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Dynamics in Financial Networks

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Highlights

- An overview on dynamic networks is provided
- Two cases of study are presented: the dynamics on the network of world's stock exchanges and the dynamics of the network of Interlocking Directorates in Italy.
- The World's Stock Exchanges Network is modeled according to Vitting Andersen et al. [58]: the model has been implemented using another data source and validated.
- The non-linear price dynamics in the model gives a new way to quantify and study disruptions propagating across financial markets: "price-quakes" are defined in terms of avalanches of price movements.
- Empirical data show power law distributions of the sizes of the avalanches which is the hallmark of Self Organized Critical systems. Moreover our detailed analysis of the price dynamics of the avalanches sheds new light on how different markets influence each other.
- In order to study the evolution of Italian corporate boards by means of interlocking, two methods have been applied: one already applied by Alfarano et al. on the German stock exchange and a second original proposed in this research
- The new method points out the presence of stable ties between companies of the Italian stock exchange. It shows that the Italian Stock Exchange is characterized by some family owned companies.

Abstract

The aim of this work is to examine the dynamics in financial networks. We propose two cases of study: the network of world's stock exchanges and the network of Interlocking Directorates in Italy.

In the first case we study the dynamics *on* the network and in the second one the dynamics *of* the network, i.e. in the first case the network topology does not change and we study the dynamic information that passes through the structure, instead, in the second case, we study how the network topology evolves over time.

In 'Prices-Quakes Shaking the World's Stock Exchanges' (2011) Vitting Andersen et al. propose a model of the World's Stock Exchanges that predicts how an individual stock exchange should be priced in terms of performance of the global exchange market. In the present work this model is adopted to describe the financial network.

Understanding how disruption can propagate across financial markets is indeed of the utmost importance, hence our aim is to study the dynamics of the World's Stock Exchange network, inter alia we are interested in the study of the avalanches of price, disturbances propagating in the world financial network of stock exchanges.

In fact the model has a direct correspondence to models of earth tectonic plate movements developed in physics. In tectonic plate movement stresses are slowly build up over centuries only to be released in a quick snap, lasting from seconds to at most some few minutes, that we feel as an earth quake.

The main idea is to describe a similar slow build-up of 'stresses' in the world's financial network of stock exchanges where stresses can be thought of arising from for example business cycles in the real economy. Just like earthquakes such a slow build up of 'stress' is then followed by a quick release in terms of domino effects where the major part of the world's stock exchanges resonate with big up or down price movements. The main innovative part of the model that we will introduce is therefore a separation of time scales just as seen in earthquakes.

We first replicate the results of the model using empirical data using as source Bloomberg. We then verify that the price dynamics can indeed be described in terms of avalanche dynamics, and we find that such dynamics show power law behavior in agreement with what is found in other Self Organized Critical (SOC) models. We extend such studies and give a quantitative as well as qualitative description of the details in the dynamics of the propagation of "price-quakes" (avalanches).

The second case is the Interlocking Directorates in Italy, i.e. the situation that occurs

when a person affiliated with one company sits on the board of directors of another organization, analyzed by using network theory. We first analyzed the Italian case in order to investigate the presence of a persistent core. Applying the same methodology used by Milaković et al. [35], from 1998 to 2010 we found quite different results: the persistent sub-graphs are not connected and so we could not find a core. Instead in the German stock exchange Milaković et al. have found a small core of directors densely connected among themselves.

In order to capture the persistent structure of the Interlocks Network we propose a different approach that allows us to assess the stability of links between companies in Italy.

We describe the dynamic board networks by means of a static graph in which an edge is related with the persistence over time of an interlock between two companies.

The results lead to affirm that in the Italian board network a set of stable links is observable, nevertheless a presence of a large turnover between the directors. There are strong ties among firms in the overall period. Most of them are ties due to the ownership of family firms: Berlusconi, Benetton, Agnelli, Caltagirone, De Benedetti.

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Introduction

Most real systems can be modeled by complex networks and network tools are widely employed in various fields. Examples, initially found in sociology and in Social Network Analysis [59], are further extended to other areas of applications, such as physics [6], biology [7], economics ([55]) and finance (see [23], [50], [51]).

Network tools are useful especially in studying the dynamical evolution of a complex system, therefore researcher interest has recently been addressed to studying networks evolving over time, known in literature as *dynamic networks*. Our work fits within this framework and the aim is to examine the dynamics in financial networks.

Afterwards recalling the basic definitions on graph theory (Chapter 1), in Chapter 2 an overview on the dynamic networks, in terms of types, measures and how representing them, is presented.

The illustration of two cases of study follows in Chapter 3 and Chapter 4.

The two presented cases are very different even if both are financial networks. In the first one, the study of World's Stock Exchange network, the aim is to capture the features of the avalanches, i.e. how the financial markets infect each other.

Hence the analysis regards some dynamic traits passing through the network. In fact the topology of the network does not change over time but the information, flowing across the markets, fluctuate hour by hour. Also the avalanches can be represented by a graph (a type of sub-graph of the original network) and their features give us information about where the avalanches start, how many markets are involved, etc.

Indeed, in the second case of study (see Chapter 4), the Interlocks Directorates network of the Italian Stock Exchange, the aim is analyze the topology of the network that is varying year by year. Thus, in this case, we are interested in describing the dynamic structural changes and the adopted techniques are based on graph theory. A static graph, describing the network evolution, is proposed.

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Chapter 1

Fundamental definitions on graph theory

A graph is the mathematical object underlying the network structure. We shortly recall some definitions on graph theory which are important for understanding the following discussion.

Let $G = (V, E)$ be a *graph* where V is the set of n *nodes* (or *vertices*) and E is the set of m pairs of nodes of V ; the pair $(i, j) \in E$ is called an *edge* (or *link*) of G , and i and j are called *adjacent* nodes ($i \sim j$).

In an *undirected* graph edges have no orientation, $(j, i) \in E$ whenever $(i, j) \in E$. On the other hand in a *directed* graph (or *digraph*) (i, j) is an ordered pair of nodes.

If there is no edge starting and ending on the same vertex (self-loop), and no more than one edge is allowed between two nodes the graph is called *simple*, otherwise it is called *multigraph*.

In an undirected graph the *degree* k_i of a node i ($i = 1, \dots, n$) is the number of edges incidental to it.

In a directed graph the *in-degree* k_i^{in} of a node i ($i = 1, \dots, n$) is the number of edges incoming to it and the *out-degree* k_i^{out} of a node i ($i = 1, \dots, n$) is the number of edges outgoing from it.

A *sub-graph* $H = (V', E')$ of G is a graph such that $V' \subseteq V$ and $E' \subseteq E$. A sequence of distinct adjacent vertices from i to j is an $i - j$ *path*; the *length* of a path is the number of edges in the path. A path joining vertices i and j and having the minimum length is called an $i - j$ *geodesic*.

The *distance* $d(i, j)$ between two vertices i and j is the length of the $i - j$ *geodesic*. A graph is *connected* if, for each pair of nodes i and j ($i, j = 1, 2, \dots, n$), a path from i to j exists, otherwise the graph is *disconnected*. A *connected component* of an undirected graph is a sub-graph in which any two nodes are connected to each other by paths, and which is connected to no additional vertices in the super-graph.

Therefore, a *connected* graph G has only one connected component, whereas a *disconnected* graph G has two (or more) connected components.

A node is *isolated* if it is not connected to any node.

A *clique* in an undirected graph is a sub-graph in which every pair of nodes are adjacent.

A graph is *complete* if every pair of nodes are adjacent.

A graph is *bipartite* if the set of nodes V can be partitioned into two subsets V_1 and V_2 , i.e. $V_1 \cap V_2 = \phi$ and $V_1 \cup V_2 = V$, such that every edge of the graph joins one node of V_1 and one of V_2 .

The adjacency relationship between the nodes of G are described by a non-negative n -square matrix A called the *adjacency matrix* associated with the graph which elements are:

$$a_{ij} = \begin{cases} 1 & \text{if } i \sim j \\ 0 & \text{otherwise} \end{cases}$$

If the graph is simple, the diagonal entries a_{ii} are zero. Finally, a weight $w_{ij} > 0$ can be possibly associated with every edge (i, j) , generating a *weighted* graph; in this case the adjacency relationships are described by a n -square matrix $W = [c_{ij}]$ called the *weighted adjacency matrix* which elements are:

$$c_{ij} = \begin{cases} w_{ij} & \text{if } i \sim j \\ 0 & \text{otherwise} \end{cases}$$

A *contract* graph is derived by another graph merging two or more vertices into one. The vertices in the contract graph correspond to sets of vertices in the original graph and the edges between vertices in the same set disappear in the contract graph.

A *hypergraph* is a generalization of a graph in which an edge can connect any number of vertices. Formally, a hypergraph H is a pair $H = (X, E)$ where X is a set of elements called *hypernodes* or *hypervertices*, and E is a set of non-empty subsets of X called *hyperedges*. While in a graph edges are pairs of vertices, in a hypergraph, hyperedges are arbitrary set

of vertices, because in a hypergraph the edges can connect any number of vertices. It is possible to represent a hypergraph as a family of sets drawn from the set V .

The *density* of a graph G is given by the ratio of the number of existing edges m to the number of edges in a complete graph of the same size (i.e. $n \cdot (n - 1)$), hence:

$$\text{density}_G = \frac{m}{n \cdot (n - 1)}$$

In general a graph is called more *cohesive* than another if it has a higher density.

The most important centrality measures are:

Degree Centrality:

$$C_d(i) = k_i \quad \forall i \in V$$

Closeness Centrality:

$$C_{\text{clos}}(i) = \frac{1}{\sum_{j \in V} d(i, j)} \quad \forall i \in V$$

Betweenness Centrality:

$$C_{\text{betw}}(i) = \sum_{r < s} \frac{g_{rs}(i)}{g_{rs}} \quad \forall i \in V \quad r, s \neq i$$

where g_{rs} is the number of geodesics from r to s and $g_{rs}(i)$ is the number of geodesics from r to s and passing through i .

Eigenvector Centrality:

Let $A = [a_{ij}]$ the adjacency matrix of a graph G , $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ is the set of the eigenvalues of A , $\rho = \max_i |\lambda_i|$ its spectral radius and \mathbf{x} the principal eigenvector corresponding to ρ :

$$C_{\text{eig}}(i) = x_i \quad \forall i \in V$$

where x_i is the i -th component of \mathbf{x} .

For other graph definitions, we refer to [25].

Chapter 2

Dynamic networks

A graph can model a great variety of systems in nature, society and technology.

Unlike the classical approach of the network theory, in this case we consider an additional dimension: the time.

Indeed the edges are not continuously active but their presence depends on the time. Like network topology, the temporal structure of edge activations can affect dynamics of systems interacting through the network.

In this chapter we present a concise overview on dynamic networks (the object of study has many names: dynamic networks, temporal networks, evolving graphs, time-varying graphs, time-aggregated graphs) and the methods for analyzing topological and temporal structure.

When a dynamic network is studied, it may be that the object of the study is the network itself (the vertices and the edges), rather than a dynamical system *on* the network. In this work we present in Chapter 4 an application of the first approach and in Chapter 3 an application of the second one.

2.1 Types of dynamic networks

In order to present the various applications of the dynamic network, in the following section we report some of the most utilized models (see [27]).

The communications between people are particularly suitable for dynamic networks. There is a lot of data available of e-mails contacts, phone calls, mobile phone text messages, instant messages and messages online forum. So a literature in this context was developed

(see [28], [60]).

In order to study the spreading dynamics of information, the analysis of various temporal centrality measures (see Section 2.2) is also done in [43] and [56].

Also the communication from a source to anyone has been analyzed by dynamic networks, i.e. from one node (the source) and a group of nodes (the elements connected to the source).

In [62] the geographical distribution of editors for each Wikipedia's in the globe is estimated. In fact, the authors try to characterize and to find the universalities and differences in temporal activity patterns of editors, using the cumulative data of 34 Wikipedia's in different languages and taking into account the circadian activity patterns among editors of all different languages (so they assumed a local time offset for each language).

In biology, frequently a static graph represents the interactions between proteins or lighter molecules. However, biological functionality is mostly related to connection activity at all times. So there is a large literature investigating the temporal aspects of protein interaction.

For example, in [24] the authors investigate how hubs might contribute to robustness and other cellular properties for protein-protein interactions dynamically regulated both in time and in space. They define two types of hubs: 'party' hubs, which interact with most of their partners simultaneously, and 'date' hubs, which bind their different partners at different times or locations. Hence they study the connectivity of dynamic networks in the two different cases.

The above-mentioned models are not the only potential applications of dynamic network modeling, but, in the network literature, every time that there is a temporal evolution a dynamic network approach could be used.

To describe a dynamic network there are two different ways depending on the information about the time. We have a graph $G = (V, E)$, a set V of n vertices, interacting with each other at certain times (the interactions are described by edges $e = (i, j) \in E$). If the duration of the interaction is negligible, there is a time set $T_e = \{t_1, \dots, t_n\}$ for $e \in E$, on the contrary, if the duration is relevant, a set of time intervals $T_e = \{(t_1, t'_1), \dots, (t_n, t'_n)\}$ for $e \in E$.

In the first case it is possible to represent the dynamic graph with a set of *contacts*, triples (i, j, t) , where $(i, j) \in E$ and t denote the time, or with a set of graphs G_t one for

each $t \in T$. In our work we prefer the second choice (see Section 4.5).

The adjacency matrix will depend on time t , hence it becomes

$$a_{(i,j)}^t = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected at time } t \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

In the second case (we can call it *interval graphs*) the contacts could be represented as a quadruples $(i, j, t, \delta t)$. It means that there is a contact (edge) between vertices i and j at time t during δt (for more details see [43]).

The vertices could be active intermittently, but usually it is more useful to maintain the same number of vertices for the whole period (see Section 4.5).

Sometimes the Agent-Based Model (ABM) is used to analyze dynamic networks.

An agent-based model (ABM) is a class of computational models for simulating the actions and interactions of autonomous agents with a view to assessing their effects on the system as a whole.

Most agent-based models are composed of: numerous agents specified at various scales; decision-making heuristics; learning rules or adaptive processes; an interaction topology and a non-agent environment.

We only mention these useful models because we do not use them in this work.

2.2 Measures of dynamic-topological structures

In graph theory there are various measures based on connections between nodes, path lengths, features of adjacency matrix, ect. When time is added to describe the network, a classical approach consists on applying the static measures to each static graph describing the network at time t and showing the trend on time of the measure or the average.

This approach is not always possible, for example in presence of paths depending on time. In [29] the authors call a path *time-respecting* if the time labels on its edges are non-decreasing (i.e. the edges are in succession in time) and they consider connectivity problems, in which they seek disjoint time-respecting paths between pairs of nodes.

As for the static graphs, two nodes can be strongly or weakly connected, but it is interesting to observe that connectivity is not a symmetric property for dynamic graphs. In fact if there is a time-respecting path from i to j and from j to k , it could be possible

not to find a time-respecting path from i to k .

In dynamic graphs the definition of *shortest path* changes in *fastest path*. However, we should consider not only the weights of the edges in terms of time but, all above, the sequence of the current edges and so the issue is more complex than a problem of distance on a weighted graph.

After the notion of distance has been extended to the temporal case, it is possible to generalize some centrality measures defined for a static graph, like the *closeness centrality* and *betweenness*, to the dynamic case (for further details see [43], [27]).

2.3 Collapsing a dynamic network in a static graph

An approach to analyzing dynamic graphs is to derive a static graph that captures both temporal and topological system properties. A direct way is to sum over time the edges and to obtain a weighted graph. The entries of the weighted matrix are

$$w_{(i,j)} = \sum_t a_{(i,j)}^t \quad (2.2)$$

where the terms of the sum are the entries of the adjacency matrix (see Equation 2.1).

This is a trivial way of projecting out the temporal dimension and sometimes it can discard too much information. However it is useful when the topological aspects are more important than the temporal.

In a static graph it is possible to combine the topological and temporal features. In Section 4.5 we propose a static weighted graph that can capture the topological features of a dynamic network. Nevertheless, the weights are not only the sum of the edges, but they give us an information about the stability of the edge during the time.

Instead in Section 3.4.3 in representing an avalanche as a static graph it is satisfactory to sum up the edges because the issue is to study topological characteristics of the avalanches.

An alternative graph representation is obtained putting an edge from node i to node j if there is a time-respecting path from i to j . The obtained graph is called *path graph* [37] and it is useful to study diffusion problem in networks.

In this work we do not use this second representation even if, for further study, it could be useful to apply it to the avalanches (see Section 3.4.3).

In conclusion, the representation of a dynamic network by means a single graph can be very useful but it should be chosen considering all aspects of the system.

Chapter 3

Case of study: World's Stock Exchanges Network

3.1 The model of Vitting-Andersen et al.

The model, proposed by Vitting Andersen et al. in [58], has been chosen to describe the World's Stock Exchange Network. The model predicts the sign of the return of a given stock exchange, using the values of the return of other stock exchanges.

It is an extension of the 2D Olami-Feder-Christensen model [42] that describes the dynamics of earthquake (see 3.4.2). In fact financial crises, like earthquakes, are able to shake up countries in a widespread and disastrous manner. While the origin of the earthquakes is well understood (the continuous build up of stress from tectonic plate movements), the causes of financial diseases are not as clear. The model proposes a top-down approach to study the contagion between the financial markets. The contagion has been analyzed from other points of view in [19] and [38] extending the study of systemic risk outside of banking system.

A key idea of the model is that humans have a tendency to place emphasis on events with big changes and to disregard events with modest information content.

Hence a large fluctuation of the return of a market impacts on other financial markets, whereas a small fluctuation does not and, on the contrary, it goes unnoticed. In fact a key principle is that a new information immediately becomes reflected in the price of an asset and thereby loses its relevance, but, we can say, only if the new information is quite significant, otherwise it is ignored.

Similarly to what happens for earthquakes, it is possible to assume that there is a "stick-slip" motion of the indices: only a large movement of a given index has a direct impact on the pricing of the remaining indices.

Applying the 2D Olami-Feder-Christensen model [42] the markets are represented as blocks and two blocks are linked by a spring of variable strength. The aim is to study how the stresses increase and spread on the global financial system. The impact from e.g. a slowly varying business cycle on the financial markets can then be studied similarly to the slow build up of stresses seen in tectonic plate movements. The fast response in the tectonic plate system seen in terms of earthquakes, then corresponds to dominos of price movements seen in the network of the worlds stock exchanges. The separation of time scales into a slowly varying variable (say impact from business cycle) and the response in terms of a fast variable (daily large price co-movements across markets) is one of the main features of the model introduced below.

The model assumes that a trader in a given stock market i prices an index in according to 3.1.

$$P_i(t) = P_i(t-1)e^{R_i(t)} \quad (3.1)$$

where $R_i(t)$ is the index return of the stock market i at time t . Hence the trader has to estimate $R_i(t)$.

A universal behavioral mechanism in the pricing done by traders is postulated. The return $R_i(t)$ is composed of two terms (see 3.2): one depending on local economic news ($\eta_i(t)$ - a bias in $\eta_i(t)$ corresponds to a slowly build of stresses in the system) and other one ($R_i^{transfer}(t)$ - with $R_i^{transfer}(t)$ corresponding to the fast variable of daily price co-movements in the network of stock exchanges) on big cumulative changes from other stock exchanges weighted by their importance (in terms of capitalization) and relatedness (in terms of geographical positioning). The model in formulas is (see also [58]):

$$R_i(t) = R_i^{transfer}(t) + \eta_i(t) \quad (3.2)$$

$$R_i^{transfer}(t) = \frac{1}{N_i^*} \sum_{j \neq i}^N \alpha_{ij} \Theta(|R_{ij}^{cum}(t-1)| > R_C) \times R_{ij}^{cum}(t-1) \beta_{ij} \quad (3.3)$$

$$R_{ij}^{cum}(t) = [1 - \Theta(|R_{ij}^{cum}(t-1)| > R_C)] \cdot R_{ij}^{cum}(t-1) + R_j(t) \quad (3.4)$$

$$\alpha_{ij} = 1 - e^{-\frac{K_j}{K_i \gamma}} \quad (3.5) \quad \beta_{ij} = e^{-\frac{|z_i - z_j|}{\tau}} \quad (3.6)$$

where K_i, K_j are the capitalizations of the indices i and j ,

z_i, z_j are the time zones of the markets i and j .

$$N_i^* = \sum_{j \neq i}^N \Theta(|R_{ij}^{cum}(t-1)| > R_C) \quad (3.7) \quad \Theta(x) = \begin{cases} 1 & \text{if } x = \text{true} \\ 0 & \text{if } x = \text{false} \end{cases} \quad (3.8)$$

Let's focus on time t in (3.3) : it is the time in which the market i , for example, opens, whereas $t-1$ (note that is not the time in which the market i had closed) is the time of the last known information (close or open) of exchange j known at time t in the market i . The definition states a time t *relative* to market i in which the model is applied. In practice, it is necessary to find a *global* time in order to apply the model for each market i .

This fact is not trivial also because the sequence of open/close turns depend on the time zones and their daylight saving time changes. Furthermore it needs to state, for each market i , which are the information of exchange j known at time t in the market i taking in account the holidays of every country.

In the model there are five parameters: N number of markets, R_C the threshold, τ the time scale of the impact across time zone, γ the impact scale from capitalization and σ the standard deviation of noise η_i .

More in detail, in (3.2) η_i represents the internal economic news relevant for the specific market i and $R_i^{transfer}$ represents the influence on market i of the other markets. In equation (3.3) the contribution of the return of market j on market i (R_{ij}^{cum}) is added only if it is greater, in absolute value, than the threshold R_C which corresponds to the fact the 'information' (in terms a large cumulative price movement of another stock index)

becomes priced in. This explains the Heaviside function Θ (see 3.8). On the other hand R_{ij}^{cum} is nullified after passing the threshold R_C (see (3.4)). The equation (3.4) means that the variations are considered also if they are cumulative over time.

Let's observe that in (3.4) the index i (compared to the original version [58]) has been added to underline that the time t referring to the market i , hence $R_{ij}^{cum}(t)$ is computable only knowing the index i . For example, if the U.S. market gains 4% and the Italian Stock Exchange is closed for a holiday, the Italian market will be influenced only when it opens, whereas the other markets have already priced the information. Let us remember that the model takes in account only of open/close prices and no intra-daily ones as in Econometrics models (see [10]).

In (3.3) the contribution of market j is multiplied by two weights: α_{ij} describes the influence of stock j on stock index i in terms of relative capitalization value of the two indices K_i and K_j . Obviously $\alpha_{ij} \neq \alpha_{ji}$. The parameter γ is used to weigh the coefficients α_{ij} .

β_{ij} describes the economic interdependence of the countries that are geographically close, z_i and z_j are the time zones of the countries i and j . The parameter τ is used to weigh the coefficients β_{ij} . In this case $\beta_{ij} = \beta_{ji} \forall i, j$.

In 3.3 N^* is the number of markets in which R_{ij}^{cum} is, in absolute value, higher than the threshold R_C .

3.2 Data and methodology

In [58] the model was applied to empirical data in order to calculate its predictive ability. The number of markets N was 24, for each market an index was chosen and data were downloaded from finance.yahoo.com.

The other four parameters of the model were calculated using maximum likelihood analysis: $\gamma = 0.8$, $\tau = 20$, $R_C = 0.03$, $\sigma^2 = 0.0006$.

In order to validate the model, in our work we replicated the application, with the same parameters, same indices but retraining data from another source. In fact the data were downloaded from Bloomberg.

The list of indices (from the two sources) is shown in Table 3.3.

The data considered are the opening and closing price of the 24 stock exchanges from

1/1/2000 to 31/12/2008. The data referring to Saturday and Sunday were not taken in account because the markets are closed (except Telaviv and Cairo on Sunday).

The results we obtained are the same as in [58], thus confirming the validity of the empirical application.

The model is able to predict the sign of the return of the open/close of a given stock exchange with a success rate of 63%.

We have computed the conditional probability that the close-open return (i.e. the overnight return) R_i of a given stock exchange following an U.S. open-close, has the same sign as the U.S. open-close return. The result is shown in Figure 3.1. In fact U.S. market has the largest capitalization and it allows to check how large movements of large capital indices impact on smaller capital indices. From Figure 3.1 it can be noticed that this effect is non-linear.

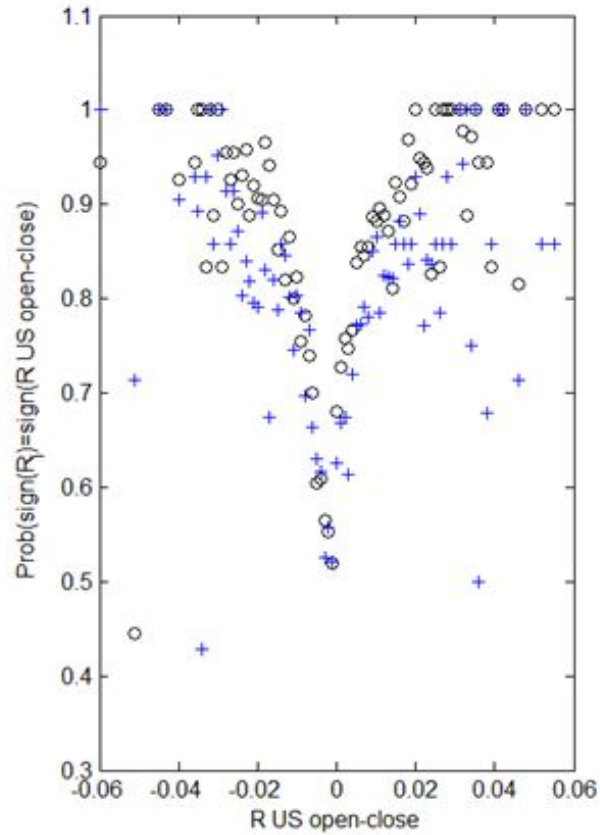


Figure 3.1: Conditional probability that the close-open return R_i of a given stock exchange (+: European markets,; circles: Asian markets) following an U.S. open-close, has the same sign as the U.S. open-close return

Moreover, in Figure 3.2 the distribution of the return R_i and its two components

$R^{transfer}$ and η_i are shown. Figure 3.2 shows that the price movements due to external (random) news, i.e. η_i , do indeed fit a normal distribution. It depends on the choice of the parameters calculated using maximum likelihood analysis.

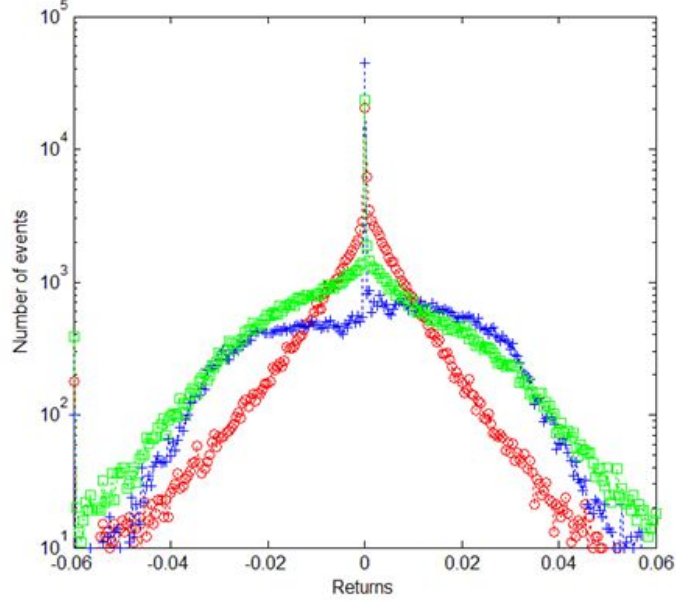


Figure 3.2: Red circles: observed return R_i ; green squares: $R^{transfer}$; blue +: the difference η_i . A logarithmic scale is on the y-axis

The data were processed using Matlab. To manage the sequence of opening/closing of the 24 markets it was necessary to know the changes of Daylight Saving Time from 2000 to 2008 in the considered 24 countries and the source is www.timeanddate.com.

Table 3.4 shows an example of a sequence of opening and closing of the 24 markets: the events are 48 per day and, more in detail, the events are the return of the close/open, i.e. $R = \ln(\frac{P_{open}}{P_{close}})$, and the return of the open/close, i.e. $R = \ln(\frac{P_{close}}{P_{open}})$ where the P_{close} and the P_{open} are respectively the index price when the market closes and opens.

In the column 'clock of the model' it is possible to observe that the events happening in the same hour are considered contemporaneous and the interactions between them are disregarded. In this way in a day the model has only about 16 different times, called 'clocks', even if it processes 48 different events.

3.3 Interpretation of the model as a network

The aim of this work is to study the dynamics of the World's Stock Exchange Network. To do this we consider the 24 markets as nodes of a network. The network is represented

by a complete graph (see Figure 3.3) because all nodes are connected to each other.

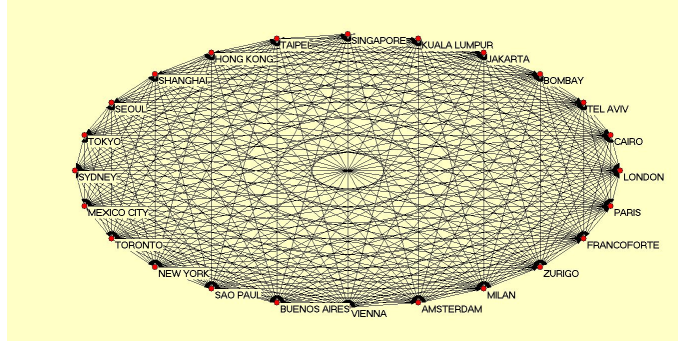


Figure 3.3: The World's Stock Exchange Network is a complete graph

The interesting thing is not the topology of the network, that is trivial, but the information passing through the links, i.e. $R^{transfer}$. Each node can be considered as a seismograph which at any clock measures the influence on itself caused by the large price movements of the other stock exchanges world-wide.

The links are weighted by α_{ij} and β_{ij} and so an information, starting from a node, will produce different impacts on the other nodes. It is possible to show the relationship

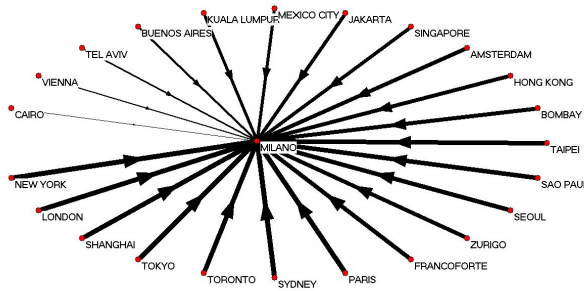


Figure 3.4: A star representing the relationships between a node (MILAN) and the others in term of weights α : thicker edges indicate a higher value of α

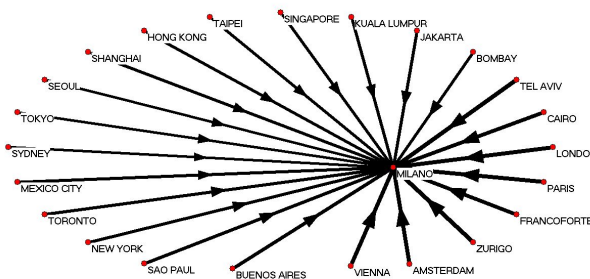


Figure 3.5: A star representing the relationships between a node (MILAN) and the others in term of weights β : thicker edges indicate a higher value of β

between the nodes by picking out a market and putting it in the center of a star. The

weighted (by α_{ij} and β_{ij}) and directed links, entering in it, show how the network influence it. If we split in two different graphs the contribution of α_{ij} and β_{ij} we can visualize the relation of a node with the others in terms of capitalization ratio (see Figure 3.4) or geographical distance (see Figure 3.5).

3.4 Study of the network dynamics

3.4.1 The SOC model

Self-organized critical models (SOC) are a class of dynamical systems which have a critical point as an attractor. Their macroscopic behavior thus displays the spatial and/or temporal scale-invariance characteristic of the critical point of a phase transition, but without the need to tune control parameters to precise values.

The concept was put forward by Bak-Tang- Wiesenfeld (BTW) in [5] and is considered to be one of the mechanisms by which complexity arises in nature. Its concepts have been largely applied across diverse fields (see [3], [4], [57], [2]) and also in finance (see [44]) .

A system is *critical* if it is in transition between two phases; for example, water at its freezing point is a critical system. A variety of critical systems demonstrate common behaviors: long-tailed distributions of some physical quantities, fractal geometries and variations in time that exhibit pink noise, i.e.: the noise is a time series with many frequency components and in a pink noise, low-frequency components have more power than high-frequency components, specifically, the power at frequency f is proportional to $1/f$. (Visible light with this power spectrum looks pink, hence the name).

SOC is typically observed in slowly driven non-equilibrium systems with extended degrees of freedom and a high level of nonlinearity.

In [5] the authors showed that the observed complexity emerged in a robust manner that did not depend on finely tuned details of the system: variable parameters in the model could be changed widely without affecting the emergence of critical behavior (hence, self-organized criticality). A visualization of the model is a sand-pile on which new sand grains are being slowly sprinkled to cause avalanches. There are no known general conditions under which a system displays SOC but the power law distribution is a common feature (a distribution is a power law if its probability distribution $p(x)$ is $p(x) = C(\alpha)x^{-\alpha}$ where $C(\alpha)$ is a normalization constant)

There is a large literature in applying SOC in finance: in [8] the authors study the Stock Market as a complex self-interacting system, characterized by an intermittent behavior. They investigate empirically the possibility that the market is in a self-organized critical state and show a power law behavior in the avalanche size, duration and laminar times (i.e. the waiting times between avalanches), but the memory process of the laminar times is not consistent with classical conservative models for self-organized criticality. So they argue that a “near-SOC” state or a time dependence in the driver, which may be chaotic, can explain this behavior.

In [21] the authors explore a simple lattice field model intended to describe statistical properties of high frequency financial markets. They simulated with time series of gains, prices, volatility, applying a standard GARCH(1,1) fit to the lattice model and they found the emergence of a self-organized critical state.

In [12] they study a few dynamical systems composed of many components whose sizes evolve according to multiplicative stochastic rules. In the case of the stock market, the distribution of the investors wealth is related to the ratio between the new capital invested in stock and the rate of increase of the stock index. They compare them with respect to the emergence of power laws in the size distribution of their components. They show that the details specifying and enforcing the smallest size of the components are crucial as well as the rules for creating new components and they present a new model with variable number of components that converges to a power law for a very wide range of parameters.

It should be noted that in our model there are some characteristics of the SOC: the memory effects, that is the cumulative ‘stress’ that determines when a block ‘slip’, and the size of the avalanches has a power law distribution (see 3.4.3 and 3.4.3)

3.4.2 The 2D Olami-Feder-Christensen model (OFC)

In 1967, Burridge and Knopoff [13] introduced a one-dimensional (1D) system of springs and blocks to study the role of friction along a fault in earthquakes. Since then, many other researchers have investigated similar dynamical models of many-body systems with friction, ranging from propagation and rupture in earthquake so to the fracture of over-layers on a rough substrate.

In 1992, a model was proposed by Olami-Feder-Christensen in [42] as an extension of Burridge and Knopoff in the two-dimensional case (2D). Their model is a non-conservative

self-organized critical model. In fact the authors showed a robust power law behavior with an exponent that depends on the level of conservation.

In the model, the fault is represented by a two-dimensional network of blocks interconnected by springs (a lattice). Additionally, each block is connected to a single rigid driving plate by another set of springs as well as connected frictionally to a fixed rigid plate.

The blocks are driven by the relative movement of the two rigid plates. When the force on one of the blocks is larger than some threshold value (the maximal static friction), the block slips. They assume that the moving block slips to the zero force position. Slip of one block will redefine the forces on its nearest neighbors. This may lead to instabilities of the neighboring blocks and thus, as a result, in further slips and a chain reaction (earthquake) can evolve.

The total number of slips following a single initial slip event is a measure of the size (seismic moment) of the earthquake.

The model was extended to other topology like small world network¹ [17], scale free network² [16], or with a variable level of conservation [63].

In [33] to the OFC model on a square lattice, some rewired long-range connections are added and the resulting network has the properties of small world networks. Also in this work power-law behavior are founded and the authors underline that the connectivity topologies are very important to models avalanche dynamical behaviors.

In our model the topology of the avalanches depends on weighted and directed links. In fact the only presence of a link between two nodes does not ensure that an information passes through it because it is filtered by the weights α and β .

3.4.3 The avalanches

As observed in [32], the avalanche dynamics is an peculiar feature of complex systems. So we study the Self-Organized Critical dynamics of avalanches on our model. We expect that the avalanche size has a power-law distribution according to the features of SOC.

In our model at first we have to determine which are the elements that compose an avalanche. In agreement with the constraints imposed by the model, we propose that

¹ A small world is a network where the distance between two randomly chosen nodes grows proportionally to the logarithm of the number of nodes n of the network

² A scale-free network is a network whose degree distribution follows a power law



Figure 3.6: An avalanche is like a wave that moves according to the times of market's open/close

only the markets that, opening or closing, have a return, in absolute value, greater the threshold, also if cumulative over time, may belong to an avalanche. The next definitions follow:

Definition 3.1 (critical node).

A node i is *positive critical* (respectively *negative critical*) at time t if and only if

- at time t the node opens or closes
- $R_i^{cum}(t) > R_C$ (respectively $R_i^{cum}(t) < -R_C$)

Definition 3.2 (avalanche).

An avalanche is a set of critical nodes (all positive or all negative) whose criticality depends to each other. A node whom criticality depends by another nodes is called *influenced*.

We can define different kinds of avalanche if we state different definitions of *influenced*. We propose two kind of avalanches: avalanche caused by single nodes and avalanche caused by a set of nodes. This second kind of avalanche could be more in agreement with the model. In fact a market i , through $R^{transfer}$, is influenced by every node j of the network in which $|R_j^{cum}| > R_C$, so the "influence" is made by a set of nodes. On the other hand in [5] the iteration is defined element by element. In order to verify which definition is more correct, at first we propose two different kinds of avalanche.

The Single Node Avalanches (SNA)

Definition 3.3 (node influenced by a single node).

A critical node i is influenced by node j at time t if and only if

- at time t the node i opens or closes
- $|R_i^{cum}(t-1) + \alpha_{ij}\beta_{ij}R_j^{cum}(\tau)| > R_C$ where $t-1$ is the previous time in which node i opened or closed
- $t-1 \leq \tau < t$
- node j is critical (with the same sign of node i) at time τ

A node i may be influenced by more than one node. A node i is not influenced by a node that is critical at time t (contemporaneous events) (in fact $\tau < t$ and not $\tau \leq t$)

Definition 3.4 (Single Nodes Avalanche (SNA)).

When

- (1) a critical node i , at time t , is influenced by another critical node j (with the same sign)
- (2) the node j has not been influenced by another node
- (3) the node j opens or closes at time τ
- (4) τ is the first time in which (1), (2) and (3) happen

we can define that an avalanche starts at time τ .

A node k belongs to the avalanche if:

- the node k is critical (with the same sign of node i) at time $t > \tau$
- or node k influences node i
- or a chain of nodes (influencing each other) exists starting from node i or from a node that influenced node i and arrives in node k .

The avalanche stops at time T if

- at time T a node belonging to the avalanche opens or closes
- for every node s at time $t > T$
 - or node s is not critical
 - or node s is critical but is not influenced by any node belonging to the avalanche.

For every avalanche it is possible to build a graph that represents it. The markets are the nodes and the links are defined in the following way: when a node i is influenced by a node j we put a link from node j to node i . The node j could or could not be influenced by another node.

Figure 3.7 shows a simple example of negative SNA: the 24 nodes are on columns and times on rows. In the model there is an event when a market opens or closes, so there are 24 markets and 48 events per day. The markets that open or close in the same hour are considered contemporaneous. The model doesn't take in account the interactions between contemporaneous markets. In this way for every day the model doesn't manage 48 different times but only, in average, 16. In Figure 3.7 the negative critical nodes are represented by a red rectangle, whereas the other nodes are represented by a yellow rectangle. Finally, in the last column the red line indicates the presence of a negative avalanche. So we can

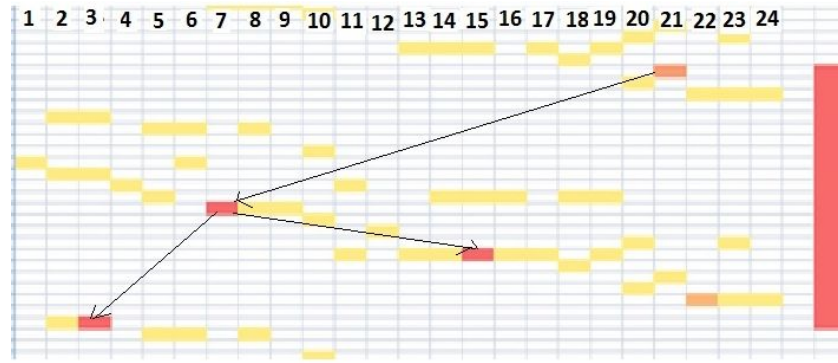


Figure 3.7: Example1: Single Node Avalanche

observe that node 21 influences node 7, node 7 influences node 3 and node 15. Node 22 is critical but it is not influenced by any other. We can say that the negative avalanche starts from node 21 and stops to node 3 after 23 clocks (little more than a day). The number of nodes belonging to the avalanche are 4.

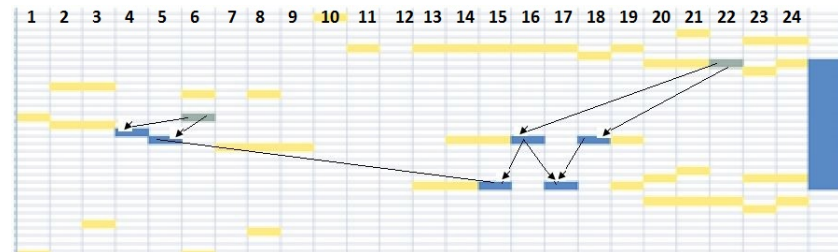


Figure 3.8: Example2: Single Node Avalanche

Figure 3.8 shows a positive avalanche composed by 8 nodes in which there are two nodes influencing but not influenced: node 6 and node 22. These kind of nodes are interesting

because they play the role of source of avalanches (in the graph these nodes have zero in-degree). The Figure 3.8 also shows some critical nodes on the same line (node 16 and 18, or node 15 and 17): they are contemporaneous and, as mentioned above, the model does not take in account the influence between the contemporaneous events. So we never put a link between these critical nodes.

Now it is possible to analyze the avalanches which are present in our model. The data are those presented in Section 3.2. The study of the avalanches leads to set of graphs (one for each avalanche): if we sum, over time, their adjacency matrices we have an information about more present links (see Equation 2.2) . If we do it for positive SNA we obtain the data show in Table 3.7, for negative SNA in Table 3.6, for negative and positive in Table 3.5.

Because the European market are all contemporaneous, with the exception of Netherlands that opens and closes one hour later, we can observe the influence from European market to Netherlands market and we cannot observe the influence between the other European markets. Also in the Asian market: for a long period of the year the markets of China, Hong Kong, Taiwan, Singapore and Malaysia are contemporaneous, but the market of South Korea opens and closes before them. So we can observe the influence of this market on the others and not the mutual influence.

The Table 3.8 shows the number of times in which a market is critical. We can distinguish between critical nodes influenced (in Figure 3.7 node 3,7,15), critical node influencing but not influenced (in Figure 3.7 node 21) and node critical but not belonging to an SNA. The sum of critical statuses for each market gives an information about the activity of each node. Figure 3.9 shows the correlation between this activity (see Table 3.8) and the volatility (variance) of the market, where $r=0,8856$. The presence of correlation suggests that the information: *how much a node is critical*, is not very interesting in order to investigate the dynamics of the network. In fact we are interested in analyzing the different roles that the different markets play. It is not certain that a market with high volatility has an important role in the dynamics of the avalanches. It could not have a large influence on the network. So we analyze the percentage of the different statuses of the markets (see Table 3.9). In this table the second column shows where the SNA starts more frequently. This role is particularly important: when in a market a large number of avalanche starts, it means that it is able to influence the network. Vice versa, the market

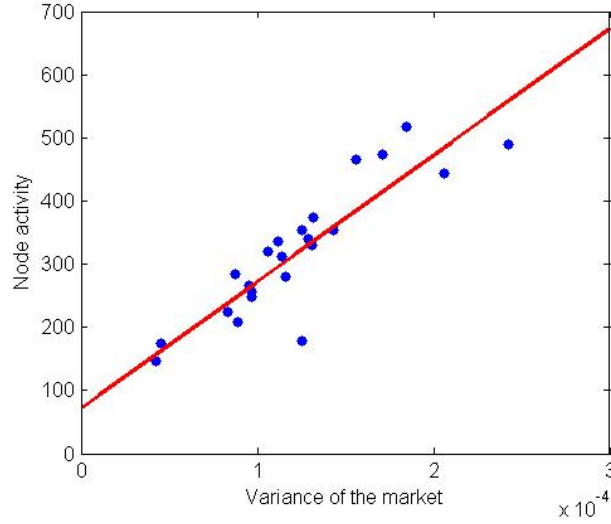


Figure 3.9: Correlation between the activity (critical node) and the volatility of the markets

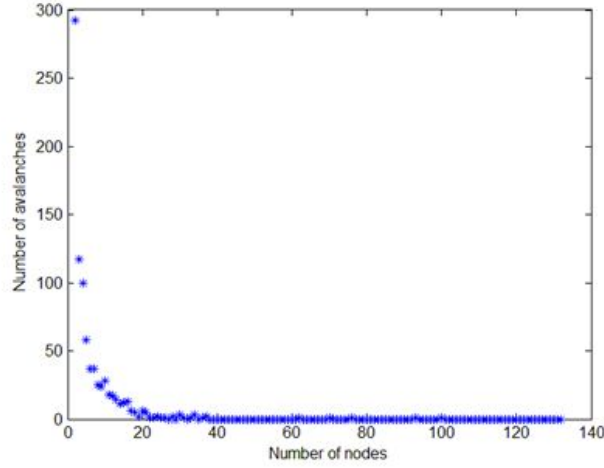


Figure 3.10: Distribution of the number of nodes belonging to a SNA

in which no avalanche starts plays a secondary role in the dynamics. The SOC models are characterized by a power law distribution. In our case we have to verify if the number of nodes belonging to an avalanche follows a power law distribution. The distribution is shown in Figure 3.10 and Figure 3.11. Since an avalanche needs time to *overwhelm* many nodes, the distribution of the duration of an avalanche also follows a power law distribution as shown in Figure 3.12 and Figure 3.13. In the model a new time t is processed if there is a new event (a market opens or closes): we call it *clock*. However it must be taken into account that between a clock and the succeeding there is not the same interval of real time and the duration is relative to the internal clock of the model.

In the Table 3.1 we can observe the average data of the SNA. Before analyzing in more

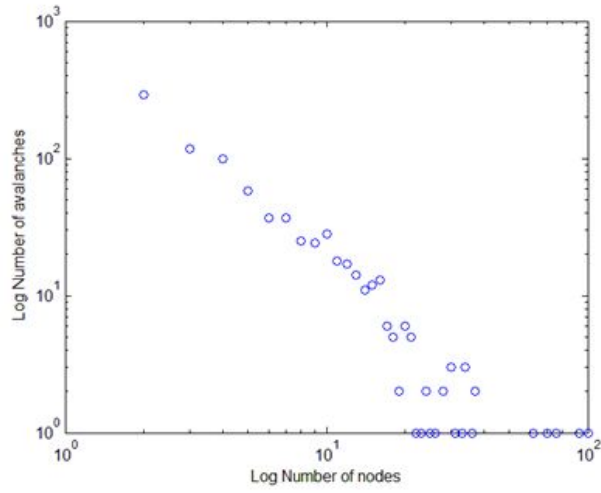


Figure 3.11: Distribution of number of nodes belonging to a SNA: log/log graphic

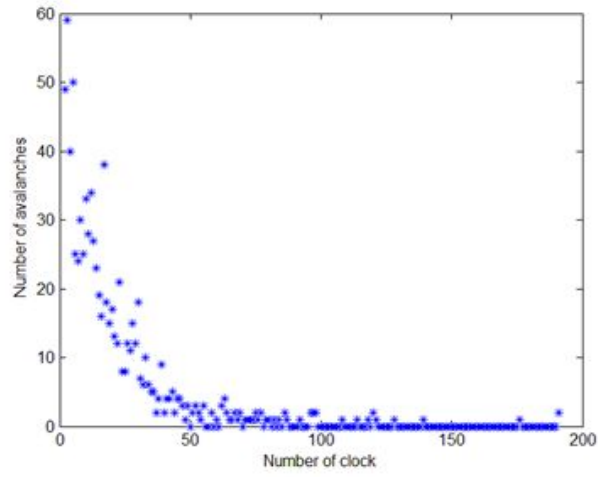


Figure 3.12: Distribution of the number of clock during a SNA

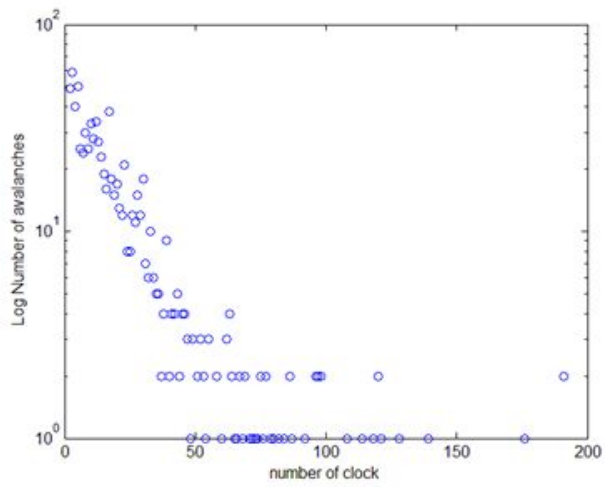


Figure 3.13: Distribution of the number of clock during a SNA: log/log graphic

	number of SNA	node average	clock average	time average(days)
Negative SNA	337	8.32	22.34	1.39
Positive SNA	515	5.27	19.65	1.22
Total SNA	852	6.46	20.49	1.28

Table 3.1: SNA Statistics

detail the obtained results we introduce a different kind of avalanche.

The Cloud Avalanches (CA)

The second kind of avalanches is based on another definition of *influenced*. The model splits R_i in two different terms: one (η_i) caused by local economic news and other ($R_i^{transfer}$) caused by a big cumulative change from other stock exchanges weighted by their importance. If $R_i^{transfer}$ is able to make critical a node i (considering only the contribute of critical nodes of the same sign of node i) we can identify a set of nodes influencing node i .

Definition 3.5 (node influenced by a cloud of nodes).

A critical node i is influenced by a set of nodes, at time t if and only if

- at time t the node i opens or closes
- $|R_i^{cum}(t-1) + \frac{\sum_{j \neq i}^{N^*} \alpha_{ij} \beta_{ij} R_{ij}^{cum}(\tau_j)}{N^*}| > R_C$
 - where $t-1$ is the previous time in which node i opened or closed
 - for every $j : t-1 \leq \tau_j < t$
 - node j is critical (with the same sign of node i) at time τ_j
 - N^* is the number of critical nodes j

The set of nodes node j , all critical in τ_j is called the *cloud* that influences the node i .

We can observe that the nodes belonging to the cloud that influences node i , are critical (so they open or close) in time between $t-1$ and t where t refers to node i (at time $t-1$ and t node i opens or closes).

Definition 3.6 (Cloud Avalanche (CA)).

When

- a critical node i , at time t , is influenced by a cloud of nodes j

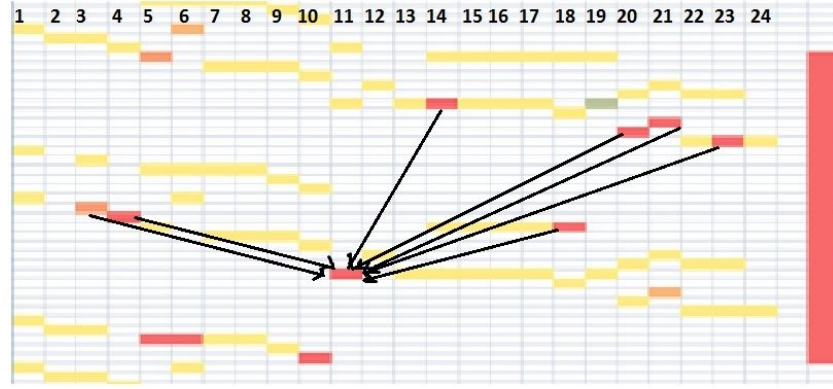


Figure 3.14: Example of Cloud Avalanche

- every node j belonging to the cloud has not been influenced by another node
- every node j belonging to the cloud opens or closes at time τ_j
- τ is $\min_j\{\tau_j\}$

we can define that an avalanche starts at time τ .

A node k belongs to the avalanche if:

- the node k is critical (with the same sign of node i) at time $t > \tau$
 - or node k belongs to the cloud that influenced node i
 - or exists a chain of nodes (influencing each other) that starts from node i and arrives to node k

The avalanche stops at time T if

- at time T opens or closes a node belonging to the avalanche
- for every node s at time $t > T$
 - or node s is not critical
 - or node s is critical but is not influenced by any node belonging to the avalanche

As done previously we can build a graph representing a CA: when a node i is influenced by a cloud of nodes we put a link from every node, belonging to the cloud, to node i . In Figure 3.14 node 11 is influenced by the cloud of nodes 3,4,14,18,20,21,23. They are all critical nodes between the times in which node 11 opens and closes. Node 19 is critical but with opposite sign, so it does not influence node 11. Node 5 and 6 are critical before node 11 opens, so they do not influence node 11 when it closes.

As for SNA we can sum, over time, the adjacency matrices representing the CA and we have an information about more present links. If we do it for positive CA we obtain the data shown in Table 3.12 , for negative CA in Table 3.11, for negative and positive in Table 3.10. The Table 3.13, shows the number of times in which a market is critical. We can distinguish between critical nodes influenced, critical nodes influencing but not influenced and nodes critical but not belonging to a CA.

Comparing to the SNA does not change the number of critical status but it changes the kind of critical status. Table 3.14 shows the percentage of the different statuses of the markets. In this table the second column shows where the CA starts more frequently. The distributions of the number of nodes present in the CA and of the number of clocks (in a day there are about 16 clocks) in the CA are showed in Figure 3.15 , Figure 3.16 , Figure 3.17 , Figure 3.18. In Table 3.2 we can observe the average data of the CA.

	number of CA	node average	clock average(clocks)	time average(days)
Negative CA	334	8.72	23.19	1.45
Positive CA	497	6.13	21.22	1.33
Total CA	831	7.17	22.01	1.38

Table 3.2: CA Statistics

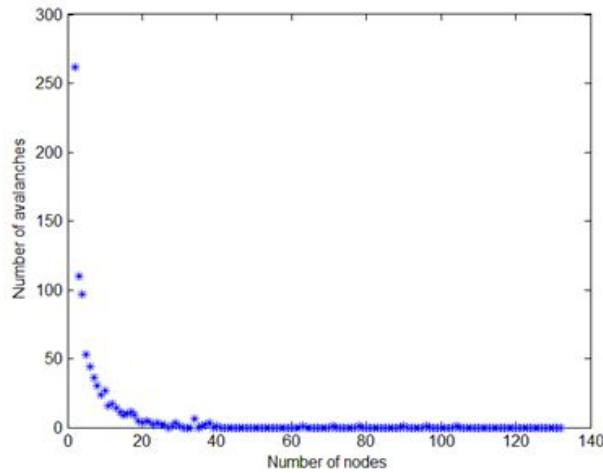


Figure 3.15: Distribution of the number of nodes belonging to a CA

Comparing SNA's and CA's

The number of nodes of CA's is greater than that of SNA's, but not very significantly. The differences in the definition of SNA and CA could lead to expect a more significant difference in terms of number of nodes. In reality the nodes interesting in a SNA or CA

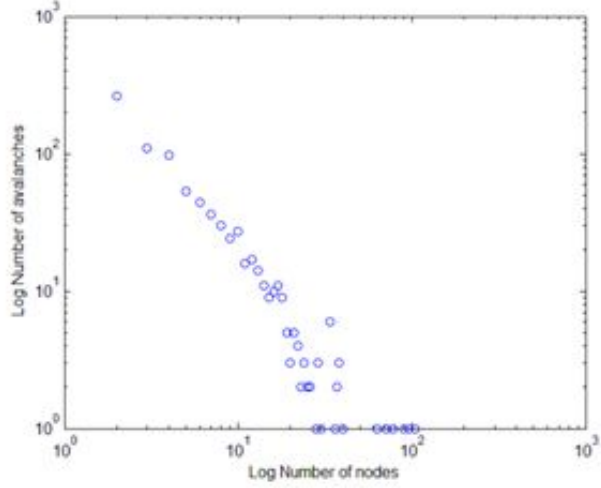


Figure 3.16: Distribution of number of nodes belonging to a CA: log/log graphic

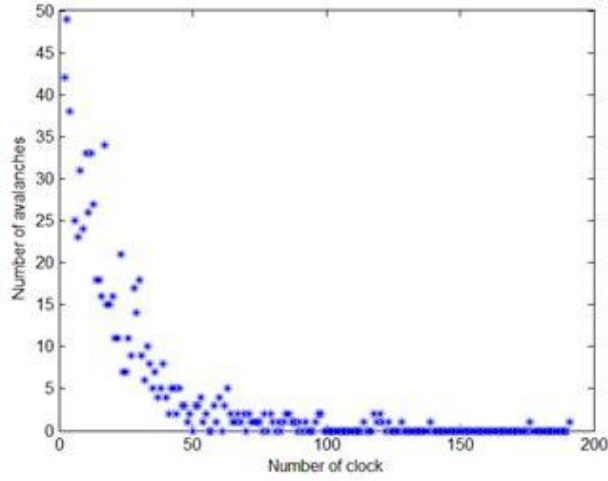


Figure 3.17: Distribution of the number of clock during a CA

are quite the same because the relation of influence between nodes is relative. In fact considering, in a CA, a cloud of nodes influencing another set of nodes, when we consider the same nodes in a SNA it may be that most of the nodes are also influenced by one or more nodes of the cloud (because of the presence of the weights α and β in the formula). So they also result influenced and also belong the SNA.

The biggest difference between CA and SNA lies in the number of links. If we compute the average in-degree and out-degree over all the avalanches for each nodes we obtain the results shown in Table 3.15 and Table 3.16.

We may observe that:

- the number of links is higher in the negative avalanche (as the number of nodes): so the negative avalanches are composed by more nodes with more links.

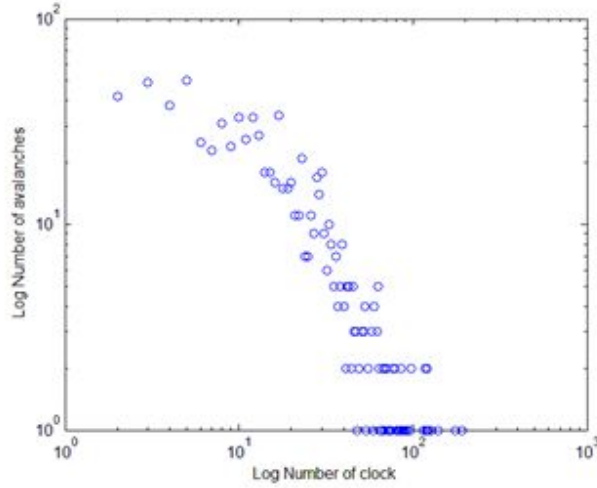


Figure 3.18: Distribution of the number of clock during a CA: log/log graphic

- the average degree is larger in the CA: this fact agrees with the definition of CA.
- it is possible to compute a balance for each node ($\Delta(\text{in-degree}) - (\text{out-degree})$): if it is negative we have a node that is more influencing than influenced, if it is positive is the other way around. This balance is more significant in the SNA where it is possible to observe the dominant role of strong markets as US and Germany (all above in negative SNA) and, on the other hand, the negligible role of markets as Egypt.

In order to investigate if there are topological differences in CA's and SNA's, we can search for some features of the links (in fact the nodes are the same). For example we can verify if the graphs representing the avalanches are assortative. A network is said to show assortative mixing if the nodes in the networks that have many connections tend to be connected to other nodes with many connections. We use the measure r of assortative mixing for networks defined by Newman in [39].

Let us consider the CA's and SNA's with almost 6 nodes (for small CA's and SNA's the topology is quite the same) and let us compute the coefficient r for each avalanche. The assortativity coefficient is the Pearson correlation coefficient of degree between pairs of linked nodes. To simplify let us consider the graphs as undirected. The distribution of r for the CA's and for SNA's is showed in Figure 3.19. We can observe that both CA's and SNA's are, for the most part, disassortative, but for the CA's this feature is stronger. Anyhow both distributions are quite Gaussian so we can conjecture that the avalanches are not characterized by a particular assortativity. In these graphs there is not the proclivity to link nodes with the same degree and, not even to link nodes with very

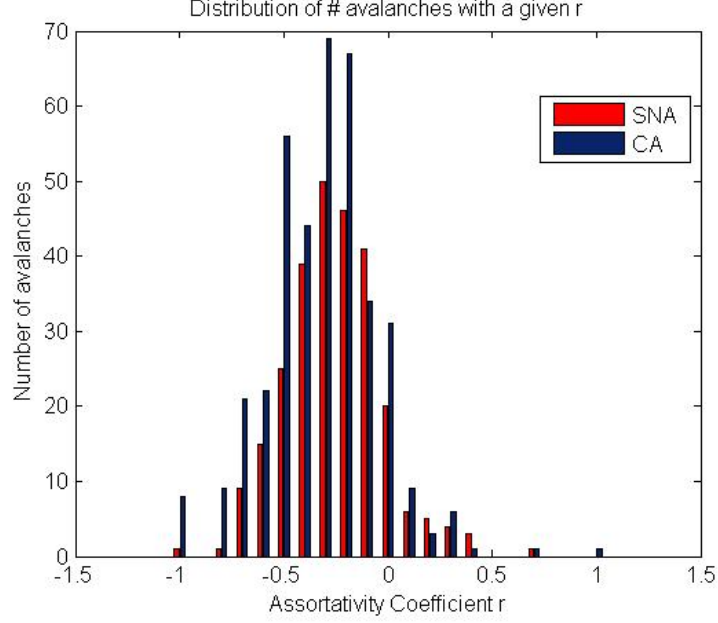


Figure 3.19: Distribution of the coefficient r of CA and SNA: histogram

different degree.

3.4.4 The dynamics of the SNA

After we compared the CA's and the SNA's we conclude that the two kinds of avalanches, although if they are based on different assumptions, do not give such various results. The SNA have the advantage of emphasizing the interaction between the single markets. Also SNA are more in agreement with the SOC model, who are the reference models of this study. For these reasons, we chose to study more thoroughly the dynamics of the SNA.

In order to do this we put our attention on the different roles that the markets have in the presence of an avalanche. The most important role is assumed by those markets in which the avalanches start. Let be a node i in which a SNA starts: the node i becomes critical, not because the network influences it, but because a local news (η_i), or a series of local news cumulated over time, is able to make the node i critical. The processing of news is a key building of the model. Furthermore the node i is able to influence other nodes. This is caused by the relationship between the node i and the other nodes. In the model this relationship is due to the weights α and β .

If we compute, for each node, the times in which a node is source of a SNA and we divide it by the total amount of SNA's, we obtain the ranking shown in Table 3.17.

Actually in the top positions there are very important markets like Germany, U.S. and Japan, vice versa in the low positions there are very weak markets like Israel, Egypt and Australia. This fact confirms that the weights α and β shape the dynamics of the network. On the other hand the presence, in the top ten, of less capitalized market like Argentina is due to the high volatility of these markets. The U.S. market has a central position: in fact, even if it is the largest capitalized market, it does not have a high volatility.

A question that arises when a market has a big loss or gain is: is it starting an avalanche? And if it is true, how many nodes does it "overwhelm"? In this regard we can compute the average number of nodes belonging to an avalanche that starts in a given market. The Table 3.18 shows this result. The primacy of Netherlands is easily explicable: this market is not contemporaneous to the other European markets but it opens and closes with an hour of difference. Because the model does not take in account of interaction between contemporaneous markets, the strong conditioning between European markets does not emerge, except in relation to the Dutch market. Therefore this primacy is caused by a limit of the model.

By inspection of Table 3.18 let consider the other nodes that are able to influence a large number of nodes: they are U.K., U.S., Switzerland. Less firmly the Asian markets like Japan, China, Hong Kong and Taiwan.

Let show an example: on 23rd of May 2013 Tokio closed with a loss of 7%, an avalanche started and all European market suffered both opening and closing. When U.S. market closed in the evening basically with no changes, the avalanche stopped without further damage. We speculate the reason: the influence between Asia and Europe is strong, but this influence becomes a "worldwide" influence only if U.S. is involved. A further analysis on the interaction between Europe, U.S., Asia, etc. has been developed in Section 3.4.5.

3.4.5 The hyper-graph of the Worlds Stock Exchanges Network

The model gives some importance to the interactions between countries belonging to the same geographical area (using the weight β). On the other hand the model does not take in account of the intra-daily interactions between markets that open or close at the same hour. So any interactions between countries in the same time zone are not considered. It could be interesting to underline the interaction between sets of nodes, making a set of countries that belong to the same macro-geographical area. A possible choice may

be:

AUSTRALIA \rightarrow **AUSTRALIA**

JAPAN, SOUTH KOREA , CHINA, HONG KONG, TAIWAN, SINGAPORE, MALAYSIA, INDONESIA, INDIA \rightarrow **ASIA**

ISRAEL, EGYPT \rightarrow **MIDDLE EAST**

UK, FRANCE, GERMANY, SWITZERLAND, ITALY, NETHERLANDS, AUSTRIA \rightarrow **EUROPE**

ARGENTINE, BRAZIL, MEXICO \rightarrow **SOUTH-CENTRAL AMERICA**

US, CANADA \rightarrow **NORTH AMERICA**

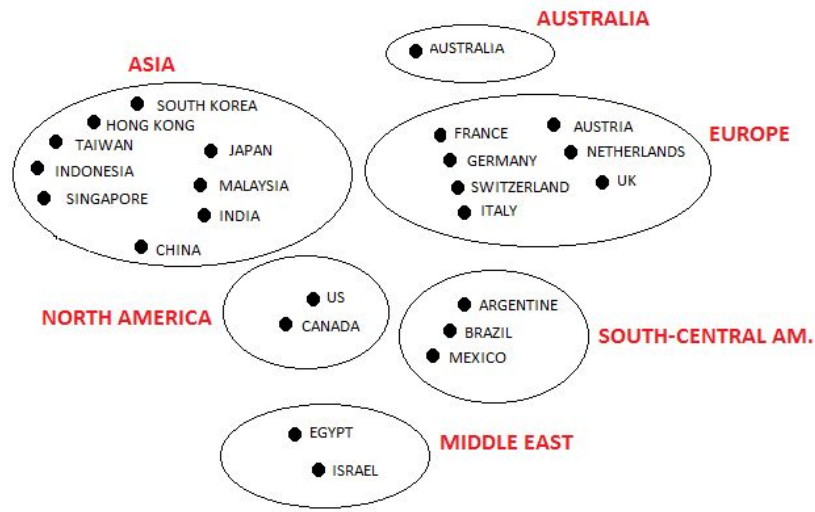


Figure 3.20: The hypergraph representing the World Stock Exchange Network

In graph theory, we are passing by a graph to an hypergraph graph (see Figure 3.20). If we consider the graph obtained by contraction of nodes, the contract graph obtained replacing the set of nodes with the geographical area, we have a new graph composed by 6 nodes (Figure 3.21).

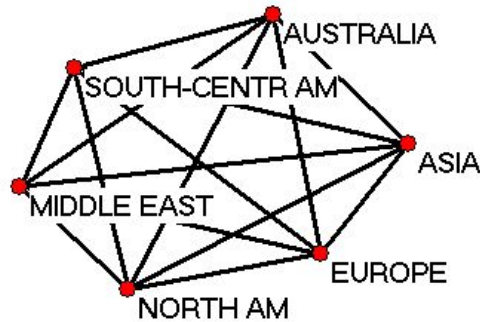


Figure 3.21: The contract graph representing the World Stock Exchange Network

3.4.5.1 The dynamics of the SNA on the hyper-graph

We can study the avalanches (SNA) on this network. So the movements inside the macro-area are not considered: in fact only the links between the 6 nodes are maintained. From 2000 to 2008 are found:

	Positive avalanches	Negative avalanches
In the network	508	336
In the contract network	420	276
Δ	-17.3%	-17.9%

This result proves that more than 82% of the avalanches move from a macro-area to another macro-area. There are only a few number of local avalanches. The contract network is more concise and studying it allows us to capture some aspects of the network dynamics.

Studying the avalanches in the contract graph we can consider (see Table 3.20) the frequency of each link on the total amount of the avalanches. The links are oriented and a link means that the node, from which the link starts, influences the node in which the link enters. The links more frequent in the SNA are the interactions between ASIA and EUROPE. Let's remember that when European markets are opening, the Asian markets are closing and vice versa. Because the most important news is the closing value of the stock is evident that they are influencing each other. Furthermore both ASIA and EUROPE have markets with high volatility, it is likely there are links outgoing from these nodes. The NORTH-AMERICA has no market with high volatility and so there are less links from this node. It seems that NORTH-AMERICA influences more ASIA than EUROPE: this fact is explained in [58], i.e. the European markets are still open when U.S. market opens up and they have access to part of the history of the open-close of the U.S. market, on the contrary the Asian markets are still closed.

By inspection of the bottom of the ranking we find the links less present: it means that there is almost no interaction between AUSTRALIA and the rest of the world. Instead the MIDDLE-EAST, as shown below, is almost not able to influence AMERICA (North, South and Central) and AUSTRALIA.

It could be interesting to underline, for each node, the frequency, on the total amount of avalanches, of links incoming and outgoing and the balance between them. A negative balance means that the node is more influenced, than influencing, by the others; a positive

balance vice versa. In the Figure 3.22, Figure 3.23, Figure 3.24, Figure 3.25, Figure 3.27, Figure 3.26, we can see, for each node, a graph showing the balance of the link frequency and in the Table 3.19 the relative values.

There are two nodes that are only influenced or influencing the others and they are NORTH AMERICA (Figure 3.24), that is only influencing the rest of the world, and MIDDLE EAST (Figure 3.26) that is only influenced. It is evident that this fact is caused by the different importance of markets present in NORTH AMERICA and in MIDDLE EAST, the other nodes play both the roles even if the strength of the interaction is very different for every Macro Area.

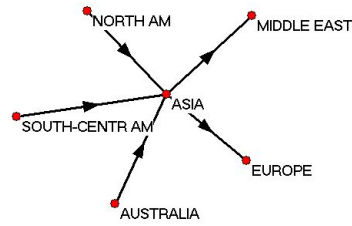


Figure 3.22: The link's balance of ASIA

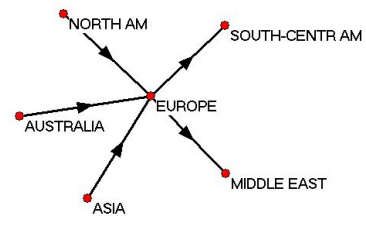


Figure 3.23: The link's balance of EUROPE

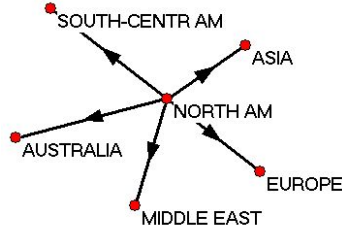


Figure 3.24: The link's balance of NORTH AMERICA

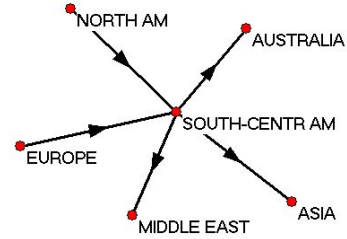


Figure 3.25: The link's balance of SOUTH and CENTRAL AMERICA

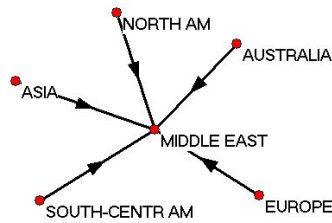


Figure 3.26: The link's balance of MIDDLE EAST

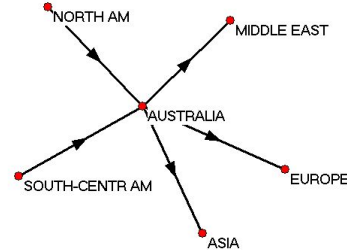


Figure 3.27: The link's balance of AUSTRALIA

Market	From finance.yahoo.it	From Bloomberg
AUSTRALIA	AORD	AS30 INDEX
JAPAN	N225	NKY INDEX
SOUTH KOREA	KS11	KOSPI INDEX
CHINA	SSSEC	SSE50 INDEX
HONG KONG	HSI	HSI INDEX
TAIWAN	TWII	TWSE INDEX
SINGAPORE	STI	FSSTI INDEX
MALAYSIA	KLSE	FBMKLCI INDEX
INDONESIA	JKSE	JCI INDEX
INDIA	BSESN	SENSEX INDEX
ISRAEL	TA100	TA-100 INDEX
EGYPT	CCSI	EGX70 INDEX
U.K.	FTSE	UKX INDEX
FRANCE	FCHI	CAC INDEX
GERMANY	GDAX	DAX INDEX
SWITZERLAND	SSMI	SMI INDEX
ITALY	MIBTEL	FTSEMIB INDEX
NETHERLANDS	AEX	AEX INDEX
AUSTRIA	ATX	ATX INDEX
ARGENTINE	MERV	MERVAL INDEX
BRASIL	BVSP	IBOV INDEX
U.S.	GSPC	INDEX
CANADA	GSPTSE	SPTSX INDEX
MEXICO	MXX	MEXBOL INDEX

Table 3.3: List of Indices

Market	Event	Time of event	Clock of the model
AUSTRALIA	C/O	0:00	1
SOUTH KOREA	C/O	1:00	2
JAPAN	C/O	1:00	2
MALAYSIA	C/O	2:00	3
SINGAPORE	C/O	2:00	3
TAIWAN	C/O	2:00	3
HONG KONG	C/O	2:15	3
CHINA	C/O	2:15	3
INDONESIA	C/O	3:28	4
INDIA	C/O	4:30	5
AUSTRALIA	O/C	6:20	6
TAIWAN	O/C	6:35	6
SOUTH KOREA	O/C	7:05	7
JAPAN	O/C	7:30	7
CHINA	O/C	8:05	8
ISRAEL	C/O	8:30	8
NETHERLANDS	C/O	9:00	9
AUSTRIA	C/O	9:00	9
FRANCE	C/O	9:00	9
GERMANY	C/O	9:00	9
ITALY	C/O	9:00	9
SWITZERLAND	C/O	9:00	9
U.K.	C/O	9:00	9
HONG KONG	O/C	9:15	9
EGYPT	C/O	9:30	9
MALAYSIA	O/C	10:00	10
SINGAPORE	O/C	10:15	10
INDONESIA	O/C	10:15	10
INDIA	O/C	11:45	11
EGYPT	O/C	13:30	12
BRASIL	C/O	14:00	12
CANADA	C/O	14:30	12
U.S.	C/O	14:30	12
ARGENTINA	C/O	15:00	12
MEXICO	C/O	15:30	12
ISRAEL	O/C	17:00	13
SWITZERLAND	O/C	17:35	13
ITALY	O/C	17:40	13
AUSTRIA	O/C	17:45	13
FRANCE	O/C	17:45	13
GERMANY	O/C	17:45	13
U.K.	O/C	17:50	13
NETHERLANDS	O/C	18:30	14
ARGENTINA	O/C	21:00	15
BRASIL	O/C	21:15	15
U.S.	O/C	21:15	15
CANADA	O/C	21:30	15
MEXICO	O/C	22:00	16

Table 3.4: Sequence of events in World's Stock Exchange Network

	AUSTRALIA	JAPAN	SOUTH KOREA	CHINA	HONG KONG	TAIWAN	SINGAPORE	MALAYSIA	INDONESIA	INDIA	ISRAEL	EGYPT	UK	FRANCE	GERMANY	SWITZERLAND	ITALY	NETHERLANDS	AUSTRIA	ARGENTINE	BRAZIL	US	CANADA	MEXICO
AUSTRALIA	0	24	24	8	20	15	34	12	38	19	27	17	8	18	4	17	15	15	4	8	0	0	8	7
JAPAN	18	0	35	33	57	43	54	21	54	42	50	25	39	40	18	37	41	47	10	11	5	7	16	11
SOUTH KOREA	35	40	0	24	77	83	95	31	76	77	51	31	28	37	16	29	38	45	3	16	7	6	12	20
CHINA	8	7	17	0	34	22	28	16	31	31	22	40	16	15	17	21	24	20	16	9	4	3	7	6
HONG KONG	45	52	66	32	0	47	67	30	69	82	52	34	30	40	41	41	52	52	30	16	8	6	21	11
TAIWAN	27	30	57	17	40	0	65	31	66	58	49	29	18	23	8	27	34	30	6	18	6	4	6	13
SINGAPORE	19	12	16	7	17	24	0	10	32	30	36	28	7	8	11	15	19	20	31	12	2	2	4	8
MALAYSIA	5	5	2	5	5	10	13	0	20	17	17	14	0	4	7	8	3	9	16	2	3	0	1	3
INDONESIA	6	6	13	7	10	15	28	14	0	27	37	35	5	13	12	15	14	14	36	11	2	1	3	10
INDIA	30	21	49	20	31	47	60	17	51	0	54	46	13	42	27	37	37	46	47	21	9	5	14	13
ISRAEL	5	5	10	2	8	17	19	9	30	13	0	31	1	10	5	17	17	18	15	12	6	1	3	19
EGYPT	0	0	0	0	0	2	2	2	4	1	2	0	1	0	0	0	0	1	7	2	0	0	0	2
UK	16	16	37	13	34	29	37	9	27	21	28	21	0	18	7	23	24	56	2	53	67	15	28	51
FRANCE	22	23	44	21	37	34	52	17	46	38	44	24	54	0	36	36	47	96	28	43	52	10	35	51
GERMANY	20	32	63	19	33	38	50	10	34	32	44	27	21	37	0	31	41	96	12	58	67	8	37	60
SWITZERLAND	9	16	33	7	24	27	47	16	37	32	38	30	16	21	19	0	39	66	24	45	36	5	26	48
ITALY	13	14	33	8	30	33	38	13	35	16	35	26	18	16	16	23	0	76	23	43	43	4	18	49
NETHERLANDS	17	16	39	7	31	43	53	22	49	31	46	29	25	27	26	42	51	0	29	42	34	4	30	59
AUSTRIA	5	4	6	3	5	8	18	9	21	10	32	33	2	9	5	8	5	16	0	19	11	0	5	36
ARGENTINE	10	12	34	10	15	42	45	12	32	22	52	27	13	28	10	25	30	33	14	0	8	0	17	38
BRAZIL	19	17	51	12	35	46	54	12	43	35	56	35	18	35	11	34	35	44	7	39	0	7	28	53
US	16	19	49	17	50	43	48	14	29	36	37	15	31	36	11	29	35	29	5	11	4	0	15	19
CANADA	16	17	22	9	24	24	30	9	16	28	36	19	24	26	11	28	30	34	18	10	17	4	0	27
MEXICO	7	2	11	4	9	22	34	10	24	18	39	31	8	9	3	19	10	14	2	10	4	2	6	0

Table 3.5: Positive and negative SNA: number of links

	AUSTRALIA	JAPAN	SOUTH KOREA	CHINA	HONG KONG	TAIWAN	SINGAPORE	MALAYSIA	INDONESIA	INDIA	ISRAEL	EGYPT	UK	FRANCE	GERMANY	SWITZERLAND	ITALY	NETHERLANDS	AUSTRIA	ARGENTINE	BRAZIL	US	CANADA	MEXICO
AUSTRALIA	0	13	12	7	12	9	21	8	24	10	17	8	7	10	4	12	12	9	2	2	0	0	5	4
JAPAN	9	0	16	22	40	24	37	13	32	24	31	15	26	25	11	19	28	23	5	6	4	1	11	9
SOUTH KOREA	21	24	0	17	38	48	51	21	42	42	25	14	22	21	10	13	18	17	2	8	3	2	5	8
CHINA	2	1	8	0	18	14	16	9	17	17	13	16	9	11	6	10	12	11	10	5	1	0	2	2
HONG KONG	28	31	34	20	0	28	40	19	44	42	32	18	21	23	23	26	31	27	20	9	4	0	9	7
TAIWAN	15	20	28	10	24	0	40	14	40	30	26	16	12	13	6	12	23	12	5	9	5	1	3	6
SINGAPORE	12	6	6	5	7	13	0	7	18	15	18	16	5	5	7	8	13	11	16	7	1	1	3	4
MALAYSIA	4	1	1	4	3	6	11	0	8	10	11	8	0	2	4	7	1	5	9	2	1	0	1	1
INDONESIA	6	2	7	1	4	8	18	12	0	12	19	16	3	6	5	7	5	8	16	5	1	1	1	4
INDIA	17	12	27	10	14	27	34	12	29	0	31	23	9	18	14	19	22	28	31	8	5	1	6	7
ISRAEL	2	2	4	2	6	8	11	3	16	9	0	18	1	5	2	12	9	10	9	6	2	0	2	9
EGYPT	0	0	0	0	0	1	1	1	1	0	1	0	1	0	0	0	0	1	1	1	0	0	0	0
UK	11	8	21	5	20	20	22	8	18	14	15	12	0	11	3	11	16	38	1	33	39	9	14	32
FRANCE	13	8	24	12	20	19	32	11	28	18	27	16	30	0	20	18	25	63	18	27	29	4	21	30
GERMANY	9	16	34	12	20	21	30	9	24	16	25	17	15	22	0	15	21	56	10	33	33	6	17	31
SWITZERLAND	5	9	20	5	11	15	30	11	20	14	21	15	10	12	8	0	23	44	14	28	21	4	16	30
ITALY	6	6	18	4	11	18	23	11	24	7	23	18	10	11	8	13	0	45	15	25	23	2	8	30
NETHERLANDS	10	9	21	6	17	24	37	20	27	21	30	16	16	18	13	23	29	0	18	25	19	1	21	37
AUSTRIA	3	2	2	2	2	5	13	8	10	3	20	22	1	4	2	3	3	14	0	13	6	0	2	22
ARGENTINE	6	5	20	5	7	24	30	11	24	11	27	15	7	12	6	15	23	16	6	0	6	0	7	22
BRAZIL	10	8	28	8	16	23	33	10	27	21	27	16	11	15	8	18	18	19	4	16	0	1	11	23
US	8	11	28	13	26	25	30	10	16	22	27	10	19	20	7	15	22	13	2	6	2	0	8	12
CANADA	10	9	17	4	14	15	20	7	10	16	22	11	15	16	7	16	21	21	12	4	9	0	0	16
MEXICO	2	0	8	2	5	11	19	8	15	9	21	14	3	5	2	8	6	6	1	3	1	0	4	0

Table 3.6: Negative SNA: number of links

	AUSTRALIA	JAPAN	SOUTH KOREA	CHINA	HONG KONG	TAIWAN	SINGAPORE	MALAYSIA	INDONESIA	INDIA	ISRAEL	EGYPT	UK	FRANCE	GERMANY	SWITZERLAND	ITALY	NETHERLANDS	AUSTRIA	ARGENTINE	BRAZIL	US	CANADA	MEXICO
AUSTRALIA	0	11	12	1	8	6	13	4	14	9	10	9	1	8	0	5	3	6	2	6	0	0	3	3
JAPAN	9	0	19	11	17	19	17	8	22	18	19	10	13	15	7	18	13	24	5	5	1	6	5	2
SOUTH KOREA	14	16	0	7	39	35	44	10	34	35	26	17	6	16	6	16	20	28	1	8	4	4	7	12
CHINA	6	6	9	0	16	8	12	7	14	14	9	24	7	4	11	11	12	9	6	4	3	3	5	4
HONG KONG	17	21	32	12	0	19	27	11	25	40	20	16	9	17	18	15	21	25	10	7	4	6	12	4
TAIWAN	12	10	29	7	16	0	25	17	26	28	23	13	6	10	2	15	11	18	1	9	1	3	3	7
SINGAPORE	7	6	10	2	10	11	0	3	14	15	18	12	2	3	4	7	6	9	15	5	1	1	1	4
MALAYSIA	1	4	1	1	2	4	2	0	12	7	6	6	0	2	3	1	2	4	7	0	2	0	0	2
INDONESIA	0	4	6	6	6	7	10	2	0	15	18	19	2	7	7	8	9	6	20	6	1	0	2	6
INDIA	13	9	22	10	17	20	26	5	22	0	23	23	4	24	13	18	15	18	16	13	4	4	8	6
ISRAEL	3	3	6	0	2	9	8	6	14	4	0	13	0	5	3	5	8	8	6	6	4	1	1	10
EGYPT	0	0	0	0	0	1	1	1	3	1	1	0	0	0	0	0	0	0	6	1	0	0	0	2
UK	5	8	16	8	14	9	15	1	9	7	13	9	0	7	4	12	8	18	1	20	28	6	14	19
FRANCE	9	15	20	9	17	15	20	6	18	20	17	8	24	0	16	18	22	33	10	16	23	6	14	21
GERMANY	11	16	29	7	13	17	20	1	10	16	19	10	6	15	0	16	20	40	2	25	34	2	20	29
SWITZERLAND	4	7	13	2	13	12	17	5	17	18	17	15	6	9	11	0	16	22	10	17	15	1	10	18
ITALY	7	8	15	4	19	15	15	2	11	9	12	8	8	5	8	10	0	31	8	18	20	2	10	19
NETHERLANDS	7	7	18	1	14	19	16	2	22	10	16	13	9	9	13	19	22	0	11	17	15	3	9	22
AUSTRIA	2	2	4	1	3	3	5	1	11	7	12	11	1	5	3	5	2	2	0	6	5	0	3	14
ARGENTINE	4	7	14	5	8	18	15	1	8	11	25	12	6	16	4	10	7	17	8	0	2	0	10	16
BRAZIL	9	9	23	4	19	23	21	2	16	14	29	19	7	20	3	16	17	25	3	23	0	6	17	30
US	8	8	21	4	24	18	18	4	13	14	10	5	12	16	4	14	13	16	3	5	2	0	7	7
CANADA	6	8	5	5	10	9	10	2	6	12	14	8	9	10	4	12	9	13	6	6	8	4	0	11
MEXICO	5	2	3	2	4	11	15	2	9	9	18	17	5	4	1	11	4	8	1	7	3	2	2	0

Table 3.7: Positive SNA: number of links

	critical positive			critical negative			Total amount
	no influenced no influencing	no influenced influencing	influenced	no influenced no influencing	no influenced influencing	influenced	
AUSTRALIA	17	19	44	6	8	52	146
JAPAN	47	37	72	50	45	70	321
SOUTH KOREA	74	59	134	66	57	128	518
CHINA	84	49	51	80	30	60	354
HONG KONG	34	41	114	28	40	116	373
TAIWAN	66	64	100	74	58	103	465
SINGAPORE	9	13	109	9	5	122	267
MALAYSIA	22	22	47	14	12	58	175
INDONESIA	40	29	118	36	13	100	336
INDIA	67	53	141	58	36	118	473
ISRAEL	24	22	100	24	17	94	281
EGYPT	25	0	81	17	0	56	179
U.K.	14	29	59	8	28	70	208
FRANCE	28	62	83	30	61	90	354
GERMANY	31	66	69	30	69	65	330
SWITZERLAND	20	35	88	20	39	82	284
ITALY	12	26	82	10	26	100	256
NETHERLANDS	11	22	129	12	18	148	340
AUSTRIA	46	25	65	22	24	66	248
ARGENTINE	116	54	92	97	37	94	490
BRASIL	80	69	88	75	54	77	443
U.S.	40	38	23	48	45	15	209
CANADA	29	24	65	22	28	57	225
MEXICO	45	30	102	23	17	95	312

Table 3.8: Activity in SNA: number of times in which a node is critical

	critical no influenced and no influencing	critical no influenced but influencing	critical influenced
AUSTRALIA	15.1%	16.5%	68.9%
JAPAN	27.6%	24.1%	50.1%
SOUTH KOREA	26.8%	25.3%	60%
CHINA	43.1%	19.1%	38.8%
HONG KONG	14.4%	20.8%	72.7%
TAIWAN	30.8%	28.8%	47.4%
SINGAPORE	6.7%	6.7%	92%
MALAYSIA	18.9%	22.4%	62.1%
INDONESIA	22.5%	13.7%	68.7%
INDIA	26.5%	20.5%	63.1%
ISRAEL	15.6%	16.7%	72.1%
EGYPT	20.6%	2.2%	77%
U.K.	7.2%	17.7%	75%
FRANCE	16.7%	33.2%	57.9%
GERMANY	18%	38.6%	46.1%
SWITZERLAND	13.7%	26.7%	65%
ITALY	6.1%	19.4%	80.1%
NETHERLANDS	5%	11.7%	91.4%
AUSTRIA	24.1%	20.5%	55.2%
ARGENTINE	40.3%	19.8%	43.3%
BRASIL	31.4%	27.6%	41.1%
U.S.	26.4%	35.5%	38.4%
CANADA	18.5%	22.8%	65.7%
MEXICO	18%	17.6%	64.6%

Table 3.9: Different roles of the node in SNA: influencing or influenced?

	AUSTRALIA	JAPAN	SOUTH KOREA	CHINA	HONG KONG	TAIWAN	SINGAPORE	MALAYSIA	INDONESIA	INDIA	ISRAEL	EGYPT	UK	FRANCE	GERMANY	SWITZERLAND	ITALY	NETHERLANDS	AUSTRIA	ARGENTINE	BRAZIL	US	CANADA	MEXICO
AUSTRALIA	0	30	24	21	24	17	35	12	40	23	32	22	22	27	5	22	22	21	5	9	1	2	17	10
JAPAN	18	0	35	37	61	46	54	21	54	49	57	26	46	44	18	38	48	52	11	15	7	18	21	15
SOUTH KOREA	38	55	0	45	98	85	98	31	80	89	60	36	58	56	18	37	51	55	7	20	13	20	27	28
CHINA	8	7	17	0	36	22	29	16	34	32	28	42	28	17	19	23	25	23	22	12	7	5	15	10
HONG KONG	47	58	66	45	0	47	68	30	71	85	62	37	53	49	48	44	61	57	32	19	10	18	31	18
TAIWAN	35	51	64	41	53	0	68	32	68	73	58	33	50	41	11	37	54	43	8	26	12	20	17	20
SINGAPORE	29	35	28	28	35	31	0	11	34	65	47	29	46	37	37	36	52	44	38	16	8	9	25	18
MALAYSIA	14	14	8	18	12	17	20	0	23	35	25	16	14	15	16	18	18	20	22	5	3	6	9	8
INDONESIA	16	26	34	28	24	27	33	16	0	57	43	36	38	33	29	34	33	34	45	19	5	9	18	15
INDIA	38	40	62	47	52	53	62	18	54	0	56	50	49	65	42	46	51	60	49	25	14	16	27	20
ISRAEL	13	13	36	16	40	32	37	13	38	33	0	31	22	27	21	28	39	41	17	34	27	21	28	38
EGYPT	5	3	17	16	14	16	18	3	13	11	12	0	17	7	10	6	9	7	21	9	8	4	8	10
UK	18	16	41	21	39	33	44	10	29	24	29	21	0	19	7	23	24	63	2	60	67	42	30	59
FRANCE	23	23	55	30	47	43	62	17	49	43	46	24	65	0	39	36	47	101	28	51	63	36	44	57
GERMANY	23	32	70	32	53	49	60	10	36	38	47	28	36	43	0	31	41	101	12	69	81	50	51	72
SWITZERLAND	13	19	47	24	48	37	60	18	42	40	40	33	45	33	28	0	46	75	24	51	58	35	41	56
ITALY	17	23	51	24	51	40	42	13	36	29	37	27	31	22	23	24	0	87	24	56	62	40	32	57
NETHERLANDS	26	21	56	26	54	54	59	23	56	45	49	31	58	38	35	46	58	0	31	54	60	46	50	67
AUSTRIA	10	14	39	30	34	34	33	9	29	27	36	35	19	21	8	18	17	41	0	52	55	27	20	52
ARGENTINE	21	18	55	32	49	59	56	14	36	37	54	30	39	43	11	32	43	41	15	0	13	19	29	42
BRAZIL	28	25	68	36	69	61	66	12	48	45	58	38	39	48	15	37	44	48	8	45	0	27	38	54
US	17	19	67	27	61	58	59	15	36	43	40	17	35	41	12	32	40	34	5	11	5	0	15	19
CANADA	20	19	41	21	45	36	41	11	24	36	44	20	42	35	19	32	36	40	24	10	22	18	0	29
MEXICO	22	16	62	27	48	54	60	12	36	46	47	36	32	31	9	30	29	27	6	15	8	7	16	0

Table 3.10: Positive and negative CA: number of links

	AUSTRALIA	JAPAN	SOUTH KOREA	CHINA	HONG KONG	TAIWAN	SINGAPORE	MALAYSIA	INDONESIA	INDIA	ISRAEL	EGYPT	UK	FRANCE	GERMANY	SWITZERLAND	ITALY	NETHERLANDS	AUSTRIA	ARGENTINE	BRAZIL	US	CANADA	MEXICO
AUSTRALIA	0	18	12	15	14	11	21	8	26	13	20	13	13	16	5	15	16	11	3	3	0	1	9	7
JAPAN	9	0	16	25	40	27	37	13	32	30	36	16	31	28	11	19	32	24	5	9	5	10	13	13
SOUTH KOREA	23	32	0	31	48	49	53	21	45	51	30	18	36	30	12	18	27	22	4	10	6	9	12	15
CHINA	2	1	8	0	19	14	16	9	19	18	17	18	17	12	8	11	13	12	14	7	3	1	8	4
HONG KONG	29	36	34	26	0	28	40	19	45	44	40	20	35	28	27	27	37	30	22	12	5	9	16	12
TAIWAN	19	33	35	28	32	0	40	15	42	39	35	18	31	23	9	21	35	22	6	14	9	11	11	10
SINGAPORE	18	21	15	18	19	17	0	8	20	40	25	17	32	21	25	22	37	26	22	8	5	5	17	13
MALAYSIA	11	9	5	11	7	10	18	0	11	23	17	10	11	10	9	15	11	13	13	2	1	4	8	4
INDONESIA	12	16	22	14	14	16	21	13	0	34	22	17	24	18	16	22	20	22	23	7	3	7	10	9
INDIA	21	21	37	24	27	29	34	12	31	0	33	25	31	32	23	26	32	36	32	11	7	5	13	12
ISRAEL	8	5	19	11	23	19	27	6	23	22	0	18	12	15	8	20	25	26	11	23	15	10	16	22
EGYPT	2	2	11	11	9	13	8	1	9	6	9	0	9	4	7	6	7	6	13	6	7	2	5	7
UK	12	8	23	11	23	22	26	9	20	15	16	12	0	12	3	11	16	43	1	36	39	24	15	37
FRANCE	13	8	31	18	26	24	39	11	31	22	29	16	36	0	20	18	25	65	18	32	36	19	27	32
GERMANY	12	16	37	19	31	29	36	9	26	18	27	18	24	26	0	15	21	60	10	40	40	28	26	41
SWITZERLAND	8	9	28	15	26	22	36	12	25	20	22	17	21	19	11	0	26	50	14	32	33	21	25	38
ITALY	9	10	27	15	27	24	26	11	25	15	25	19	18	16	12	14	0	51	16	33	39	23	17	35
NETHERLANDS	17	11	31	19	34	32	40	20	31	27	31	16	31	25	16	24	32	0	19	35	38	24	34	44
AUSTRIA	8	6	22	18	21	26	22	8	18	15	23	23	13	11	5	10	12	32	0	37	33	19	14	32
ARGENTINE	14	8	30	20	24	35	38	12	27	18	28	17	21	21	6	19	30	21	6	0	7	8	13	24
BRAZIL	16	12	36	21	35	33	40	10	31	23	29	18	25	23	12	21	25	21	5	21	0	14	20	24
US	9	11	37	19	32	35	39	11	20	24	28	11	21	21	7	17	25	15	2	6	3	0	8	12
CANADA	13	11	24	12	27	24	26	8	15	19	26	12	24	22	12	19	25	25	16	4	13	8	0	18
MEXICO	11	7	33	15	26	31	36	9	25	26	27	17	15	19	7	16	18	13	4	6	3	3	9	0

Table 3.11: Negative C/A: number of links

	AUSTRALIA	JAPAN	SOUTH KOREA	CHINA	HONG KONG	TAIWAN	SINGAPORE	MALAYSIA	INDONESIA	INDIA	ISRAEL	EGYPT	UK	FRANCE	GERMANY	SWITZERLAND	ITALY	NETHERLANDS	AUSTRIA	ARGENTINE	BRAZIL	US	CANADA	MEXICO
AUSTRALIA	0	12	12	6	10	6	14	4	14	10	12	9	9	11	0	7	6	10	2	6	1	1	8	3
JAPAN	9	0	19	12	21	19	17	8	22	19	21	10	15	16	7	19	16	28	6	6	2	8	8	2
SOUTH KOREA	15	23	0	14	50	36	45	10	35	38	30	18	22	26	6	19	24	33	3	10	7	11	15	13
CHINA	6	6	9	0	17	8	13	7	15	14	11	24	11	5	11	12	12	11	8	5	4	4	7	6
HONG KONG	18	22	32	19	0	19	28	11	26	41	22	17	18	21	21	17	24	27	10	7	5	9	15	6
TAIWAN	16	18	29	13	21	0	28	17	26	34	23	15	19	18	2	16	19	21	2	12	3	9	6	10
SINGAPORE	11	14	13	10	16	14	0	3	14	25	22	12	14	16	12	14	15	18	16	8	3	4	8	5
MALAYSIA	3	5	3	7	5	7	2	0	12	12	8	6	3	5	7	3	7	7	9	3	2	2	1	4
INDONESIA	4	10	12	14	10	11	12	3	0	23	21	19	14	15	13	12	13	12	22	12	2	2	8	6
INDIA	17	19	25	23	25	24	28	6	23	0	23	25	18	33	19	20	19	24	17	14	7	11	14	8
ISRAEL	5	8	17	5	17	13	10	7	15	11	0	13	10	12	13	8	14	15	6	11	12	11	12	16
EGYPT	3	1	6	5	5	3	10	2	4	5	3	0	8	3	3	0	2	1	8	3	1	2	3	3
UK	6	8	18	10	16	11	18	1	9	9	13	9	0	7	4	12	8	20	1	24	28	18	15	22
FRANCE	10	15	24	12	21	19	23	6	18	21	17	8	29	0	19	18	22	36	10	19	27	17	17	25
GERMANY	11	16	33	13	22	20	24	1	10	20	20	10	12	17	0	16	20	41	2	29	41	22	25	31
SWITZERLAND	5	10	19	9	22	15	24	6	17	20	18	16	24	14	17	0	20	25	10	19	25	14	16	18
ITALY	8	13	24	9	24	16	16	2	11	14	12	8	13	6	11	10	0	36	8	23	23	17	15	22
NETHERLANDS	9	10	25	7	20	22	19	3	25	18	18	15	27	13	19	22	26	0	12	19	22	22	16	23
AUSTRIA	2	8	17	12	13	8	11	1	11	12	13	12	6	10	3	8	5	9	0	15	22	8	6	20
ARGENTINE	7	10	25	12	25	24	18	2	9	19	26	13	18	22	5	13	13	20	9	0	6	11	16	18
BRAZIL	12	13	32	15	34	28	26	2	17	22	29	20	14	25	3	16	19	27	3	24	0	13	18	30
US	8	8	30	8	29	23	20	4	16	19	12	6	14	20	5	15	15	19	3	5	2	0	7	7
CANADA	7	8	17	9	18	12	15	3	9	17	18	8	18	13	7	13	11	15	8	6	9	10	0	11
MEXICO	11	9	29	12	22	23	24	3	11	20	20	19	17	12	2	14	11	14	2	9	5	4	7	0

Table 3.12: Positive CA: number of links

	critical positive			critical negative		Total amount	
	no influenced	no influenced	influenced	no influenced	no influenced	influenced	
	no influencing	influencing		no influencing	influencing		
AUSTRALIA	16	17	47	6	7	53	146
JAPAN	43	36	77	44	40	81	321
SOUTH KOREA	66	59	142	58	58	135	518
CHINA	76	42	66	75	25	70	354
HONG KONG	26	36	127	24	36	124	373
TAIWAN	63	65	102	71	60	104	465
SINGAPORE	9	11	111	8	6	122	267
MALAYSIA	19	25	47	13	13	58	175
INDONESIA	38	31	118	34	13	102	336
INDIA	61	54	146	53	34	125	473
ISRAEL	20	26	100	22	19	94	281
EGYPT	21	3	82	16	1	56	179
U.K.	9	20	73	6	17	83	208
FRANCE	26	56	91	29	53	99	354
GERMANY	29	61	76	29	63	72	330
SWITZERLAND	19	34	90	18	38	85	284
ITALY	9	24	87	6	23	107	256
NETHERLANDS	8	18	136	8	19	151	340
AUSTRIA	40	29	67	20	22	70	248
ARGENTINE	105	56	101	86	38	104	490
BRASIL	75	66	96	64	56	86	443
U.S.	27	33	41	28	41	39	209
CANADA	22	24	72	17	24	66	225
MEXICO	40	33	104	16	22	97	312

Table 3.13: Activity in CA: number of times in which a node is critical

	critical no influenced and no influencing	critical no influenced but influencing	critical influenced
AUSTRALIA	15.1%	16.5%	68.9%
JAPAN	27.6%	24.1%	50.1%
SOUTH KOREA	26.8%	25.3%	60%
CHINA	43.1%	19.1%	38.8%
HONG KONG	14.4%	20.8%	72.7%
TAIWAN	30.8%	28.8%	47.4%
SINGAPORE	6.7%	6.7%	92%
MALAYSIA	18.9%	22.4%	62.1%
INDONESIA	22.5%	13.7%	68.7%
INDIA	26.5%	20.5%	63.1%
ISRAEL	15.6%	16.7%	72.1%
EGYPT	20.6%	2.2%	77%
U.K.	7.2%	17.7%	75%
FRANCE	16.7%	33.2%	57.9%
GERMANY	18%	38.6%	46.1%
SWITZERLAND	13.7%	26.7%	65%
ITALY	6.1%	19.4%	80.1%
NETHERLANDS	5%	11.7%	91.4%
AUSTRIA	24.1%	20.5%	55.2%
ARGENTINE	40.3%	19.8%	43.3%
BRASIL	31.4%	27.6%	41.1%
U.S.	26.4%	35.5%	38.4%
CANADA	18.5%	22.8%	65.7%
MEXICO	18%	17.6%	64.6%

Table 3.14: Different roles of the node in CA: influencing or influenced?

	positive CA			negative CA			positive and negative CA		
	in-degree	out-degree	$\Delta(\text{IN-OUT})$	in-degree	out-degree	$\Delta(\text{IN-OUT})$	in-degree	out-degree	$\Delta(\text{IN-OUT})$
AUSTRALIA	0.41	0.34	0.06	0.89	0.81	0.08	0.65	0.58	0.07
JAPAN	0.53	0.62	-0.09	0.93	1.44	-0.51	0.73	1.03	-0.3
SOUTH KOREA	0.94	1.01	-0.07	1.72	1.8	-0.09	1.33	1.41	-0.08
CHINA	0.51	0.45	0.06	1.25	0.75	0.49	0.88	0.6	0.28
HONG KONG	0.93	0.87	0.06	1.75	1.86	-0.11	1.34	1.36	-0.03
TAIWAN	0.76	0.75	0.01	1.71	1.61	0.1	1.23	1.18	0.05
SINGAPORE	0.89	0.58	0.32	2.15	1.35	0.8	1.52	0.96	0.56
MALAYSIA	0.22	0.24	-0.02	0.76	0.7	0.07	0.49	0.47	0.02
INDONESIA	0.72	0.53	0.19	1.79	1.14	0.64	1.26	0.84	0.42
INDIA	0.88	0.88	0	1.68	1.66	0.02	1.28	1.27	0.01
ISRAEL	0.83	0.52	0.31	1.78	1.15	0.63	1.3	0.84	0.47
EGYPT	0.62	0.17	0.46	1.16	0.48	0.68	0.89	0.32	0.57
UK	0.71	0.58	0.13	1.59	1.3	0.29	1.15	0.94	0.21
FRANCE	0.68	0.87	-0.19	1.35	1.78	-0.43	1.02	1.33	-0.31
GERMANY	0.41	0.91	-0.5	0.81	1.82	-1.01	0.61	1.37	-0.76
SWITZERLAND	0.61	0.77	-0.16	1.22	1.59	-0.37	0.91	1.18	-0.26
ITALY	0.68	0.68	0.01	1.64	1.52	0.12	1.16	1.1	0.06
NETHERLANDS	0.94	0.83	0.11	1.93	1.89	0.04	1.44	1.36	0.08
AUSTRIA	0.36	0.47	-0.11	0.84	1.28	-0.45	0.6	0.87	-0.28
ARGENTINE	0.56	0.68	-0.12	1.18	1.34	-0.16	0.87	1.01	-0.14
BRAZIL	0.52	0.89	-0.37	1.05	1.54	-0.49	0.78	1.21	-0.43
US	0.46	0.59	-0.13	0.79	1.24	-0.44	0.63	0.91	-0.29
CANADA	0.53	0.53	0	1.04	1.21	-0.17	0.78	0.87	-0.08
MEXICO	0.62	0.59	0.03	1.39	1.13	0.27	1.01	0.86	0.15
average	0.64	0.64	0	1.35	1.35	0	0.99	0.99	0

Table 3.15: Average in-degree and out-degree in CA

	positive SNA			negative SNA			positive and negative SNA		
	in-degree	out-degree	$\Delta(\text{IN-OUT})$	in-degree	out-degree	$\Delta(\text{IN-OUT})$	in-degree	out-degree	$\Delta(\text{IN-OUT})$
AUSTRALIA	0.31	0.26	0.05	0.62	0.62	0	0.47	0.44	0.03
JAPAN	0.37	0.56	-0.19	0.6	1.28	-0.68	0.49	0.92	-0.43
SOUTH KOREA	0.64	0.8	-0.15	1.14	1.4	-0.26	0.89	1.1	-0.21
CHINA	0.21	0.4	-0.18	0.52	0.62	-0.1	0.37	0.51	-0.14
HONG KONG	0.57	0.76	-0.19	0.99	1.59	-0.6	0.78	1.18	-0.39
TAIWAN	0.6	0.57	0.04	1.2	1.1	0.11	0.9	0.83	0.07
SINGAPORE	0.73	0.33	0.41	1.78	0.61	1.17	1.25	0.47	0.79
MALAYSIA	0.2	0.13	0.07	0.72	0.3	0.42	0.46	0.22	0.25
INDONESIA	0.68	0.32	0.35	1.53	0.5	1.03	1.1	0.41	0.69
INDIA	0.65	0.65	0	1.14	1.2	-0.06	0.89	0.93	-0.03
ISRAEL	0.74	0.25	0.49	1.51	0.44	1.07	1.12	0.34	0.78
EGYPT	0.58	0.03	0.55	1.04	0.03	1.01	0.81	0.03	0.78
UK	0.28	0.49	-0.21	0.75	1.13	-0.38	0.52	0.81	-0.3
FRANCE	0.45	0.74	-0.3	0.85	1.52	-0.68	0.65	1.13	-0.49
GERMANY	0.28	0.74	-0.46	0.52	1.46	-0.94	0.4	1.1	-0.7
SWITZERLAND	0.52	0.54	-0.03	0.89	1.15	-0.26	0.7	0.84	-0.14
ITALY	0.51	0.51	0	1.13	1.07	0.07	0.82	0.79	0.03
NETHERLANDS	0.74	0.58	0.17	1.47	1.36	0.12	1.11	0.97	0.14
AUSTRIA	0.31	0.21	0.1	0.67	0.48	0.19	0.49	0.35	0.15
ARGENTINE	0.44	0.44	0	0.83	0.91	-0.07	0.64	0.67	-0.04
BRAZIL	0.35	0.7	-0.34	0.64	1.1	-0.46	0.5	0.9	-0.4
US	0.12	0.48	-0.37	0.1	1.04	-0.94	0.11	0.76	-0.65
CANADA	0.32	0.37	-0.05	0.53	0.87	-0.34	0.42	0.62	-0.19
MEXICO	0.53	0.28	0.24	1.03	0.45	0.57	0.78	0.37	0.41
average	0.46	0.46	0	0.93	0.93	0	0.69	0.69	0

Table 3.16: Average in-degree and out-degree in SNA

Percentage of times in which in a market starts a SNA	
GERMANIA	15.8%
FRANCE	14.4%
BRAZIL	14.4%
TAIWAN	14.3%
SOUTH KOREA	13.6%
ARGENTINE	10.7%
INDIA	10.4%
US	9.7%
JAPAN	9.6%
HONG KONG	9.5%
CHINA	9.3%
SWITZERLAND	8.7%
UK	6.7%
ITALY	6.1%
CANADA	6.1%
AUSTRIA	5.8%
MEXICO	5.5%
INDONESIA	4.9%
NETHERLANDS	4.7%
ISRAEL	4.6%
MALAYSIA	4.0%
AUSTRALIA	3.2%
SINGAPORE	2.1%
EGYPT	0%

Table 3.17: The ranking of the markets in which a SNA starts

Average number of nodes belonging to an avalanche that starts from a market			
	Positive SNA	Negative SNA	SNA
AUSTRALIA	12.3	7.5	9.9
JAPAN	9.4	5.6	7.5
SOUTH KOREA	10.5	9.4	10
CHINA	5.9	9.2	7.5
HONG KONG	8.6	7.1	7.8
TAIWAN	8	8.6	8.3
SINGAPORE	8.7	5	6.8
MALAYSIA	4.5	5.2	4.8
INDONESIA	5.8	8.2	7
INDIA	9.2	6.8	8
ISRAEL	8.8	5.6	7.2
EGYPT	0	0	0
UK	15.2	5.1	10.2
FRANCE	12.8	6.8	9.8
GERMANY	10.8	7.5	9.1
SWITZERLAND	10.2	12	11.1
ITALY	11.3	6.2	8.7
NETHERLANDS	10.5	19	14.8
AUSTRIA	9	5.8	7.4
ARGENTINE	7.4	7.5	7.5
BRAZIL	9.8	6.8	8.3
US	14	7.3	10.6
CANADA	7.3	7.3	7.3
MEXICO	9.3	9.3	9.3

Table 3.18: The number of nodes, in average, swept away by an avalanche SNA starting from a given market

ASIA versus	link's balance
AUSTRALIA	-0.8%
EUROPE	6.4%
MIDDLE EAST	18%
NORTH AM	-14.6%
SOUTH-CENTR AM	-17.9%
AUSTRALIA versus	link's balance
ASIA	0.8%
EUROPE	0.1%
MIDDLE EAST	4.4%
NORTH AM	-2.8%
SOUTH-CENTR AM	-2.3%
MIDDLE EAST versus	link's balance
ASIA	-18%
AUSTRALIA	-4.4%
EUROPE	-7.6%
NORTH AM	-9.1%
SOUTH-CENTR AM	-12.7%
SOUTH-CENTR AM versus	link's balance
ASIA	17.9%
AUSTRALIA	2.3%
EUROPE	-10.5%
MIDDLE EAST	12.7%
NORTH AM	-1.3%
NORTH AM versus	link's balance
ASIA	14.6%
AUSTRALIA	2.8%
EUROPE	3.4%
MIDDLE EAST	9.1%
SOUTH-CENTR AM	1.3%
EUROPE versus	link's balance
ASIA	-6.4%
AUSTRALIA	-0.1%
MIDDLE EAST	7.6%
NORTH AM	-3.4%
SOUTH-CENTR AM	10,5%

Table 3.19: Link's balance in the Macro Area Network

		# Pos.SNA =420		# Neg.SNA =276		
		Links in Pos.SNA		Links in Neg.SNA		Average
		#	%	#	%	%
ASIA	EUROPE	183	43.5%	144	52.1%	47.8%
EUROPE	ASIA	161	38.3%	123	44.5%	41.4%
EUROPE	SOUTH-CENTR AM	125	29.7%	104	37.6%	33.7%
SOUTH-CENTR AM	ASIA	128	30.4%	99	35.8%	33.1%
ASIA	MIDDLE EAST	125	29.7%	87	31.5%	30.6%
SOUTH-CENTR AM	EUROPE	93	22.1%	67	24.2%	23.2%
NORTH AM	ASIA	76	18%	75	27.1%	22.6%
EUROPE	MIDDLE EAST	70	16.6%	52	18.8%	17.7%
NORTH AM	EUROPE	62	14.7%	57	20.6%	17.7%
SOUTH-CENTR AM	MIDDLE EAST	74	17.6%	47	17%	17.3%
ASIA	SOUTH-CENTR AM	61	14.5%	44	15.9%	15.2%
EUROPE	NORTH AM	52	12.3%	45	16.3%	14.3%
MIDDLE EAST	ASIA	48	11.4%	38	13.7%	12.5%
AUSTRALIA	ASIA	40	9.5%	32	11.5%	10.5%
MIDDLE EAST	EUROPE	32	7.6%	35	12.6%	10.1%
ASIA	AUSTRALIA	35	8.3%	31	11.2%	9.7%
NORTH AM	MIDDLE EAST	30	7.1%	34	12.3%	9.7%
ASIA	NORTH AM	37	8.8%	20	7.2%	8%
NORTH AM	SOUTH-CENTR AM	27	6.4%	24	8.6%	7.5%
SOUTH-CENTR AM	NORTH AM	27	6.4%	17	6.1%	6.2%
AUSTRALIA	EUROPE	16	3.8%	21	7.6%	5.7%
EUROPE	AUSTRALIA	15	3.5%	21	7.6%	5.5%
AUSTRALIA	MIDDLE EAST	16	3.8%	18	6.5%	5.1%
MIDDLE EAST	SOUTH-CENTR AM	19	4.5%	13	4.7%	4.6%
SOUTH-CENTR AM	AUSTRALIA	16	3.8%	13	4.7%	4.2%
NORTH AM	AUSTRALIA	11	2.6%	15	5.4%	4%
AUSTRALIA	SOUTH-CENTR AM	7	1.6%	6	2.1%	1.9%
AUSTRALIA	NORTH AM	3	0.7%	5	1.8%	1.2%
MIDDLE EAST	AUSTRALIA	3	0.7%	2	0.7%	0.7%
MIDDLE EAST	NORTH AM	2	0.4%	2	0.7%	0.6%

Table 3.20: Link in SNA on the Macro Area Network

Chapter 4

Case of study: Interlocking directorates

4.1 Introduction

Interlocking Directorates are links that are established between companies when a director of a company sits on the board of directors of other companies. They have been defined by Mirzuchi in [36].

The interlocking directorates may be direct or indirect. The first situation arises when two companies share an administrator, the latter happens if two companies have at least one director who is on the board of a third company.

For this reason an approach based on network theory is suitable to describe these horizontal linkages between companies: the companies become the nodes of the network and shared directors represent the links.

Large part of literature has been developed on this topic: using the network theory in order to explain the interlock phenomenon and empirical studies on various countries have shown that this kind of network is characterized by a topological structure that tends to persist over time while still maintaining its structural properties (see Section 4.2).

The object of our case of study is the dynamic evolution of the interlocking directorate network referring to the Italian case.

To discover how the structure evolves over time (14 years) we examine an extensive database: in particular, we are interested in determining the role played by directors in the network structure.

In spite of recent improvements in corporate governance that should limit interlocks, they are also an important characteristic of Italian capitalism.

Using the same methodology proposed by [34], we could assess if in Italy, as in the

German case, a connected and stable structure emerges, due to the presence of directors with multiple mandates. As a result, we find a network structure characterized by a "core", due to the presence of a few directors with multiple assignments and this core is stable (over time), but, unlike the German case, not connected. This result is consistent with the literature on the Italian case, as explained below.

Also, we investigate the nature of the stable links over time; to this end we propose an alternative approach based on dynamic networks that allows us to investigate the stability of the links also identifying their percentages of stability as time passes.

The analysis is performed by quantifying the variation of links in a time period. Then we construct a unique cumulative network, where nodes are companies and the existence of an edge is related with the persistence over time of an interlock between two companies.

We think it is of interest to determine whether the ties persisting between firms are due to a stable presence of directors with the same mandate for years, or the persistence occurs despite a turnover effect.

For instance looking at the stability in all time period (from 1998 to 2011 with 100% of stability) the resulting emerging structure has few connected components, each very cohesive, i.e. with a high density.

This states, on one hand, that Italy is characterized since many years back by a governance structure at serving of the ownership, which cannot be separated from shareholding control of a few important family groups. On the other hand, there is a component in which large Italian firms both in financial sector and industry are connected to each other by sharing their directors. Our work offers a further contribution to the literature on dynamic networks. Indeed, although the time dimension is projected on a unique static network, the methodology that we propose captures the time variability of the links better than other dynamical networks that project the time dimension into a static structure (see Section 2.3).

4.2 Overview of literature

In the literature many articles refer to the study of network's properties in the corporate board context, at first we focus attention to the Italian case.

Gambini et al. [22] present an analysis of the Italian interlocks, in particular they distinguish interlocks between banks, banks/firms, firms. They underline the presence of

many ties between banks: direct and indirect. They also detail the interlocks distinguishing whether the interlock is due to an executive director or to a non-executive director.

They state that interlock continues to be one of the most important channels of link between businesses in Italy: the number of links between the companies listed in Italian Stock Exchange is very high, as is the case in other countries. In particular, they found that the Blue Chips are a more homogeneous and connected network, while the network formed by banks has a almost linear structure. The connections do not affect all companies equally, but there is a considerable degree of concentration within groups of companies that share part of their directors.

Carbonai and Di Bartolomeo [15] present a paper that investigates the Italian insurance system by analyzing the interlocks. The investigation follows a two-step procedure: first, it analyzes the network of the insurance industry by focusing on interlocks; second, network statistics are combined in synthetic indices through principal component analysis in order to verify a correlation between indices and companies' market shares. The analysis is limited to non-life insurances, which are indeed the least competitive and the most closed compared to the competition of other financial agents (life insurances compete with other forms of financial investments). The study is limited to only one year (2004) and uses the data of the ANIA, the Italian Association of Insurance Companies. They state that interlocks are indicators of potential power relationships between companies and it cannot be inferred that directors exploit networks of board memberships merely because such potential exists. Interlocking directors seem to be used by insurance companies to support a large cartel that dominates the market. By placing a director on a cartel partner's board, each cartel member has an observer in place who can monitor activities such as plans to reduce price, expand capacity, or introduce new products that could undermine the cartel agreement. Interlocking directorates can help minimize trust problems by putting insiders in places where they can both monitor and affect what other companies are doing. Thus, in Italy the interlocking directors seem to be the instrument used by insurance cartel to maintain its stability.

An extensive work has been done by Rinaldi and Vasta ([45], [46], [47], [48]). Their studies describe in detail the structure of Italian capitalism and, the latter, during the 1913-2001 period by using network analysis techniques, focused on seven benchmark years (1913,1927,1936,1960,1972,1983 and 2011). Each benchmark year is analyzed by means

of the network composed by the top 250 companies and it is integrated by a historical and structural analysis. The paper highlights some different phases in the evolution of the Italian interlocking directorates and they are considered a consequence to some major institutional break-ups: the crisis of the German-type universal banks and the creation of large state-owned sector of the economy in the early 1930s, the nationalization of the electricity industry in 1962; a massive privatization of state-owned enterprises in the 1990s and the emergence of the technological trajectory of the third industrial revolution in the 1983s.

They found that the network is very cohesive from 1913 to 1960 with a maximum in 1927 because of the influence of the German-type universal banks. After the nationalization of the electricity industry in 1972 the network decreased its cohesion, but the fall in the degree of cohesion appears in 1983 and 2001. They measure the cohesion of the network by using two centrality indices: degree centrality and betweenness centrality.

Santella et al. in a first paper [54] analyze the Italian situation from 1998 to 2006 and in a second [53] from 1998 to 2007. In the first paper they focus on stability over time in the number of directorships: directors who have just one, two or three appointments, unlikely enter in the director category with multiple appointments. They conclude that a great number of Italian companies are connected to each other by a small number of directors, whom maintain permanently their appointment over time and are defined "Lords of the Italian Stock Market" by the authors.

In the second paper they focus on the relationship between interlocking directorships and company performance for the main companies listed on the Italian stock market. They use a unique dataset that includes two distinct groups of variables: corporate governance variables related to board size and interlocking directorships and a group of variables related to the economic and financial performance of the companies considered. They find that the corporate governance reforms introduced over the period considered have shown some effectiveness by slightly dispersing the network of companies and that interlocking directorships are negatively related with company performance.

Larcker and al. seem to have a different opinion [31]: they assess that firms with central boards of directors earn superior risk-adjusted stock returns. Using data referring to the U.S. stock exchange from 2000 to 2007 they map the board network and they compute four centrality measures: degree, closeness, betweenness and eigenvector. Then

they do a regression analysis with these measures, stock return and other measures of firm performance. They find the balance of the potential costs and benefits associated with board networks and they establish several regularities regarding the relation between board centrality and multiple measure of firm performance.

Also Yeo et al. ([61]) find a positive relationship between the number of director interlocks and their firms' performance measured by ROA. They use a sample consisting of 246 firms from major sectors of the French economy in 1999. They investigate factors explaining interlocking directorates: their results indicate that directors of larger firms hold more interlocks. Finally they find evidence that directors hold more interlocks when a block-holder is present on the board.

Santella et al. [54] also propose a comparison of Italian, French, German, UK and US listed Blue Chips. An analysis of the interlocking directorships among the five countries is performed. They found that two models stand out. On the one hand a model made of a high number of companies linked to each other through a small number of shared directors who serve on several company boards at the time (France, Germany, and Italy). On the other hand, in the UK much fewer companies are connected to each other and essentially through directors who have no more than two board positions at a time. An intermediate case is represented by the US, where a high number of companies are connected to each other just like Germany, France, and Italy. However, just like the UK, such connections are made through directors who tend to have just two board positions at time, a sign that, unlike Italy, Germany, and France, the UK and US networks might not be functional to systemic collusion.

In agreement with Santella, the work relating to the German stock market proposed by Alfarano et al. is [1]. They analyze the corporate board network in the case of Germany and their methodology has been later extended to a dynamical approach by Milakovic et al. [35]: they found, in both director and board network, a "persistent core", despite the great turnover in the identity of core directors over time. They suggest that both the reconstruction of broken ties among large corporations, as well as their preference for recruiting experienced directors with multiple board memberships, are responsible for the time persistence of a network core.

Heemskerk et al. [26] extend the network analysis to Europe. They consider the companies of Eurofirst top 300 index and build the board network in 2005 and 2010.

They show that in 2010 the European board network is stronger than five years earlier, i.e. the network increases its density. They suggest that this structure can be a basis for overcoming the present euro crisis.

We only mention a part of literature (see [9], [14], [20],[18],[49]) that analyze the small world structure of the board or director network, i.e. a network where the distance between two randomly chosen nodes grows proportionally to the logarithm of the number of nodes N of the network.

4.3 Data set and networks construction

We study the composition of boards in Italy from 1998 to 2011. All the information concerning the composition of the board of the companies are listed on Borsa Italiana, the Italian Stock Exchange and are provided by *Assonime*, the Association of Joint Stock Companies incorporated in Italy¹. The governance of the listed companies in Italy generally adopted the traditional model which is characterized by a board of directors (*Consiglio di Amministrazione*) and a board of statutory auditors (*Collegio sindacale*), both appointed by the shareholders. Generally, the majority of the board is composed by non-executive directors (about the roles of latter see [11]); unless decided by internal regulatory choices, the board of a firm stays for three years, but the appointment of a director can be confirmed for more one mandate. Only few Italian companies adopted the dualistic model, with two separate committees (Supervisory and Management), similar to the model adopted by Germany.

Using this data set, for each year of the sample we study the interlocking structure in Italy constructing two networks (that we will call *board network* and *director network*). Let n and q be respectively the number of companies and directors; the board network is a weighted graph of n nodes/companies, in which every pair of nodes is connected by a weighted edge if they have at least one common director. In this network, nodes are the companies and two nodes i and j are connected by an edge of weight b_{ij} (b_{ij} integer, $b_{ij} \geq 1$) if they have b_{ij} common directors. Similarly, the director network is a weighted graph of q nodes/directors in which every pair of directors i, j is connected by a weighted edge (of weight d_{ij}) if they sit in d_{ij} company's boards.

The above mentioned graphs can be associated to suitable matrices obtained in the

¹Associazione tra le Società Italiane per Azioni.

following way. Let M be a matrix of order $q \times n$ such that $m_{ij} = 1$ if director i sits on the board of company j , $m_{ij} = 0$ otherwise. We construct the n -square matrix $B' = M^T M$, where the off-diagonal entries are the weights of the edges, whereas the diagonal entry b'_{ii} is the size of the i -th company's board ($i = 1, \dots, n$), and the q -square matrix $D' = M M^T$, where the off-diagonal entries are the weights of the edges, whereas the diagonal entry d'_{jj} ($j = 1, \dots, q$) gives the total number of j th director's mandates.

The same matrices B' and D' can be constructed by the bipartite graph, representing the relationship between boards and directors, passing to its one-mode projection (see [14] and [9] for a more detailed description). We set all the diagonal entries of B' and D' equal to zero and all the weights b'_{ij} and d'_{ij} equal to 1, obtaining exactly the adjacency matrices associated with these graphs: B and D .

Some preliminary remarks can be made through an overall look at the board network structure. For every year of the sample these networks are characterized by a quite similar topology. They have many isolated nodes, few connected components (often formed by two vertices and one edge) and one component with most nodes (over 70% of the total number of the companies) which we call the *main component*. The main component is an usual presence in networks describing interlocking directorates (see [14], [9], [41]), because in general most companies tend to be connected to each other via direct and indirect relationships. This explains the special attention we reserve to the analysis of the main component.

Table 4.1 summarizes some descriptive statistics of the board network B and some information concerning the number of directors and their appointments.

The board network shows similar parameter values year by year; the total number of companies ranges from 235 to 301; board size tends to slightly increase on average, but company links (here computed by counting the number of links between companies in the un-weighted network) tend to reduce.

Focusing on the main component, some additional characteristics emerge. Network density is the sum of the degrees divided by $n(n-1)$, where n is the number of the companies/nodes of the network. A high number of links among the companies implies a high density value in the board network. This parameter therefore gives an idea of how intense the ties in the network are. The main component size tends to increase as the total number of companies in the network increases, at the same time the density of the

main component is low and it tends to decrease, whereas its size slightly increase. This fact seems suggest that the connections among the companies are not particularly intense. Other remarks concerning the trend of number of director and their appointments are reported in the next section.

4.4 Evolution of interlocking directorates

To analyze how the structure of interlocking directorates evolves, we think important to give a look at how the structure of the directorships changes over time. Concerning the case of Italy, the mainstream literature showed a structure of the board network which remained cohesive for a long time, reinforced by the presence of a small group of directors with multiple assignments (they are often CEOs in one of the companies, or important shareholders).

Referring to Table 4.1, unlike what was found in [35], in Italy we do not observe a slight and constant decrease in the number of directors and mandates. On the contrary, the number of directors, as well as the number of mandates, grows until 2009, with the exception of a slight decrease between 2001 and 2003. From 2009 the values start to slightly decrease. This trend is also due to the peculiarity of the Italian situation, because since 1998 new laws contributed to reform the corporate governance², although before 2012 there was no restrictions on multiple mandates. Only in 2012 the prohibition of accepting or exercising positions between firms or groups of competitors operating in the credit markets, insurance and finance (so-called "non-interlocking") became law³.

4.4.1 The *core* of German Stock Exchange versus the *core* of Italian Stock Exchange

Looking at the number of links between companies, which is expression of the multiple board memberships, we observe a clear decrease only after 2005, even if this phenomenon is less intense with respect to the German case (see [35]).

To assess the influence of directors with multiple directorships in the board structure we use the same methodology proposed in [35]. We construct the bipartite graph representing the connections between directors and companies; from it we extract, by 1-mode projection, the sub-graph of companies with directors whose number of interlocks exceeds

²D.L. nr. 58/1998, D.L. nr. 6/2003.

³Law nr. 214/2011.

a certain threshold b . This will also allow us to compare the Italian situation with the German case, highlighting similarities and differences. The sample analyzed in the Italian case encloses all listed companies from the Italian Stock Exchange and it is therefore wider than the sample of the German case, as in [35] the analysis is restricted to companies listed on the DAX and MDAX (equivalent to the Blue Chips in Italy). Also, we analyze the incidence of interlocks for 4 years of the sample, instead of 3 years of the German sample. Given that the number of companies taken into consideration is higher, the number of directors is much higher than that of Germany.

Focusing to the evolution of the interlock, the number of directors appointed at least once within 14 years is 6373. Among these, the total number of directors giving rise to interlocks within a time interval of 14 years is 3957. These are directors who simultaneously serve on different boards. Therefore, we construct a bipartite graph that is formed by 3957 directors and 454 companies. From it we extract the sub-graph formed by directors that have at least b interlocks, where $2 \leq b \leq 7$. - Table 4.2 shows the sub-graph extracted for different values of b in 4 years (1998, 2002, 2006, 2010). Using the same terminology as in [35] we call this sub-graph *b-core*⁴.

By inspection of Table 4.2, we observe, as expected, that the number of directors with multiple directorships decreases as b increases. This is more evident in recent years: in 2006, 7 directors have had at least 6 appointments; only one director has had 7 appointments. In 2010 only two directors have had 6 assignments (in this case it not possible to compute the density) and none has had 7 appointments. In [53], [45], and [46] the authors claim that the presence of a few directors with multiple mandates is a feature of the Italian system, but this is also basically what happens in the German being equal the number of mandates.

For small values of b ($b = 2, 3, 4$), in each of the examined years the majority of directors with multiple directorships tend to be connected together in a single large component. However already for $b = 2$ other connected components appear, each with a few nodes. Some of them are cliques (as in 2006 for $b = 3$ or in 2010 for $b = 5$). The situation is different in the German case, which is always characterized by a single, highly connected and dense component.

To complete the analysis we plot the density of the b -core. Examining the pattern for

⁴ b -core is used with the same meaning as used in [35], which is different from the definition of k -core introduced and used by Seidman in [52].

each of the years, we observe that the density increases with the b -core, similarly to the German case (Figure 4.1).

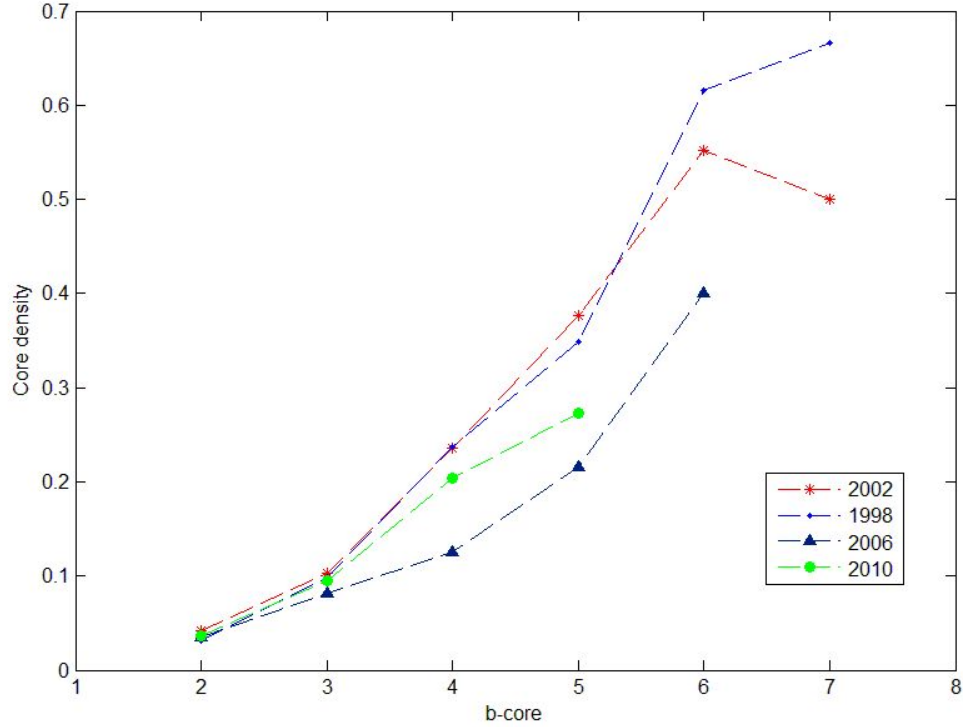


Figure 4.1: Core density trends vs b -core.

Therefore the most striking difference with respect to the analysis performed in [35] for the German case seems to be that in Italy there is a core of directors with multiple directorships which is dense, but not connected. In the German case all biggest firms are connected to each other, whereas the Italian situation is characterized by small networks (of companies in many cases also very cohesive).

The reasons can be searched in the different rules of corporate governance between the two Countries. As pointed out in [35] and in [1], where the analysis of the German case is performed for just one year, the core structure in Germany's corporate network is centered around many highly capitalized companies, whose role is then fundamental for the existence of this connected core. On the contrary, in Italy historically the presence of a few family groups influence the director appointments, affecting the management decisions of the companies. Another aspect concerns the ownership, since in the past the executive directors were strongly related to important shareholders, although the recent laws tried to reform this aspect. This could explain the presence of few dense connected components

instead of one component densely connected.

Our findings seems to confirm the central role played by a few directors in the Italian system, as it is also noticed by the literature about the Italian situation ([45],[46],[54]). Focusing on the small connected components, they are formed by directors who sit on the boards of directors of companies (Mediaset, Mediolanum, Vianini, Cementir), belonging to the Italian Blue Chips. Also, this leads us to think that our findings are not affected by having included in our analysis all the companies listed on the Italian stock exchange (let us remember that in [35] the analysis of the German case is restricted to companies listed on the DAX and MDAX).

4.5 A new model of network dynamics

The analysis performed comparing the Italian situation to the German case highlighted many differences. What does not emerge in the previous studies is which companies create stable links over time and which of these links are attributable to the presence of the same directors in multiple boards. Many companies indeed maintain the same links with others in the entire period, but they do not maintain the same directors; this aspect could be interesting to investigate in the Italian context, which is characterized by a pyramidal structure in which the family groups play an important role.

In this Section we propose another approach to study the time evolution of the board network, in order to find out which links are the most stable over time.

We build a static graph in order to capture both temporal and topological properties of the network and we propose a new method to do it.

To start with, we describe the network at time $t = 1, 2, \dots, m$ with graphs G_1, G_2, \dots, G_m . First we observe that varying the time, not only the links vary, but also the vertices of the network. Indeed, when we will apply this model to the Italian Stock Exchange in a long time period some firms can be delisted, whereas new firms become members.

Hence in the network some nodes may disappear, whereas others may appear. For this reason, we consider as a set of vertices the union of all vertices appearing in the given time interval. Therefore, for every $t = 1, 2, \dots, m$ the graph G_t will have the same number of nodes n . When a vertex v exists at time $t = i$ but it does not exist at time $t = j$ it will appear both in G_i and in graph G_j , the only difference being that in G_j it will appear as

an isolated and fictitious vertex⁵ The fact of taking into account all the appearing vertices is widely used in literature (see for instance [30],[40]), as significantly it simplifies the pattern, even though it increases its complexity.

Let $G_t = (V_t, E_t)$ with $|V_t| = n$ and $t = 1, \dots, m$ be the graph describing the network at time t . Let us consider the adjacent matrices A_t associated to graphs G_t ($t = 1, \dots, m$), and define for each time interval $\tau = [t, t + 1]$ a n -square, symmetric matrix $D_\tau = [d_{ij}^\tau]$ such that:

$$d_{ij}^\tau = \begin{cases} -1 & a_{ij}^t = a_{ij}^{t+1} = 0 \\ 1 & \text{if } a_{ij}^t = a_{ij}^{t+1} = 1 \\ 0 & \text{otherwise} \end{cases}$$

The entries of the matrix D_τ quantify the persistence of the edges in a time interval, giving the information when in a given time interval τ the edge (i, j) does not exist ($d_{ij}^\tau = -1$) or it has been subject to any variation ($d_{ij}^\tau = 0$), i.e. at time t it does not exist and at time $t + 1$ it exists (or vice versa), or if an existing link remains unchanged at all ($d_{ij}^\tau = 1$). We call *active at time t* a link (i, j) such that $d_{ij}^\tau \neq -1$.

Since the number of time intervals τ is $m - 1$, we obtain $\tau = 1, \dots, m - 1$ variation matrices D_τ . Let us now define the *link activity* v_{ij} and *link stability*⁶ s_{ij} of the edge (i, j) as:

$$v_{ij} = \sum_{\tau=1, d_{ij}^\tau \neq -1}^{m-1} 1,$$

$$s_{ij} = \sum_{\tau=1, d_{ij}^\tau \neq -1}^{m-1} d_{ij}^\tau.$$

Observe that the value v_{ij} of (i, j) counts in the whole time span the number of active links and takes into account only the intervals in which the link (i, j) persists or it has been subject to any change ($d_{ij}^\tau = 1$ or $d_{ij}^\tau = 0$): in general it quantifies the link activity for the entry (i, j) in the overall time. On the other hand, the link stability s_{ij} considers the number of persistent links, as it is equal to the number of time intervals in which an edge (i, j) remains unchanged.

⁵Since our aim is mainly to find out the persistence of links, the existence of isolated, fictitious vertices does not affect the validity of our study.

⁶In this work, the word "stability" refers to the presence over time of network links, and it is not the meaning assumed in dynamical system theory.

Finally, we quantify the relative stability for each links, defining a n -square symmetric matrix $C = [c_{ij}]$ (we will call it *cumulative matrix*) such that:

$$c_{ij} = \begin{cases} \frac{s_{ij}}{m-1} & \text{if } v_{ij} \neq 0 \\ -1 & \text{otherwise} \end{cases}$$

Observe that for an active link (i, j) , $0 \leq c_{ij} \leq 1$, whereas for a non-active link (i, j) , $c_{ij} = -1$. By the previous definition, if the link (i, j) always exists in the network, then $c_{ij} = 1$, this being the maximum link stability. If the link (i, j) has been activated (at least once) in a time interval but in the following time it has been cancelled, then $c_{ij} = 0$, hence it has not acquired stability. The other values of c_{ij} , with $0 < c_{ij} < 1$ indicate the percentage of relative link stability. In the following example we illustrate our approach to study the network dynamics.

The network stability can be further illustrated by the construction of a new network, representing the stable links in the overall time period. In order to do this, we define a threshold θ of stability and the graph S_θ whose adjacency matrix is:

$$a_{ij}^\theta = \begin{cases} 1 & c_{ij} \geq \theta \\ 0 & \text{otherwise} \end{cases}.$$

The resulting graph S_θ shows the most stable links, e.g. with percentage of relative link stability more than θ . For this reason, we will call it *stability graph*.

Example

Let us consider the evolution of a graph G with $n = 5$ for $t = 11$ time periods; the graph evolves by changing its links over time. Figure 4.2 shows how G evolves in the overall period. We observe that the link between node 1 and node 2 is active and always stable, so that $c_{12} = \frac{10}{10} = 100\%$; the link between node 2 and node 3 at first is stable, whereas for $t = 7$ disappears, so that $c_{23} = \frac{5}{10} = 50\%$. The link between node 4 and node 5 is active but not stable, as it disappears and afterwards appears ($c_{45} = \frac{0}{10} = 0\%$). The link between node 1 and node 3 is not active, and therefore $c_{13} = -1$. The weighted graph representing the no negative values of the cumulative matrix C is shown in Figure 4.3; this graph illustrates which links are active and stable.

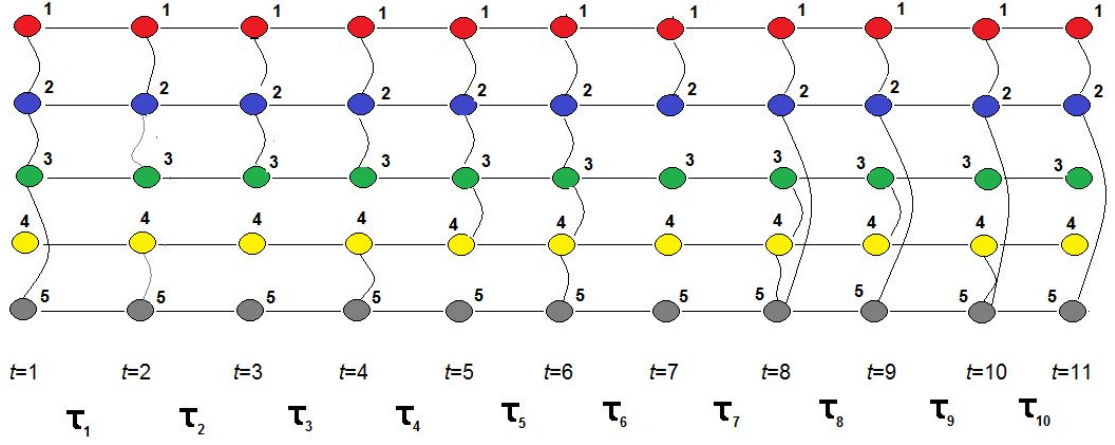


Figure 4.2: Dynamics of a graph G with 5 vertices for $t = 11$ time periods; graph evolves by changing its links over time.

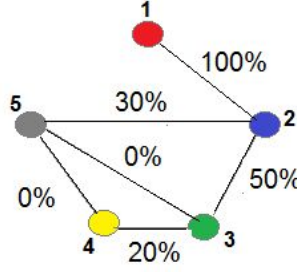


Figure 4.3: The dynamics of the graph represented in Figure 4.2 is modeled by the cumulative matrix C , represented by a weighted graph, weights representing the entries $c_{ij} \neq -1$. The weighted graphs describes the activity of the links, stating the percentage of stability. For instance, the link between node 1 and node 2 is always stable and active, so that $c_{12} = \frac{10}{10} = 100\%$; the link between node 2 and node 3 at first initially is present, whereas for $t = 7$ disappears, so that $c_{23} = \frac{5}{10} = 50\%$. The link between node 4 and node 5 is active but not stable, as at first it disappears, reappearing afterwards ($c_{45} = \frac{0}{10} = 0\%$).

4.6 Application to the board network

If we apply the previously described model to the board networks from 1998 up to 2011, we obtain 13 variation matrices D_{I_t} from which the cumulative matrix C is derived. In this case, for each year, a square matrix of 454 nodes must be used, since we take into account all the companies that, throughout 14 years, have been listed in the Italian Stock Exchange. Thus, a certain company will hold the same position in the matrix as time passes and no ambiguities will arise in establishing where interlocks are placed. Therefore, assessing the time stability of an interlock between two boards is equivalent to assess the stability of a given link. At this level of the analysis, however, we do not know yet whether the interlock is due to the same director or more different directors who have been subsequently appointed. Initially, we state that the threshold is $\theta = 100\%$. Obviously,

this represents an extreme case where interlocks are always existent in time.

By inspection of Figure 4.4 it's evident that there are 8 connected stable components. As a result, there are strong ties among firms in the overall period. Most of them are ties due to the ownership of family firms. Indeed, the connected component formed by Caltagirone, Cementir, Vianini Ind., Vianini Lavori, Banca Finnat represents the Caltagirone family; the component formed by Sogefi, Cir, Cofide, Gruppo Editoriale L'Espresso represents the De Benedetti family; the component formed by Fiat and Exor represents the Agnelli family; the component formed by Carraro, Autogrill, Benetton represents the Benetton family; finally the component formed by Mediolanum, Arnaldo Mondadori, Mediaset represents the Berlusconi family.

The component with the most number of nodes reflects the result of the cross-shareholding among the most historical and important Italian companies, as RCS, Pirelli SpA, Italcementi and Mediobanca. This was essentially noticed also in the analysis of [22], in which the authors point out the existence of a cohesive group of firms, in a central position with respect to most centrality measures, belonging to the *salotto buono*⁷ of the Italian companies. Moreover it becomes the main component if we decrease the threshold, namely in Figure 4.6 we can observe that this component has incorporated the components of Benetton family, Agnelli family, De Benedetti family, Berlusconi family. Instead the Caltagirone family continues to remain a separate component.

It is worth wondering whether the stable links, shown in Figure 4.4 are caused by the step-in/out of different directors or not. In order to find this out, we can use the bipartite network which represents the relationships among directors and the boards they sit on, and we state appointment stability. Hence, it will be possible to assess the nature of interlock stability. On the one hand, we could observe those interlocks, whose stability derives from the stability of director appointment/s, on the other we could find those interlocks that are stable regardless of appointment stability. If we take the list of directors being appointed at least once within 14 years, this amounts to 6373 directors. Among these, the total number of directors giving rise to interlocks within a time interval of 14 years is 3957. These are directors who simultaneously serve on different boards. Therefore, we need to construct a bipartite network that is formed by 3957 directors and 454 companies.

⁷This is a typical Italian expression to identify a the well-heeled elite of bankers, industrialists and politicians that still dominate Italian economic life.

As it happened previously for every single year analyzed, only some vertices are active and many vertices will be fictitious. If we apply our methodology to the bipartite network, we obtain a cumulative matrix C' , which means that each of its elements represents the percentage of link stability in time, i.e. in this case the appointment stability.

From the analysis of all the above mentioned data, we can establish that the first significant threshold, that is the one in which at least a link exists, is approximately 69%. This means that no director has had an appointment, giving rise to an interlock, for more than a 10 year total period.

On the contrary, in the board network the presence of stable links occurs also with a threshold $\theta = 100\%$. This means that in such a network interlock stability higher than 69% is caused by the step-in/out of different directors, i.e. an interlock persist but the director that causes the edge is not always the same.

Establishing a threshold $\theta = 65\%$ we are able to identify the kind of links in the board network by means of the bipartite network. In fact, the stability of a link in the board network is due to the appointment stability if the corresponding appointment is present in the bipartite network. In Figure 4.5 we can observe the bipartite graph obtained fixing the threshold of 65%: on the left of the edges there are the directors and on the right the companies in which board they sit in.

While in the board network, with the same threshold, the active vertices amount to 77 companies, only 36 companies (out of the 77) have appointment stable links and they appear in bipartite network too.

Figure 4.6 summarizes the analysis of the stability on the board network, by including the information extracted by the bipartite network. The information about appointment stability is represented in the graph by red thick edges, whereas the simple stability is represented by black edges.

From the inspection of this graph, it is evident that in most cases the link stability is not due by the appointment stability. Nevertheless we have to notice the presence of stable links among the most important firms and then we can suppose that firms tend to maintain their connections over time, despite of a remarkable turnover among directors. In some components the stability of appointments is due to the family ownership; the most evident are the cases of Caltagirone and De Benedetti families. Caltagirone family controls many industrial firms; this ownership structure is reflected also in the governance:

these firms are linked in a very cohesive connected component, where links are due to the presence of the same directors for most years of the sample (quite similar is the analysis of De Benedetti group).

Our analysis reinforce what we have found in the previous Section: the italian case is characterized by a stable disconnected small structure with intense ties and every components have a cohesive structure, often submitted to a ownership control of few important families.

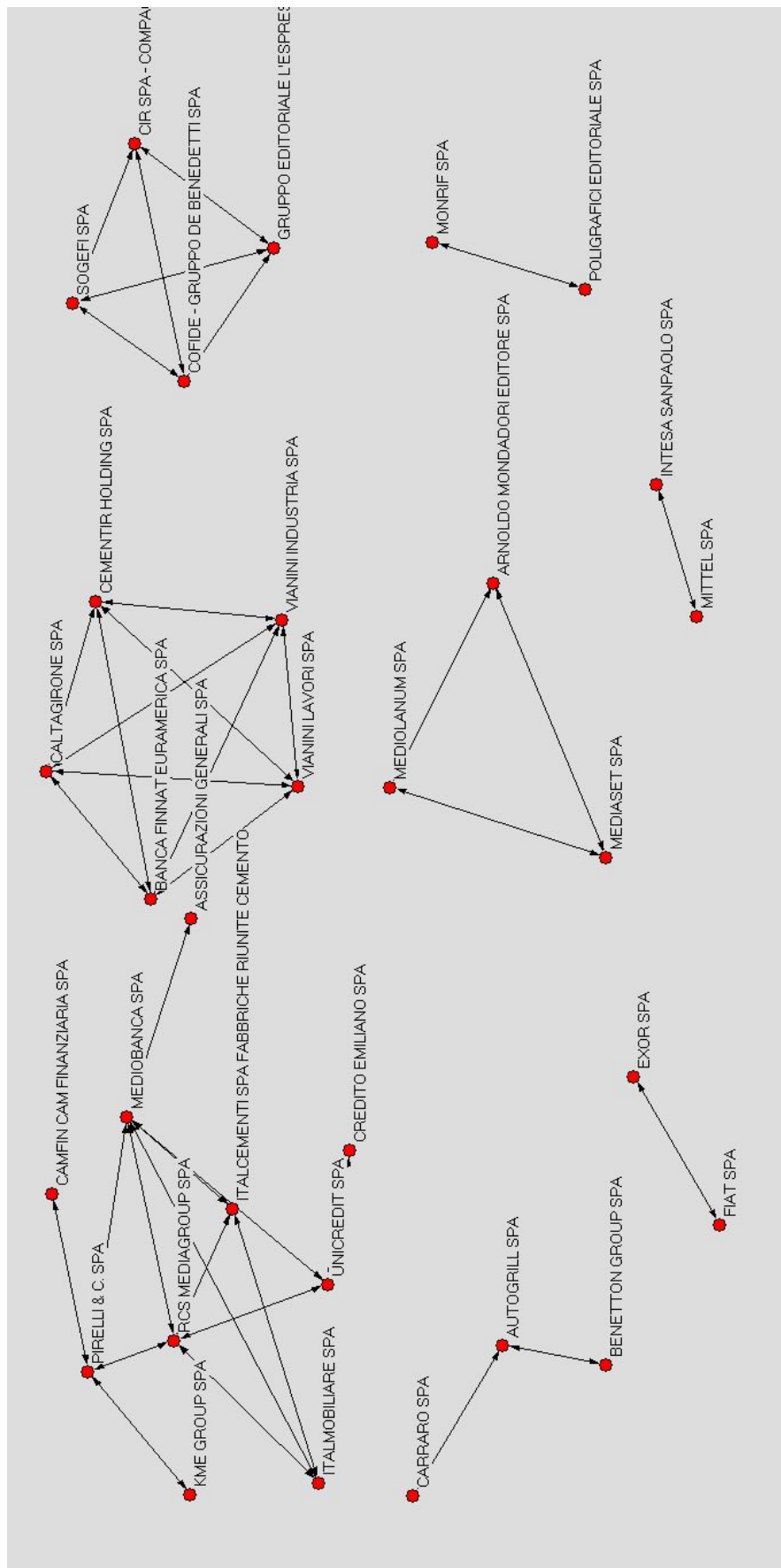


Figure 4.4: The stability graph with $\theta = 100\%$

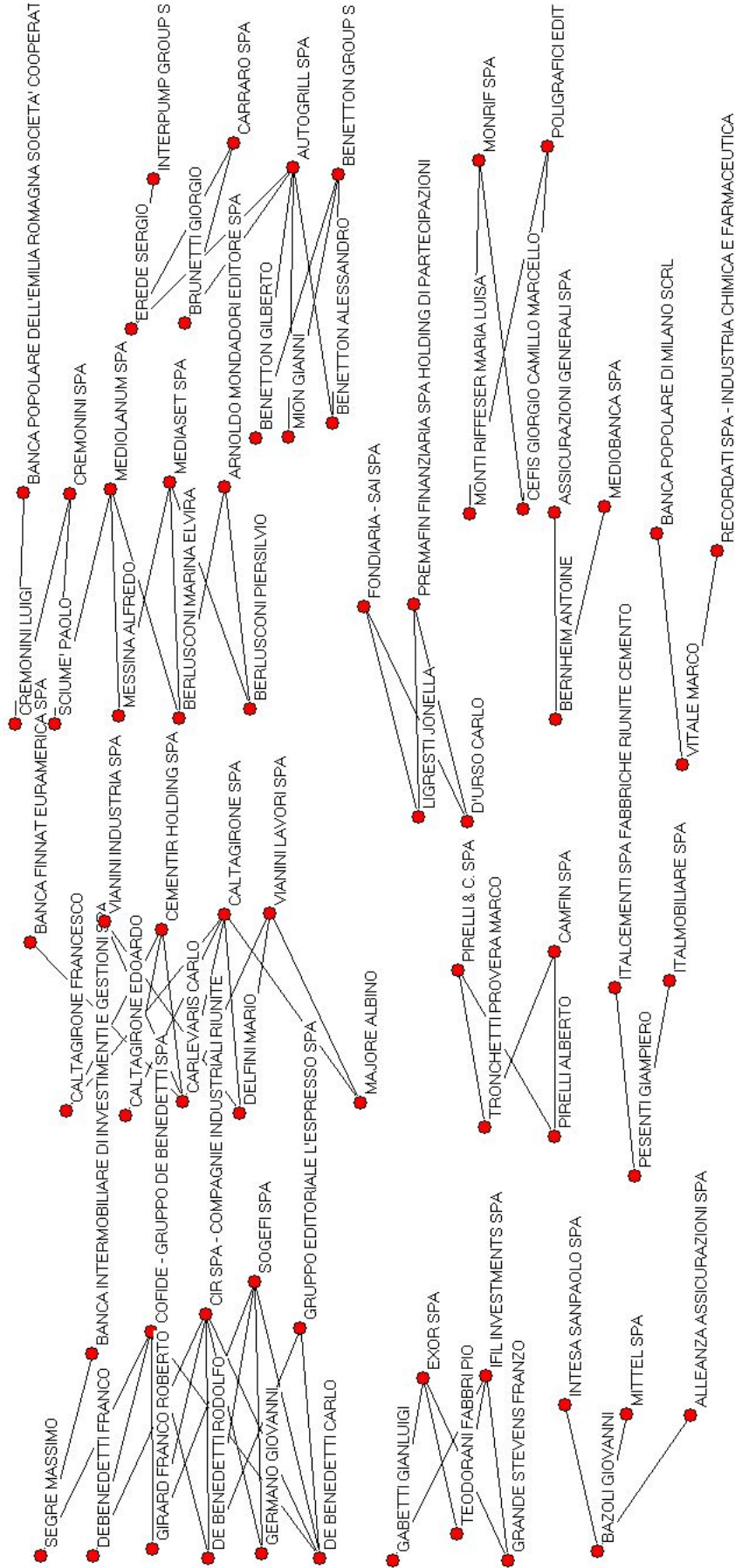


Figure 4.5: The bipartite graph with $\theta = 65\%$: on the left of the edges there are the directors and on the left the companies in which board they sit in.

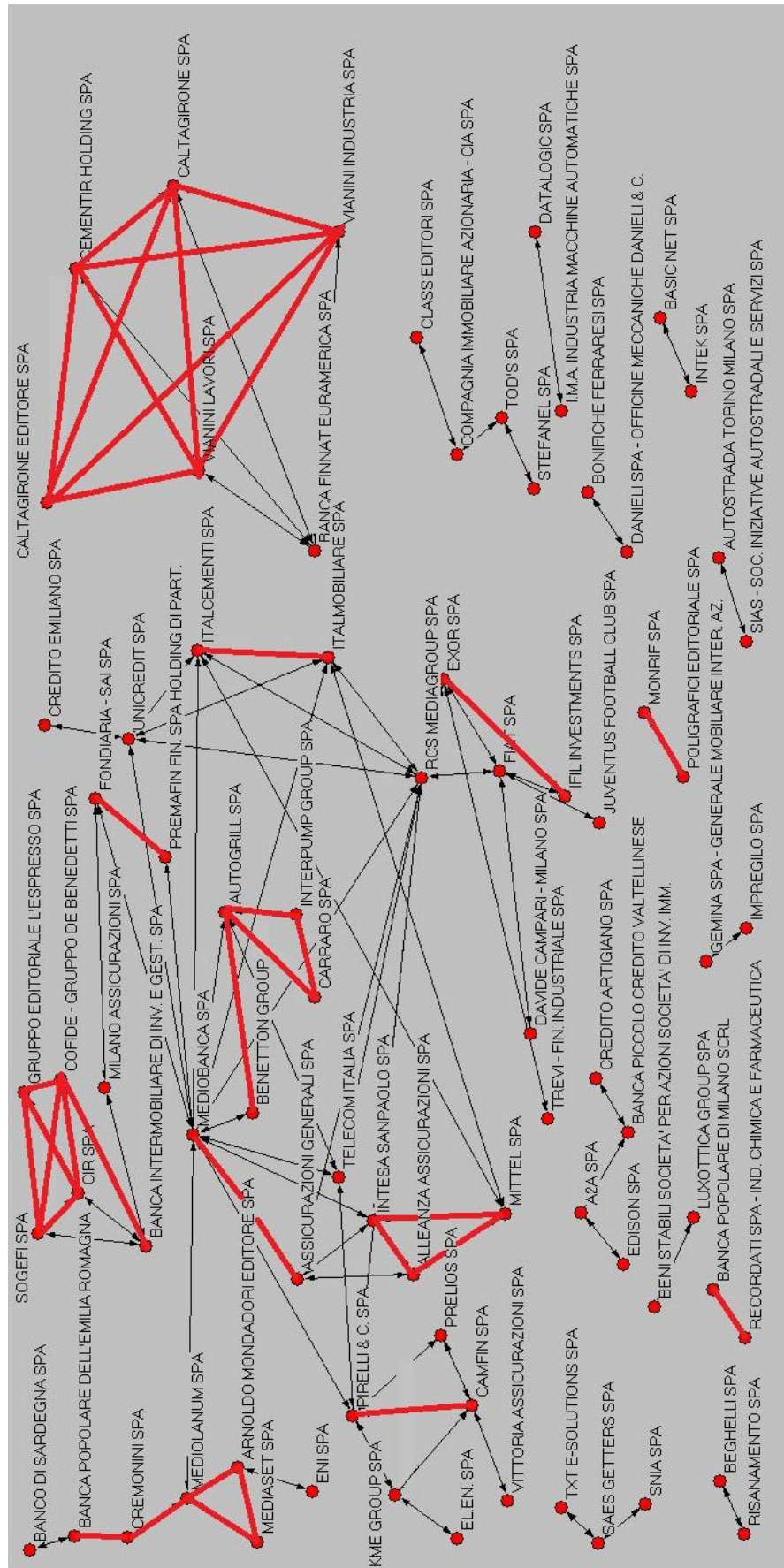


Figure 4.6: Analysis of the stability on the board network with $\theta = 65\%$. Links are 102; the information about appointment stability is represented by the red thick edges (36), whereas the simple stability is represented by the black edges (66).

Years	1998	1999	2000	2001	2002	2003	2004
#of companies	235	255	291	301	295	262	268
# directors	1782	1913	2212	2309	2306	2106	2171
# mandates	2267	2434	2781	2899	2875	2636	2719
Aver mandates	1.27	1.27	1.26	1.26	1.25	1.25	1.25
#of isolate nodes	53	56	61	52	63	53	52
#of components	6	8	9	6	6	6	9
Company links	845	654	718	740	689	661	673
Aver. board size	9.65	9.58	9.57	9.64	9.75	10.06	10.14
Main C size	170	182	205	236	221	197	195
Network density (Main C)	0.04017	0.03873	0.03309	0.02643	0.02814	0.03382	0.03479
Company links (Main C)	577	638	692	733	682	653	658

Years	2005	2006	2007	2008	2009	2010	2011
#of companies	277	283	293	283	278	262	258
# directors	2236	2290	2403	2356	2381	2263	2227
# mandates	2836	2845	2949	2871	2866	2728	2671
Aver mandates	1.27	1.24	1.23	1.22	1.20	1.21	1.20
#of isolate nodes	49	57	66	61	68	61	64
#of components	11	8	9	8	4	5	5
Company links	701	651	639	577	536	493	470
Aver. board size	10.24	10.05	10.06	10.14	10.30	10.41	10.35
Main C size	204	210	210	207	203	191	184
Network density (Main C)	0.03299	0.02916	0.02866	0.02664	0.02590	0.02678	0.02750
Company links (Main C)	683	640	629	568	531	486	463

Table 4.1: Descriptive statistics of the interlock network. Number of isolated nodes. of components and company links are referred to the board network. Network density is the sum of the degrees divided by $n(n-1)$, where n is the number of the nodes of the network.

	b=2	b=3	b=4	b=5	b=6	b=7
1998						
2002						
2006						
2010						

Table 4.2: Network structure constructed by considering only directors with an increasing threshold of board memberships b .

Conclusions

We have presented an overview on dynamic networks: these models are suitable to describe the evolution of a system in different fields of applications.

In the first case of study we have implemented the model of Vitting Andersen et al. by a large data base: the daily returns of 24 financial indices worldwide from 2000 to 2008. After first validating known empirical properties of the network of stock exchanges (e.g. directional price movements of stock exchanges following a large US open/close price movement) we have studied non-linear price dynamics within the network of stock exchanges in general, with a focus of propagation of avalanches of "price-quakes" in particular.

Two different definitions of avalanches have been proposed: both have as size distribution a power law in agreement with the hypothesis that empirical data can be described in terms of a Self Organized Critical model.

In particular we have focused our attention on the avalanche caused by a single node, i.e. when a large cumulative movement of the price of a single index is able to influence the price of other indices.

The avalanches have been represented by means of graphs. The features of these graphs provide us with additional information of financial contagion.

In general the perturbation of small avalanches of price movements across markets is a very frequent phenomenon, on average every two days and half there is a contagion between markets in different countries.

However larger events with a contagion propagating throughout the network of the worlds stock exchanges are less frequent, just like earth quakes where a Gutenberg-Richter scale defines the likelihood for a small versus large quake.

The factors that can give rise to the onset and the spread of an avalanche are: the volatility of the markets, the temporal sequence of open/close of the stock exchanges worldwide, the capitalization and geographical position. In order to visualize the role of

influence between the markets is suitable to assemble them in macro geographical areas.

Our method enables us to highlight the major role played by North America markets and the negligible role of e.g. Middle East markets. Conversely the other markets play a bivalent role. The statistical features of the system will be analyzed through stylized facts.

In the second case of study we have examined how interlocking directorate evolves from 1998 to 2011, in Italy. We are interested to establish whether the governance of the Italian companies is based on a stable structure over time; our results have revealed a network structure stable over time characterized by a "core", due to the presence of few directors with multiple assignments. Unlike the German case, this core is stable but not connected.

This result is consistent with most previous findings in literature.

Finally we have proposed an alternative approach based on temporal networks in order to investigate the nature of the links which are stable over time.

The analysis is performed by quantifying the variation of links in a time period, by means of a unique cumulative network, where the nodes are the companies and the existence of an edge is related with the persistence over time of an interlock between two companies.

A weighted graph represents the activity of the links, stating the percentage of stability. Our findings reveal that the Italian case is characterized by a small structure, with ties that are stable over time.

These persistent links reflect on the one hand the ownership control of few important families, on the other hand they are due to cross-shareholdings between companies, that tend to maintain their connections despite a turnover effect.

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