet I_t^j \quad (\text{E-2})$$

where A_y and A_π are the vectors of parameters of the linear functions of individual consumption and inflation, respectively. Aggregating (E-1) and (E-2) and applying the equilibrium conditions (2.59), (2.61), (2.62) and (2.64) we get the following aggregate linear functions

$$\hat{y}_t = A_y^a \hat{a}_t + A_y^\eta \eta_t + A_y^g Z_t + A_y^Z Z_t + A_y^s 0 + A_y^b 0 \equiv A_y \bullet I_t \quad (\text{E-3})$$

$$\hat{q}_t = \frac{\theta}{1-\theta} (A_\pi^a \hat{a}_t + A_\pi^\eta \eta_t + A_\pi^g Z_t + A_\pi^Z Z_t + A_\pi^s 0 + A_\pi^b 0) \equiv \frac{\theta}{1-\theta} A_\pi \bullet I_t \quad (\text{E-4})$$

$$\hat{\pi}_t = A_\pi^a \hat{a}_t + A_\pi^\eta \eta_t + A_\pi^g Z_t + A_\pi^Z Z_t + A_\pi^s 0 + A_\pi^b 0 \equiv A_\pi \bullet I_t \quad (\text{E-5})$$

where $I_t = (\hat{a}_t, \eta_t, Z_t, Z_t, 0, 0)$ is the vector of state variables for the macro variables.

Given $\hat{y}_t = \hat{c}_t$, using (E-1)-(E-5) and agent j 's perception model in (2.57) we compute λ_c^Ψ and λ_q^Ψ as follows

$$\Psi_t(\hat{c}) = \int_0^1 \mathbb{E}_t^j (\hat{c}_{t+1}^j - \hat{c}_{t+1}) dj = A_y^g \int_0^1 \mathbb{E}_t^j (g_{t+1}^j - Z_{t+1}) dj = -A_y^g \lambda_Z^g Z_t \quad \implies \quad \lambda_c^\Psi = -A_y^g \lambda_Z^g$$

$$\Psi_t(\hat{q}) = \int_0^1 \mathbb{E}_t^j (\hat{q}_{t+1}^j - \hat{q}_{t+1}) dj = \frac{\theta}{1-\theta} A_\pi^g \int_0^1 \mathbb{E}_t^j (g_{t+1}^j - Z_{t+1}) dj = -\frac{\theta}{1-\theta} A_\pi^g \lambda_Z^g Z_t \quad \implies \quad \lambda_q^\Psi = -\frac{\theta}{1-\theta} A_\pi^g \lambda_Z^g$$

Using the same information we can compute Γ^π

$$\begin{aligned} \Gamma^\pi Z_t &= \int_0^1 \mathbb{E}_t^j \hat{\pi}_{t+1} dj - \mathbb{E}_t^m \hat{\pi}_{t+1} = A_\pi^a \left[\int_0^1 \mathbb{E}_t^j \hat{a}_{t+1} dj - \mathbb{E}_t^m \hat{a}_{t+1} \right] + A_\pi^\eta \left[\int_0^1 \mathbb{E}_t^j \eta_{t+1} dj - \mathbb{E}_t^m \eta_{t+1} \right] \\ &+ (A_\pi^g + A_\pi^Z) \left[\int_0^1 \mathbb{E}_t^j Z_{t+1} dj - \mathbb{E}_t^m Z_{t+1} \right] = [A_\pi^a \lambda_a^g + A_\pi^\eta \lambda_\eta^g + (A_\pi^g + A_\pi^Z) (\lambda_Z^a \lambda_a^g + \lambda_Z^\eta \lambda_\eta^g + \lambda_Z^g)] Z_t \\ &\implies \Gamma^\pi = [A_\pi^a \lambda_a^g + A_\pi^\eta \lambda_\eta^g + (A_\pi^g + A_\pi^Z) (\lambda_Z^a \lambda_a^g + \lambda_Z^\eta \lambda_\eta^g + \lambda_Z^g)] \end{aligned}$$

To compute Γ^x recall $\hat{x}_t = \hat{y}_t - \hat{a}_t$, hence $\int_0^1 \mathbb{E}_t^j \hat{x}_{t+1} dj = \int_0^1 \mathbb{E}_t^j \hat{y}_{t+1} dj - \int_0^1 \mathbb{E}_t^j \hat{a}_{t+1} dj$,

$$\begin{aligned} \Gamma^x Z_t &= \int_0^1 \mathbb{E}_t^j \hat{y}_{t+1} dj - \mathbb{E}_t^m \hat{y}_{t+1} - \int_0^1 \mathbb{E}_t^j \hat{a}_{t+1} dj + \mathbb{E}_t^m \hat{a}_{t+1} = (A_y^a - 1) \left[\int_0^1 \mathbb{E}_t^j \hat{a}_{t+1} dj - \mathbb{E}_t^m \hat{a}_{t+1} \right] \\ &\quad + A_y^\eta \left[\int_0^1 \mathbb{E}_t^j \eta_{t+1} dj - \mathbb{E}_t^m \eta_{t+1} \right] + (A_y^g + A_y^Z) \left[\int_0^1 \mathbb{E}_t^j Z_{t+1} dj - \mathbb{E}_t^m Z_{t+1} \right] \\ &= [(A_y^a - 1) \lambda_a^g + A_y^\eta \lambda_\eta^g + (A_y^g + A_y^Z) (\lambda_Z^a \lambda_a^g + \lambda_Z^\eta \lambda_\eta^g + \lambda_Z^g)] Z_t \\ &\implies \Gamma^x = [(A_y^a - 1) \lambda_a^g + A_y^\eta \lambda_\eta^g + (A_y^g + A_y^Z) (\lambda_Z^a \lambda_a^g + \lambda_Z^\eta \lambda_\eta^g + \lambda_Z^g)] \end{aligned}$$

Finally, to compute Γ^{p^s} recall the stock price linear equilibrium function, as for (E-3)-(E-5), is

$$\hat{p}_t^s = A_{p^s}^a \hat{a}_t + A_{p^s}^\eta \eta_t + A_{p^s}^g 0 + A_{p^s}^Z Z_t + A_{p^s}^s 0 + A_{p^s}^b 0 \equiv A_{p^s} \bullet I_t \quad (\text{E-6})$$

and similarly to the computations above

$$\begin{aligned} \Gamma^{p^s} Z_t &= \int_0^1 \mathbb{E}_t^j \hat{p}_{t+1}^s dj - \mathbb{E}_t^m \hat{p}_{t+1}^s = A_{p^s}^a \left[\int_0^1 \mathbb{E}_t^j \hat{a}_{t+1} dj - \mathbb{E}_t^m \hat{a}_{t+1} \right] + A_{p^s}^\eta \left[\int_0^1 \mathbb{E}_t^j \eta_{t+1} dj - \mathbb{E}_t^m \eta_{t+1} \right] \\ &\quad + A_{p^s}^Z \left[\int_0^1 \mathbb{E}_t^j Z_{t+1} dj - \mathbb{E}_t^m Z_{t+1} \right] = [A_{p^s}^a \lambda_a^g + A_{p^s}^\eta \lambda_\eta^g + A_{p^s}^Z (\lambda_Z^a \lambda_a^g + \lambda_Z^\eta \lambda_\eta^g + \lambda_Z^g)] Z_t \\ &\implies \Gamma^{p^s} = [A_{p^s}^a \lambda_a^g + A_{p^s}^\eta \lambda_\eta^g + A_{p^s}^Z (\lambda_Z^a \lambda_a^g + \lambda_Z^\eta \lambda_\eta^g + \lambda_Z^g)] \end{aligned}$$

□